UCMERCED MESA LAB

Fractional Order Thinking

– from control, signal processing to energy informatics and beyond

YangQuan Chen, Ph.D., Director,
MESA**(Mechatronics, Embedded Systems and Automation)**LAB
ME/EECS/SNRI/UCSolar, School of Engineering,
University of California, Merced
E: yqchen@ieee.org; *or*, yangquan.chen@ucmerced.edu
T: (209)228-4672; **O**: SE1-254; **Lab**: Castle #22 (**T**: 228-4398)

Sept. 27, 2013. Friday 11:00-12:00. TOK2013, Malatya, Turkey

Thinking on "… thinking"

- Computational Thinking (CPS)
- Control Thinking
- System Thinking
- Multidisciplinary Thinking
- Cyber-Physical Thinking (CPS)
- Lumped Parameter Thinking
- …
- **Fractional Order Thinking**

My submission - "Computational" can be put in front of almost every thing

- Computational intelligence
- Computational material
- Computational neuron science
- Computational psychology
- Computational fluid dynamic
- Computational biology
- Computational chemistry
- Computational ecology
- Computational social science
- Computational virology
- ….

My submission - “Control” can be put after almost every thing

- Speed Control
- Diet Control
- Weight Control
- Emotion Control
- Arm Control
- Microclimate Control
- Machine Control
- Human Gait Control
- Blood-pressure Control
- Aging Control
- Evacuation Control/Traffic Control/Conggestion Control
- ….

My submission – "Fractional Order Thinking" should be everywhere

- AKA "Fractional order dynamic system thinking"
- Fractional order dynamics in either spatial evolution axis or temporal evolution axis.
- **Due to the complexity of the system, fractional thinking is essential to obtain insights and conclude rationally.**
  – Bruce J. West. ***Where Medicine Went Wrong: Rediscovering the Path to Complexity***. World Scientific 2006.
  – Yurij Baryshev and Pekka Teerikorpi. "*Discovery Of Cosmic Fractals*". World Scientific 2002. Foreword by: Benoit Mandelbrot
  – ME280 "***Fractional Order Mechanics***" @ UC Merced (Fall 13)

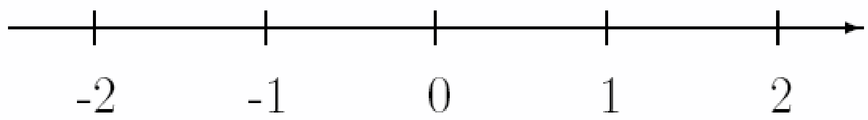
Outline

- Fractional Calculus and Fractional Order Thinking
- From Control, Signal Processing to Energy Informatics and Beyond
- Concluding Remarks

Outline

- Fractional Calculus and Fractional Order Thinking
- From Control, Signal Processing to Energy Informatics and Beyond
- Concluding Remarks

… from integer to non-integer …



$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n}$$

$$x^n = e^{n \ln x}$$

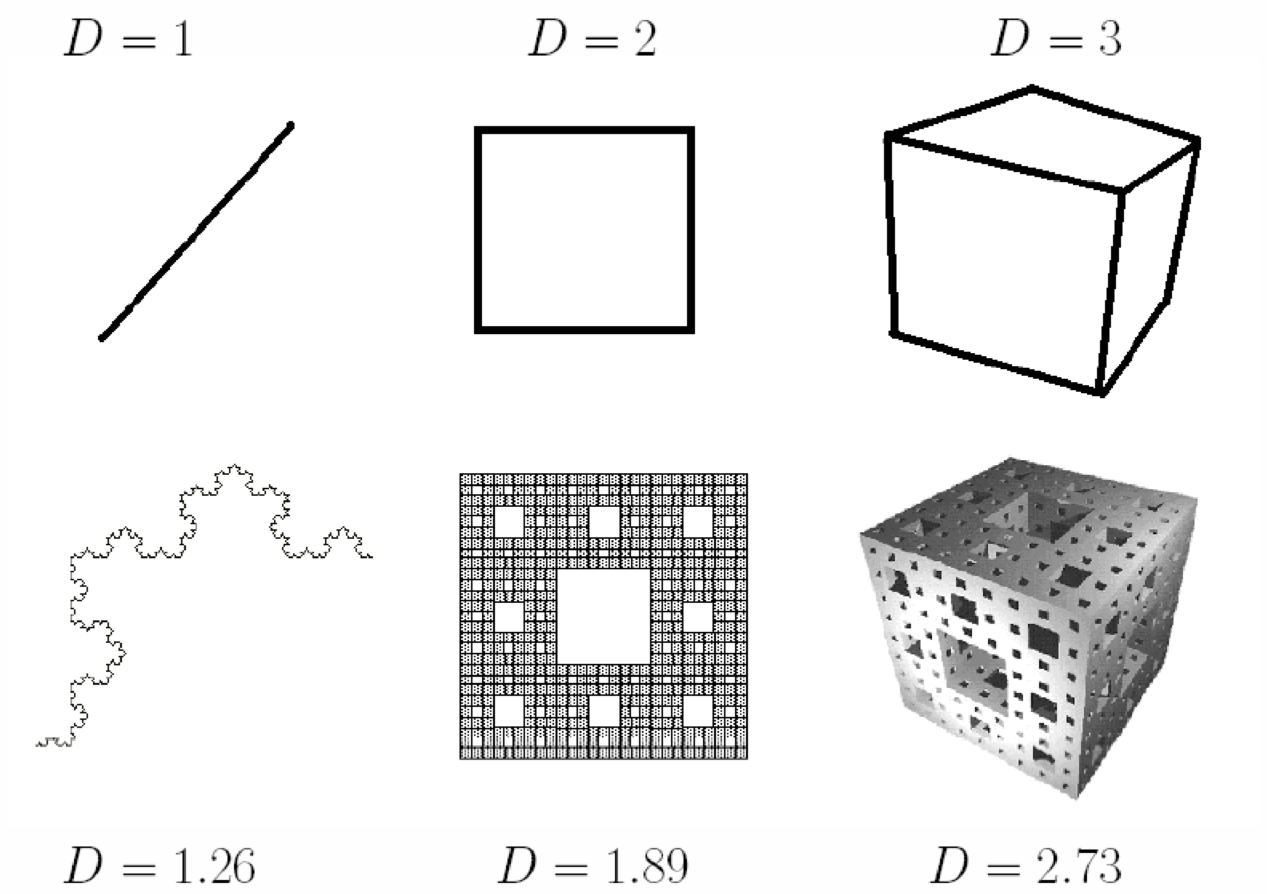
$$n! = 1 \cdot 2 \cdot 3 \cdot \ \ldots \cdot (n-1) \cdot n,$$

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \qquad x > 0,$$

$$\Gamma(n+1) = 1 \cdot 2 \cdot 3 \cdot \ \ldots \cdot n = n!$$

Slide credit: Igor Podlubny

… from integer to non-integer …



Slide credit: Igor Podlubny

Interpolation of operations

$$f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \frac{d^3 f}{dt^3}, \quad \ldots$$

$$f, \quad \int f(t)dt, \quad \int dt \int f(t)dt, \quad \int dt \int dt \int f(t)dt, \quad \ldots$$

$$\ldots, \quad \frac{d^{-2} f}{dt^{-2}}, \quad \frac{d^{-1} f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \ldots$$

Slide credit: Igor Podlubny

"Fractional Order Thinking" or, "In Between Thinking"

- For example
  - Between integers there are non-integers;
  - Between logic 0 and logic 1, there is the fuzzy logic;
  - Between integer order splines, there are "fractional order splines"
  - Between integer high order moments, there are noninteger order moments (e.g. FLOS)
  - Between "integer dimensions", there are fractal dimensions
  - Fractional Fourier transform (FrFT) – in-between time-n-freq.
  - Non-Integer order calculus (fractional order calculus – abuse of terminology.) (FOC)


NEURO FUZZY
Made in Japan
NS-ZCC10
KEEP WARM
EXTENDED KEEP WARM
WHITE RICE
REG/SUSHI
SOFTER
HARDER
QUICK COOKING
18:34
MIXED
PORRIDGE
SWEET
SEMIBROWN
BROWN
COOKING
REHEAT
RESET
MENU
TIMER
TIME SETTING
PRE-WASHED
ZOJIRUSHI
0
1

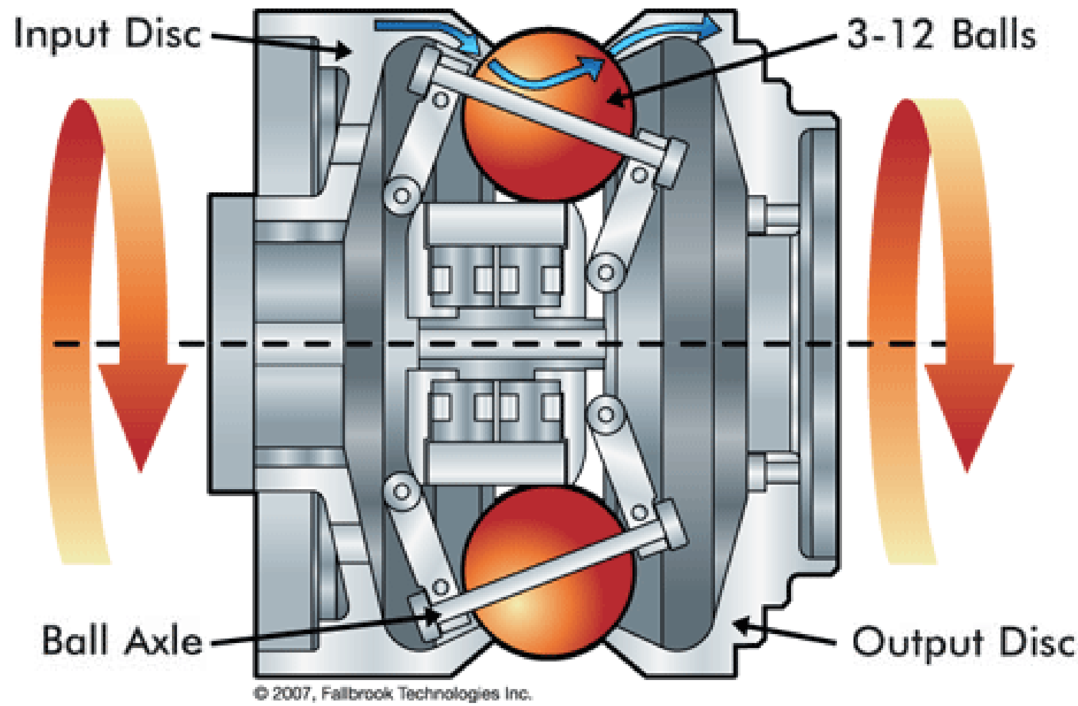
Conclusion of Talk

Integer-Order Calculus

Fractional-Order Calculus

Slide credit: Richard L. Magin, ICCC12

- Integer-Order Calculus
- Fractional-Order Calculus



Discrete gears vs. constantly-variable transmission

http://spectrum.ieee.org/energywise/energy/renewables/could-mechanics-best-power-electronics-in-evs

Slide credit: Calvin Coopmans, 2/28/2013 email comment

Fractional Calculus was born in 1695

$$\frac{d^n f}{dt^n}$$

What if the order will be $n = ½$?

It will lead to a paradox, from which one day useful consequences will be drawn.

G.F.A. de L'Hôpital
(1661–1704)

G.W. Leibniz
(1646–1716)

Slide credit: Igor Podlubny

G. W. Leibniz (1695–1697)

In the letters to J. Wallis and J. Bernulli (in 1697) Leibniz mentioned the possible approach to fractional-order differentiation in that sense, that for non-integer values of n the definition could be the following:

$$\frac{d^n e^{mx}}{dx^n} = m^n e^{mx},$$

L. Euler (1730)

$$\frac{d^n x^m}{dx^n} = m(m-1)\ldots(m-n+1)x^{m-n}$$

$$\Gamma(m+1) = m(m-1)\ldots(m-n+1)\,\Gamma(m-n+1)$$

$$\frac{d^n x^m}{dx^n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)} x^{m-n}.$$

Euler suggested to use this relationship also for negative or non-integer (rational) values of n. Taking $m = 1$ and $n = \frac{1}{2}$, Euler obtained:

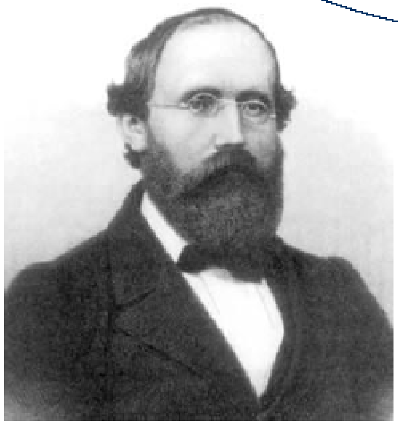
$$\frac{d^{1/2}x}{dx^{1/2}} = \sqrt{\frac{4x}{\pi}} \qquad \left(= \frac{2}{\sqrt{\pi}} x^{1/2} \right)$$

Slide credit: Igor Podlubny

Riemann–Liouville definition

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)\,d\tau}{(t-\tau)^{\alpha-n+1}} $$

$$ (n-1 \le \alpha < n) $$



G.F.B. Riemann
(1826–1866)

J. Liouville
(1809–1882)

Slide credit: Igor Podlubny

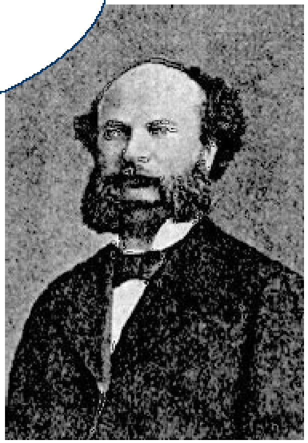
Grünwald–Letnikov definition

$$ {}_aD_t^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t - jh) $$

$[x]$ – integer part of x

A.K. Grünwald

A.V. Letnikov



Slide credit: Igor Podlubny

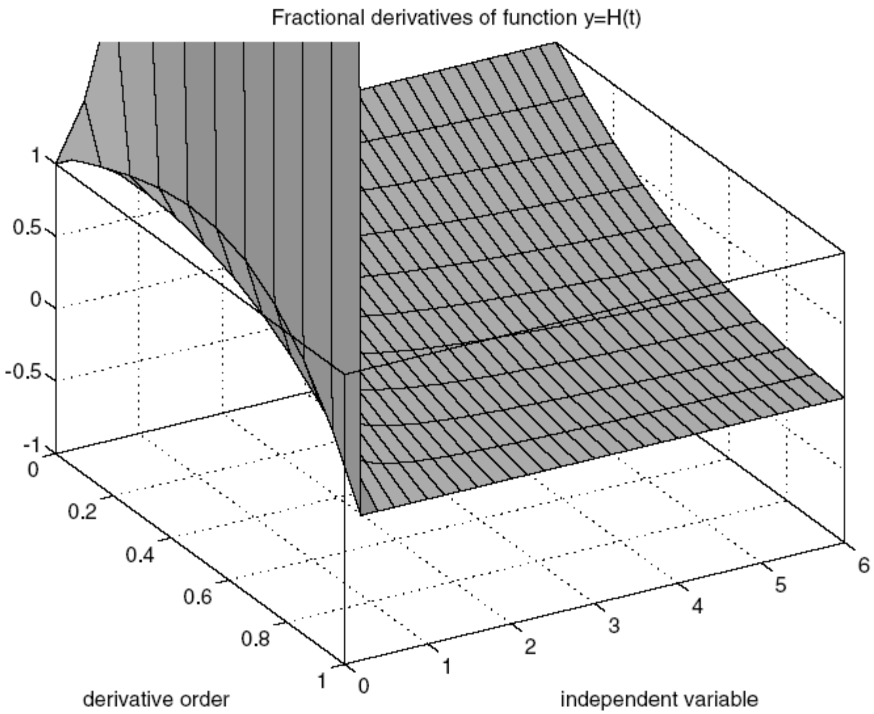
Operator ${}_a\mathrm{D}_t^\alpha$

A generalization of differential and integral operators:

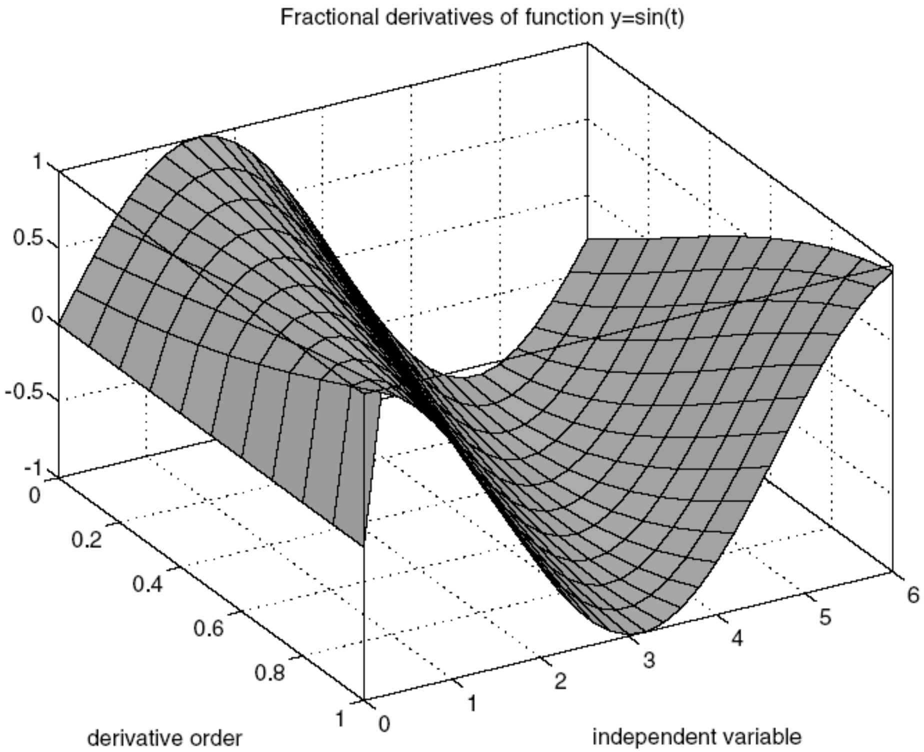
$$ {}_a\mathrm{D}_t^\alpha = \begin{cases} \mathrm{d}^\alpha/\mathrm{d}t^\alpha & \mathbb{R}(\alpha) > 0, \\ 1 & \mathbb{R}(\alpha) = 0, \\ \int_a^t (\mathrm{d}\tau)^{-\alpha} & \mathbb{R}(\alpha) < 0. \end{cases} \tag{7} $$

There are two commonly used definitions for the general fractional order differentiation and integral, i.e., the **Grünwald-Letnikov definition** and the **Riemann-Liouville definition**.

Example: Heaviside's unit step

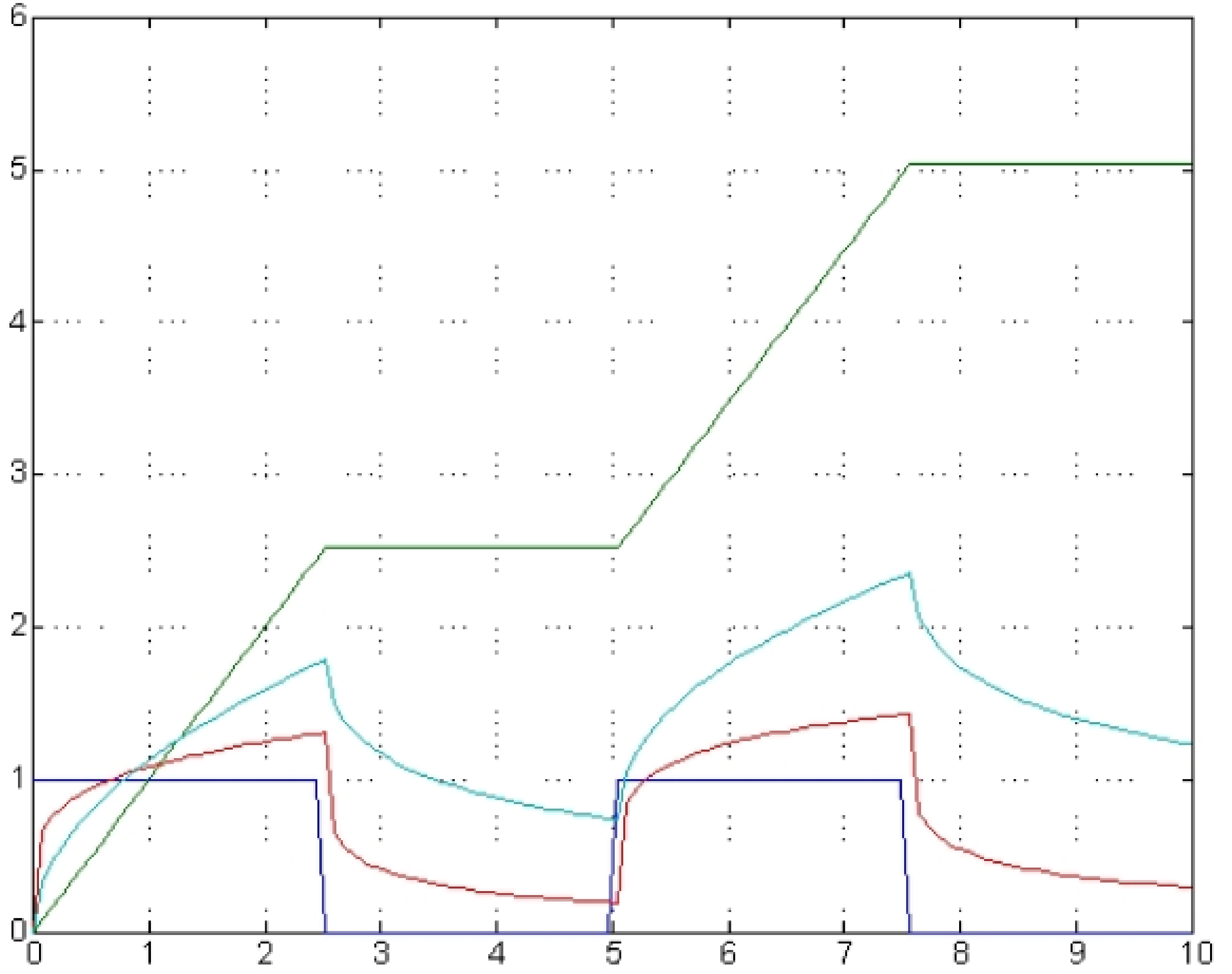


Example: $\sin(t)$



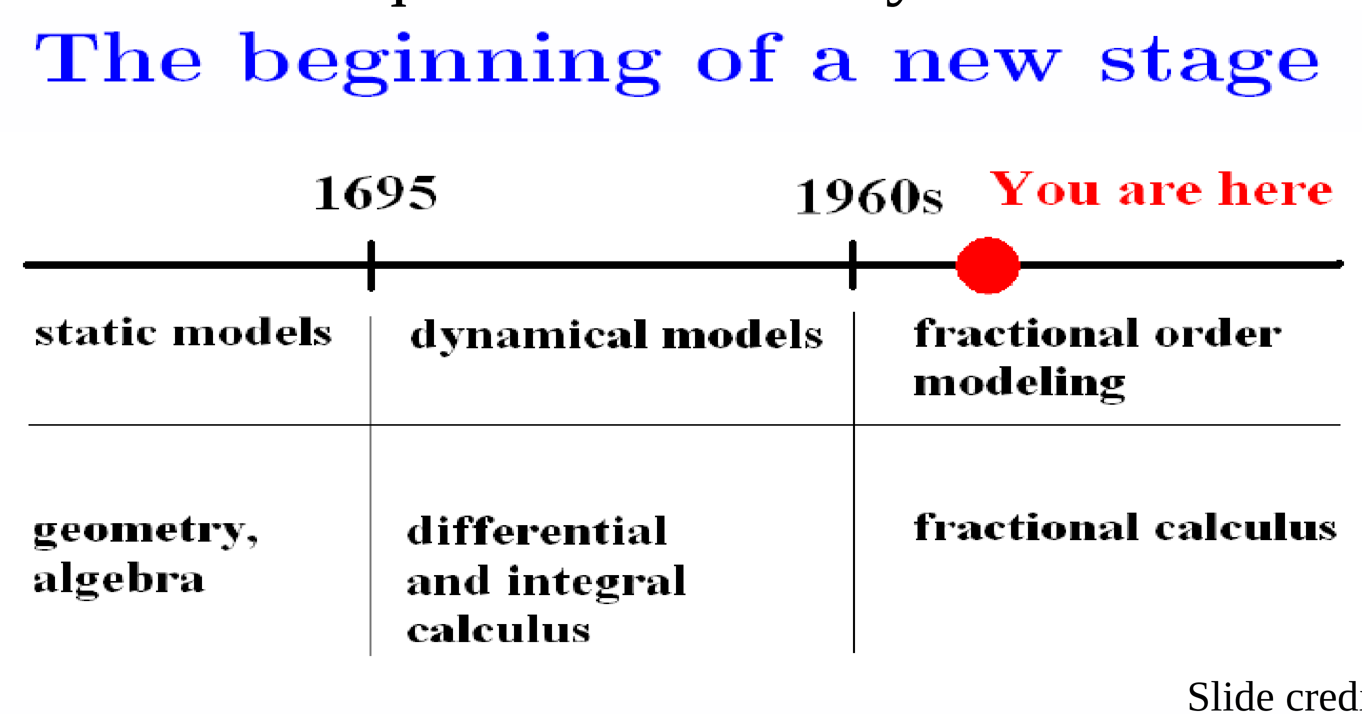
Slide credit: Igor Podlubny

Fractional derivatives of ramp function.



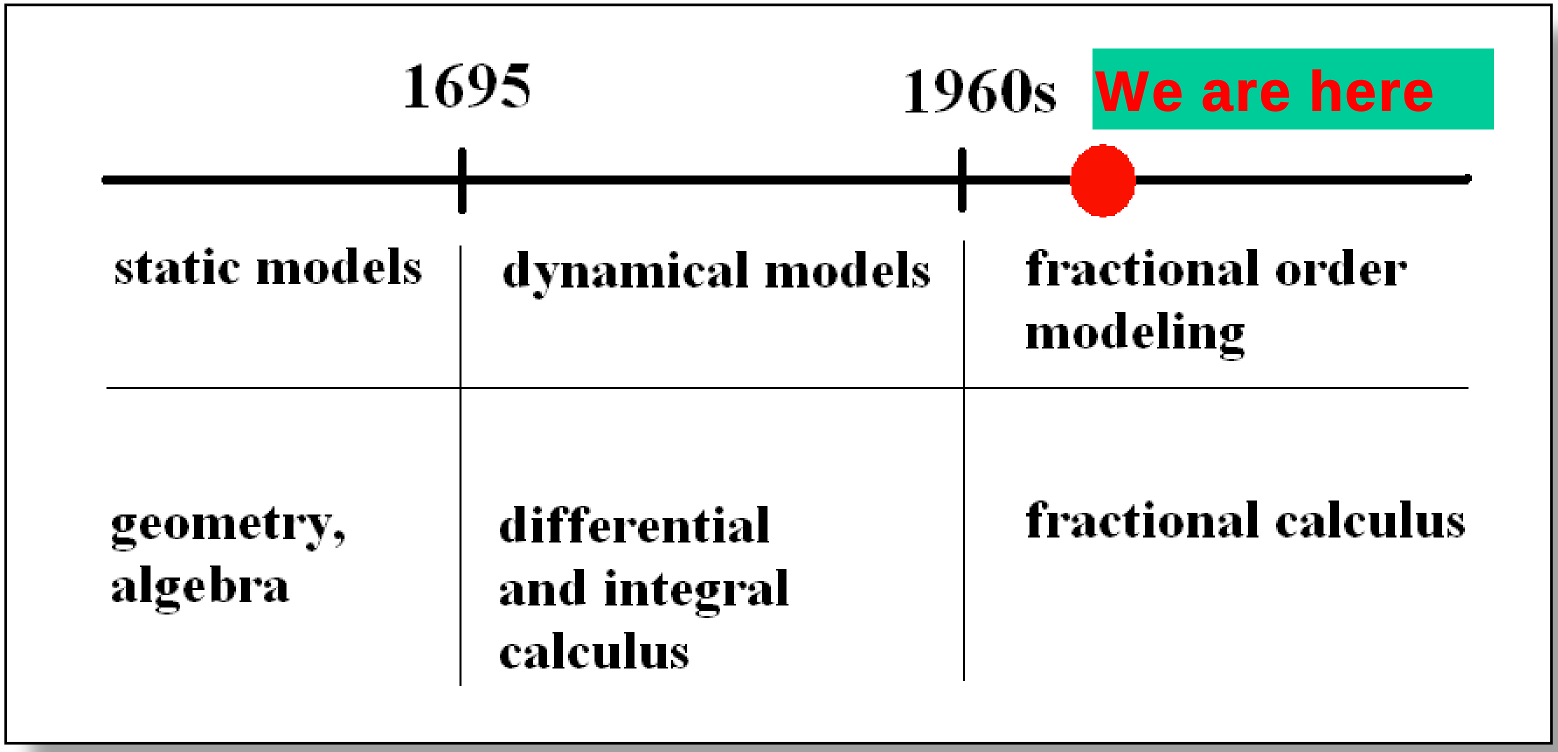
Why and How and When

•Why – Many reasons. Dynamic systems modeling and controls. Better characterization, better control performance
How – Analog versus digital realization methods. Many.
When – **Now**. Ubiquitous. Take a try since we have the new tool.



Slide credit: Igor Podlubny

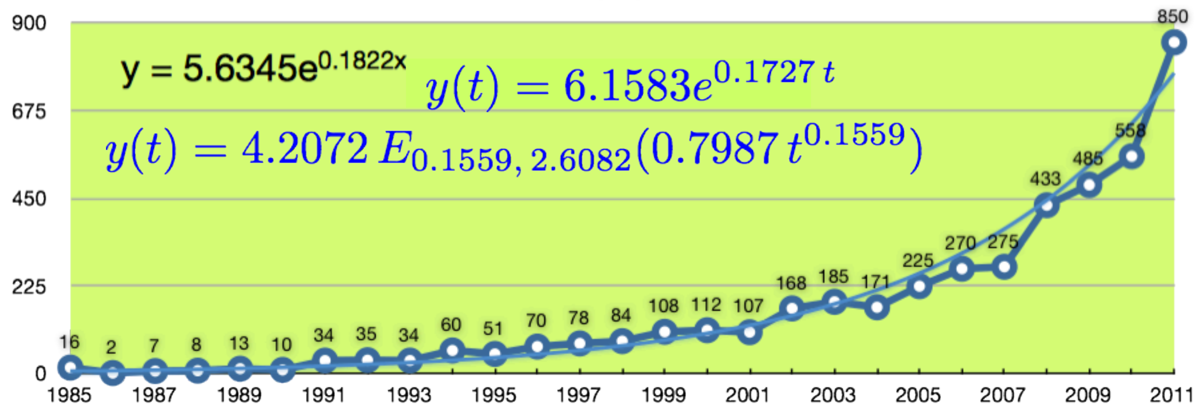
Fractional Calculus: a response to more advanced characterization of our more complex world at smaller scale



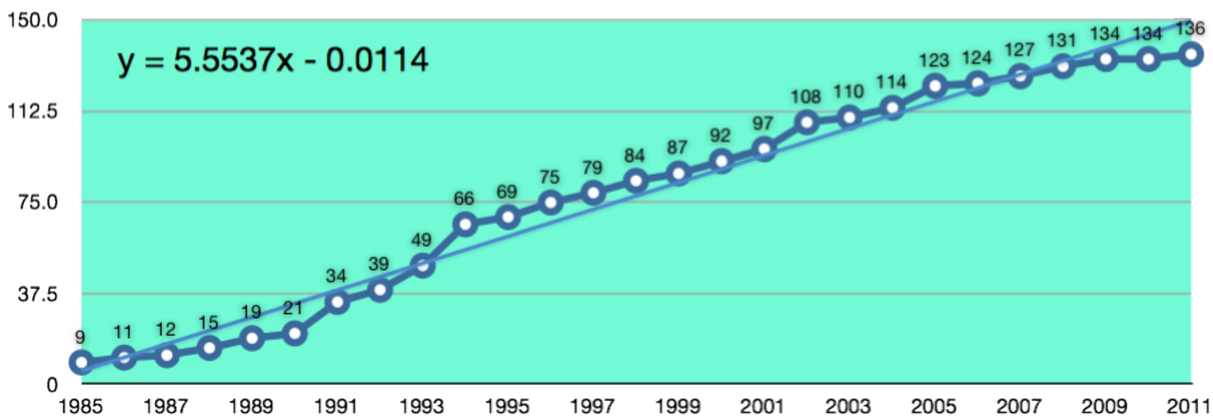
Slide credit: Igor Podlubny

Rapid development and numerous applications

Number of papers in WoS



Cumulative number of different WoS Subject areas



Slide credit: Igor Podlubny

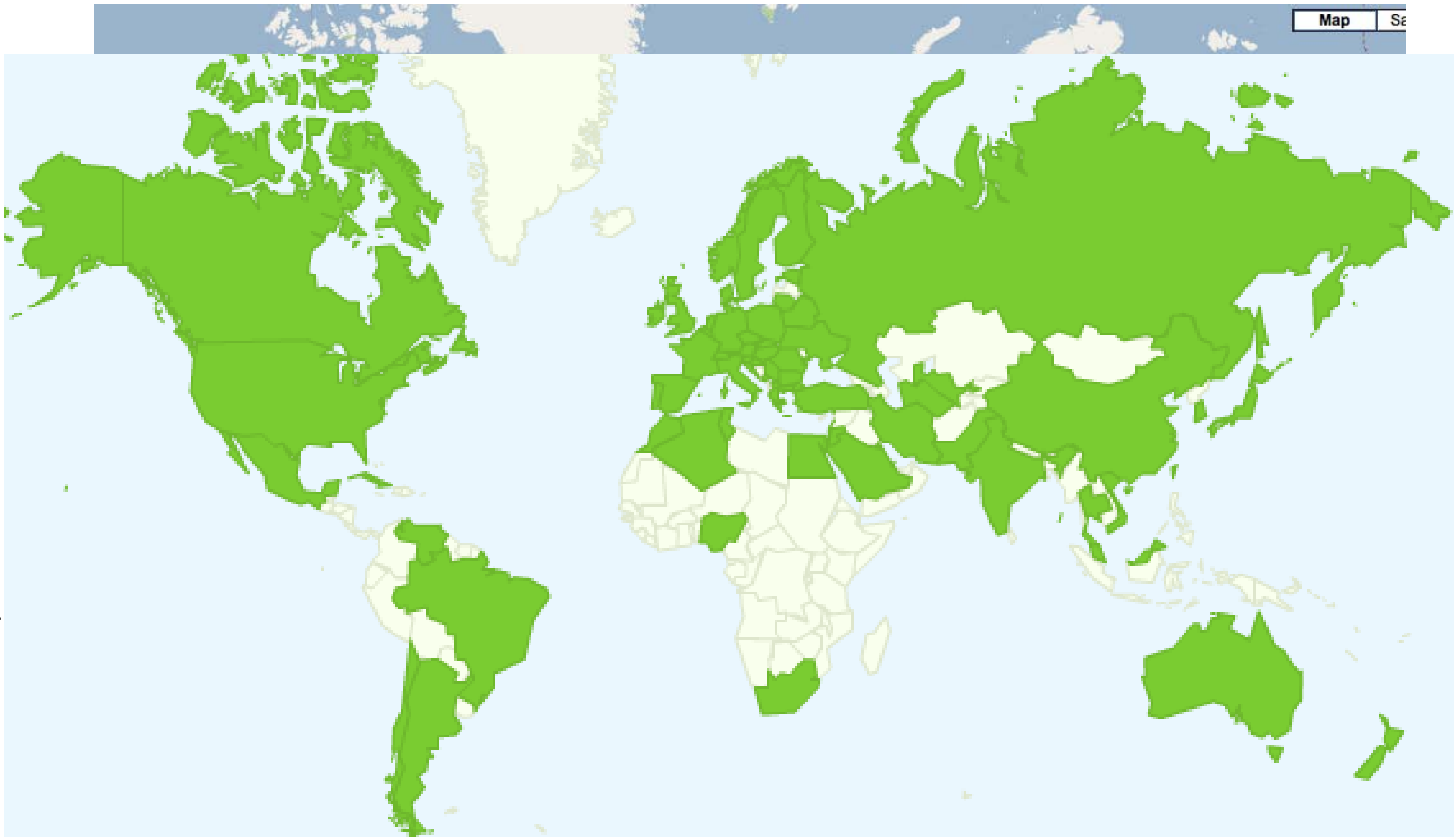
Fractional Calculus in WoK: 136 subject areas (applications)

ACOUSTICS
AGRICULTURAL ECONOMICS & POLICY
AGRICULTURAL ENGINEERING
AGRONOMY
ANESTHESIOLOGY
ASTRONOMY & ASTROPHYSICS
AUTOMATION & CONTROL SYSTEMS
BIOCHEMICAL RESEARCH METHODS
BIOCHEMISTRY & MOLECULAR BIOLOGY
BIOLOGY
BIOPHYSICS
BIOTECHNOLOGY & APPLIED MICROBIOLOGY
BUSINESS
BUSINESS, FINANCE
CARDIAC & CARDIOVASCULAR SYSTEMS
CELL BIOLOGY
CHEMISTRY, ANALYTICAL
CHEMISTRY, APPLIED
CHEMISTRY, INORGANIC & NUCLEAR
CHEMISTRY, MULTIDISCIPLINARY
CHEMISTRY, ORGANIC
CHEMISTRY, PHYSICAL
COMPUTER SCIENCE, ARTIFICIAL INTELLIGENCE
COMPUTER SCIENCE, CYBERNETICS
COMPUTER SCIENCE, HARDWARE & ARCHITECTURE
COMPUTER SCIENCE, INFORMATION SYSTEMS
COMPUTER SCIENCE, INTERDISCIPLINARY APPLICATIONS
COMPUTER SCIENCE, SOFTWARE ENGINEERING
COMPUTER SCIENCE, THEORY & METHODS
CONSTRUCTION & BUILDING TECHNOLOGY
CRIMINOLOGY & PENOLOGY
CRYSTALLOGRAPHY
DENTISTRY, ORAL SURGERY & MEDICINE
ECOLOGY
ECONOMICS
EDUCATION & EDUCATIONAL RESEARCH
EDUCATION, SCIENTIFIC DISCIPLINES
ELECTROCHEMISTRY
ENERGY & FUELS
ENGINEERING, AEROSPACE
ENGINEERING, BIOMEDICAL
ENGINEERING, CHEMICAL
ENGINEERING, CIVIL
ENGINEERING, ELECTRICAL & ELECTRONIC
ENGINEERING, ENVIRONMENTAL

Slide credit: Igor Podlubny

The current map of the fractional calculus

ALGERIA
ARGENTINA
AUSTRALIA
AUSTRIA
BELGIUM
BRAZIL
BULGARIA
BYELARUS
CANADA
CHILE
CROATIA
CUBA
CZECH REP
DENMARK
EGYPT
ENGLAND
ESTONIA
FINLAND
FRANCE
GERMANY
GREECE
GUADELOUPE
HUNGARY
INDIA
IRAN
IRELAND
ISRAEL
ITALY
JAPAN
JORDAN
KUWAIT
LEBANON
LITHUANIA
MACEDONIA
MALAYSIA
MEXICO
MOROCCO



NETHERLANDS
NEW ZEALAND
NIGERIA
NORTH IRELAND
NORWAY
PAKISTAN
P.R. CHINA
POLAND
PORTUGAL
QATAR
ROMANIA
RUSSIA
SAUDI ARABIA
SCOTLAND
SERBIA
MONTENEG
SINGAPORE
SLOVAKIA
SLOVENIA
SOUTH AFRICA
SOUTH KOREA
SPAIN
SWEDEN
SWITZERLAND
TAIWAN
THAILAND
TUNISIA
TURKEY
TURKMENISTAN
U ARAB EMIRATES
UKRAINE
USA
UZBEKISTAN
VENEZUELA
VIETNAM
WALES
YUGOSLAVIA

Turkey is in top 10!

Slide credit: Igor Podlubny

Data Source: Web of Knowledge

Modeling: heat transfer

$$\frac{\partial^2 y(x,t)}{\partial x^2} = k^2 \frac{\partial y(x,t)}{\partial t},$$
$$(t > 0, \quad 0 < x < \infty)$$

$$\begin{aligned} y(0,t) &= m(t) \\ y(x,0) &= 0 \\ \left|\lim_{x\to\infty} y(x,t)\right| &< \infty \end{aligned}$$

Transfer function:

$$\begin{aligned} \frac{\mathrm{d}^2 Y(x,s)}{\mathrm{d}x^2} &= k^2 s Y(x,s) \\ Q(0,s) &= M(s) \\ \left|\lim_{x\to\infty} Y(x,s)\right| &< \infty \end{aligned}$$

$$Y(x,s) = A(s)e^{-kx\sqrt{s}} + B(s)e^{kx\sqrt{s}}$$

$$\begin{aligned} A(s) &= Y(0,s) = M(s) \\ B(s) &= 0 \end{aligned}$$

$$Y(x,s) = M(s)e^{-kx\sqrt{s}}$$

$$G(s) = \frac{Y(x,s)}{M(s)} = e^{-kx\sqrt{s}}$$

think about transfer function $e^{-\sqrt{s}}$!

FO Controller + IO Plant

Fractional order speed control of DC motor

System transfer function $G(s) = \frac{k}{Js(Ts+1)}$ J being the payload inertia. Phase margin of controlled system:

$$\Phi_m = \arg\left[C(j\omega_g)G(j\omega_g)\right] + \pi$$

Controller: $C(s) = k_1 \frac{k_2 s+1}{s^\alpha}$, $k_2 = T$ giving a **constant phase margin**:

$$\begin{aligned}\Phi_m &= \arg\left[C(j\omega)G(j\omega)\right] + \pi = \arg\left[\frac{k_1 k}{(j\omega)^{(1+\alpha)}}\right] + \pi \\ &= \arg\left[(j\omega)^{-(1+\alpha)}\right] + \pi = \pi - (1+\alpha)\frac{\pi}{2}\end{aligned}$$

Step response:

$$y(t) = \pounds^{-1}\left\{\frac{kk_1/J}{s\left(s^{1+\alpha} + kk_1/J\right)}\right\} = \left(\frac{kk_1}{J}\right) t^{1+\alpha} E_{1+\alpha,2+\alpha}\left(-\frac{kk_1}{J}t^{1+\alpha}\right) \quad (63)$$

Mittag-Leffler function: definition

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \quad (\alpha > 0, \quad \beta > 0)$$

$$E_{1,1}(z) = e^z,$$

$$E_{2,1}(z^2) = \cosh(z), \quad E_{2,2}(z^2) = \frac{\sinh(z)}{z}.$$

$$E_{1/2,1}(z) = e^{z^2}\operatorname{erfc}(-z);$$

$$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2} dt.$$

Slide credit: Igor Podlubny

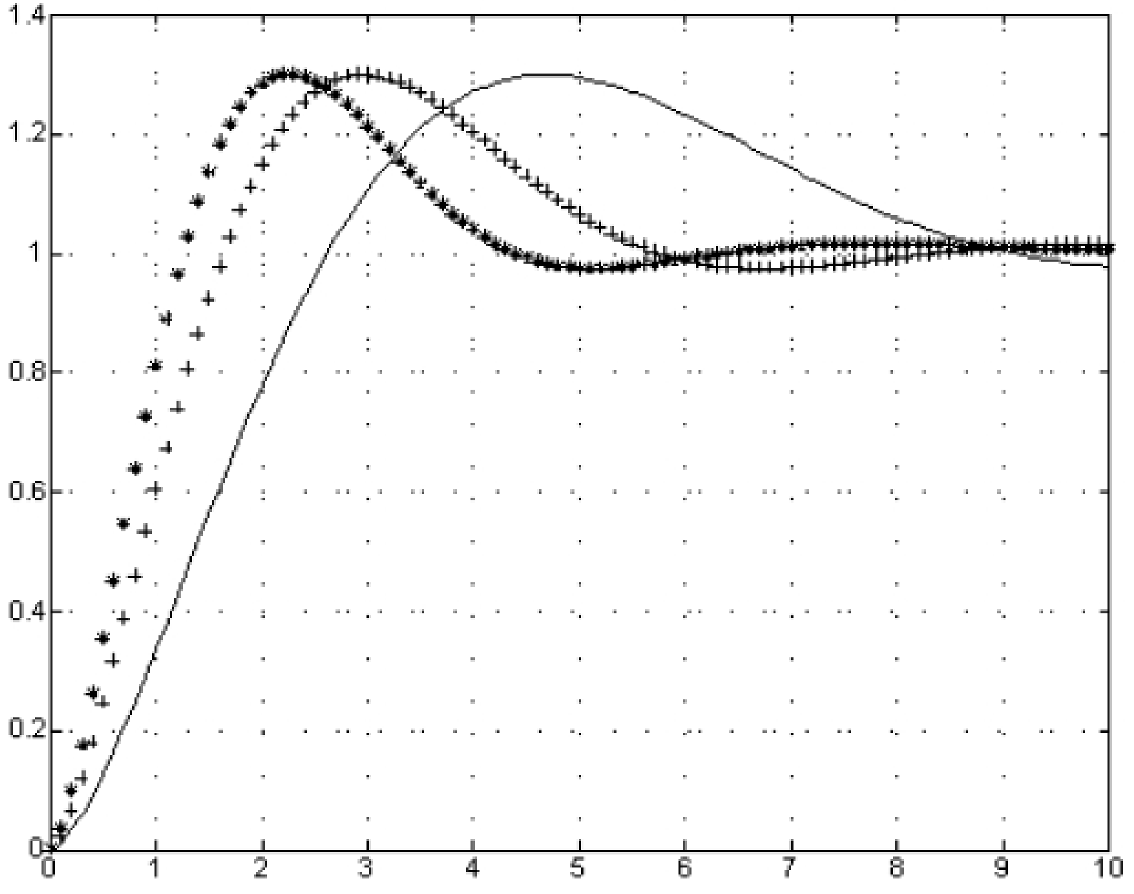
G. M. Mittag-Leffler

Professor Donald E. Knuth, creator of TEX:

"As far as the spacing in mathematics is concerned... I took *Acta Mathematica*, from 1910 approximately; this was a journal in Sweden ... Mittag-Leffler was the editor, and his wife was very rich, and they had the highest budget for making quality mathematics printing. So the typography was especially good in *Acta Mathematica*."

(Questions and Answers with Prof. Donald E. Knuth, Charles University, Prague, March 1996)

Slide credit: Igor Podlubny



Note the iso-damping (similar overshoot!)

Fractional operator

- First order differentiator: *s*
- First order integrator: *1/s*

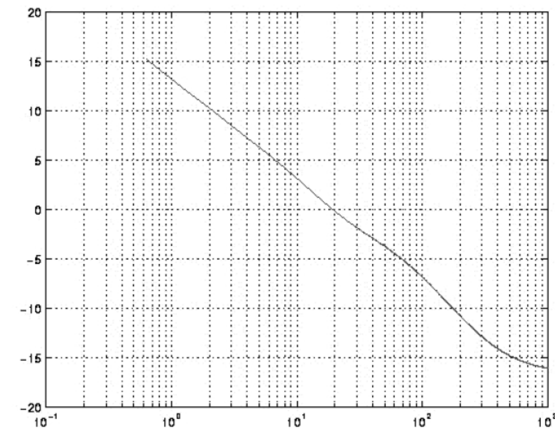
What is s^α when α is a *non-integer*?

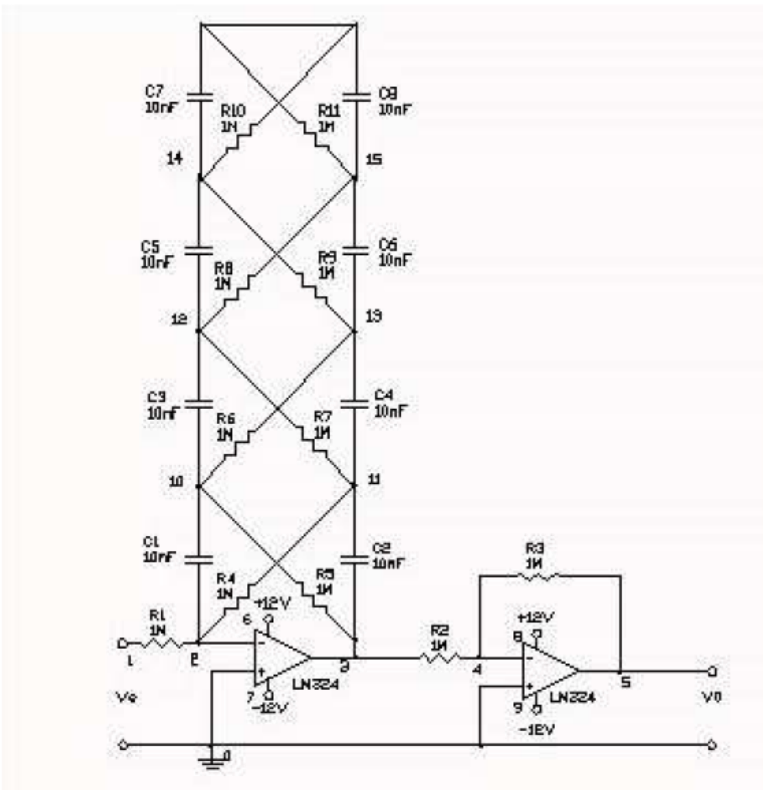
UCMERCED

MESA LAB

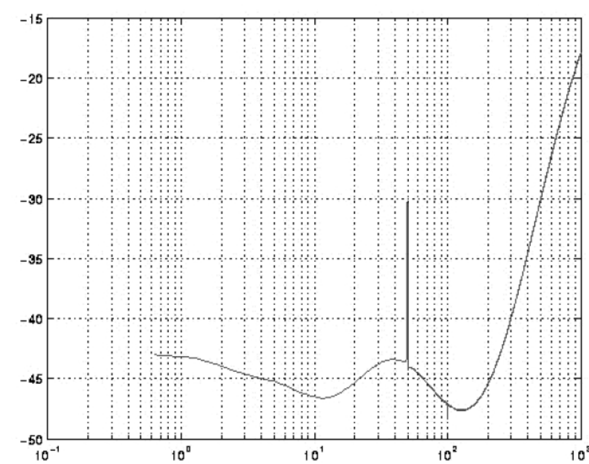
Possible? Possible!

Magnitude plot (dB vs. rad./sec.)





Phase plot (deg. vs. rad./sec.



Analog $1/\sqrt{s}$ using op-amps.

I. Petras, I. Podlubny, P. O'Leary, L. Dorcak, and Vinagre B. "**Analogue Realization of Fractional Order Controllers**". FBERG, Technical University of Kosice, Kosice, Slovak, **ISBN 8070996277** edition, 2002.

Fractor: Analogue device

Fractional Calculus Day at USU, April 19, 2005

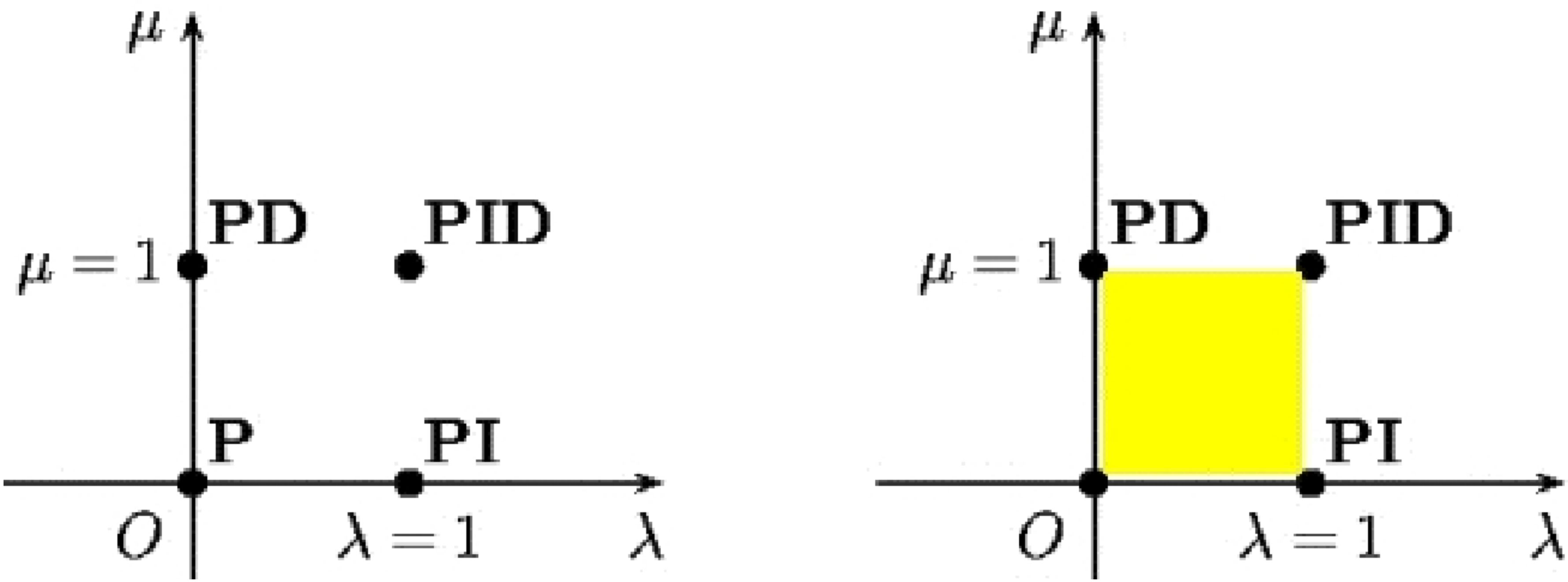
Photo credit: Igor Podlubny

UCMERCED MESA LAB

Fractional order PID control

- 90% are PI/PID type in industry. (**Ubiquitous**)

$$u(t) = K_p\left(e(t) + T_i \mathrm{D}_t^{-\lambda} e(t) + \frac{1}{T_d} \mathrm{D}_t^{\mu} e(t)\right). \qquad (\mathrm{D}_t^{(*)} \equiv {}_0\mathrm{D}_t^{(*)}).$$

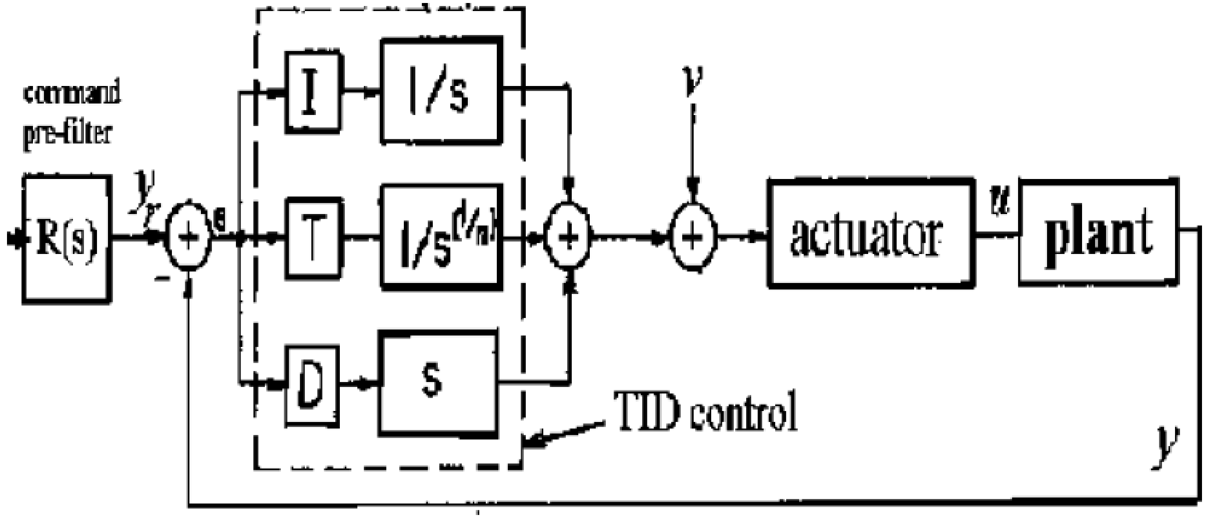


Igor Podlubny. "Fractional-order systems and $PI^{\lambda}D^{\mu}$-controllers". IEEE Trans. Automatic Control,44(1): 208–214, 1999.

*YangQuan Chen, Dingyu Xue, and Huifang Dou. "**Fractional Calculus and Biomimetic Control**". IEEE Int. Conf. on Robotics and Biomimetics (RoBio04), August 22-25, 2004, Shengyang, China.*

UCMERCED

MESALAB

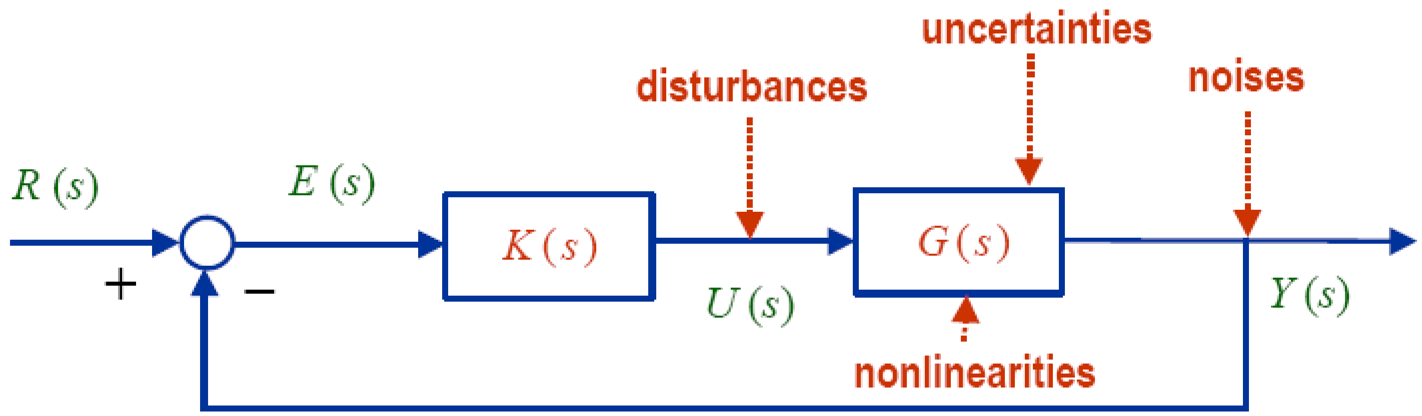


US05371670 on TID by B. J. Lurie, 1994

"3-param. tunable tilt-integral-deriv. controller"

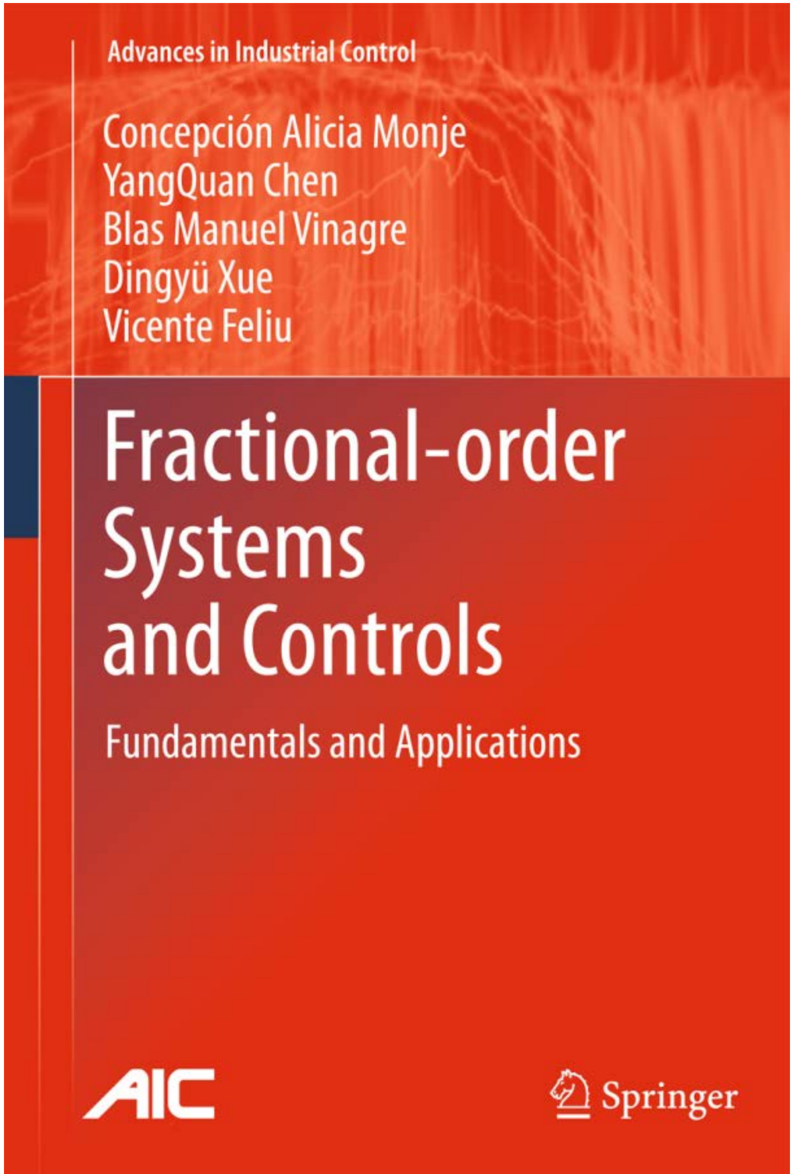
Fractional Order Controls

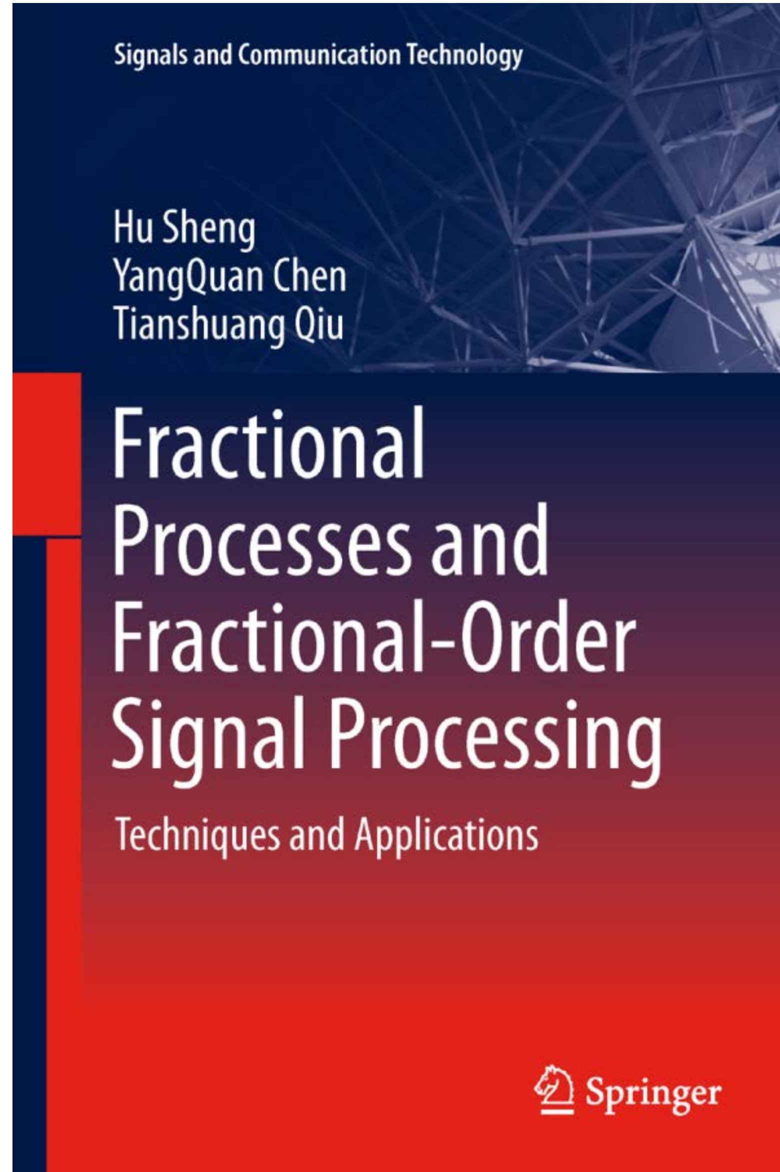
- **IO Controller + IO Plant**
- **FO Controller + IO Plant**
- **FO Controller + FO Plant**
- **IO Controller + FO Plant**



D. Xue and **Y. Chen***, "**A Comparative Introduction of Four Fractional Order Controllers**". Proc. of The 4th IEEE World Congress on Intelligent Control and Automation (WCICA02), June 10-14, 2002, Shanghai, China. pp. 3228-3235.

Think “fractional order”

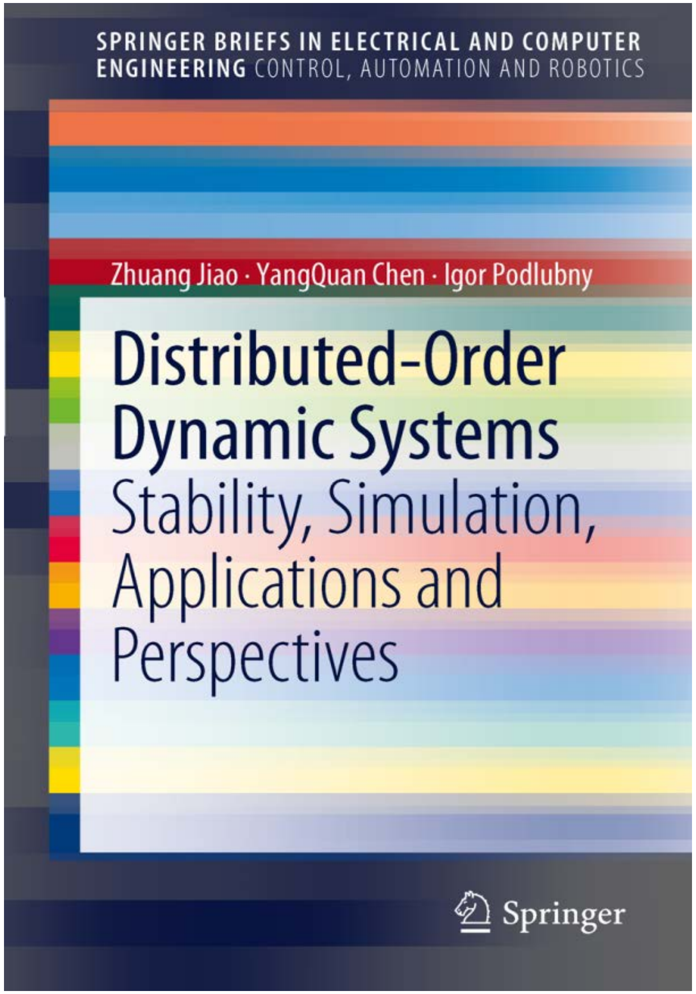


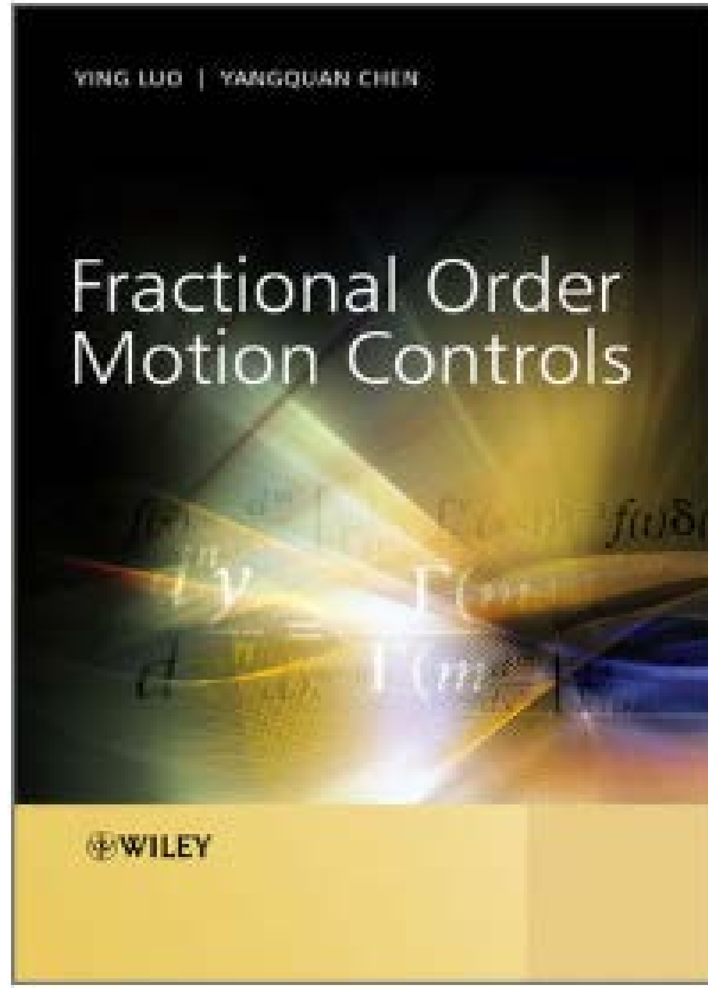


2001-2010

2005-2011

Think "fractional order"





Fractional Order Signal Processing

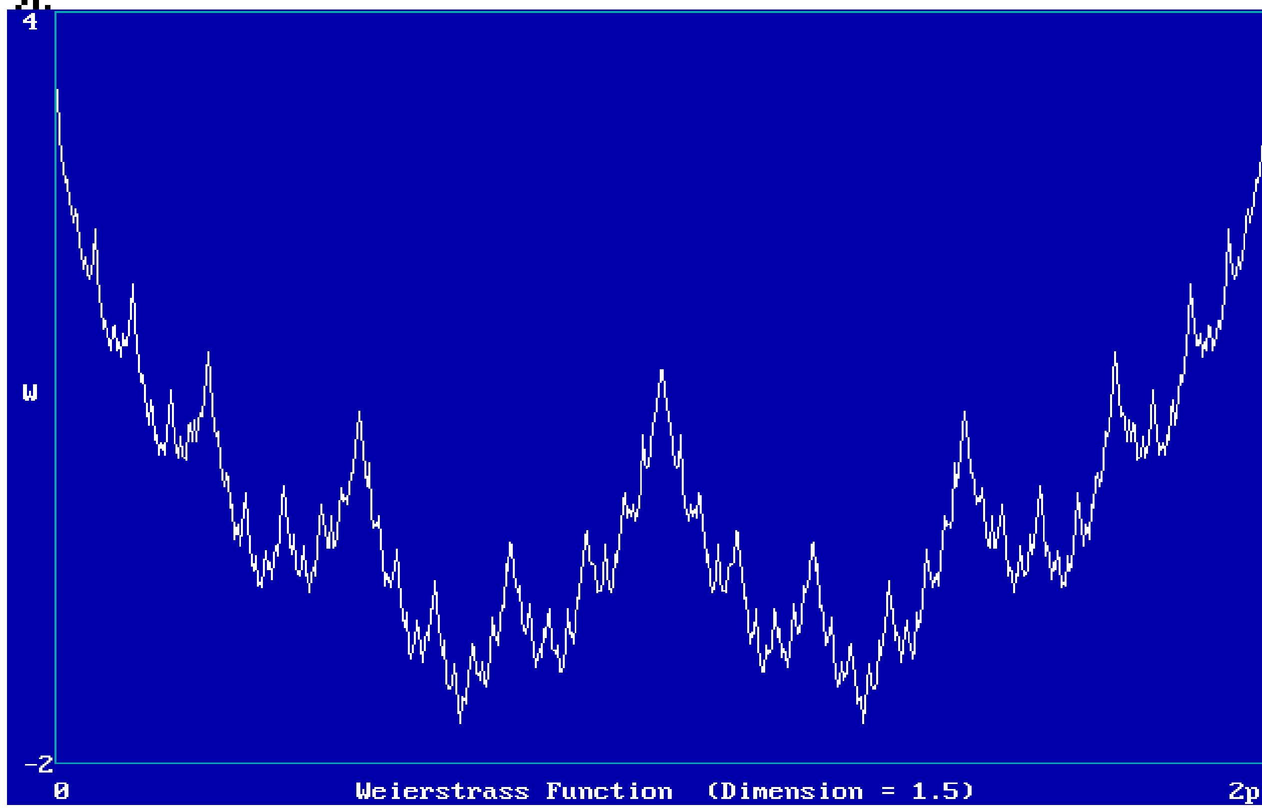
- Additional characterization
- Infinite variance issue (2^{nd} order moment)
- Long range dependence
- Time-frequency approach (FrFT)

Example-1: Weierstrass function

$$f_a(x) = \sum_{k=1}^{\infty} \frac{\sin(\pi k^a x)}{\pi k^a}$$

- Nowhere differentiable!

Fractional order derivative exists

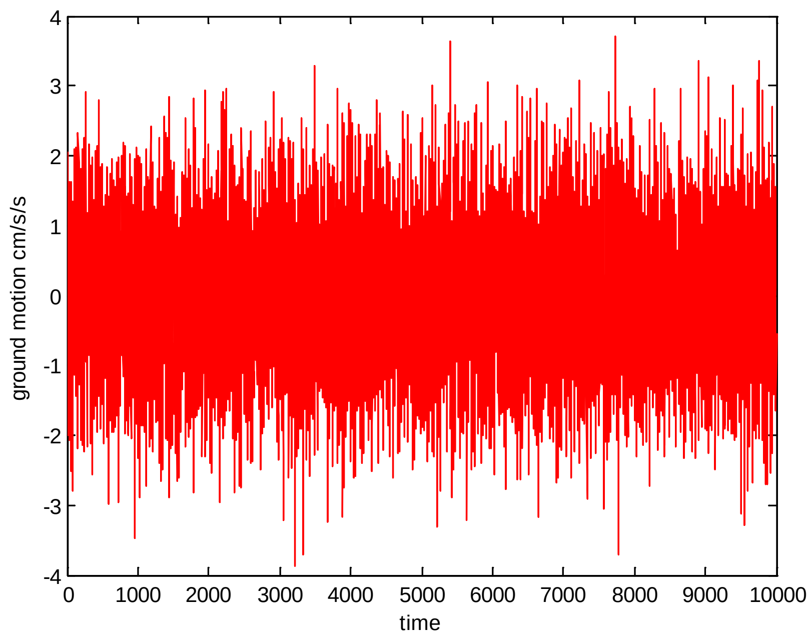


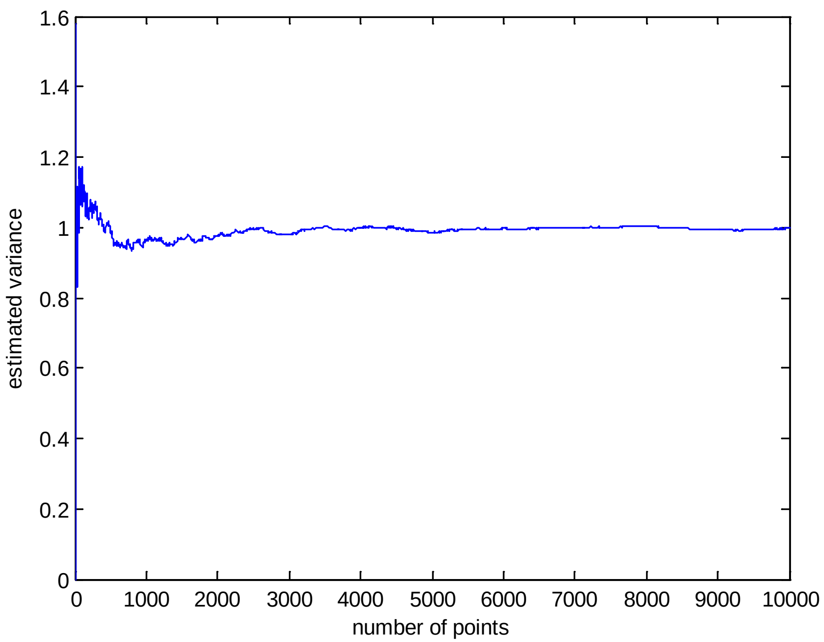
differentiability order 0.5 or less

sprott.physics.wisc.edu/phys505/lect11.htm

Wen Chen. "Soft matters". Slides presented at 2007 FOC_Day @ USU.

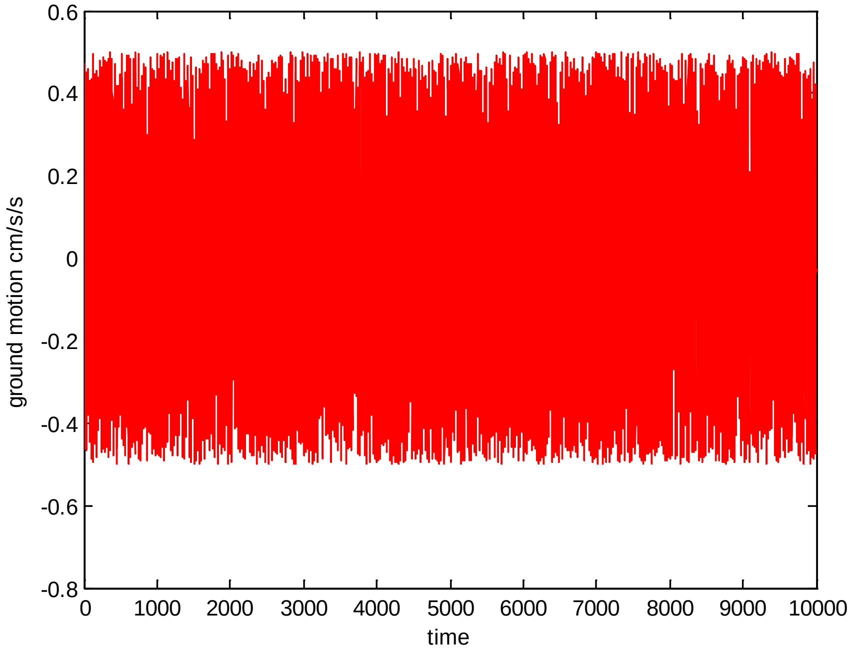
Noise - 1

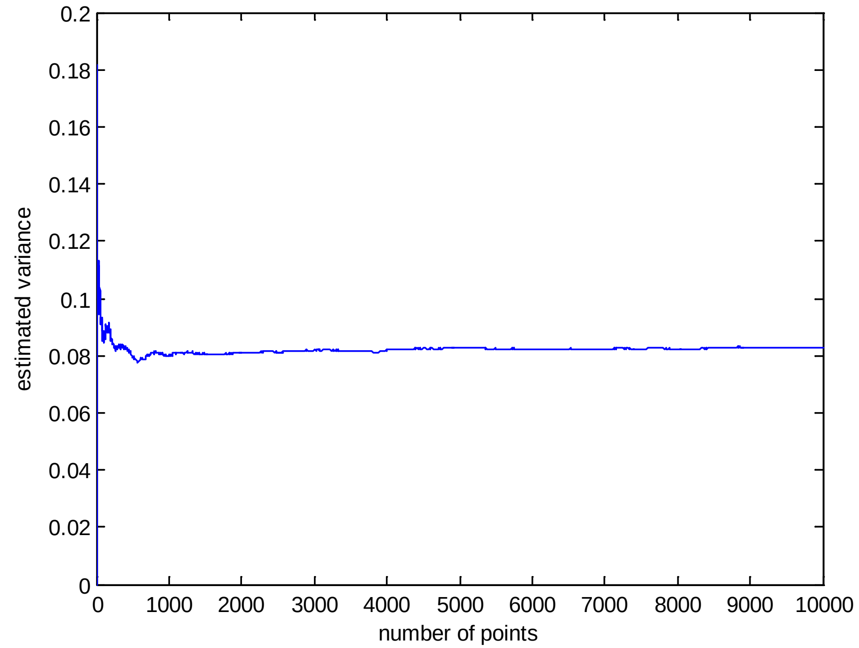




Normal distribution N(0,1) Sample Variance

Noise - 2





Uniformly distributed   Sample Variance

Fractional Lower Order Statistics (FLOS) or Fractional Lower Order Moments (FLOM)

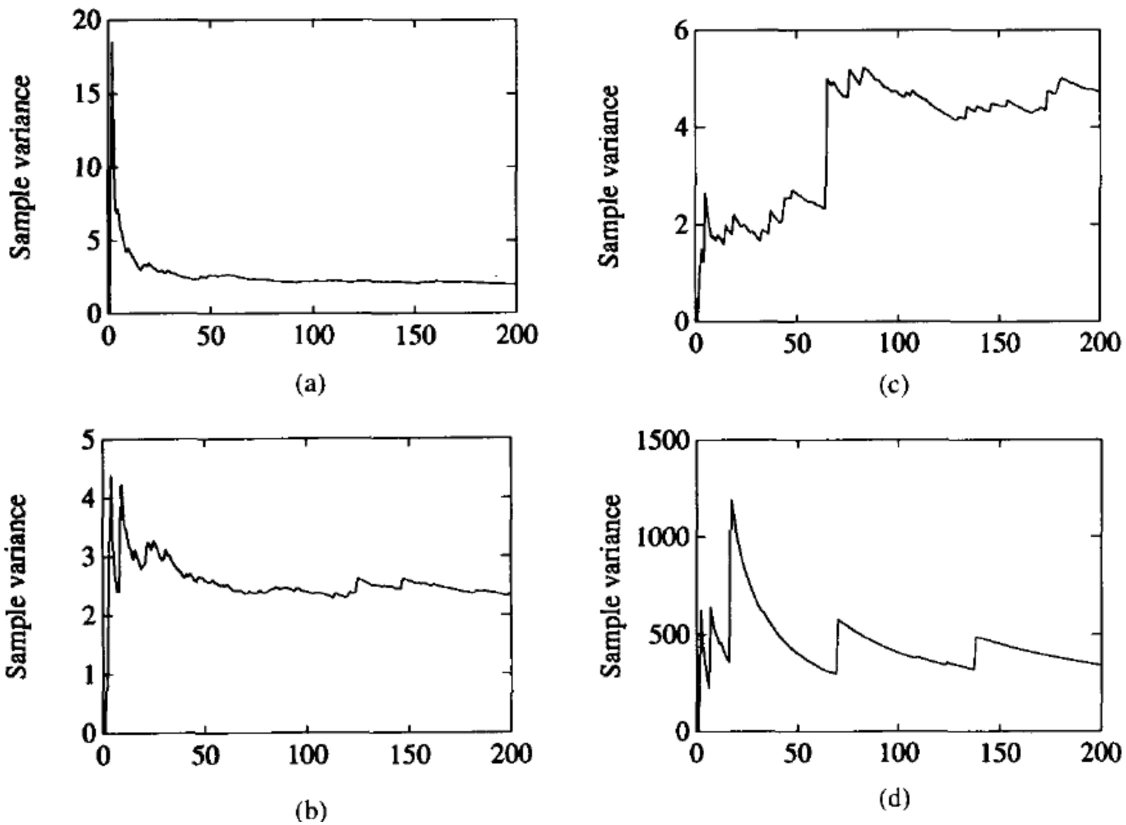


Fig. 2. Running sample variances for four different values of α: (a) $\alpha = 2.0$; (b) $\alpha = 1.9$; (c) $\alpha = 1.5$; (d) $\alpha = 1.1$.

Shao, M., and Nikias, C. L., 1993. “Signal processing with fractional lower order moments: stable processes and their applications”. Proceedings of the IEEE, 81 (7) , pp. 986 – 1010.

Important Remarks

A simple test of infinite variance is to plot the running sample variance estimate S_n with respect to number of points n where $S_n^2 = (\sum_{k=1}^{n}(x_k - \bar{x}_n)^2)/(n-1)$ and $\bar{x}_n = \sum_{k=1}^{n} x_k/n$. For finite variance processes x_k, S_n will converge to a constant value as n increases. If S_n does not converge to a constant value, x_k is a non-Gaussian infinite-variance process with fractional lower order $\alpha < 2$.

In fact, for a non-Gaussian stable distribution with characteristic exponent α, only the moments of orders less than α are finite. Therefore, variance can no longer be used as a measure of dispersion and in turn, many standard signal processing techniques such as spectral analysis and all least squares (LS) based methods may give misleading results.

Long-range dependence

- History: The first model for long range dependence was introduced by Mandelbrot and Van Ness (1968)
- Value: financial data

  communications networks data

  video traffic, biocorrosion data, …

  signals from nature and man-made systems

Long-range dependence

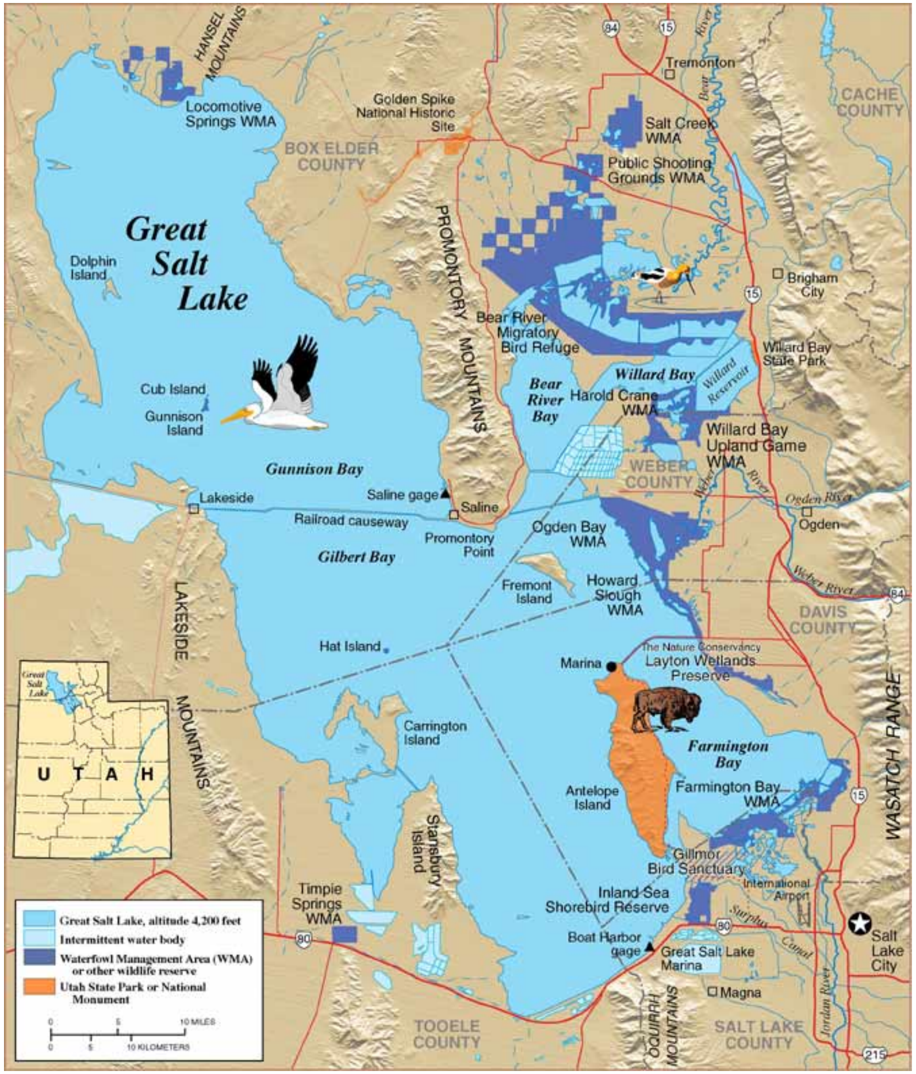
- Consider a second order stationary time series $Y = \{Y(k)\}$ with mean zero. The time series Y is said to be long-range dependent if

$$r_Y(k) = EY(k)Y(0) \sim c_Y |k|^{-\gamma}, k \to \infty, 0 < \gamma < 1$$

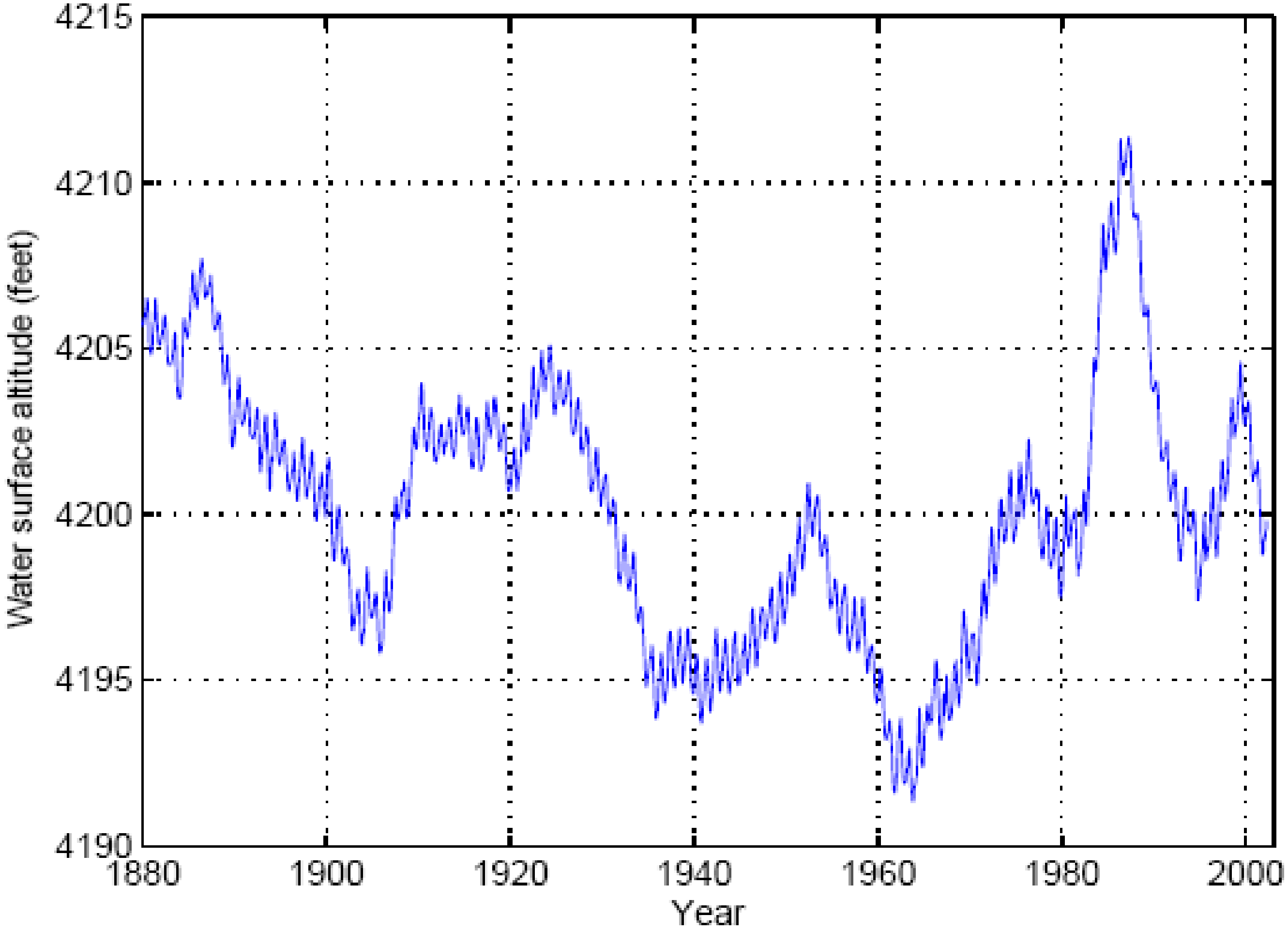
$$s_Y(\xi) \sim c_s |\xi|^{-\alpha}, 0 < \alpha < 1,$$

MESA LAB

GSL: Do you care about it?



Long-term water-surface elevation graphs of the Great Salt Lake



Elevation Records of Great Salt Lake

- The Great Salt Lake, located in Utah, U.S.A, is the fourth largest terminal lake in the world with drainage area of 90,000 km^2.
- The United States Geological Survey (USGS) has been collecting water-surface-elevation data from Great Salt Lake since 1875.
- The modern era record-breaking rise of GSL level between 1982 and 1986 resulted in severe economic impact. The lake levels rose to a new historic high level of 4211:85 ft in 1986, 12.2 ft of this increase occurring after 1982.
- The rise in the lake since 1982 had caused 285 million U.S. dollars worth of damage to lakeside.
- According to the research in recent years, traditional time series analysis methods and models were found to be insufficient to describe adequately this dramatic rise and fall of GSL levels.
- This opened up the possibility of investigating whether there is long-range dependence in GSL water-surface-elevation data so that we can apply FOSP to it.

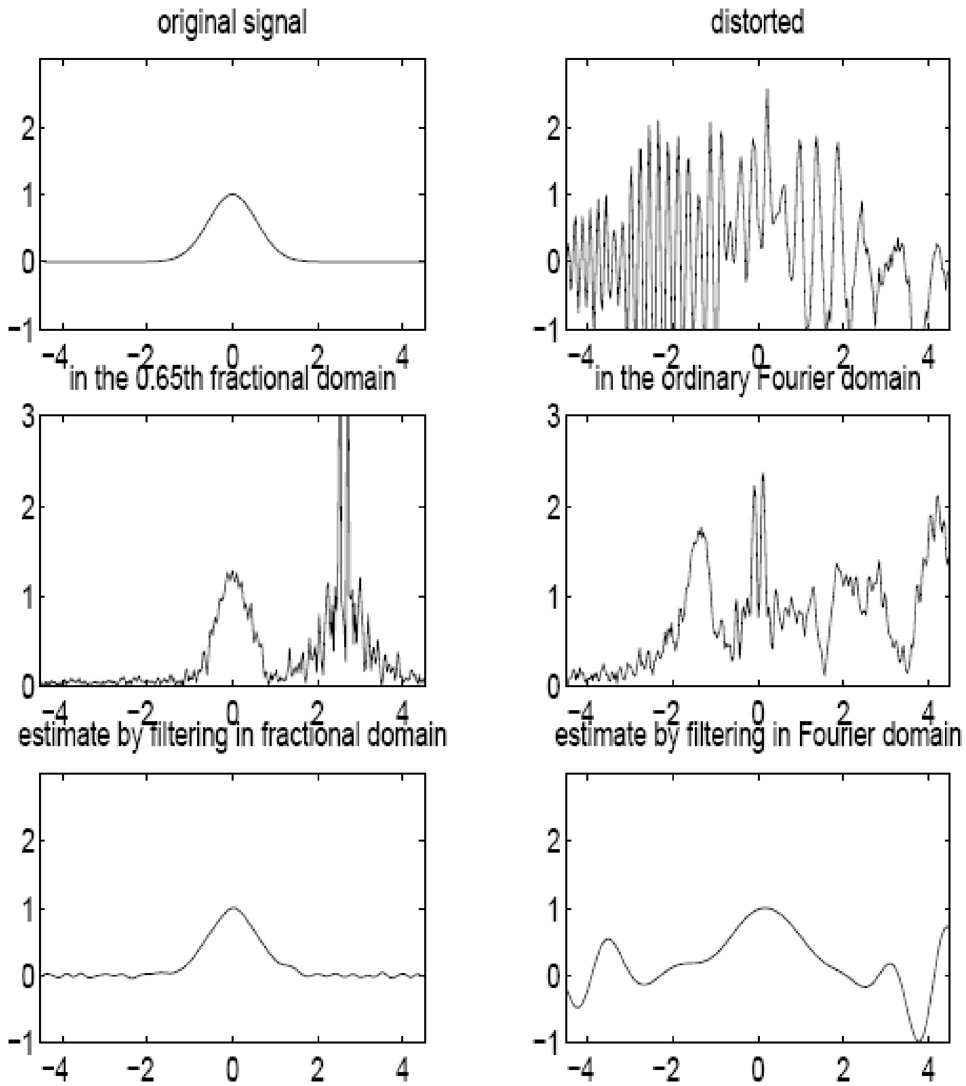
A recent paper

- Hu Sheng, YangQuan Chen “**FARIMA with stable innovations model of Great Salt Lake elevation time series**” Signal Processing, Volume 91, Issue 3, March 2011, Pages 553-561

UCMERCED

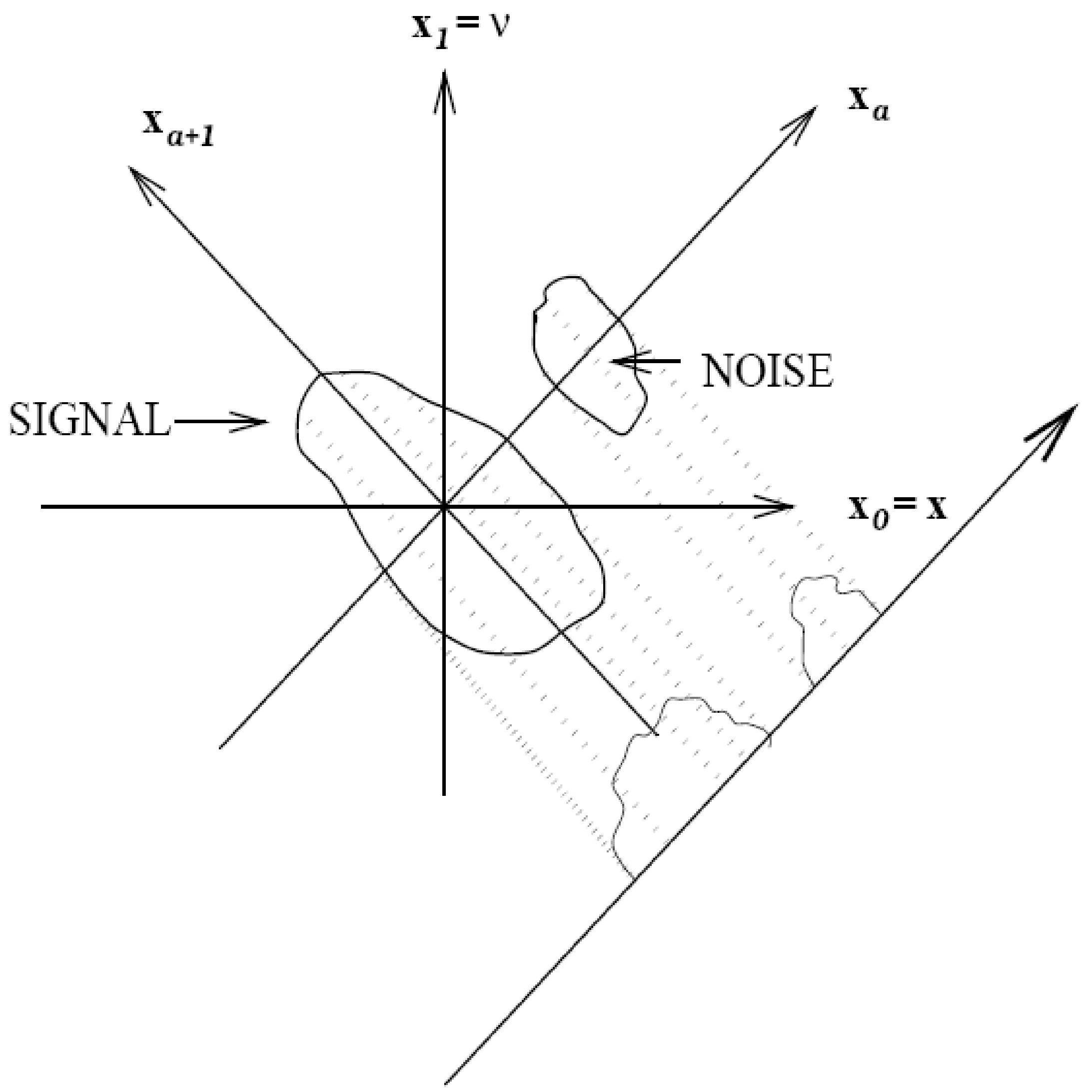
MESA LAB

Optimal filtering in fractional order Fourier domain



Slide credit: HALDUN M. OZAKTAS

Optimal filtering in fractional Fourier domain



Slide credit: HALDUN M. OZAKTAS

A main reference (101 references cited)

- YangQuan Chen* and Rongtao Sun+ and Anhong Zhou. "An Overview of Fractional Order Signal Processing (FOSP) Techniques". DETC2007-34228 in *Proc. of the ASME Design Engineering Technical Conferences*, Sept. 4-7, 2007 Las Vegas, NE, USA, 3rd ASME Symposium on Fractional Derivatives and Their Applications (FDTA'07), part of the 6th ASME International Conference on Multibody Systems, Nonlinear Dynamics, and Control (MSNDC). 18 pages.

FOSP Techniques

- Fractional derivative and integral
- Fractional linear system
- Autoregressive fractional integral moving average
- 1/f noise
- Hurst parameter estimation
- Fractional Fourier Transform
- Fractional Cosine, Sine and Hartley transform
- Fractals
- Fractional Splines
- Fractional Lower Order Moments (FLOM) and Fractional Lower Order Statistics (FLOS)

Fractional Calculus, LRD, Power Law,

$u(t)$ → $H(s) = \dfrac{1}{s^{\alpha}}$ → $y(t)$

White Noise

$$h(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}$$

$$Ryy(\tau) = \sigma^2 \frac{|\tau|^{2\alpha-1}}{2\Gamma(2\alpha)\cos\alpha\pi}$$

y(t) is a Brownian motion when α=1, i.e., $1/f^2$ process.

$1/f^{2\alpha}$ noise (signal) generation via fractional dynamic system

Power laws in

- Signal/Systems
- Probability distribution
- Random processes (correlation functions)

Rule of thumb for Fractional Order Thinking

- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- *1/f* ^{a} noise
- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …

Power law and power law Lyapunov

- "Power law is ubiquitous" – John Doyle 2001 IEEE CDC Plenary Talk http://www.cds.caltech.edu/~doyle/CDC2001/index.htm
- "When you talk about power law, you are talking actually about fractional order calculus!" – YangQuan Chen 2006 IFAC FDA06 Plenary Talk
- "Lyapunov is ubiquitous in control literature" – ibid.

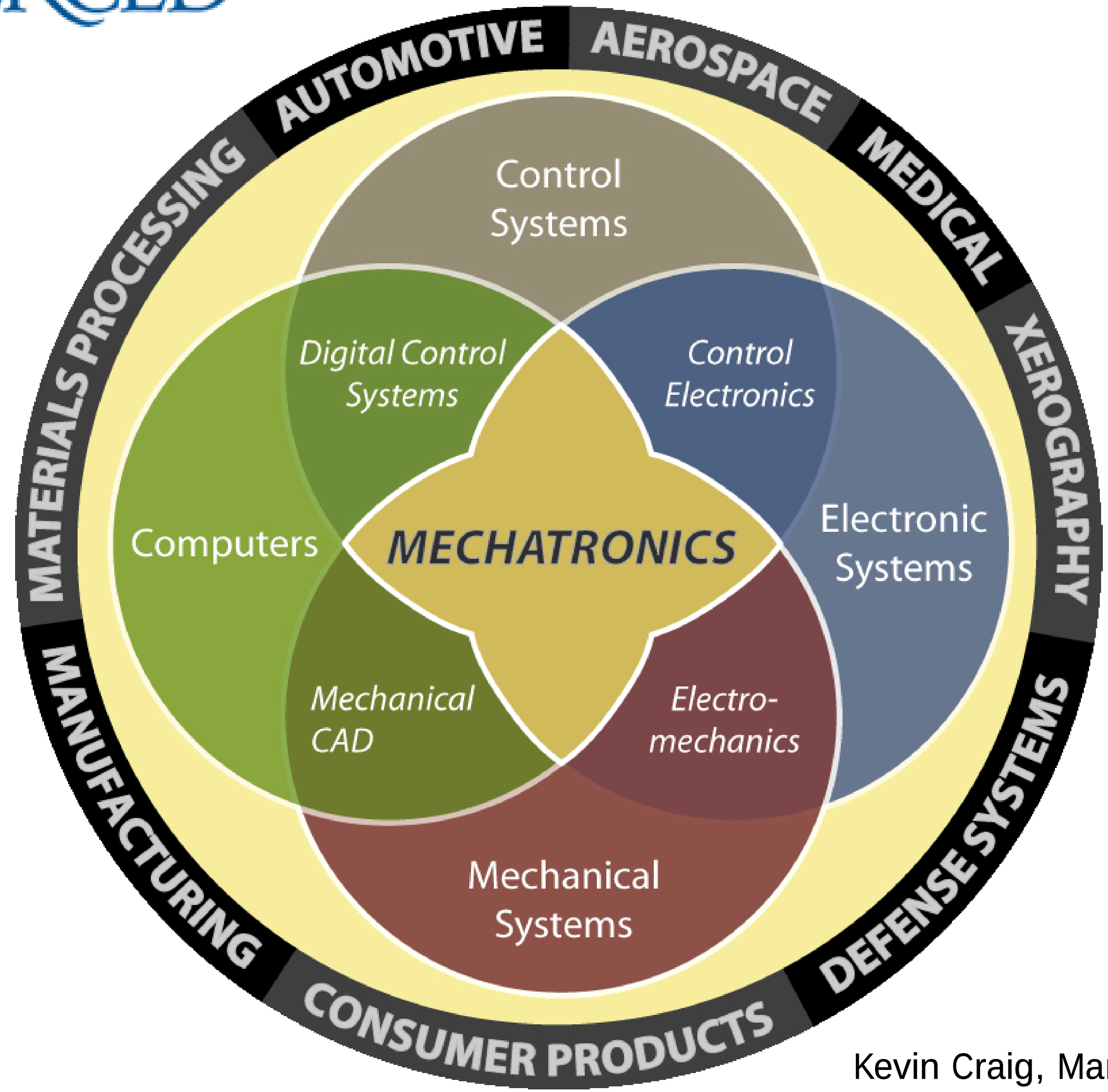
Intuitions

- One does not have to be rich to be smart.
- One does not have to be smart to use fractional order calculus.
- **A dynamic system does not have to make the "*generalized energy*" decay exponentially to be stable!**

Y. Li, Y. Q. Chen and I. Podlubny. "Mittag-Leffler Stability of Fractional Order Nonlinear Systems", *Automatica*, 45(8): 965-1969, 2009. DOI: 10.1016/j.automatica.2009.04.003

Outline

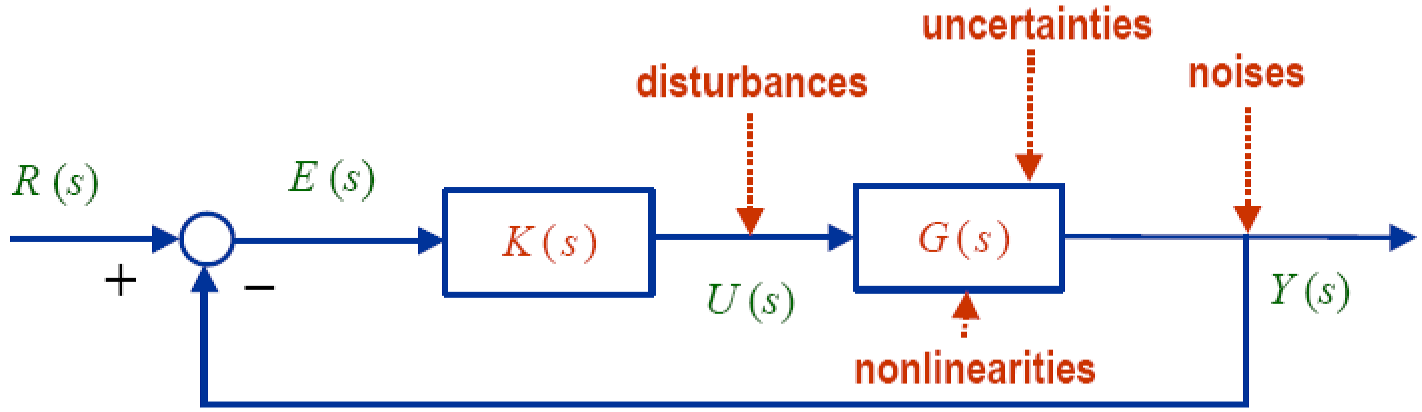
- Fractional Calculus and Fractional Order Thinking
- From Control, Signal Processing to Energy Informatics and Beyond
- Concluding Remarks



Kevin Craig, Marquette Univ.

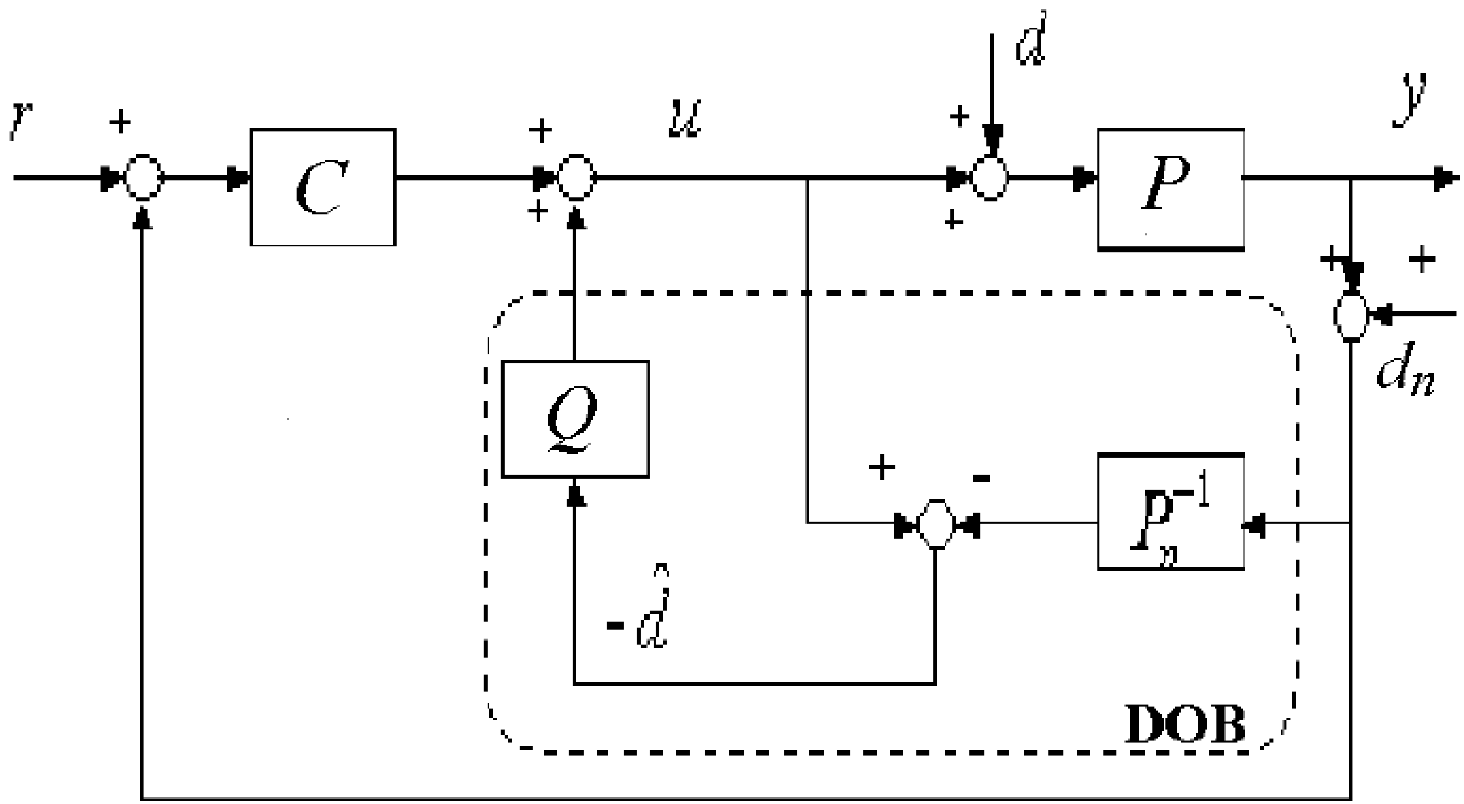
Fractional Order Controls

- **IO Controller + IO Plant**
- **FO Controller + IO Plant**
- **FO Controller + FO Plant**
- **IO Controller + FO Plant**



D. Xue and **Y. Chen***, "**A Comparative Introduction of Four Fractional Order Controllers**". Proc. of The 4th IEEE World Congress on Intelligent Control and Automation (WCICA02), June 10-14, 2002, Shanghai, China. pp. 3228-3235.

YangQuan Chen*, Blas M. Vinagre and Igor Podlubny. "Fractional order disturbance observer for vibration suppression", (Kluwer) Nonlinear Dynamics , Vol. 38, Nos. 1-4, December 2004, pp. 355-367.



Attacked topics

- Fractional order adaptive control
- Fractional order PI/D control
- Most recently
  - Fractional order conditional integrator (e.g. Clegg integrator) (JPC)
  - Fractional order consensus seeking (IEEE SMC-B 10)
  - Fractional order optimal control (MATLAB Toolbox)
  - Fractional order model predictive control (??)

How to design/tune FOC for motion control?

$$C(s) = K_p(1 + K_d s^{\mu})$$

$$P(s) = \frac{1}{s(Ts+1)}$$

(i) Phase margin specification

$$\mathrm{Arg}[G(j\omega_c)] = \mathrm{Arg}[C(j\omega_c)P(j\omega_c)] = -\pi + \phi_m,$$

(ii) Robustness to variation in the gain of the plant

$$\left(\frac{\mathrm{d}(\mathrm{Arg}(C(j\omega)P(j\omega)))}{\mathrm{d}\omega}\right)_{\omega=\omega_c} = 0,$$

with the condition that the phase derivative w. r. t. the frequency is zero, i.e., the phase Bode plot is flat, at the gain crossover frequency. It means that the system is more robust to gain changes and the overshoots of the response are almost the same.

(iii) Gain crossover frequency specification

$$|G(j\omega_c)|_{dB} = |C(j\omega_c)P(j\omega_c)|_{dB} = 0.$$

Impressive Performance!

- How about FO[PD]?

$$C_3(s) = K_{p3}[1 + K_{d3}s]^{\mu}$$

- Note: FOPD shown previously is:

$$C_2(s) = K_{p2}(1 + K_{d2}s^{\lambda})$$

Ying Luo, Y. Q. Chen "Fractional order [proportional derivative] controller for a class of fractional order systems" *Automatica*, 45(10) 2009, pp 2446-2450.

Smart Mechatronics

Biomimetic Materials and Biomimetic Actuators

- EAP (electroactive polymers), a.k.a. artificial muscle
- ferroelectric and relaxor materials
- piezoceramic and piezopolymetric materials
- liquid crystal elastomers
- electro and magnetostrictive materials
- shape memory alloys/polymers
- intelligent gels etc.

However, little has been reported on the controls of actuators made with these biomimetic materials.

Compensation of nonlinearity with memory

- e.g., hysteresis, backlash.
- My Assertion: Fractional calculus may better help us.

MESA LAB

A Hidden Evidence

IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, VOL. 9, NO. 1, JANUARY 2001 17

Phase Control Approach to Hysteresis Reduction

Juan Manuel Cruz-Hernández, *Member, IEEE,* and Vincent Hayward, *Member, IEEE,*

***Abstract*—This paper describes a method for the design of compensators able to reduce hysteresis in transducers, as well as two measures to quantify and compare controller performance. Rate independent hysteresis, as represented by the Preisach model of hysteresis, is seen as an input–output phase lag. The compensation is based on controllers derived from the "phaser," a unitary gain operator that shifts a periodic signal by a single phase angle. A "variable phaser" is shown to be able to handle minor hysteresis loops. Practical implementations of these controllers are given and discussed. Experimental results exemplify the use of these techniques.**

***Index Terms*—Compensation, hysteresis, intelligent materials, phase control, piezoelectric transducers, smart materials, transducers.**

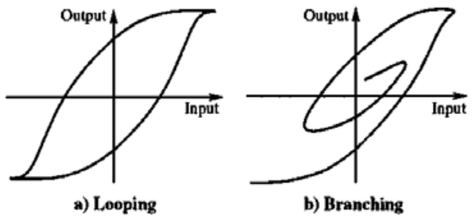


Fig. 1. Hysteresis loop and branching.

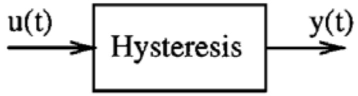


Fig. 2. A black box representation of hysteresis.

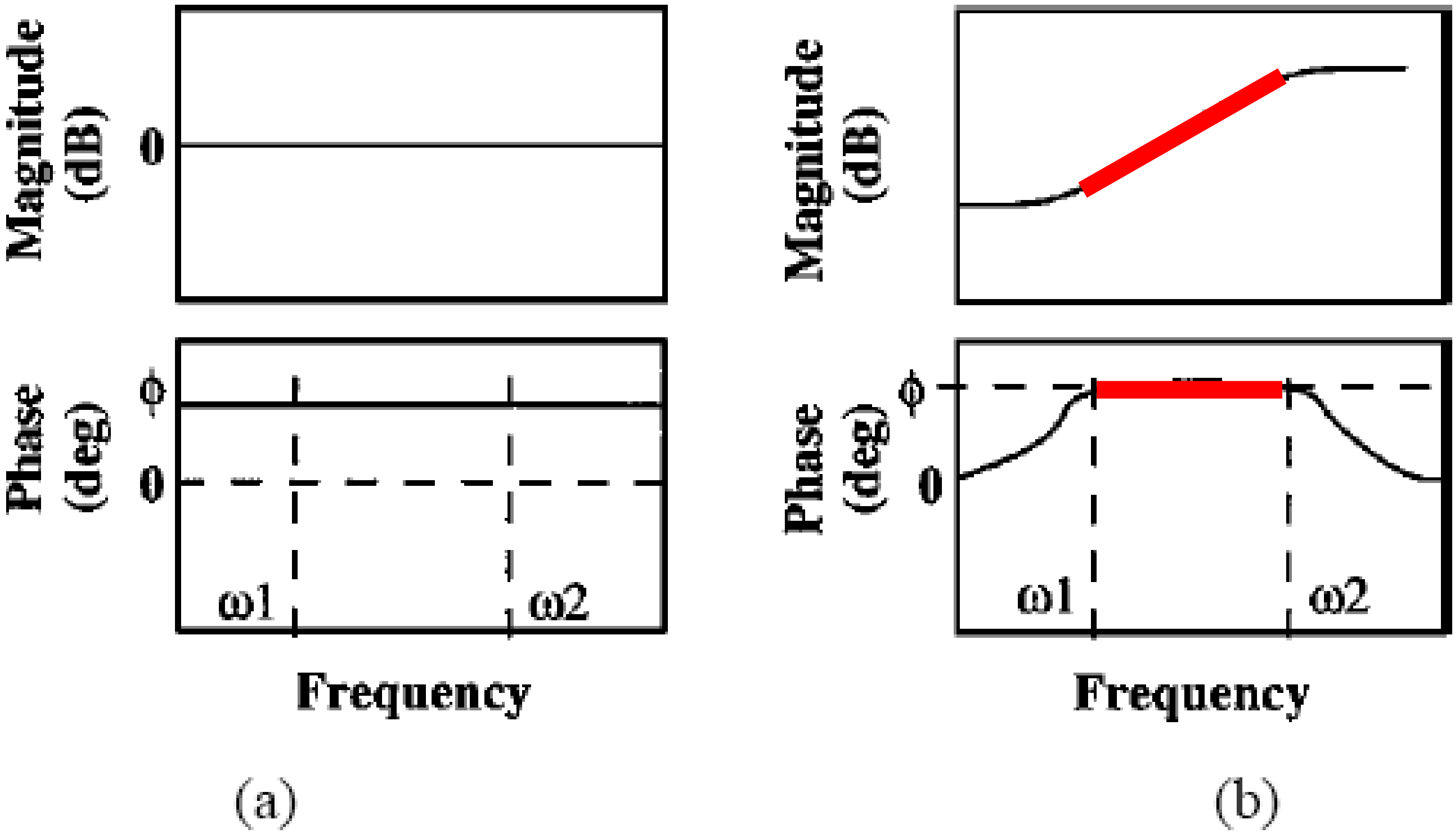


Fig. 10. Frequency response. (a) Ideal phaser. (b) Approximation.

UCMERCED MESA LAB

"smart material" based Fractor™

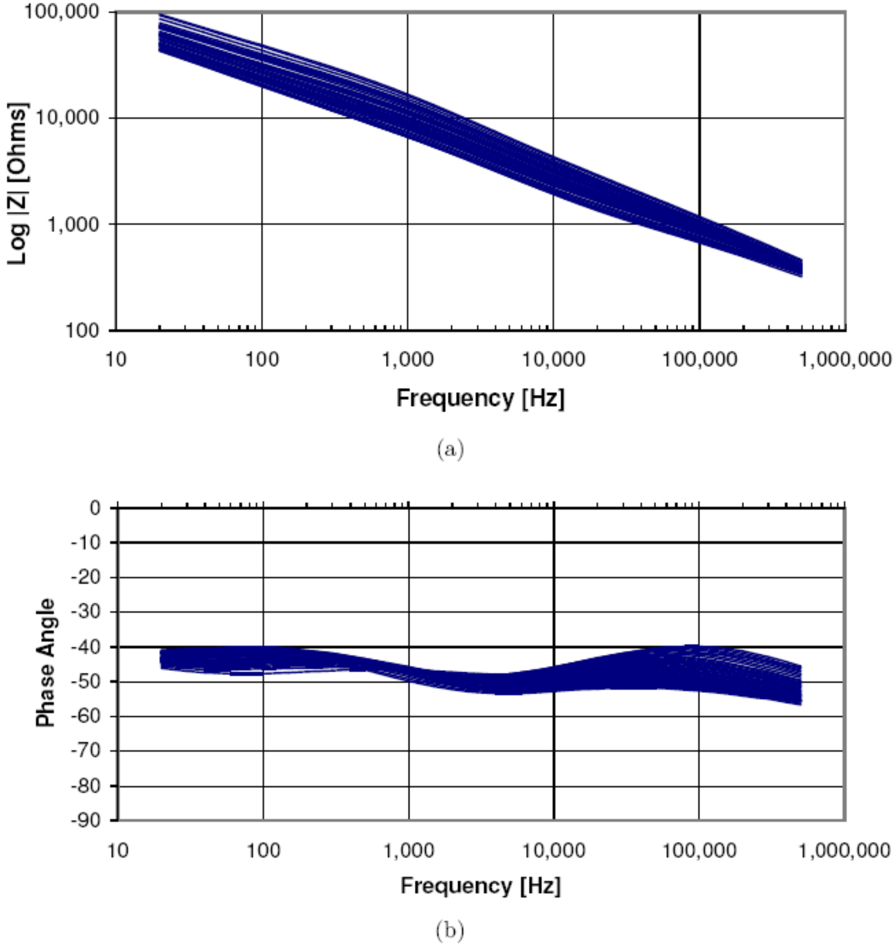


Fig. 1. Spectral response of the Fractor™ used in this demonstration project; (a) the impedance magnitude and (b) impedance phase. The multiple lines show the variation over 26 impedance measurement scans.

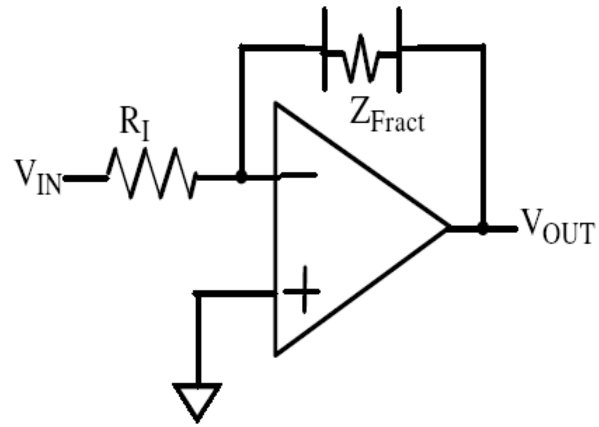


Fig. 2. Schematic for a fractional order integrator. Z_F represents the Fractor™ element. The schematic symbol for the Fractor™ was designed to give the impression of a generalized Warburg impedance; a mixture of resistive and capacitive characteristics.

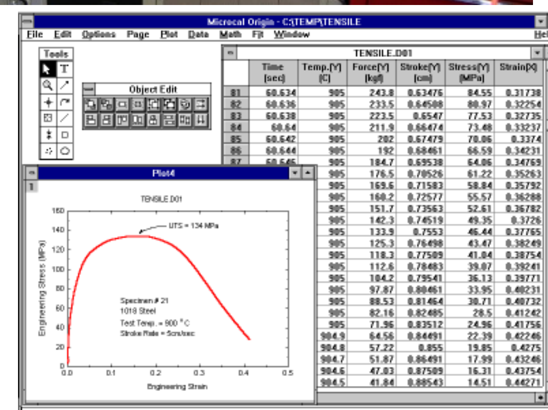
Gary W. Bohannan "**Analog Fractional Order Controller in a Temperature Control Application**". Proc. of the 2nd IFAC FDA06, July 19-21, 2006, Porto, Portugal.

Big Picture, or, *The take-home message*

- The big picture for the future is the intelligent control of biomimetic system using biomimetic materials with fractional order calculus embedded. In other words, it is definitely worth to have a look of the notion of ``*intelligent control of intelligent materials using intelligent materials*."

USU Material Research Laboratory

- **Materials Processing, Heat Treating, Materials Joining, and Powder Metallurgy Studies using the Gleeble 1500D System**



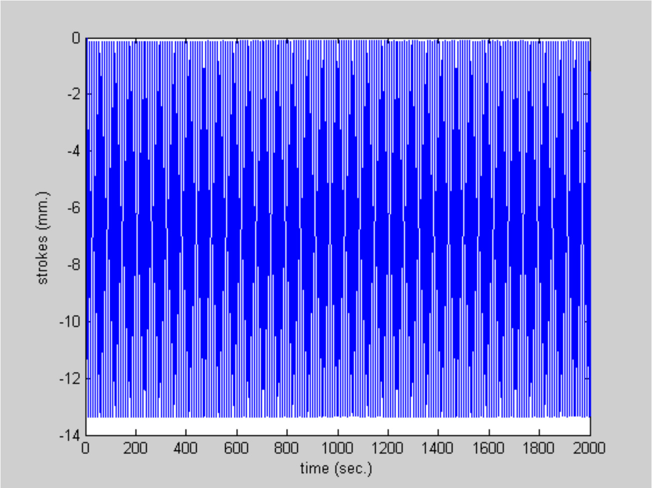
Source: http://www.mae.usu.edu/faculty/leijun/gleeble.html

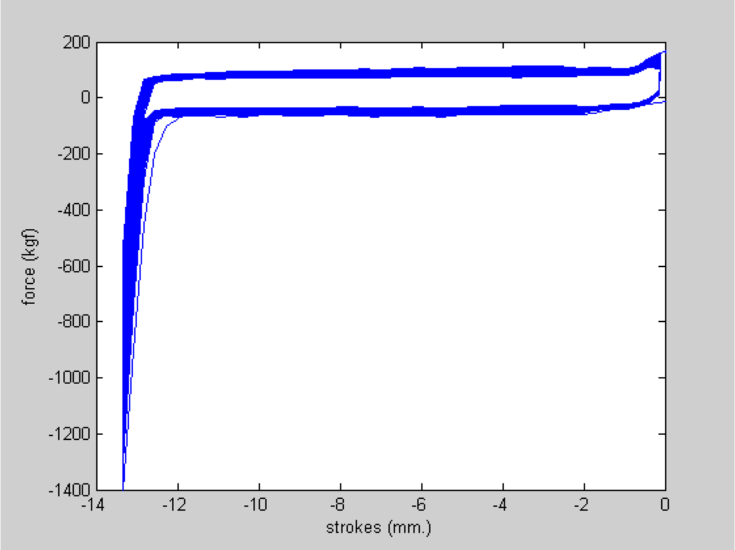
NSF NER: Solid-state synthesis of nano-scale hydrogen storage materials by bulk mechanical alloying

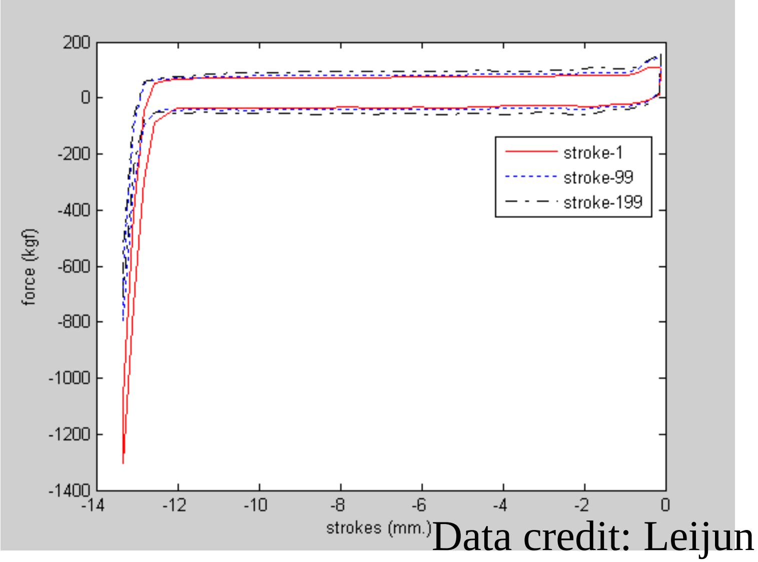
http://www.mae.usu.edu/faculty/leijun/

Fractional order calculus?

- Dynamic force measurements vs. strokes

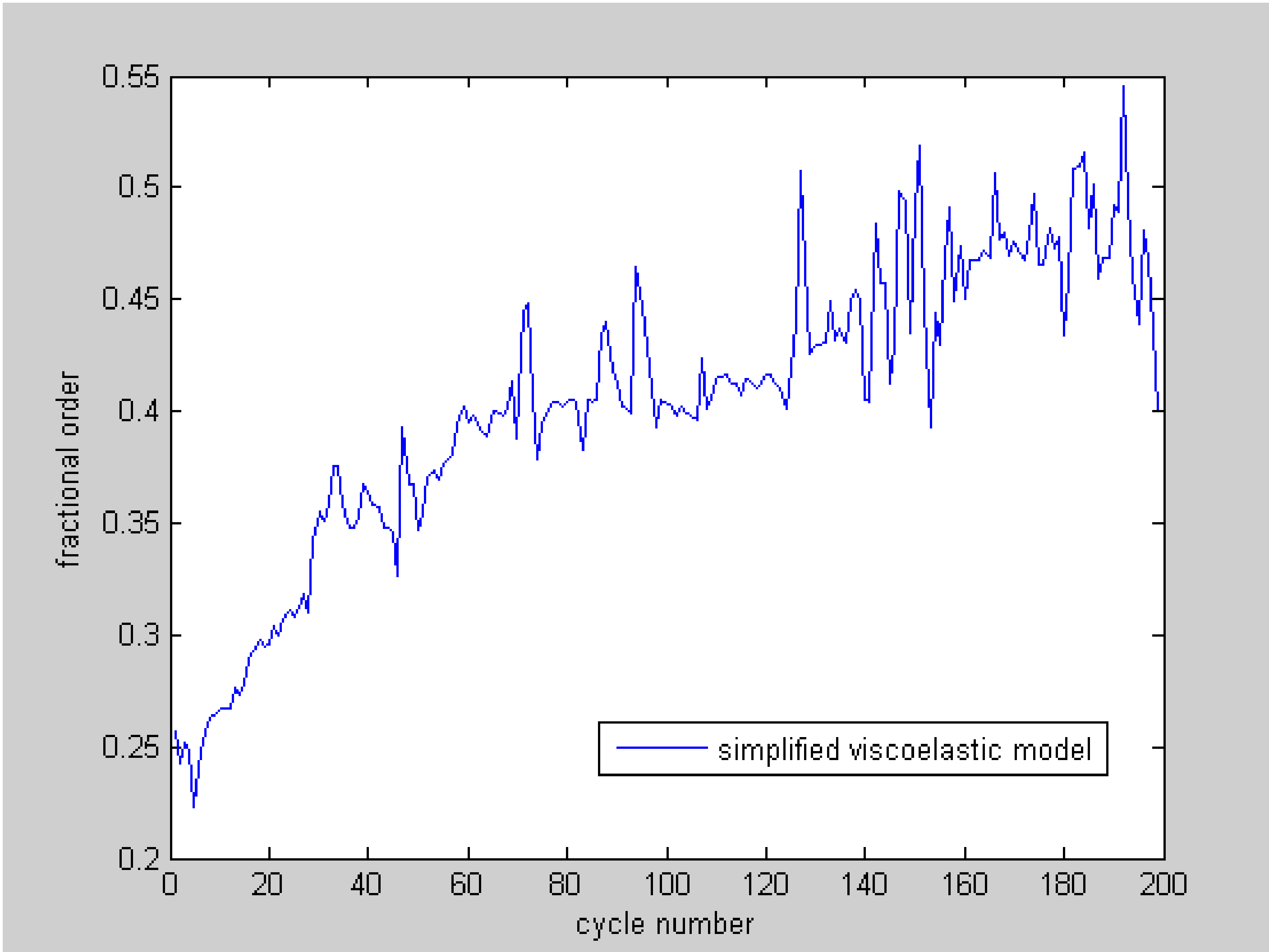






Data credit: Leijun Li

Fractional order vs. strokes



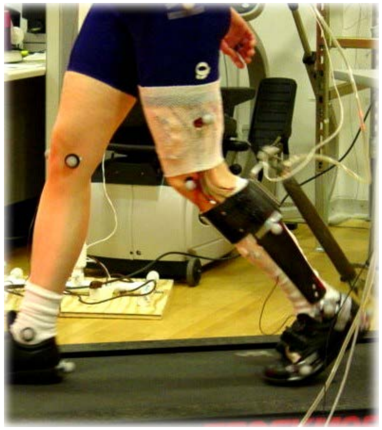
Big picture of nanoparticle manufacturing

- Now: given cycles, given stroke profile, see how particulate process evolves.
- Future: Production process development – given final particle grain size distribution, how to achieve this by using minimum number of cycles with possible cycle-to-cycle, or run-to-run (per several cycles) adaptive learning control with variable stroke profiles.

Fractional order ILC (iterative learning control)?

- D-alpha type ILC with a (really good) reason?!
  - YangQuan Chen and Kevin L. Moore. ``*On D^{α}-type Iterative Learning Control*". Presented at the IEEE Conference on Decision and Control (CDC'01), Dec. 3-7, 2001, Orlando, FL, USA. pp.4451-4456. http://www.csois.usu.edu/publications/pdf/pub054.pdf

Biomechatronics



Electronically controlled leg and hand prosthesis, neural prosthesis, retinal implants, assistive and rehabilitative robots ...

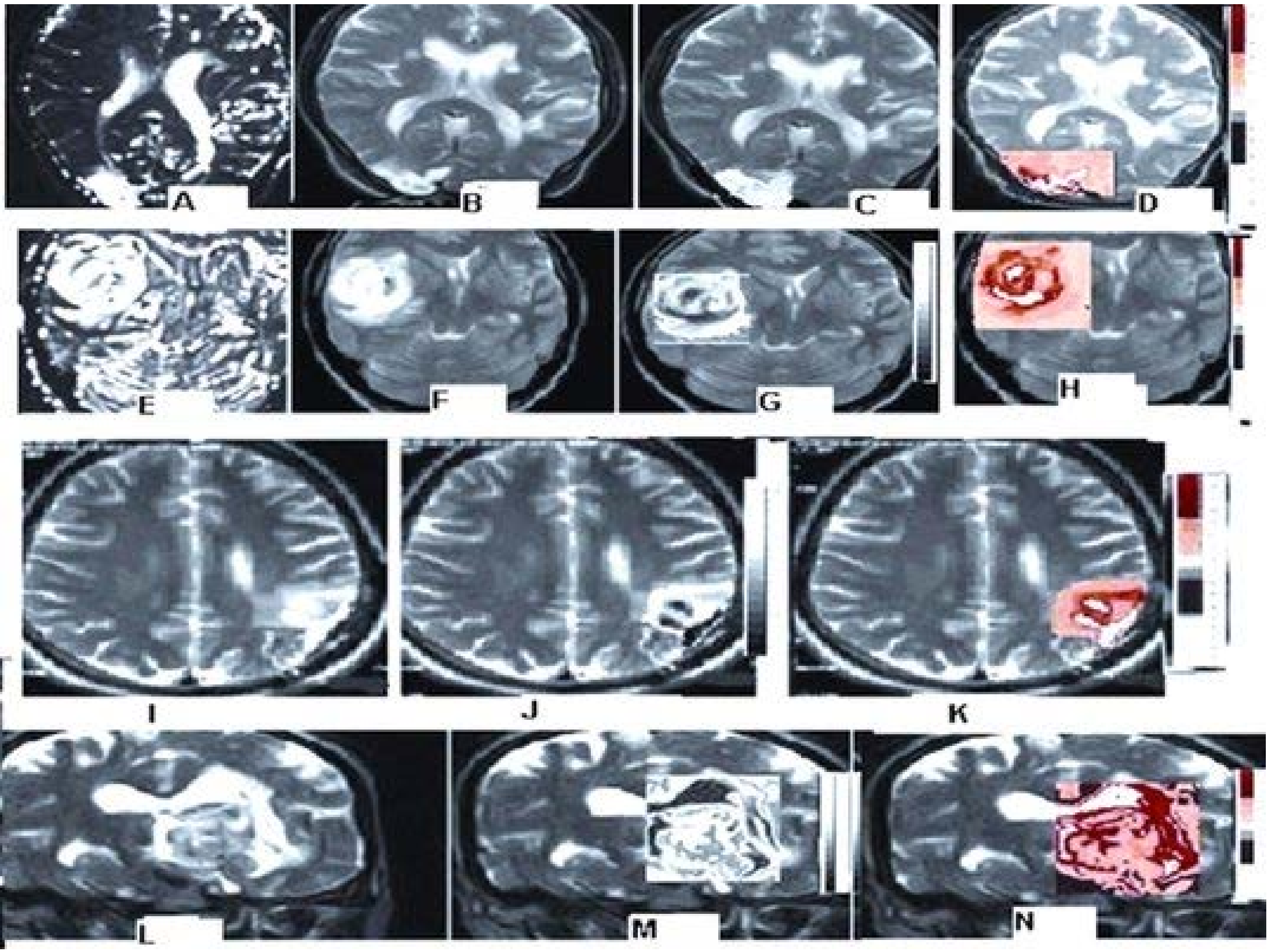
-- "Emerging Trends and Innovations in Biomechatronics" by Frost & Sullivan

Source: http://www-personal.umich.edu/~ferrisdp/NSF/research.htm

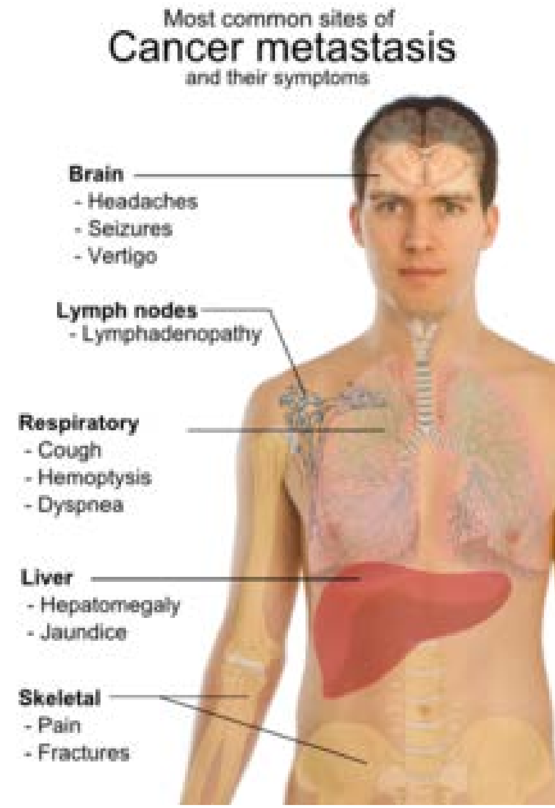
Biomechatronics

- Biomechatronics is the interdisciplinary study of biology, mechanics, and electronics. Biomechatronics focuses on the interactivity of biological organs (including the brain) with electromechanical devices and systems.
- Universities and research centers worldwide have taken notice of biomechatronics in light of its potential for development of advanced medical devices and life-support systems.

Complex relaxation in NMR

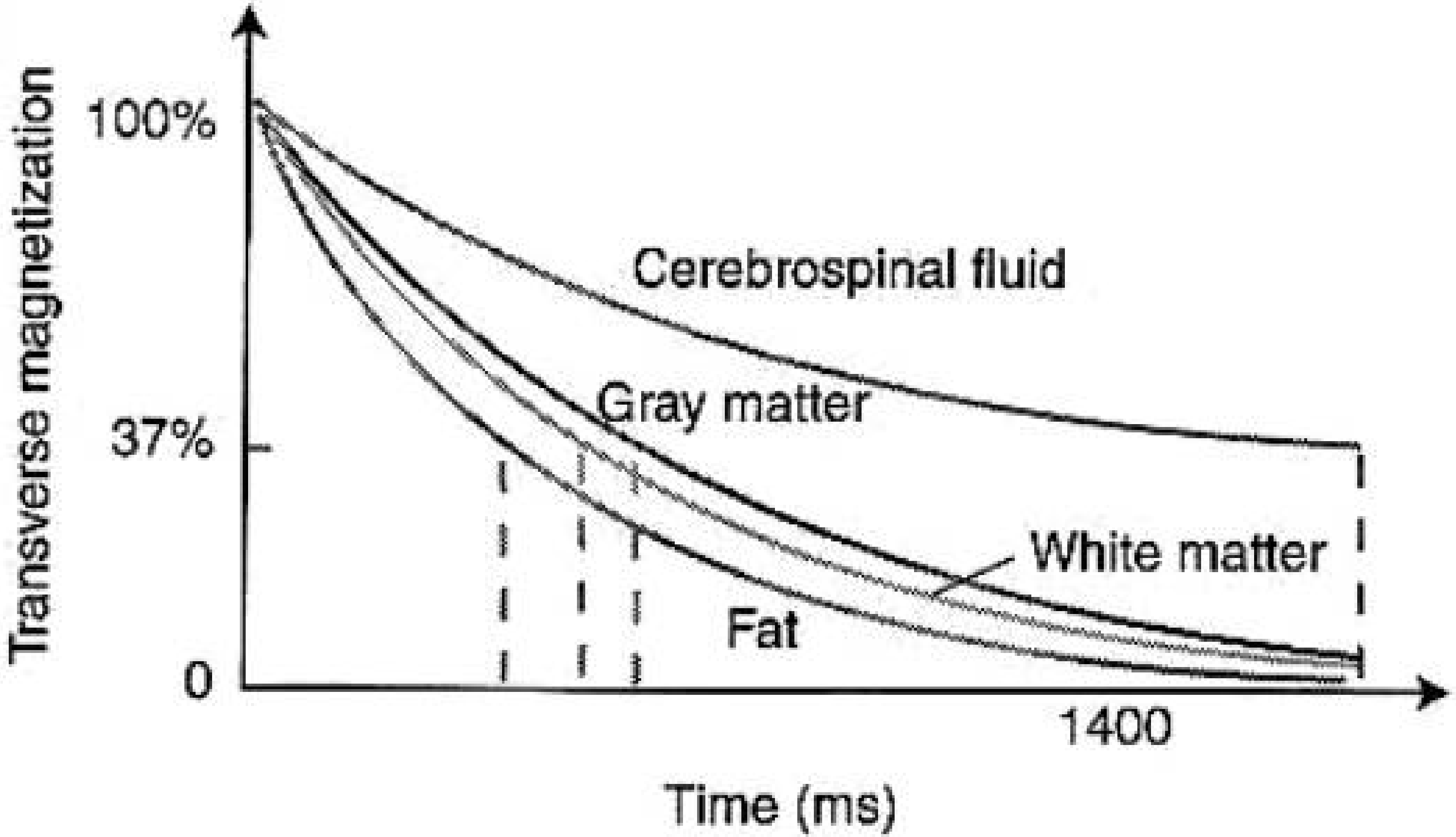


http://en.wikipedia.org/wiki/Metastasis



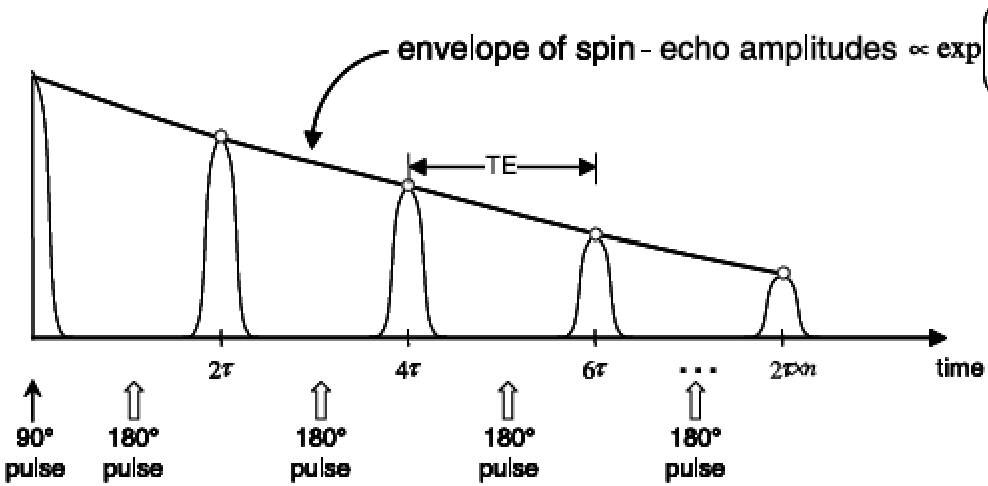
http://www.ispub.com/journal/the-internet-journal-of-radiology/volume-13-number-1/in-vivo-mr-measurement-of-refractive-index-relative-water-content-and-t2-relaxation-time-of-various-brain-lesions-with-clinical-application-to-discriminate-brain-lesions.article-g08.fs.jpg

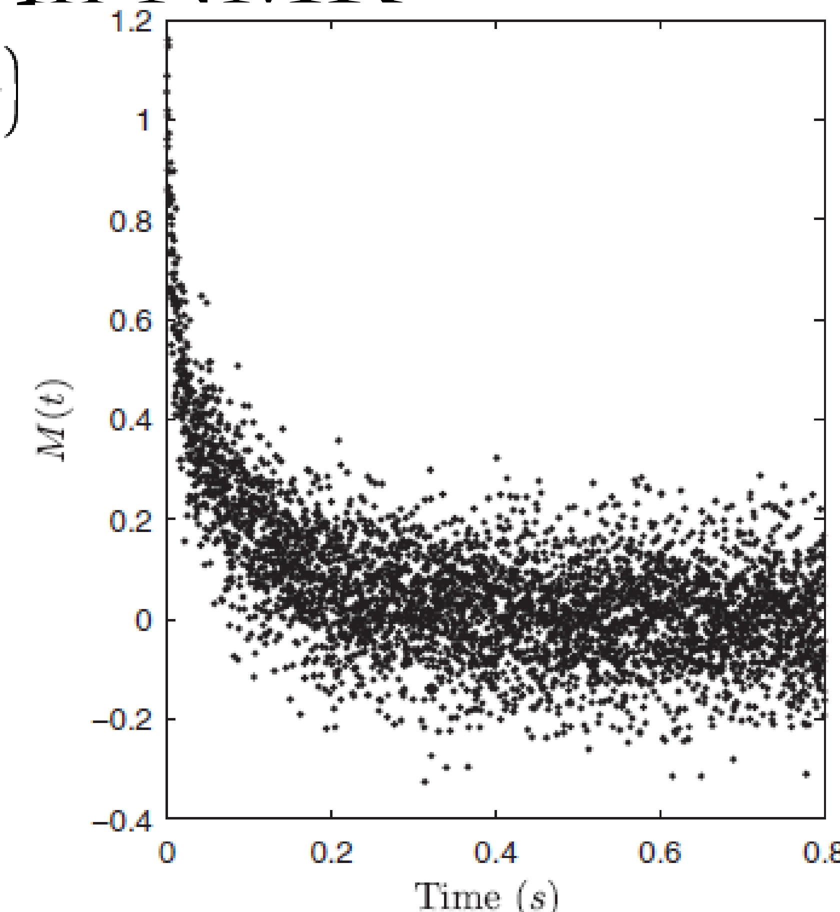
T2 relaxation in NMR



http://hs.doversherborn.org/hs/bridgerj/DSHS/apphysics/NMR/T2.htm

T2 relaxation in NMR



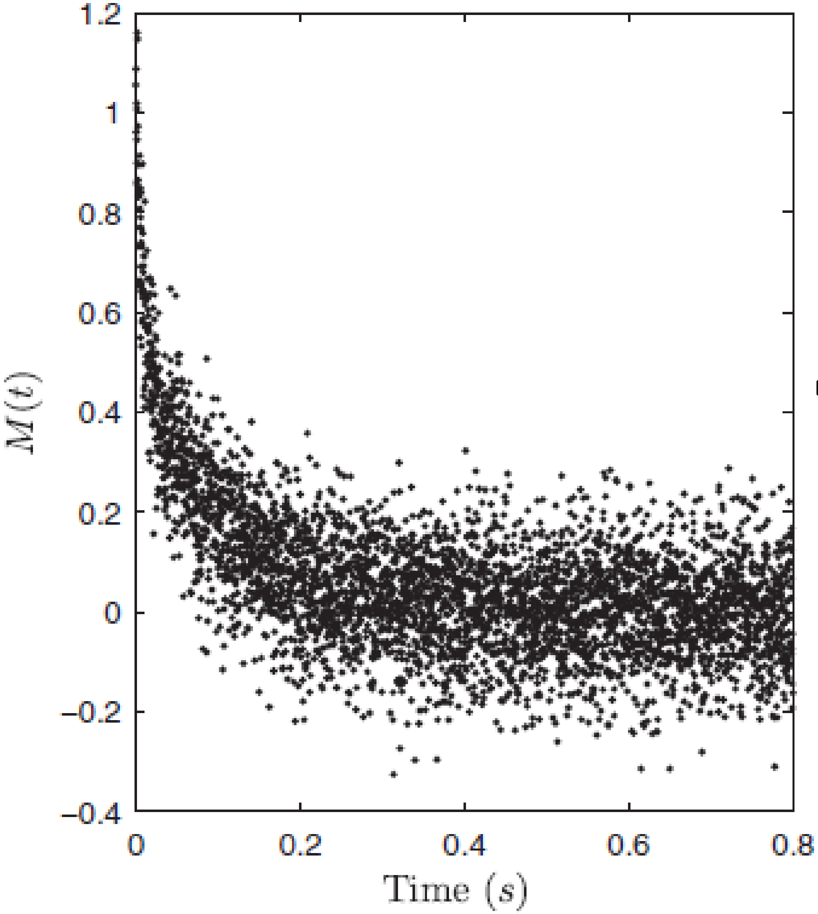


Carr–Purcell–Meiboom–Gill (*CPMG*) pulse sequence, as shown in Fig. 1, is widely used to measure spin–spin *relaxation* time *T2*

magnetization decay data *M(t) with poor SNR*

UCMERCED MESA LAB

Complex relaxation: How to characterize or model it?



- Debye relaxation $\exp(-t/\tau)$

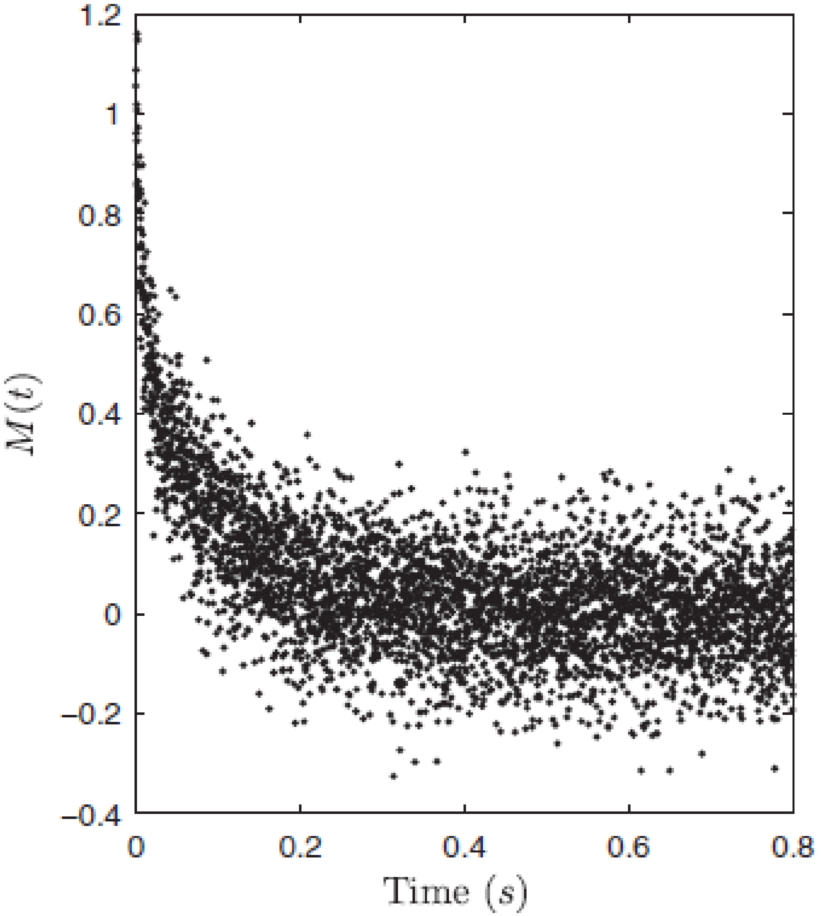
$$1/(1+\tau s)$$

- Distributed-parameter (infinite # of time constants)

$$\int_0^T \frac{f(\tau)}{\tau s+1}\, d\tau$$

(H. Fröhlich, 1949)

Complex relaxation: How to better characterize or model it?



- Cole-Cole (1941)

$$1/\left(1+\tau s^{\alpha}\right)$$

- Distributed-parameter (infinite # time constants)

$$\int_0^T \frac{f(\tau)}{\tau s^{\alpha}+1} d\tau$$

(Hu, Li and Chen, IEEE CDC 2010)

More complex relaxation models

- Cole-Davidson

$$H_{\text{C-D}}(s) = \int_0^T \frac{f(\tau)}{(1+\tau s)^{\beta}} \mathrm{d}\tau$$

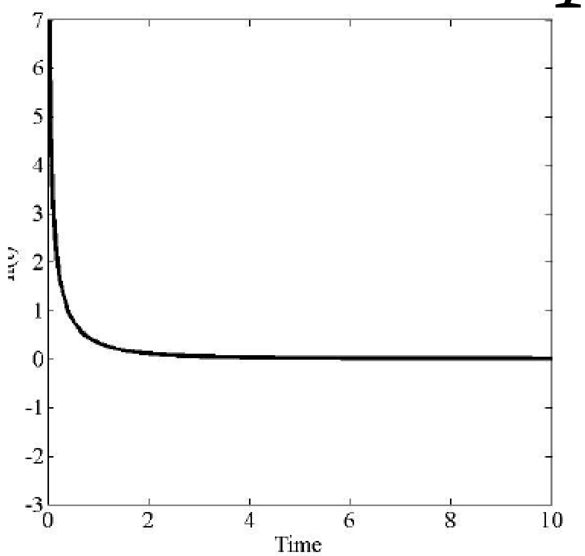
- Havriliak-Negami

$$H_{\text{H-N}}(s) = \int_0^T \frac{f(\tau)}{(1+\tau s^{\alpha})^{\beta}} \mathrm{d}\tau$$

- **Distributed-order case? Sure!**

$$H(s) = \int_0^1 \frac{f(\gamma)}{\tau s^{\gamma}+1} \mathrm{d}\gamma$$

An illustration

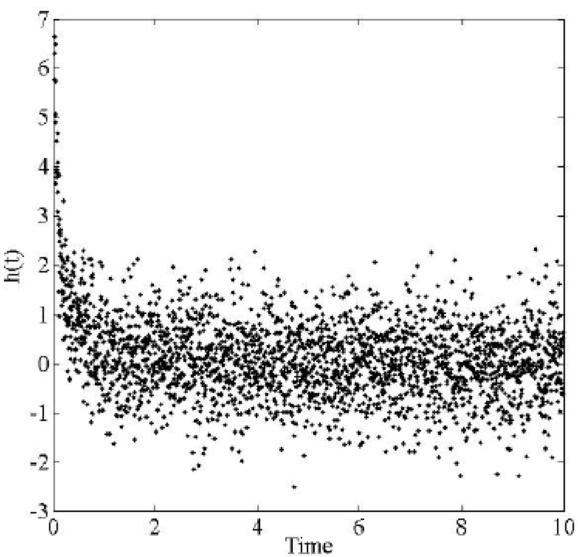


- Distributed-parameter (infinite time constants)

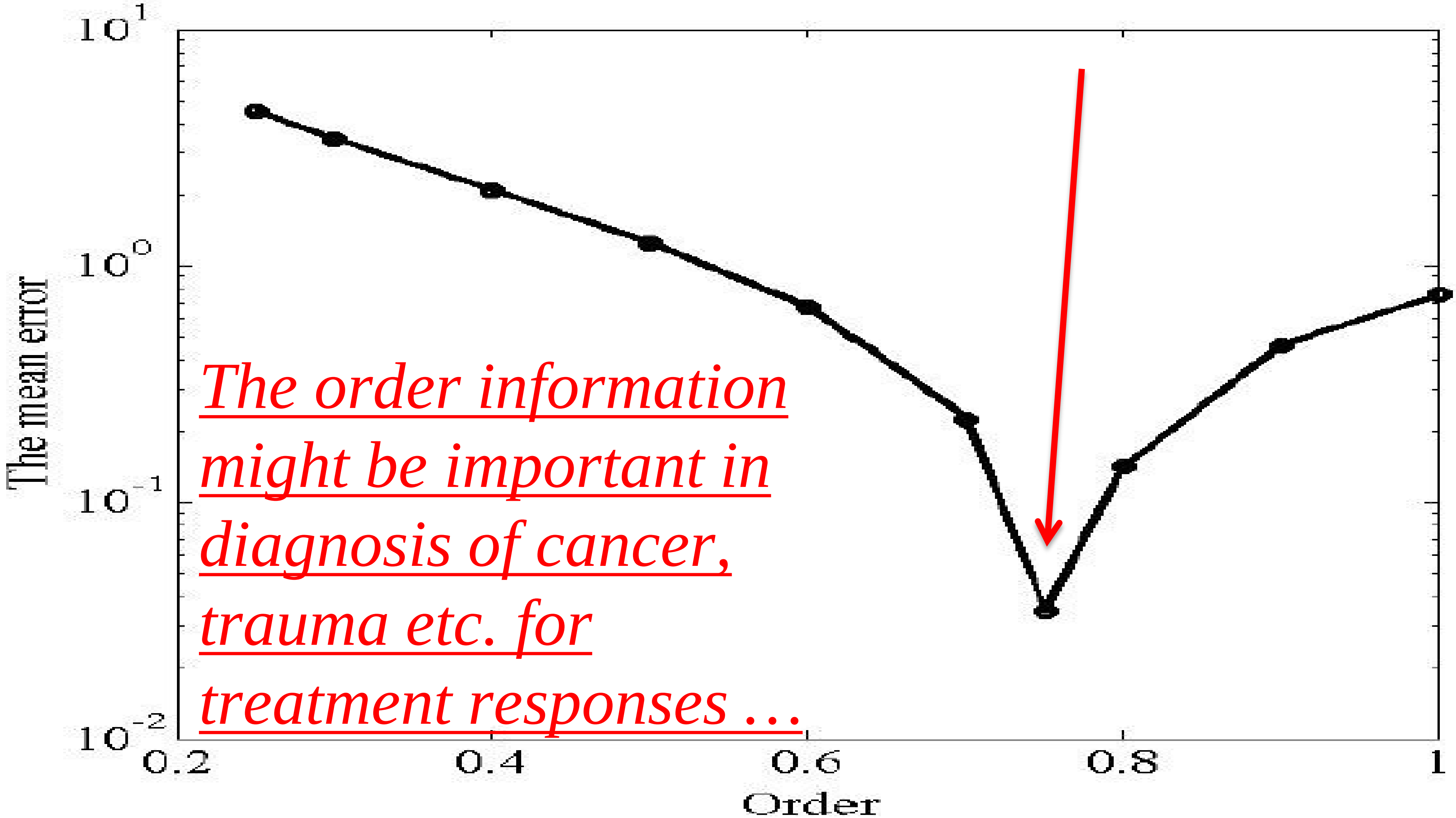
$$\int_0^T \frac{f(\tau)}{\tau s^{\alpha}+1}\,\mathrm{d}\tau$$

- $\alpha = 0.75$, T=1 sec.

$$f(\tau) = 1 + 2\tau$$

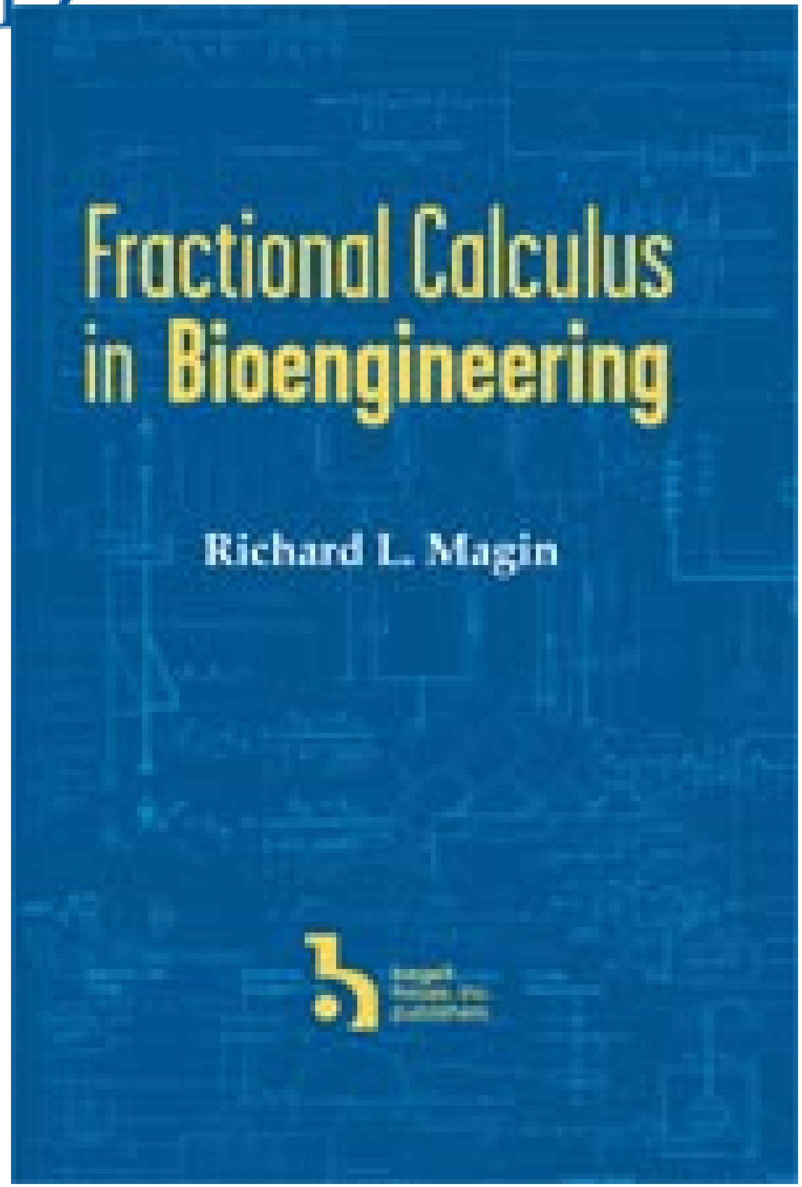


Scanning the “order” and fitting



Applications – C-FOSE Proposal (Center for Fractional Order Systems Engineering)

1. Human-augmentation
2. Human Nerve System
3. Robotic equipment
4. Electric drive systems
5. Power Converters
6. Disk drive servo
7. Audio signal processing
8. Aircraft
9. Automobiles
10. Fuel cells
11. Lidar, radar, sonar, ultrasonic imaging
12. Battery chargers
13. Nuclear reactors
14. Temperature Control
15. Biosensor signal processing



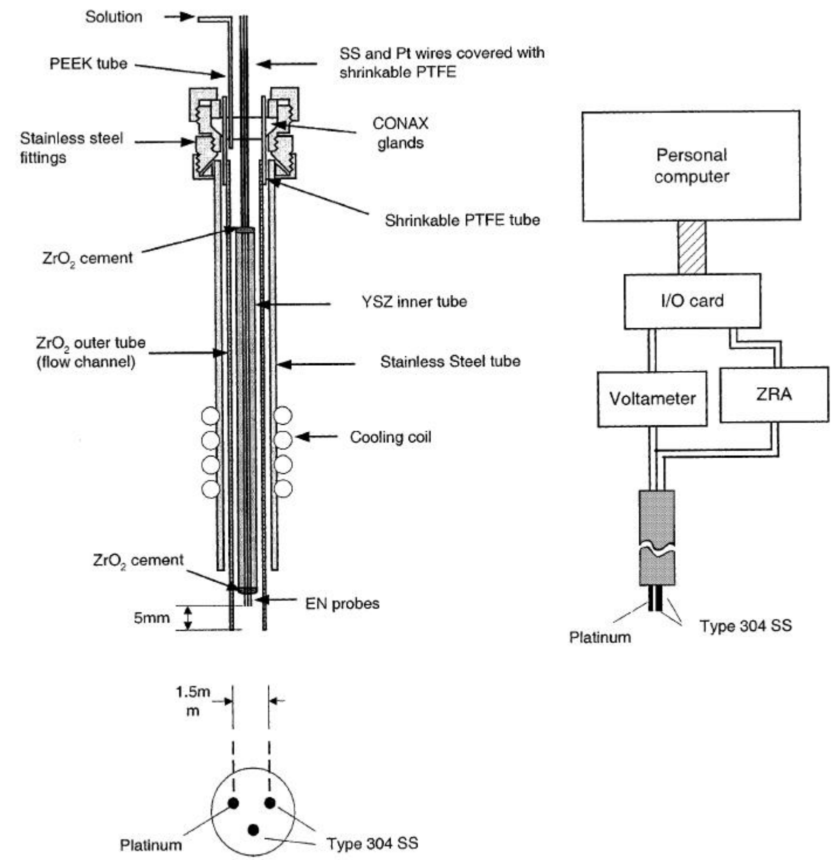
ISBN 1-56700-215-3 ; 978-1-56700-215-7

Books by Bruce J. West

- 2006. **Where Medicine Went Wrong: Rediscovering the Path to Complexity (Studies of Nonlinear Phenomena in Life Science)**
- 2003. Biodynamics: Why the Wirewalker Doesn't Fall
- 1995. **The Lure of Modern Science: Fractal Thinking (Studies of Nonlinear Phenomena in Life Sciences, Vol 3)**
- 1994. Fractal Physiology
- 1991. Fractal Physiology & Chaos in Medicine (Studies of Nonlinear Phenomena in Life Science, vol. 1)
- 1986. An Essay on the Importance of Being Nonlinear (Lecture Notes in Biomathematics)

Biocorrosion /Bioimplants

X.Y.Zhou, et al./ *Corros. Sci*. 44(2002) 841-860

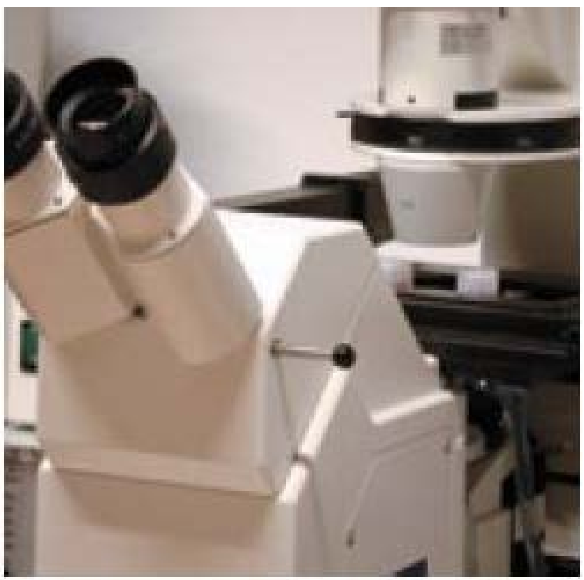


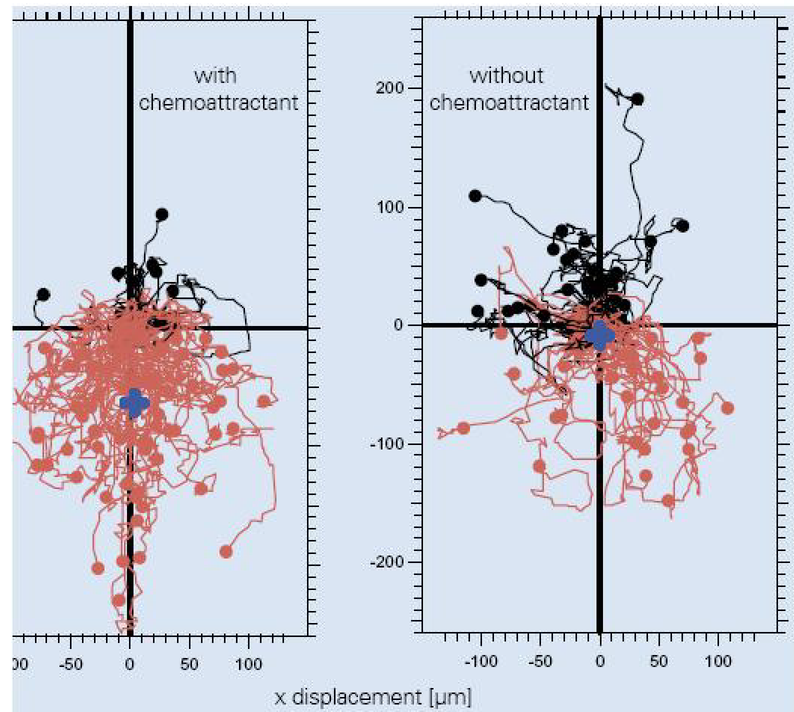
Biocompatibility study needs a non-destructive method –
Electrochemical noise (ECN) *measurement offers unique advantages !*

Slide credit: A. Zhou

Chemotaxis characterization

Zhou, A. (PI), Chen, Y.Q., Sims, R., Miller, C. NIH R15 Grant# ES013688-01A1 (2006-2009)





Slide credit: A. Zhou

Now, energy informatics

- Energy Informatics http://cei.usc.edu/
  - “Energy+Information < Energy”
    - energy experts,
    - computer scientists, and
    - social and behavioral studies experts
- Papers/articles (as of 3/3/2011)
  - ISI: 3 ; ScienceDirect: 37; ieeeXplore: 6 ; Google: 3280; GoogleScholar: 193

“data mining” @ ieeeXplore (x1000)

- Control/820; network/1100; signal/1087; comm*/1182;energy/847; power/1013; system/2117; circuit/1439; education/50;
- Kalman/15; Lyapunov/15; Kharitonov/0.328; Youla/0.337
- Observer/14; feedforward/15; feedback/105; **optim*/325**
- Adaptive/118; nonlinear/149;stability/123; linear/195; robust/83
- Fuzzy/55; neural/94; cybernetics/35; physical/82; chemical/83
- Friction/10; hyster*/19; dead*/13; vision/63; image/297;pattern/177
- PID/7; UAV/2; interval/26; anomal*/12.3; random/82; stochastic/40
- Geometrical/17; algebraic/11; **math*/667; fluctuation/30; noise/202**
- **Forecast/21.4; demand/56.6; behavi*/143; social/18.5**

Signal -> Information -> Knowledge -> Wisdom

- **Signals** are from man-made large scale, complex systems, usually LRD (long range dependent)
  - Enabled by "cyberinfrastructure", anything, anytime, anywhere
- **Information** is the third essence of the natural world supplementing matter and **energy**
  - Extract from signals considering FOSP!
- **Knowledge is in particular useful in social context (behavior, policy level)**
  - Put information in (social) context
- **Wisdom**: ?

Opportunities

- Battery Management Systems
- Demand/load forecast
  - FOSP for FLOM processes; LRD + infinite variance
- Inference from variability
  - Hypothesis in social contexts
- Social behavior modeling (energy consumption/conservation)
- Culture model (energy consumption)
- Policy implications, optimal trading/pricing etc.

So, what’s beyond?

Rule of thumb for Fractional Order Thinking – about everything

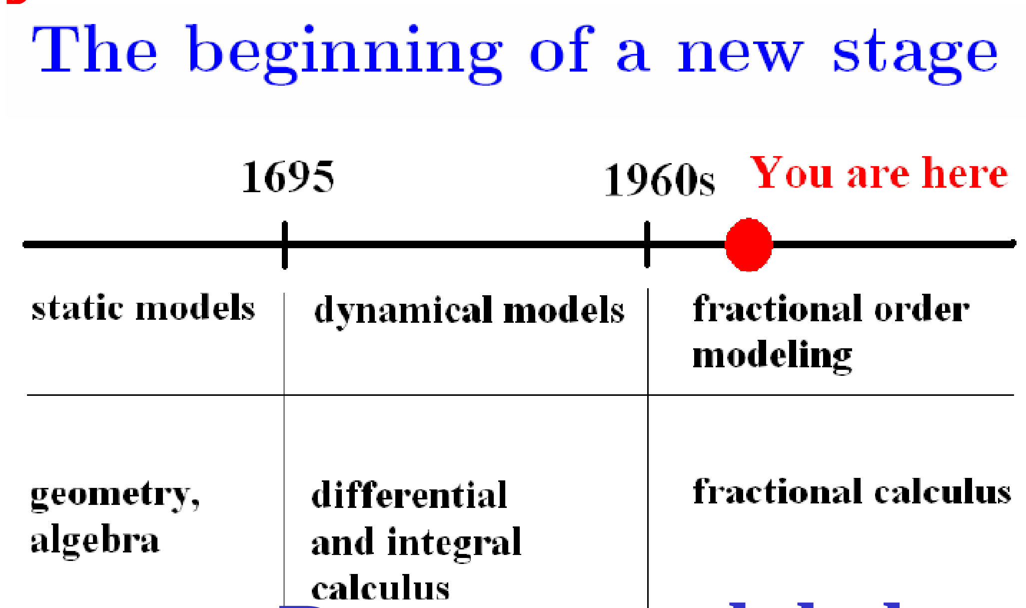
- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- $1/f^{a}$ noise
- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …

Outline

- Fractional Calculus and Fractional Order Thinking
- From Control, Signal Processing to Energy Informatics and Beyond
- Concluding Remarks

Concluding remarks

- “Go west, young man.” – Horace Greeley
- “Go Fractional.” – YangQuan Chen
- **Fractional Order Thinking enables exciting multidiscipline joint research that matters!**



Do more and do better!!

G.W. Scott Blair (1950):

"We may express our concepts in Newtonian terms if we find this convenient but, if we do so, we must realize that we have made a translation into a language which is foreign to the system which we are studying."

S. Westerlund (1991):

"Expressed differently, we may say that Nature works with fractional time derivatives."

K. Nishimoto (1989):

"The fractional calculus is the calculus of the XXI century."

To probe further

http://www.tuke.sk/podlubny/

http://mechatronics.ece.usu.edu/foc/

Slide credit: Igor Podlubny

Acknowledgements

- **TOK13! Profs Nusret Tan, Celaleddin Yeroglu, Serdar Hamamci**
- NRC Twinning Grant, 2003-2005. (Igor Podlubny, K. Moore co-PIs)
- NSF Workshop Grant, 2004 (Om Agrawal, PI)
- USU New Faculty Research Grant, 2002-2003
- USU TCO Technology Bridge Grant, 2005
- USU SDL Skunk Works Grant, 2005-2006 (Anhong Zhou, co-PI)
- NSF SBIR Phase-1 Grant, 2006 (Gary Bohannan, PI)
- Igor Podlubny, Ivo Petras, Lubomir Dorcak, Blas Vinagre, Shunji Manabe, J.T.M. Machado, J. Sabatier, Om Agrawal, Kevin L. Moore, Dingyu Xue, Anhong Zhou, Richard L. Magin, Wen Chen, Changpin Li, Yan Li.
- Concepción A. Monje, José Ignacio Suárez, Chunna Zhao, Jinsong Liang, Hyosung Ahn, Tripti Bhaskaran, Theodore Ndzana, Christophe Tricaud, Rongtao Sun, Nikita Zaveri, …

Q/A Session

- Apologize for not citing carefully math/phyx FOC papers and for not referring to more complete FOC literatures
- Check **http://mechatronics.ece.usu.edu/foc** for more information.
- Jinsong Liang. "**Control of Linear Time-Invariant Distributed Parameter Systems: from Integer Order to Fractional Order**". MS thesis, Electrical and Computer Engineering Dept. of Utah State University, 2005. (119 pages)
- Mr. Rongtao Sun. "**Fractional Order Signal Processing: Techniques and Applications**", ibid, 2007.
- Chunna Zhao. "**Research on Analysis and Design Methods of Fractional Order Systems**". PhD thesis, Northeastern University, China, 2006.
- Concepci´on Alicia Monje Micharet. "**Design Methods of Fractional Order Controllers for Industrial Applications**". PhD thesis, University of Extremadura, Spain, 2006.

Backup slides

- Youtube channels of CSOIS:
  - **http://www.youtube.com/user/MASnetPlatform**
  - **http://www.youtube.com/user/USUOSAM**
  - **http://www.youtube.com/user/FractionalCalculus**

IFAC

ELSEVIER

Control Engineering Practice

Best Paper Prize

Awarded jointly by Elsevier Ltd and the International Federation of Automatic Control (IFAC)

for the best paper published in the period 2008-2011

Awarded to

Y. Chen

for the paper

Tuning and auto-tuning of fractional order controllers for industry applications

(Vol. 16, No. 7, pp. 798-812)

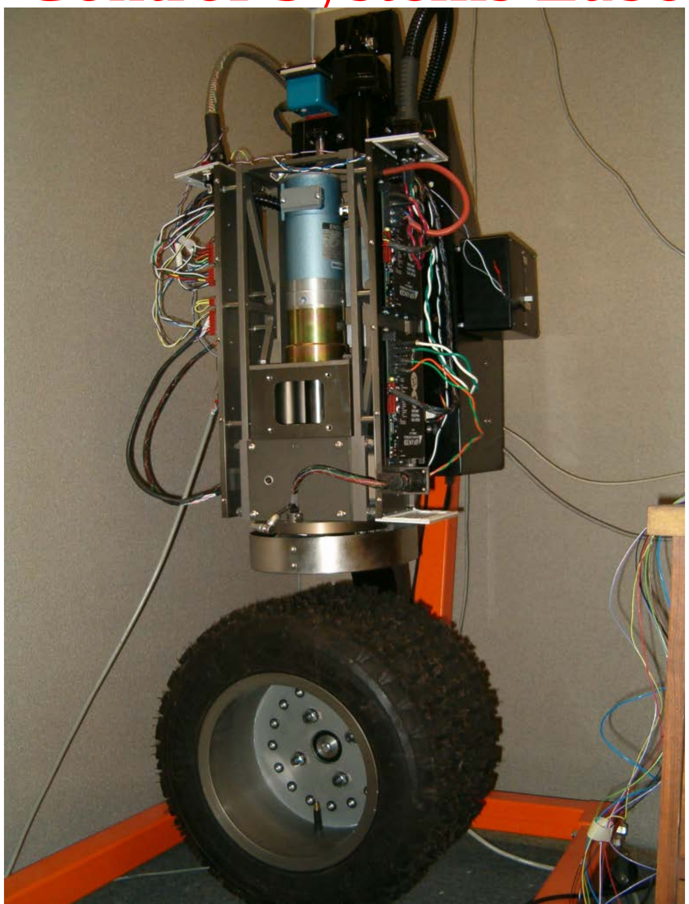
Christopher Greenwell, Publisher
Elsevier Ltd.

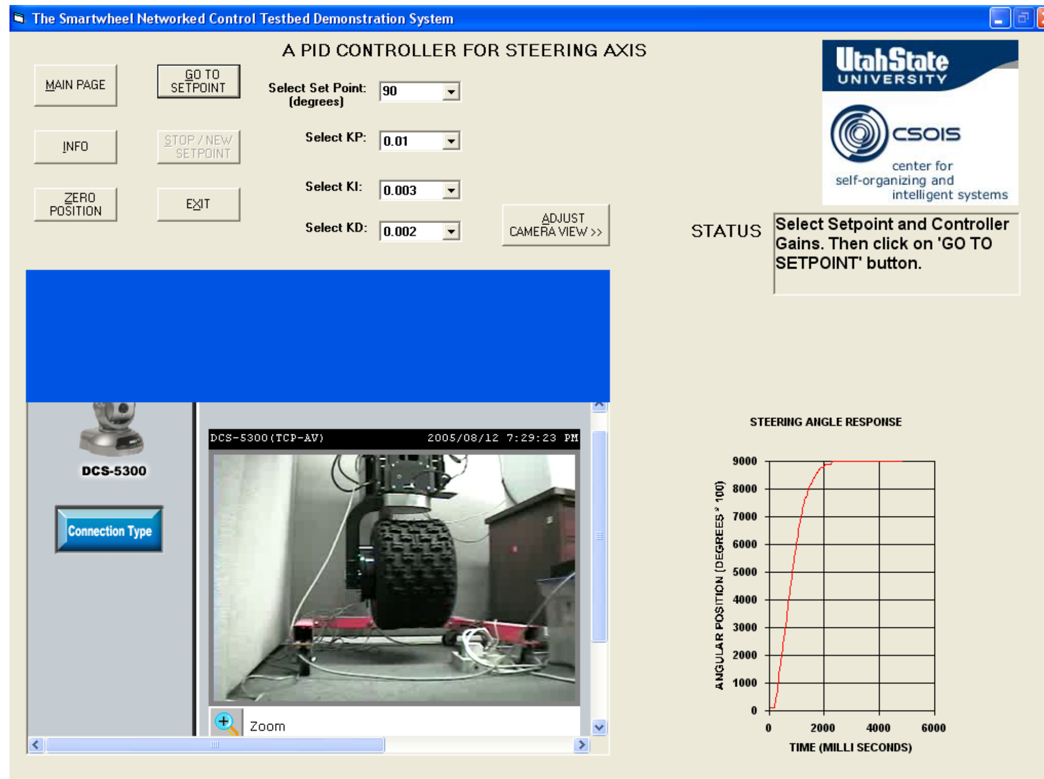
Prof. Andreas Kugi, Editor-in-Chief
Control Engineering Practice

- Fractional Order System – official keyword of IFAC
- pid12.ing.unibs.it/

USU Smart Wheel Demo Rig

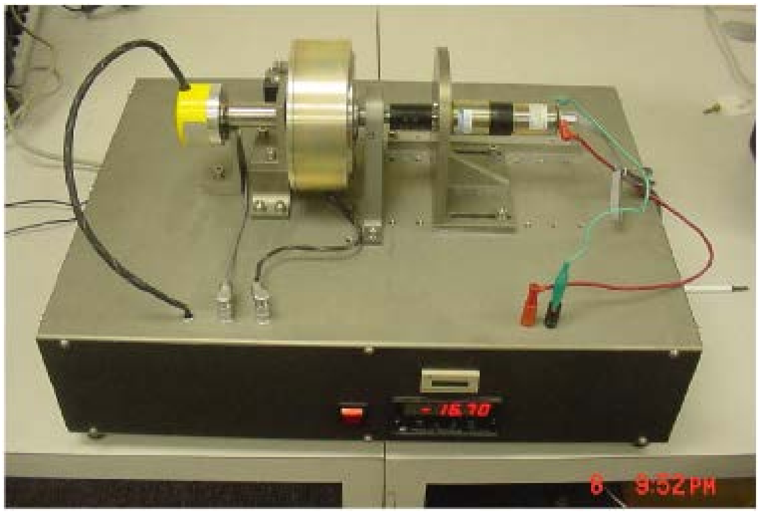
"Omni-directional Robotic Wheel - A Mobile Real-Time Control Systems Laboratory", Int. J. Eng. Edu. 2008.

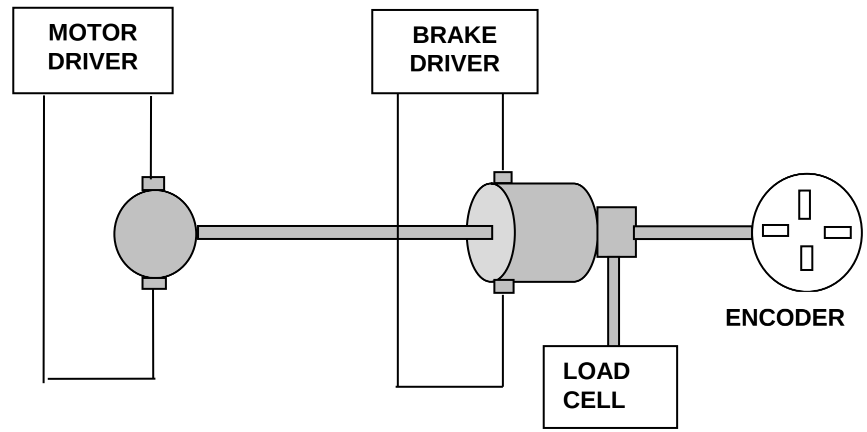




http://www.csois.usu.edu/people/smartwheel/CompleteInfoPage.htm

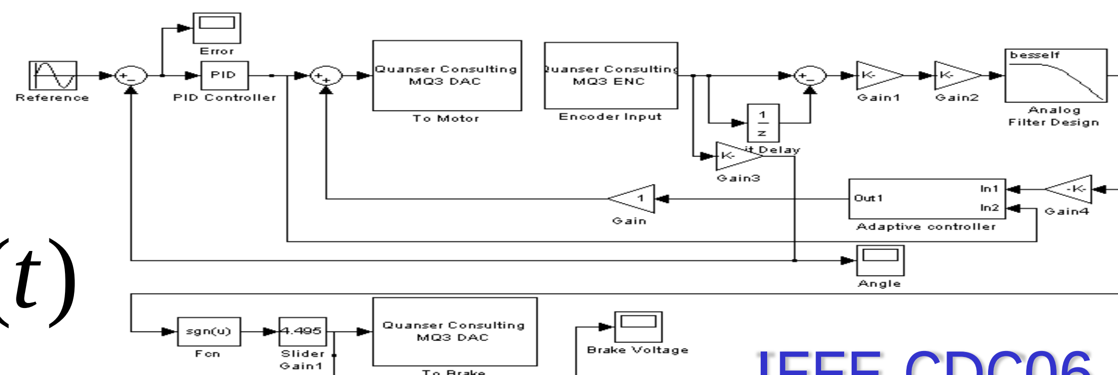
Fractional Horsepower Dynamometer – A General Purpose Hardware-In-The-Loop Real-Time Simulation Platform for Nonlinear Control Research & Education





$$\dot{x}(t) = v(t)$$

$$\dot{v}(t) = -f(t,x) + u(t)$$



IEEE CDC06