Iterative Learning Feedback Control of A Batch Chemical Process : Simulational Studies *

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Abstract

This paper presents an application of **Iterative Learning Control (ILC)** methodology to the temperature profile control of a *Batch Chemical Reactor*. The motivation and the basic ideas are briefly introduced. The D-type and P-type ILC schemes are applied and compared. The *feedback assisted (FA)* ILC and the ILC with *current iteration tracking error (CITE)* are discussed together with the *high-order* ILC schemes. The effectiveness of the proposed schemes is demonstrated by simulation studies of a simplified model of a polymerization reactor.

Keywords: Batch-Process Control; Temperature Profile Tracking; Iterative Learning Control;

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1 Introduction

Most large-scale chemical engineering processes have traditionally been operated in a continuous manner. However, batch processes, particularly batch chemical reactors, have drawn increasing attentions because of the intrinsic kinetic advantages of batch over continuous operation for some reactions. Batch processes challenge the modeling and control due to their inherent dynamic nature.

A typical batch reactor is depicted in Fig. 1. Reactant is charged into the vessel. Steam is fed into the jacket to bring the reaction mass up to a specified temperature. Then cooling water is added to the jacket to remove the exothermic heat of reaction and to make the reactor temperature follow the *prescribed temperature profile*. This temperature-time curve is fed into the temperature controller as a set-point signal.

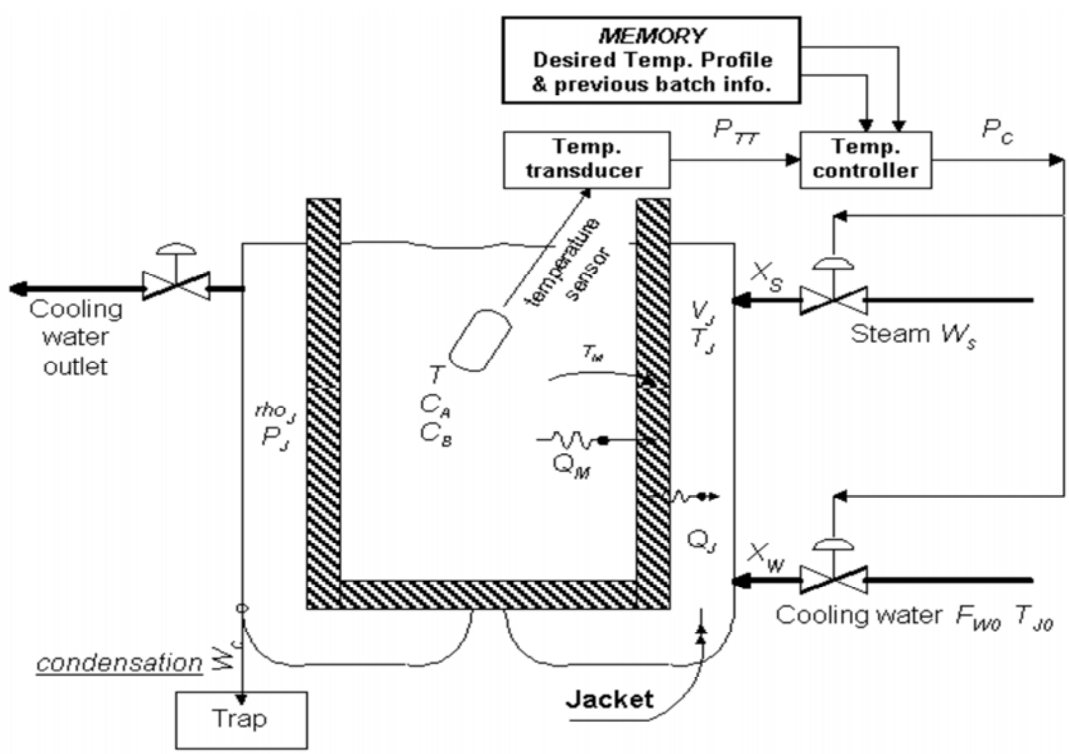

Figure 1: A Chemical Batch Reactor

Conventional control for tracking a given trajectory (pattern) in a finite time (batch duration) interval is a difficult task. As first investigated in [1, 2], new properties can be found if the plant is operated in a repeated fashion. This was formalized into *Iterative Learning Control* method[3]. The basic idea to utilize the system's repetition to compensate or reject uncertainties and disturbances. The control action of the current batch is synthesized from the control action and the resulting tracking error of previous batch. Learning can hence be achieved through the batch to batch practice.

In this paper, by intensive simulation studies, some ILC schemes are investigated for the temperature profile tracking control of a batch polymerization process. The simplified simulation plant model is given in Section 2. Among the ILC schemes, a feedback-feedforward structure is used which was called either *feedback-assisted ILC* or ILC with *current iteration tracking error*. High-order ILC is used and the effectiveness is demonstrated. It has been well demonstrated in this paper that the ILC plus feedback scheme is effective and applicable for batch reactor control.

2 A Batch Reactor Model

A simplified batch polymerization reaction model [4] is used for the simulation study. A more complete batch reactor model can be found in [5]. As illustrated in Fig. 1, the jacket effect is neglected. An equivalent thermal flow Q is from manipulating the valves for steam or cooling water flow control. The valves are under split range control so that both steam valve and cooling water valve can not be opened simultaneously. Hence Q is regarded as a total control. The reaction equations are given as follows:

$$\left\{ \begin{array}{rcl} \frac{\mathrm{d}c}{\mathrm{d}t} & = & -a_1 k_d c + Q' \\ \frac{\mathrm{d}x_M}{\mathrm{d}t} & = & -a_2 k_p \sqrt{\frac{2\eta k_d}{k_t}} \sqrt{c} x_M \\ \frac{\mathrm{d}p}{\mathrm{d}t} & = & a_3 \eta k_d c \\ \frac{\mathrm{d}T}{\mathrm{d}t} & = & H_r \frac{\mathrm{d}x_M}{\mathrm{d}t} + Q \end{array} \right. \qquad (1)$$

where c , x_M and p are the concentrations of initiator, monomer and polymer respectively; T is the temperature inside the reactor ($^\circ$K). Q' is assumed to be 0 which means that no additional initiator is added during the reaction, i.e., all reactants have been filled in the reactor at the beginning. Reaction rate constants $k_i, i \in \{d, p, t\}$ are functions of T where d, p, t represent the phases of beginning, growing and stopping.

$$k_i = k_{i0} \exp(-\frac{E_i}{RT}), \quad i = d, p, t \qquad (2)$$

where the related constants in (1) and (2) are given in [4]. The initial states $c(0) = 200$, $x_M(0) = 500$, $p(0) = 0$ and $T(0) = 300$ $^\circ$K .

The given temperature profile is pre-designed to satisfy some performance indices. For example, the time duration is as short as possible; the monomer condensation should be 0 at the end of a batch; the temperature budget should as small as possible and so on. This will be the topic of optimal batch operation pattern *planning*. We concentrate on the tracking control of a well planned temperature profile as tight as possible by the iterative learning control method. In this paper, for simulation study, desired temperature profile $T_d(t)$ is defined as

$$T_d(t) = \left\{ \begin{array}{ll} \frac{T_e}{t_m} t, & \text{if } 0 \le t \le t_m; \\ T_e, & \text{if } t_m \le t \le t_e. \end{array} \right. \qquad (3)$$

Set $t_e = 2$ hr.; $t_m = 1$ hr.; $T_e = 435$ °K .

A simple P-type feedback controller

$$Q(t) = K_p e(t), \quad e(t) \triangleq T_d(t) - T(t) \tag{4}$$

is applied and the responses are shown in Fig. 2. Clearly, a conventional controller is hard to track the temperature profile in a finite time interval.

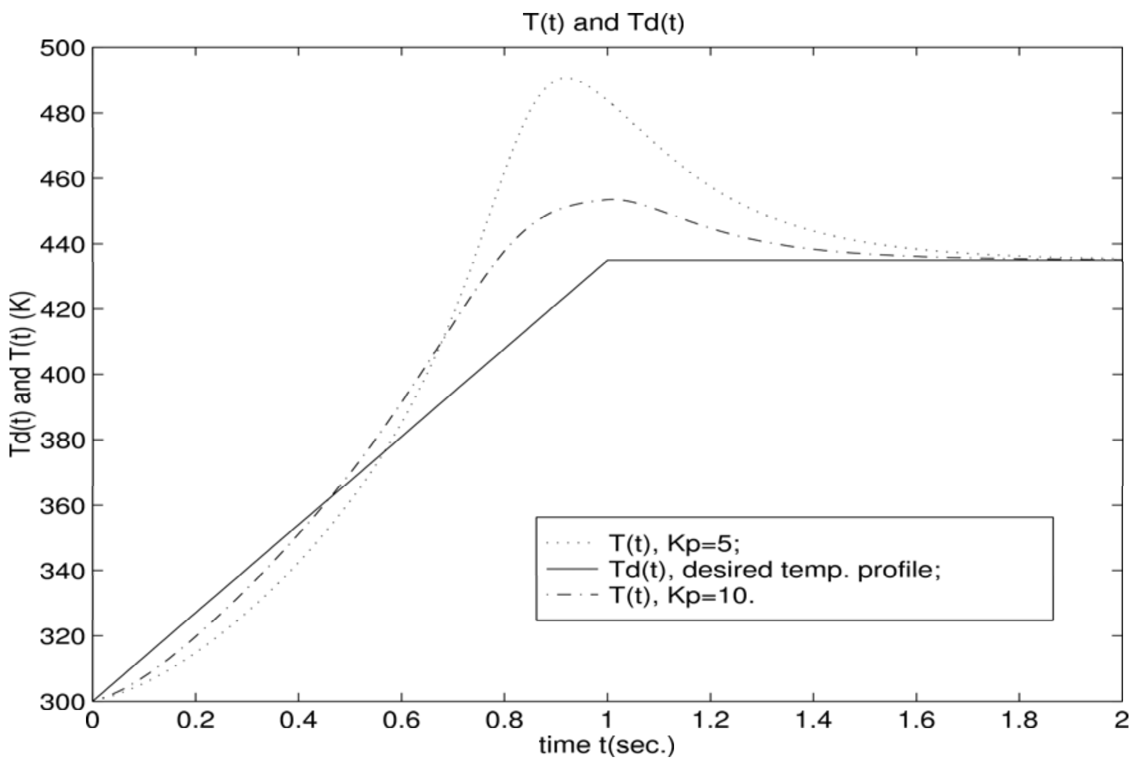

Figure 2: Responses for simple P-controllers

3 Iterative Learning Control Schemes

3.1 Basic Idea of ILC

The term 'Iterative Learning Control' (ILC) was coined by Arimoto and his associates [3] for a better control for the repetitive systems. We can intuitively find that *learning is a bridge between knowledge and experience.* That is to say, the lack of knowledge is bridged by experience. *Knowledge* and *Experience* in technical language can be obtained by 'modeling' and by 'repetitions after applying some *learning* control law'. One way to describe the *learning* is as a process where the objective of achieving a desired result is obtained by experience when only partial knowledge about the plant is available.

Roughly speaking, the purpose of introducing the ILC is to utilize the system repetitions as the *experience* to improve the system control performance even under incomplete *knowledge* of the system to be controlled. It should be pointed out that the ILC is not an open-loop control operation, although the ILC only modifies the input command for the next repetition. ILC is closed-loop in repetitions since updates are performed for the next repetition using the feedback measurements of the previous repetition, as opposed to the closed structure of conventional controllers in time which updates the control signal of the next time step using the feedback at current or past time steps. The difference is that the thinking in ILC is in the repetition domain, which makes it appears as open-loop control in the time domain. Taking Arimoto's simple D-type ILC scheme[3] as an example, it can be clearly illustrated by Figure 3.

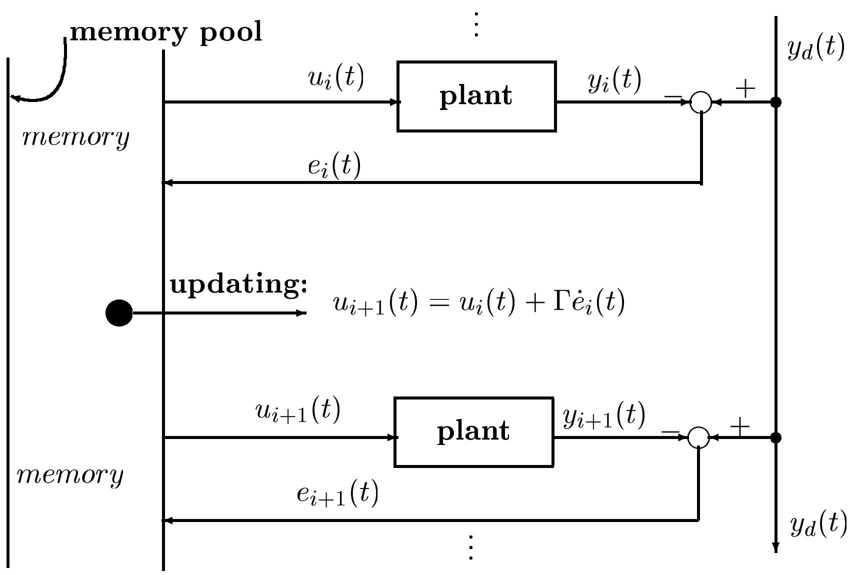

Figure 3: Block-Diagram of *Iterative Learning Control*

3.2 D-type ILC

As illustrated in Fig. 3, the control is updated by using the derivative of the tracking error in the previous iteration. In this case, the ILC updating law is given by

$$Q_{i+1} = Q_i + K_d \dot{e}_i(t), \quad \dot{e}_i(t) \triangleq \dot{T}_d - \dot{T}_i \tag{5}$$

where K_d is the learning gain which is to be properly chosen such that $e_i \to 0$ as $i \to \infty$. One of the convergence condition is that [3]

$$\|1 - CBK_d\| < 1 \tag{6}$$

where C and B are output and input matrix respectively. In this paper, $CB = 1$. Hence K_d should satisfy the condition that $\mid 1 - K_d \mid < 1$.

3.3 High-order ILC

It is quite intuitive that if more of the previous control efforts and the resulted tracking errors are used, better ILC performance can be expected.

It is interesting to investigate the ILC laws in the iteration number i-direction. If we look at the conventional ILC updating law [3]

$$u_{i+1}(t) = u_i(t) + \Gamma \dot{e}_i(t)$$

for the control of the dynamics along the ILC iteration number i-direction, it is obviously a pure integral controller. Suppose the initial control $u_0(t) = 0$, then

$$u_{i+1}(t) = \Gamma \sum_{j=0}^{i} \dot{e}_j(t)$$

which is in an integral (I) controller form in the i-direction. If we use the PI controller in the i-direction

$$u_{i+1}(t) = k'_P \dot{e}_i(t) + k'_I \sum_{j=0}^{i} \dot{e}_j(t),$$

the ILC updating law takes the form that

$$u_{i+1}(t) = u_i(t) + \Gamma \dot{e}_i(t) + \Gamma_1 \dot{e}_{i-1}(t)$$

where $\Gamma = k'_P + k'_I$ and $\Gamma_1 = -k'_P$. By using the difference $\dot{e}_i(t) - \dot{e}_{i-1}(t)$ as the approximation of the *derivative* along the i-direction, the PID controller in i-direction

$$u_{i+1}(t) = k'_P \dot{e}_i(t) + k'_I \sum_{j=0}^{i} \dot{e}_j(t) + k'_D (\dot{e}_i(t) - \dot{e}_{i-1}(t))$$

will result in the following form of the ILC updating law

$$u_{i+1}(t) = u_i(t) + \Gamma \dot{e}_i(t) + \Gamma_1 \dot{e}_{i-1}(t) + \Gamma_2 \dot{e}_{i-2}(t).$$

where $\Gamma = k'_P + k'_I + k'_D$, $\Gamma_1 = -k'_P - 2k'_D$ and $\Gamma_2 = k'_D$. This is a high-order iterative learning controller. The above arguments indicate that the high-order ILC is capable of giving better ILC performance than the traditional first-order case where only an integral controller is actually used.

In this paper, the N-th order D-type ILC updating law is written as follows:

$$Q_{i+1} = Q_i + \sum_{j=1}^{N} K_{d_j} \dot{e}_{i-j+1}(t) \tag{7}$$

where the learning gains should satisfy that the roots of (8) are inside the unit circle.

$$(1 - CBK_{d_1}) z^{-1} - \sum_{j=2}^{N} CBK_{d_j} z^{-j} = 0 \tag{8}$$

where z is one step shifting operator. According to [6], a sufficient condition is given by

$$\mid 1 - CBK_{d_1} \mid + \sum_{j=2}^{N} \mid CBK_{d_j} \mid < 1. \tag{9}$$

3.4 P-type Iterative Learning Feedback Control

From [7], it is clear that better ILC performance can be achieved by introducing a feedback loop. Then the system considered is actually controlled by an ILC controller in the iteration number direction and a feedback controller in the time direction simultaneously. This is illustrated in Figure 4.

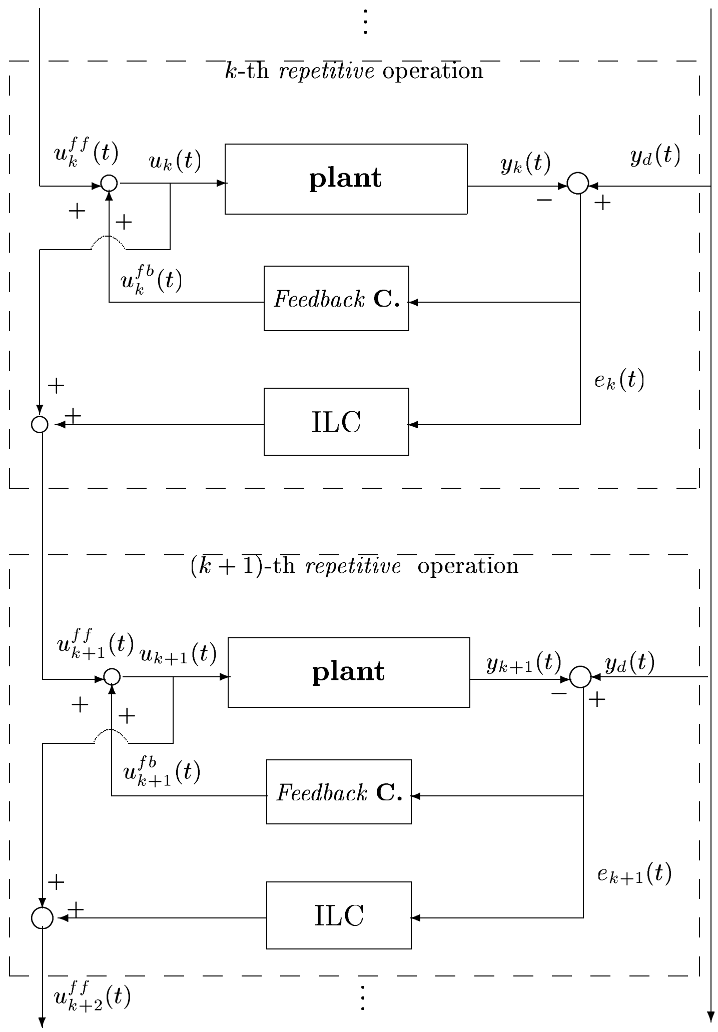

Figure 4: Block-Diagram of *Iterative Learning Feedback Control*

Iterative learning controller can also be regarded as an *intelligent feedforward controller* which is with a 'plug-in' form. In the real world, the ILC should be taken as an *additive* to the existing conventional controller. The main target of the ILC is to utilize the system operation repetition to improve the control performance, when the system executes a given task repeatedly. When analyzing the property of the ILC, considerations should be taken both in the time axis and in the repetition number direction, which is in essence in the category of the 2-D system theory. However, as the repetitive task is to be executed in a fixed finite time interval, more attentions should actually be paid in the repetition axis in the analysis of the iterative learning control property.

As the most practical systems have a finite escape time, the stability problem of ILC is mainly concentrated in the repetition direction, which sometimes is studied in terms of the ILC convergence. Some ILC convergence analysis can be found in recent literatures, e.g., [6, 8], and the references therein.

A P-type scheme is preferable because the D-type ILC (5 will introduce higher noise level. According to Fig. 4, the P-type scheme can be written as

$$\begin{cases} Q_i(t) & = & Q_i^{ff}(t) + Q_i^{fb}(t) \\ Q_i^{fb}(t) & = & K_p e_i(t) \\ Q_i^{ff}(t) & = & Q_{i-1}(t) + K_{pl} e_{i-1}(t) \\ & = & Q_i^{ff}(t) + (K_p + K_{pl}) e_{i-1}(t) \end{cases} . \tag{10}$$

A convergence condition can be found in [8] for discrete-time nonlinear systems where the role of feedback is regarded as an assistance to the ILC.

3.5 P-type ILC with CITE

Consider the PI-controller in the ILC iteration direction as follows:

$$Q_i(t) = k_I' \sum_{j=0}^{i} e_j(t) + k_P' e_i(t). \tag{11}$$

Writing (11) in an iterative form, we have

$$\begin{aligned} Q_i(t) &= Q_{i-1}(t) + k_I' e_i(t) + k_P'(e_i(t) - e_{i-1}(t)) \\ &= Q_{i-1}(t) + K_p e_i(t) + K_{pl} e_{i-1}(t) \end{aligned} \tag{12}$$

where $K_p = k_I' + k_P', K_{pl} = -k_P'$. Updating law (12) is called as the *ILC with Current Iteration Tracking Error (CITE)*. A convergence condition was given in [9] where the k_P' is assumed to be 0. It was shown in [9] that the convergence as well as the robustness of the ILC with CITE scheme (12) is independent of the choice of K_p, the CITE gain. This actually invokes a high-gain ILC as indicated in [10]. This provides another view angle for Iterative Learning Feedback Control as described in Fig. 4.

On the other hand, we can see that the schemes (12) and (10) are essentially the same but with different explanations.

One of the interesting scheme is the high-order in control term, that is,

$$Q_i(t) = p' Q_{i-1}(t) + (1 - p') Q_{i-2}(t) + K_p e_i(t) + K_{pl} e_{i-1}(t) \tag{13}$$

where p' is a positive fraction ($p' \in [0, 1]$). Improved ILC convergence property can be expected as demonstrated in the following simulation.

4 Simulation Studies

The simulations are carried out in MATLAB V4.2c. RK-4 is used to numerically integrate (1) with a fixed step $h = 0.1$ hr. Total number of integration points is $N_p = 201$.

4.1 D-type Iterative Learning Control

To obtain the D-information, a five-point formula is used for the numerical differentiation.

$$\left\{\begin{array}{l} \hat{e}(1) = \frac{-25e(1)+48e(2)-36e(3)+16e(4)-3e(5)}{12h} \\ \hat{e}(2) = \frac{-3e(1)-10e(2)+18e(3)-6e(4)+e(5)}{12h} \\ \hat{e}(j) = \frac{e(j-2)-8e(j-1)+8e(j+1)+e(j+2)}{12h} \\ \qquad\qquad (j = 2, \cdots, N_p - 2) \\ \hat{e}(N_p - 1) = \frac{1}{12h}\{-e(N_p - 4) + 6e(N_p - 3) \\ \quad -18e(N_p - 2) + 10e(N_p - 1) + 3e(N_p)\} \\ \hat{e}(N_p) = \frac{1}{12h}\{3e(N_p - 4) - 16e(N_p - 3) \\ \quad +36e(N_p - 2) - 48e(N_p - 1) + 25e(N_p)\} \end{array}\right. \tag{14}$$

Hence the derivative of the tracking error $\hat{e}$ is with lower noise level. According to (6), we choose $K_d = 0.5$ for the first order ($N = 1$). High-order schemes are also considered. Set $K_{d_1} = K_d, K_{d_2} = \pm 10\% K_d$. A set of results with the same computation conditions are presented in Fig. 5(a). Clearly, a high-order scheme may give a better ILC convergence performance. In this case, $K_{d_2} = -10\% K_d$. However, when $K_{d_2} = +10\% K_d$, the ILC performance becomes worse. This can be analyzed from (8).

It can be observed from Fig. 5(a) that the peak error is unacceptably large. A feedback loop (4) may be introduced to reduce the peak error. Set $K_p = 20$. Three similar cases are summarized in Fig. 5(b). The tracking error bound decreases more monotonically. This is because the overall system under feedback control becomes more dissipative.

4.2 P-type ILC plus Feedback Controller

As discussed in Section 3.4 and Section 3.5, ILC schemes (10) and (12) in a feedback structure which is illustrated in Fig.4 are essentially the same. We now concentrate on the P-PI scheme (11) which uses P component of tracking error in the time t-direction while PI components of tracking error in the ILC iteration number i-direction. Hence in this sense, D-type scheme (5) is a D-I one while the scheme used in Fig. 5(b) is PD-I.

We will investigate the effects of different choices of learning gains K_p and K_{pl} on the convergence performance of ILC scheme (12).

- **Case 1.** $K_{pl} = 0$.

In this case, only CITE is used, i.e.,

$$Q_i(t) = Q_{i-1}(t) + K_p e_i(t). \tag{15}$$

This can be regarded as a P-I scheme as discussed in the above. It is interesting to note that from the analysis of [9], the convergence and robustness of ILC scheme (15) are independent of the choice of K_p. However, larger K_p will give better ILC performance as also indicated in [11]. This is clearly illustrated by Fig. 6.

- **Case 2.** $K_{pl} \neq 0$.

  This is P-PI type ILC. Along the i-direction, the PI gains $k'_P = -K_{pl}$, $k'_I = K_p + K_{pl}$. It is intuitive from the conventional PID controller tuning that increasing k'_I will reduce the convergence bound of tracking error. This is similar to the effect of an integral (I) controller in the time-domain. This effect was illustrated in Fig. 6.

  We now observe the effect of K_{pl} under a fixed K_p. Qualitatively speaking, to guarantee the ILC convergence (stability in the i-direction), k'_P, i.e., K_{pl} can not be arbitrarily chosen. We considered 5 sub-cases for $K_{pl} = 2, 0.5, 0, -2$ and -4 respectively. The results are presented in Fig. 7. From the comparison in Fig. 7, the tuning of K_{pl} and K_p is possible based on the existing PID tuning method which deserves future research.

- **Case 3.** High-order in Control.

  As described in (13), the high-order can be in control terms. This in fact adds a input signal filter. In this case, a first order filter. Different choices of p' may result in different ILC convergence transients. Fig. 8 shows the results for $p' = 1, 0.9,$ and 0.7 respectively. It is interesting to observe that ILC convergence performance improves as p' slightly decreases from 1.

- **Case 4.** $K_{pl} = -4; K_p = 5, 10, 15.$

  Similar to **Case 1,** we will show that under a fixed K_{pl}, the ILC convergence improves when K_p increases. This is well illustrated by Fig. 9

The converged results are almost the same. A set of plots for the system states in the 10-th ILC is given in Fig. 10. The monomer condensation finally decreases to 0 while the polymer condensation keeps increasing, according to the predesigned temperature profile $T_d(t)$.

5 Concluding Remarks

A new method, Iterative Learning Feedback Control, is proposed for the finite-time temperature profile control of a chemical reactor system. D-type, P-type, as well as high-order ILC schemes are presented. ILC with a feedback structure is discussed in terms of FA-ILC (Feedback Assisted) and ILC with CITE (current iteration tracking error). Simulation studies have verified the effectiveness of the proposed ILC schemes. Among the proposed schemes, the CITE P-type scheme is the most preferable due to its simplicity and effectiveness. Constraints in actual field conditions such as the temperature measurement error, jacket thermal dynamics, valve actuator dynamics and so on are under investigation.

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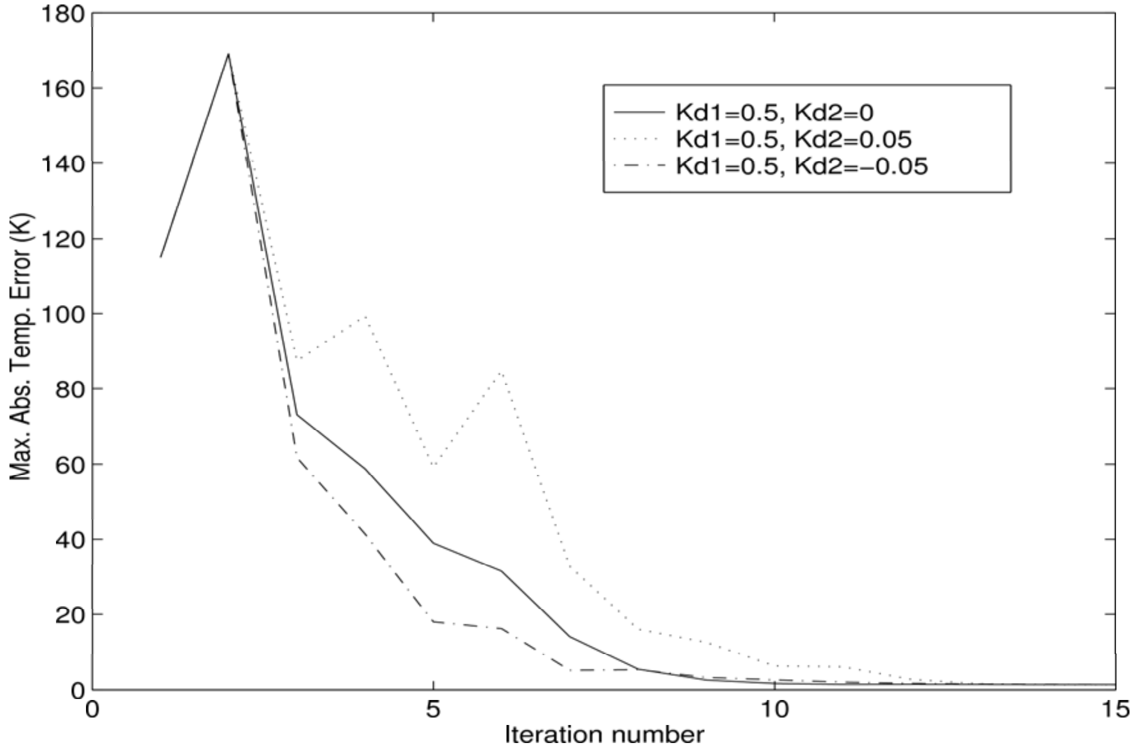
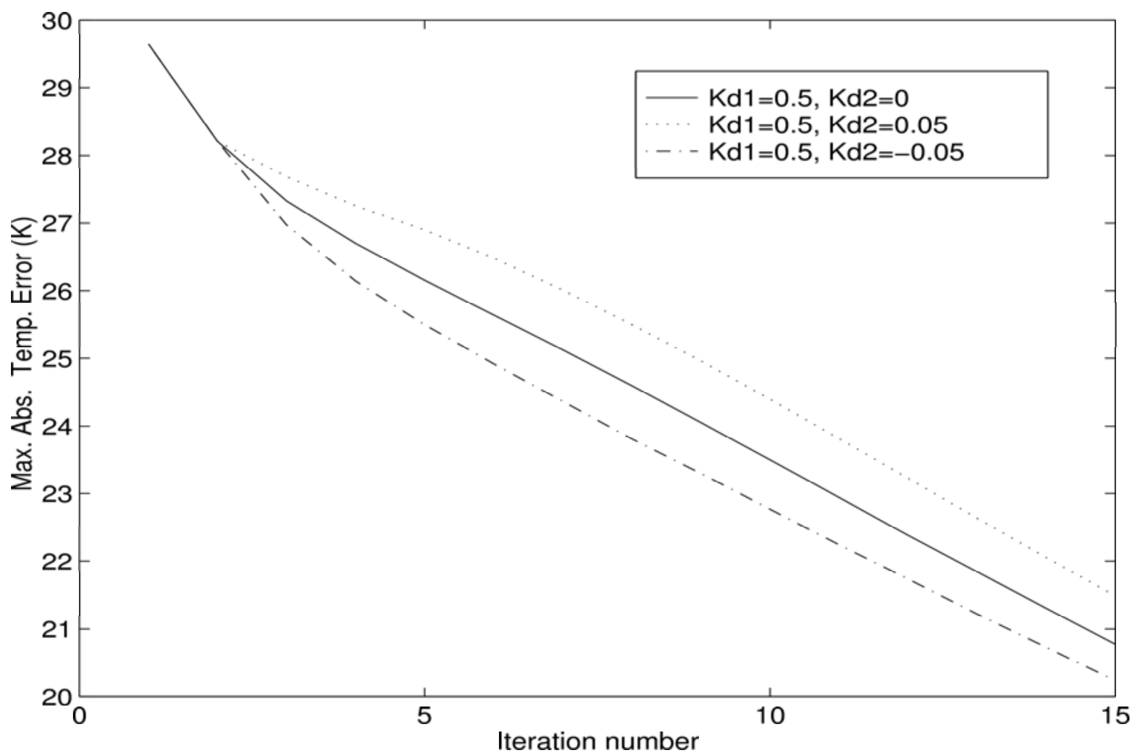


(a) $K_p = 0.0$ (D-type ILC only) (D-I scheme)



(b) $K_p(t) = 20$ (PD-I scheme)

Figure 5: Convergence comparisons for D-type ILC schemes

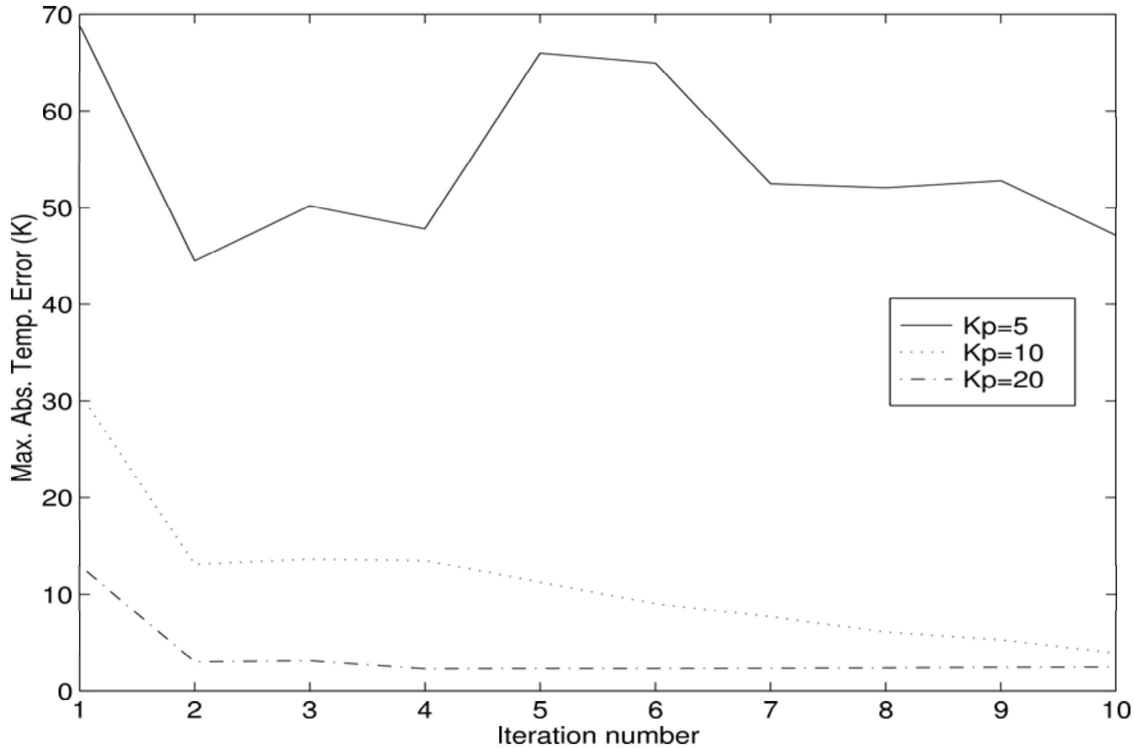


Figure 6: Convergence comparisons for P-I type (ILC+CITE) schemes

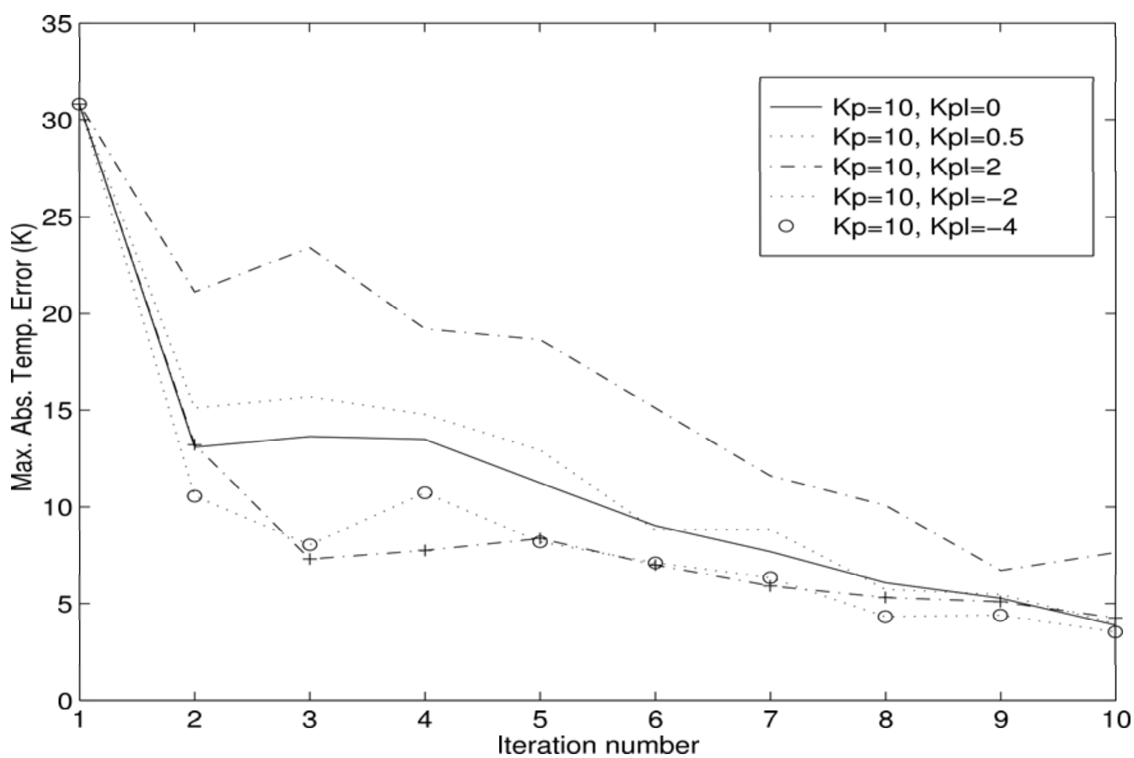


Figure 7: Convergence comparisons for P-PI type (ILC+CITE) schemes

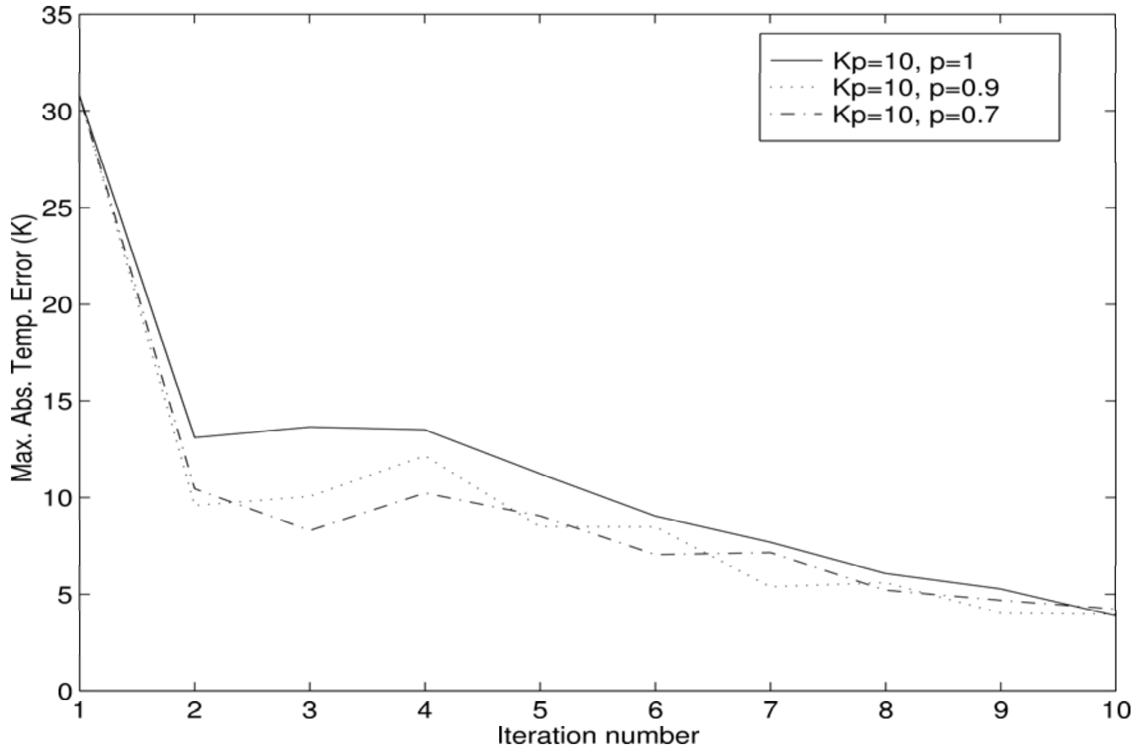


Figure 8: Convergence comparisons for P-type scheme (13)

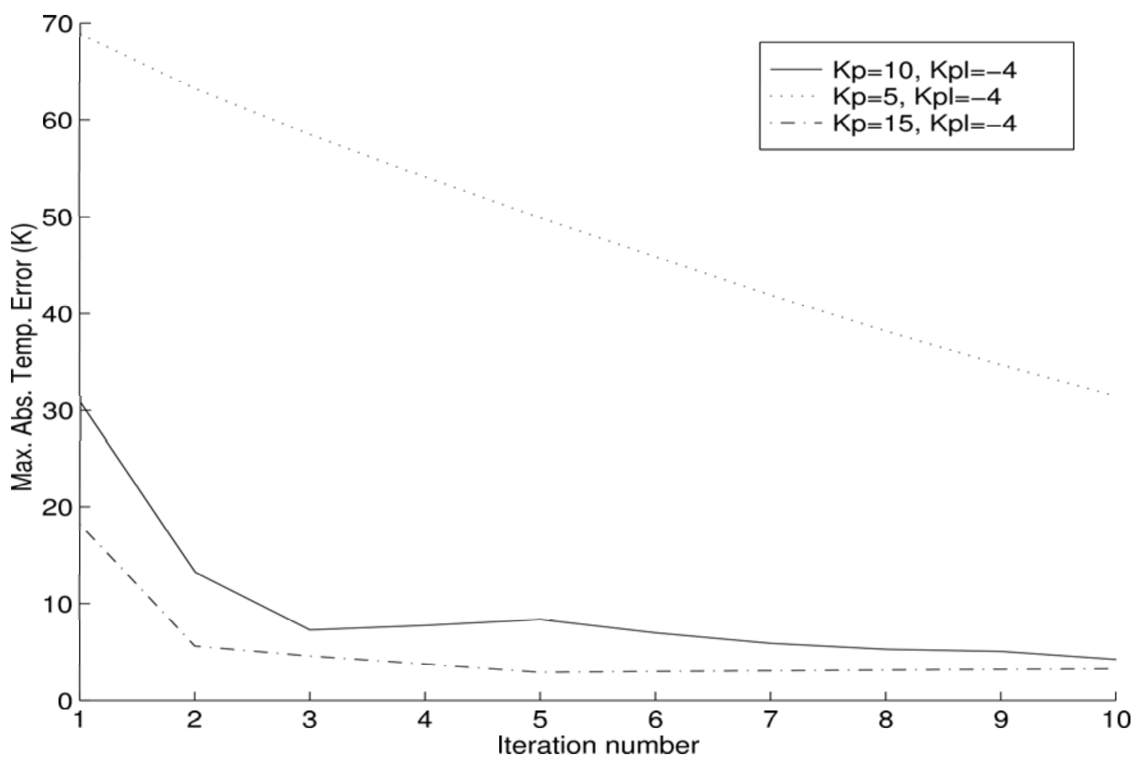


Figure 9: Convergence comparisons for P-PI type (ILC+CITE) schemes

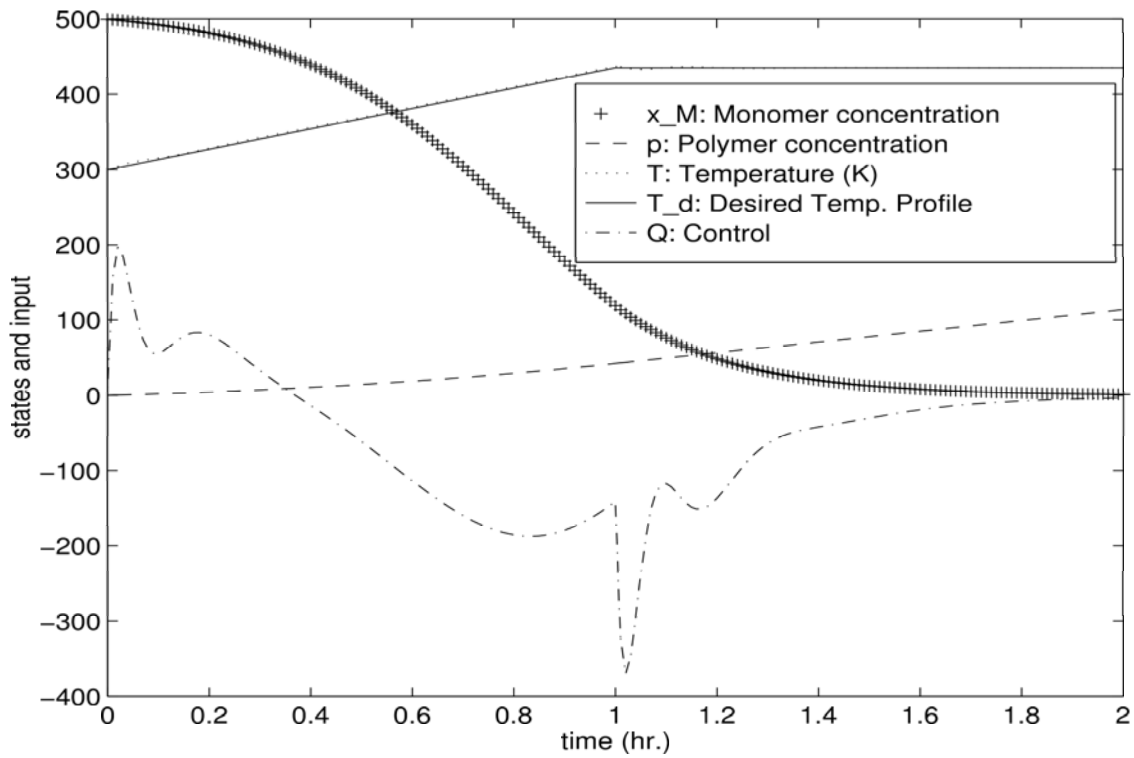


Figure 10: Converged system states and input (at the 10th ILC iteration)