Comparative Studies of Iterative Learning Control Schemes for A Batch Chemical Process *

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Abstract

This paper presents an application of **Iterative Learning Control (ILC)** methodology to the temperature profile control of a *Batch Chemical Reactor*. The D-type and P-type ILC schemes are applied and compared. The *feedback assisted (FA)* ILC and the ILC with *current iteration tracking error (CITE)* are discussed together with the *high-order* ILC schemes. The effectiveness of the proposed schemes is demonstrated by simulation studies using a simplified polymerization reactor model.
Keywords: Batch-Process Control; Temperature Profile Tracking; Iterative Learning Control;

1 Introduction

Most large-scale chemical engineering processes have traditionally been operated in a continuous manner. However, batch processes, particularly batch chemical reactors, have drawn increasing attentions from industries.

A typical batch reactor is shown in Fig. 1. Reactant is charged into the vessel. Steam is fed into the jacket to bring the reaction mass up to a specified temperature. Then cooling water is added to the jacket to remove the exothermic heat of reaction such that the reactor temperature can follow the *prescribed temperature profile.*

It is well known that it is difficult for a conventional control to track a given trajectory (pattern) in a finite time (batch duration) interval. An *Iterative Learning Control*, on the other hand, is able to utilize the system's repetition to compensate or reject uncertainties and disturbances and hence able to track the prescribed trajectory in a finite interval. In particular, the control efforts of the current batch incorporate the control efforts and tracking errors of the previous batch.

In this paper, through intensive simulation studies,

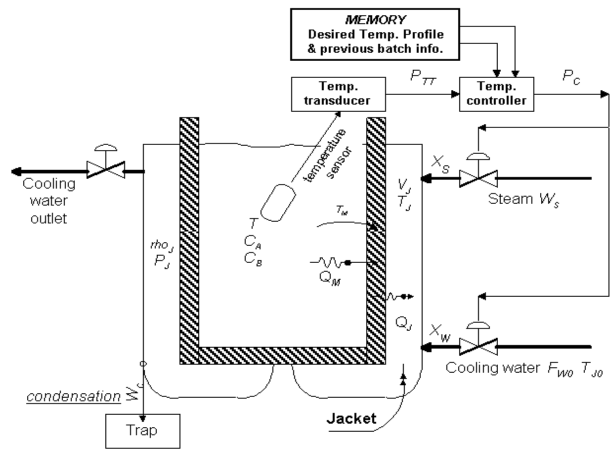

Figure 1: A Chemical Batch Reactor

a number of ILC schemes are investigated for the temperature profile tracking control of a batch polymerization process. The simplified plant model is given in Section 2. Among the ILC schemes, a feedback-feedforward structure, which is either *feedback-assisted ILC* or ILC with *current iteration tracking error*. High-order ILC is used and the effectiveness is demonstrated. It is shown in this paper that the ILC schemes with feedback or high-order updating are effective for batch reactor control.

2 A Batch Reactor Model

A simplified batch polymerization reaction model [4] is used for the simulation study in which the jacket effect is neglected. A more complete batch reactor model can be found in [5]. An equivalent thermal flow Q is from manipulating the valves for steam or cooling water flow control. The valves are under split range control so that steam valve and cooling water valve can not be opened simultaneously. Hence Q is regarded as a total control. The reaction equations are given as follows:

$$\left\{ \begin{array}{rcl} \frac{\mathrm{d}c}{\mathrm{d}t} & = & -a_1 k_d c + Q' \\ \frac{\mathrm{d}x_M}{\mathrm{d}t} & = & -a_2 k_p \sqrt{\frac{2\eta k_d}{k_t}} \sqrt{c} x_M \\ \frac{\mathrm{d}p}{\mathrm{d}t} & = & a_3 \eta k_d c \\ \frac{\mathrm{d}T}{\mathrm{d}t} & = & H_r \frac{\mathrm{d}x_M}{\mathrm{d}t} + Q \end{array} \right. \qquad (1)$$

where c , x_M and p are the concentrations of initiator, monomer and polymer respectively; T is the temper-

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ature inside the reactor (°K). Q' is assumed to be 0 which means that no additional initiator is added during the reaction, i.e., all reactants have been filled in the reactor at the beginning. Reaction rate constants $k_i, i \in \{d, p, t\}$ are functions of T where d, p, t represent the phases of beginning, growing and stopping.

$$k_i = k_{i0} \exp(-\frac{E_i}{RT}), \quad i = d, p, t. \tag{2}$$

The relevant constants in (1) and (2) are given in [4]. The initial states are $c(0) = 200$, $x_M(0) = 500$, $p(0) = 0$ and $T(0) = 300$ °K .

A well planned temperature profile $T_d(t)$ is given as

$$T_d(t) = \begin{cases} \frac{T_e}{t_m} t, & \text{if } 0 \le t \le t_m; \\ T_e, & \text{if } t_m \le t \le t_e, \end{cases} \tag{3}$$

with settings $t_e = 2$ hr.; $t_m = 1$ hr. and $T_e = 435$ °K .

For comparative purpose, a simple P-type feedback controller

$$Q(t) = K_p e(t), \quad e(t) \stackrel{\triangle}{=} T_d(t) - T(t) \tag{4}$$

is first applied and the responses are shown in Fig. 2. Clearly, a conventional controller is hard to track the temperature profile in a finite time interval.

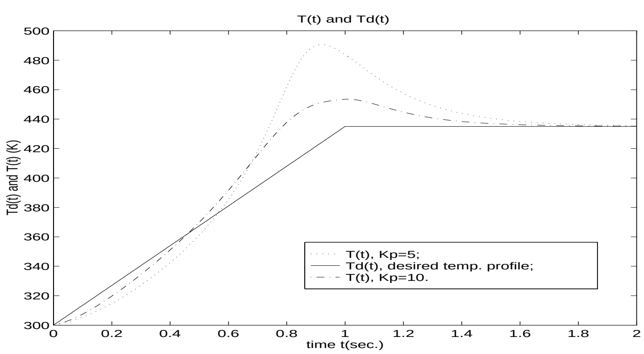


Figure 2: Responses for simple P-controllers

3 Iterative Learning Control Schemes

The basic idea of ILC is to update the control signal of the whole tracking period $[0, T]$ in a pointwise manner. In the following we briefly introduce four ILC schemes used for temperature control of the batch polymerization reaction.

3.1 D-type ILC

The D-type ILC input signals are updated by using the derivative of the tracking error in the previous iteration. In this case, the ILC updating law is given by

$$Q_{i+1} = Q_i + K_d \dot{e}_i(t), \quad \dot{e}_i(t) \stackrel{\triangle}{=} \dot{T}_d - \dot{T}_i \tag{5}$$

where K_d is the learning gain which is to be properly chosen such that $e_i \to 0$ as $i \to \infty$. The convergence condition is that [3]

$$\|1 - CBK_d\| < 1 \tag{6}$$

where B and C are input distribution matrix and output matrix respectively. In (1), the system input is Q and output is T, hence $CB = 1$. K_d should be selected to satisfy the condition $\mid 1 - K_d \mid < 1$.

3.2 High-order ILC

It is quite intuitive that if more of the previous control efforts and tracking errors are used, better ILC performance can be expected.

It is interesting to investigate the ILC laws in the iteration number i-direction. The conventional ILC updating law [3]

$$u_{i+1}(t) = u_i(t) + \Gamma \dot{e}_i(t)$$

along the ILC iteration number i-direction is obviously a pure integral controller. Suppose the initial control $u_0(t) = 0$, then

$$u_{i+1}(t) = \Gamma \sum_{j=0}^{i} \dot{e}_j(t),$$

which is an integrator of $\dot{e}(t)$ in the i-direction. If we use a PI controller in the i-direction

$$u_{i+1}(t) = k'_P \dot{e}_i(t) + k'_I \sum_{j=0}^{i} \dot{e}_j(t),$$

the corresponding ILC updating law takes the form

$$u_{i+1}(t) = u_i(t) + \Gamma \dot{e}_i(t) + \Gamma_1 \dot{e}_{i-1}(t)$$

where $\Gamma = k'_P + k'_I$ and $\Gamma_1 = -k'_P$. By using the difference $\dot{e}_i(t) - \dot{e}_{i-1}(t)$ as the approximation of the *derivative* along the i-direction, a PID controller in i-direction

$$u_{i+1}(t) = k'_P \dot{e}_i(t) + k'_I \sum_{j=0}^{i} \dot{e}_j(t) + k'_D(\dot{e}_i(t) - \dot{e}_{i-1}(t))$$

will result in the following ILC updating law

$$u_{i+1}(t) = u_i(t) + \Gamma \dot{e}_i(t) + \Gamma_1 \dot{e}_{i-1}(t) + \Gamma_2 \dot{e}_{i-2}(t).$$

where $\Gamma = k'_P + k'_I + k'_D$, $\Gamma_1 = -k'_P - 2k'_D$ and $\Gamma_2 = k'_D$. This is a high-order iterative learning controller. The above arguments indicate that the high-order ILC is capable of giving better ILC performance than the traditional integral controller.

In general, an N-th order D-type ILC updating law is

$$Q_{i+1} = Q_i + \sum_{j=1}^{N} K_{d_j} \dot{e}_{i-j+1}(t) \tag{7}$$

where the learning gains should satisfy that the roots of (8) are inside the unit circle.

$$(1 - CBK_{d_1})z^{-1} - \sum_{j=2}^{N} CBK_{d_j} z^{-j} = 0 \qquad (8)$$

where z is one step shifting operator. According to [6], a sufficient condition is given by

$$\mid 1 - CBK_{d_1} \mid + \sum_{j=2}^{N} \mid CBK_{d_j} \mid < 1. \qquad (9)$$

3.3 P-type Iterative Learning Feedback Control

From [7], it is clear that better ILC performance can be achieved by introducing a feedback loop. The control system is actually an ILC controller in the iteration number direction and a feedback controller in the time direction simultaneously (Fig. 3).

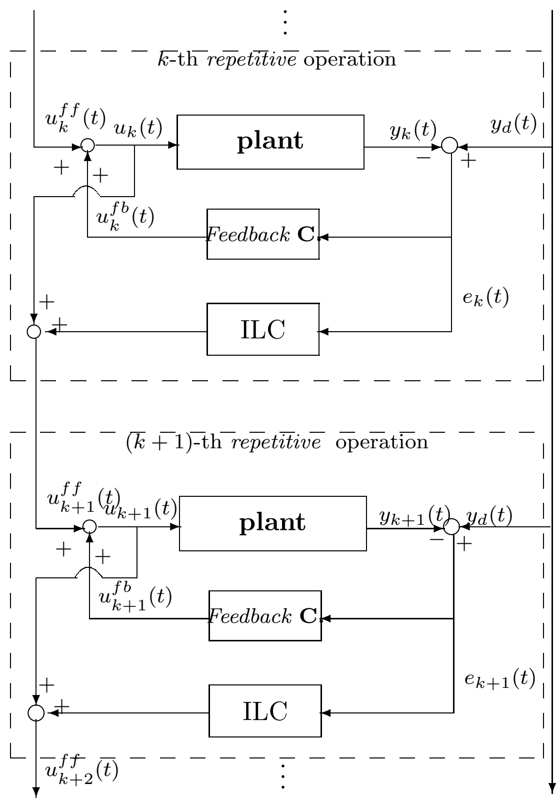


Figure 3: Block-Diagram of *Iterative Learning Feedback Control*

In practice, P-type ILC scheme is preferred because the D-type ILC (5) is sensitive to the measurement noise. According to Fig. 3, the P-type scheme can be written as

$$\begin{cases} Q_i(t) &= Q_i^{ff}(t) + Q_i^{fb}(t) \\ Q_i^{fb}(t) &= K_p e_i(t) \\ Q_i^{ff}(t) &= Q_{i-1}(t) + K_{pl} e_{i-1}(t) \\ &= Q_i^{ff}(t) + (K_p + K_{pl}) e_{i-1}(t) \end{cases} . \qquad (10)$$

A convergence condition can be found in [8] for discrete-time nonlinear systems where the role of feedback is regarded as an assistance to the ILC.

3.4 P-type ILC with CITE

Consider the PI-controller in the ILC iteration direction as follows:

$$Q_i(t) = k_I' \sum_{j=0}^{i} e_j(t) + k_P' e_i(t). \qquad (11)$$

Writing (11) in an iterative form, we have

$$\begin{aligned} Q_i(t) &= Q_{i-1}(t) + k_I' e_i(t) + k_P'(e_i(t) - e_{i-1}(t)) \\ &= Q_{i-1}(t) + K_p e_i(t) + K_{pl} e_{i-1}(t) \qquad (12) \end{aligned}$$

where $K_p = k_I' + k_P', K_{pl} = -k_P'$. Updating law (12) is called as the *ILC with Current Iteration Tracking Error (CITE)*. A convergence condition was given in [9] where k_P' is assumed to be 0. It was shown in [9] that the convergence as well as the robustness of the ILC with CITE scheme (12) is independent of the choice of K_p, the CITE gain. This actually invokes a high-gain ILC as indicated in [10].

High-order scheme can be synthesized with P-type ILC and CITE.

$$Q_i(t) = p' Q_{i-1}(t) + (1-p') Q_{i-2}(t) + K_p e_i(t) + K_{pl} e_{i-1}(t) \qquad (13)$$

where p' is a positive fraction ($p' \in [0, 1]$). Improved ILC convergence property can be expected as discussed in Sec. 3.2.

4 Simulation Studies

Simulations are carried out in MATLAB V4.2c. RK-4 is used to numerically integrate (1) with a fixed step $h = 0.1$ hr. Total number of integration points is $N_p = 201$.

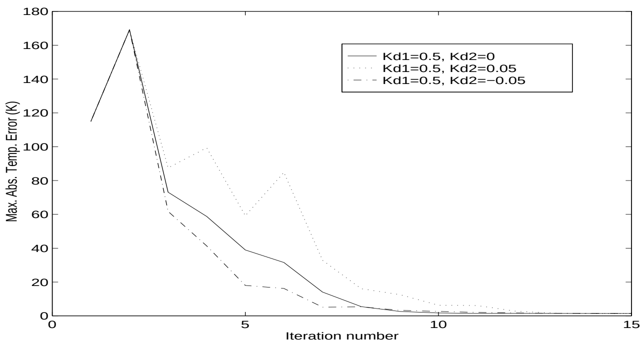
4.1 D-type Iterative Learning Control

To obtain the derivative information with low noise level, numerical differentiation is conducted using a five-point formula as follows:

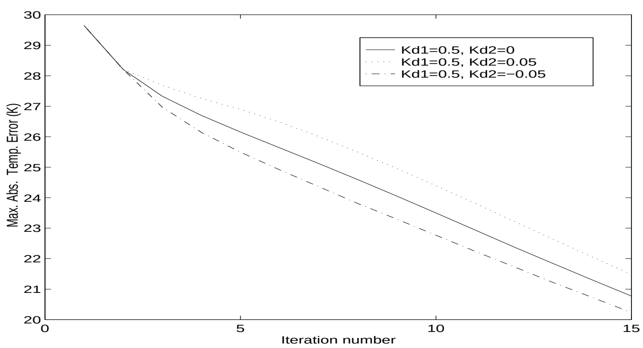
$$\begin{cases} \hat{e}(1) = \frac{-25e(1)+48e(2)-36e(3)+16e(4)-3e(5)}{12h} \\ \hat{e}(2) = \frac{-3e(1)-10e(2)+18e(3)-6e(4)+e(5)}{12h} \\ \hat{e}(j) = \frac{e(j-2)-8e(j-1)+8e(j+1)+e(j+2)}{12h} \\ \qquad (j = 2, \cdots, N_p - 2) \\ \hat{e}(N_p - 1) = \frac{1}{12h}\{-e(N_p-4) + 6e(N_p-3) \\ \quad -18e(N_p-2) + 10e(N_p-1) + 3e(N_p)\} \\ \hat{e}(N_p) = \frac{1}{12h}\{3e(N_p-4) - 16e(N_p-3) \\ \quad +36e(N_p-2) - 48e(N_p-1) + 25e(N_p)\} \end{cases} \qquad (14)$$

D-type ILC scheme (5) is first studied with $K_d = 0.5$. The high-order scheme (7) is also considered ($N = 2$) with $K_{d_1} = K_d, K_{d_2} = \pm 10\% K_d$. A set of results with the same computation conditions are presented in Fig. 4(a). Clearly, the high-order ILC scheme may give a better ILC convergence performance provided a negative K_{d_2} is used. This can be analyzed by the position of the pole from (8).

It can be observed from Fig. 4(a) that the tracking error bound is unacceptably large at the beginning. This is the overshoot of the response along i-direction. A feedback loop (4) may be introduced to reduce it, where K_p is set to 20. Three similar cases are summarized in Fig. 4(b). The tracking error bound decreases monotonically. This is because the overall system under feedback control becomes more dissipative.



(a) $K_p = 0.0$ (D-type ILC only) (D-I scheme)



(b) $K_p(t) = 20$ (PD-I scheme)

Figure 4: Convergence comparisons for D-type ILC schemes

4.2 P-type ILC plus Feedback Controller

As discussed in Section 3.3 and Section 3.4, ILC schemes (10) and (12) under a feedback structure illustrated in Fig.3 are essentially the same. We now concentrate on the P-PI scheme (11) which uses P component of tracking error in the time t-direction while PI components of tracking error in the ILC iteration number i-direction. Hence in this sense, D-type scheme (5) is a D-I one while the scheme used in Fig. 4(b) is PD-I.

We will investigate the effects of different choices of learning gains K_p and K_{pl} on the convergence performance of ILC scheme (12).

Case 1. $K_{pl} = 0$. In this case, only CITE is used, i.e.,

$$Q_i(t) = Q_{i-1}(t) + K_p e_i(t). \tag{15}$$

This can be regarded as a P-I scheme as discussed in the above. It is interesting to note that from the analysis of [9], the convergence and robustness of ILC scheme (15) are independent of the choice of K_p. However, larger K_p will give better ILC performance as also indicated in [11]. This is clearly illustrated by Fig. 5.

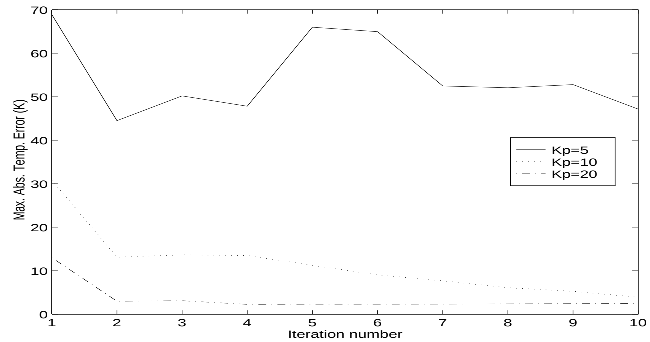


Figure 5: Convergence comparisons for P-I type (ILC+CITE) schemes

Case 2. $K_{pl} \neq 0$. This is P-PI type ILC. Along the i-direction, the PI gains $k'_P = -K_{pl}$, $k'_I = K_p + K_{pl}$. It is intuitive from the conventional PID controller tuning that increasing k'_I will reduce the convergence bound of tracking error. This is similar to the effect of an integral (I) controller in the time-domain. This effect was illustrated in Fig. 5.

We now observe the effect of K_{pl} under a fixed K_p. Qualitatively speaking, to guarantee the ILC convergence (stability in the i-direction), k'_P, i.e., K_{pl} can not be arbitrarily chosen. We considered 5 sub-cases for $K_{pl} = 2, 0.5, 0, -2$ and -4 respectively. The results are presented in Fig. 6. From the comparison in Fig. 6, the tuning of K_{pl} and K_p is possible based on the existing PID tuning method which deserves future research.

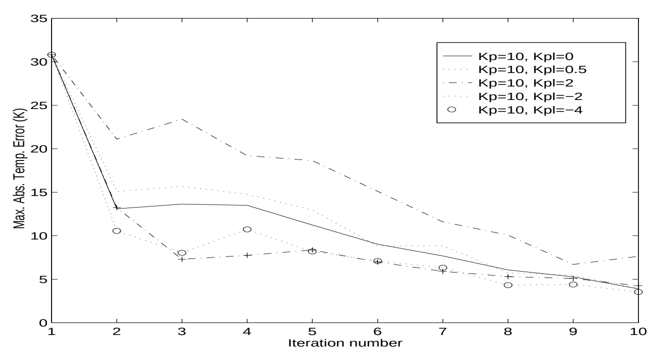


Figure 6: Convergence comparisons for P-PI type (ILC+CITE) schemes

Case 3. High-order in Control. As described in (13), the high-order can be in control terms. This in fact adds a input signal filter. In this case, a first order filter. Different choices of p' may result in different ILC convergence transients. Fig. 7 shows the results for $p' = 1, 0.9$, and 0.7 respectively. It is interesting to observe that ILC convergence performance improves as p' slightly decreases from 1.

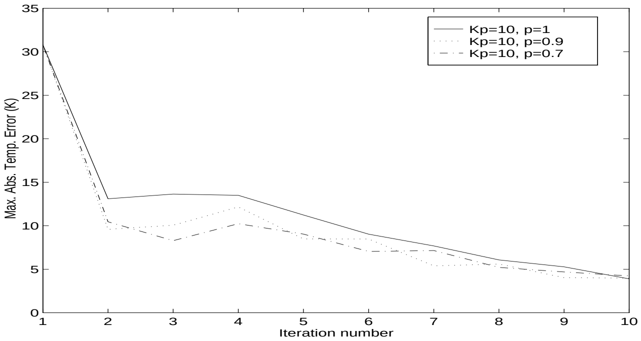


Figure 7: Convergence comparisons for P-type scheme (13)

Case 4. $K_{pl} = -4; K_p = 5, 10, 15$. Similar to **Case 1,** we will show that under a fixed K_{pl}, the ILC convergence improves when K_p increases. This is well illustrated by Fig. 8

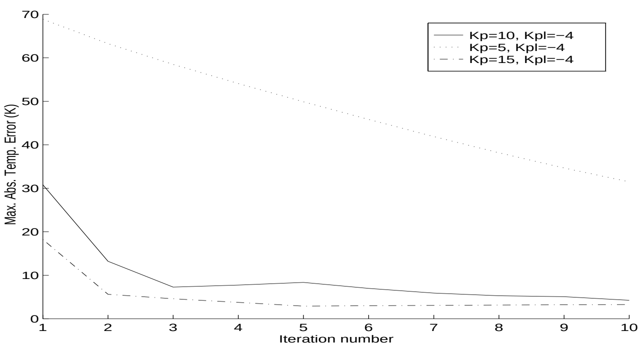


Figure 8: Convergence comparisons for P-PI type (ILC+CITE) schemes

The converged results are almost the same. A set of plots for the system states in the 10-th ILC is given in Fig. 9. The monomer condensation finally decreases to 0 while the polymer condensation keeps increasing, according to the predesigned temperature profile $T_d(t)$.

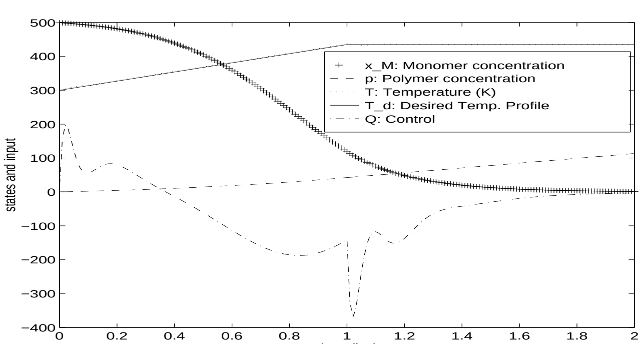


Figure 9: Converged system states and input (at the 10th ILC iteration)

5 Concluding Remarks

Iterative Learning Feedback Control method is proposed for the tracking control of the finite-time temperature profile control of a chemical reactor system. D-type, P-type, as well as high-order ILC schemes are presented. ILC with a feedback structure is discussed in terms of FA-ILC (Feedback Assisted) and ILC with CITE (current iteration tracking error). Among the proposed schemes, the CITE P-type scheme is the most preferable due to its simplicity and effectiveness. Simulation studies have illustrated the effectiveness of the ILC schemes which is consistent with analytical results and clearly indicates the applicability of ILC schemes to batch processing problems.

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