ME280 "Fractional Order Mechanics" (FOMech 4 units with labs)

Fall 2014, Instructor: YangQuan Chen (ychen53@ucmerced.edu)

Catalog Description: This course prepares students with fractional calculus (differentiation or integration of non-integer order) and fractional dynamic modeling of complex mechanical systems such as porous medias, particulate systems, soft matters etc. that have inherent nature of memory, heredity, or long-range dependence (LRD), or long range interactions at or across various scales.

Textbook: R. L. Magin (06). "*Fractional Calculus in Bioengineering*" Begell House Publishers. ISBN-978-1567002157

Course Goal: ME280 is to prepare graduate students, *not necessarily in the field of mechanical engineering*, with the basic knowledge of fractional calculus (FC: differentiation or integration of noninteger order) and working ability in using FC in modeling their respective (complex) systems related to their research topics, where, as exploring into micro and nano world, more and more "anomalous" behaviors are being observed.

Course Learning Outcomes: Upon completion of ME280, students should be able to

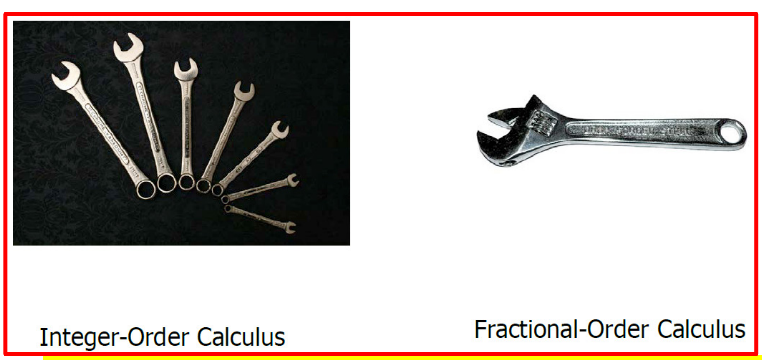
1. Perform basic fractional calculus math derivations;
2. Do fractional order modeling of relaxation processes from complex systems using both numerical experiments;
3. Do numerical solution of FO differential equations; Perform simple fractional order damping control analysis;
4. Understand the fractional mechanics in classical sense (Bagley-Torvik's stress-strain relationships etc.)
5. Appreciate fractional order calculus of variation and its role in fractional order Euler-Lagrange mechanics;
6. Acquire variable-order (CO) and distributed-order (DO) thinking in fractional order mechanics.
7. Use the theory and techniques in "fractional order mechanics" to address the modeling of non-mechanical systems in their respective domain such as thermal/fluid systems, bioengineering system, cognitive science, material science, biological systems, physiological systems, networked systems, human-centric man-made systems etc.
8. Agree that "being anomalous is normal" when FC is used as the modeling tool.

Prerequisites: Instructor approval (via email). ODE, Laplace transform, basic modeling techniques etc.

Class size: 10 maximum registered for credits. Send email request for enrollment first. Graduate students only.

Course Outline:

1) Course admin, general motivations of fractional calculus, and real world needs (soft-matter, bioengineering, etc); (2 weeks)
2) FOMech motivations: FOT (fractional order thinking) and fractional stochasticity; (2 weeks)
3) Fractional mechanics in classical sense (Bagley-Torvik) (1 week)
4) Fundamentals of fractional calculus and geometrical/physical interpretations (1 week)
5) CTRW (continuous time random walk) and anomalous diffusion (2 weeks)
6) Fractional order system modeling; (1 weeks);
7) Fractional order damping control(1 week)
8) Variable-order (VO) and distributed-order (DO) mechanics (1 week)
9) Fractional-order analytical mechanics (2 weeks)
   Integer-order (IO) analytical mechanics and a dark cloud; IO optimal control, IO calculus of variation and RIOTS_95; fractional order analytical mechanics (FO Euler-Lagrange mechanics & fractional variational principle)
10) FISP (focused independent study and presentation) – Weeks 15, 16 and 17 (final exam week) plus guest lectures. **Sample Topics:** Battery system models, biological and physiological signal processing, nanomaterial modeling of particles with long range interactions, extreme event dynamics, complexity quantification, hysteresis modeling and compensation, fractional order stochastic mechanics for evolving complex networks, controlled cognition etc.



Fractional Order Mechanics!

Hooke's law: $F = kx$

Newton's fluid: $F = kx'$ → $F(t) = k\,x^{(\alpha)}(t)$

Newton's 2nd law: $F = kx''$

Going in-between: interpolation of operators:

$$\ldots,\ \frac{d^{-2}f}{dt^{-2}},\ \frac{d^{-1}f}{dt^{-1}},\ f,\ \frac{df}{dt},\ \frac{d^2f}{dt^2},\ \ldots$$

For more information, please visit http://www.asmeconferences.org/IDETC2013/Workshops.cfm (click **W6**)

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APPLIED FRACTIONAL CALCULUS (AFC)

… from integer to non-integer …

$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n}$$

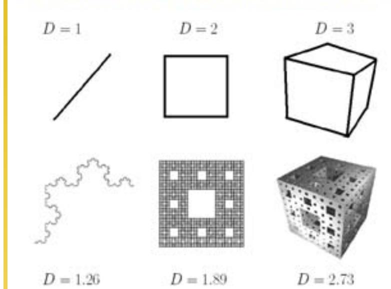
$$x^n = e^{n \ln x}$$

$$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n,$$

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \qquad x > 0,$$

$$\Gamma(n+1) = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n = n!$$

… from integer to non-integer …

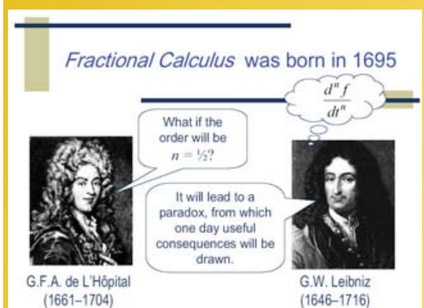


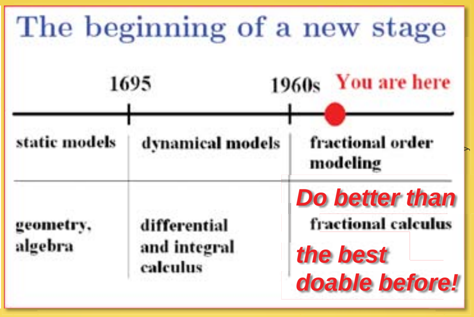
Interpolation of operations

$$f, \quad \frac{df}{dt}, \quad \frac{d^2f}{dt^2}, \quad \frac{d^3f}{dt^3}, \quad \ldots$$

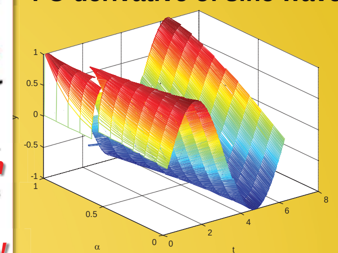
$$f, \quad \int f(t)dt, \quad \int dt \int f(t)dt, \quad \int dt \int dt \int f(t)dt, \quad \ldots$$

$$\ldots, \quad \frac{d^{-2}f}{dt^{-2}}, \quad \frac{d^{-1}f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2f}{dt^2}, \quad \ldots$$





FO derivative of sine wave

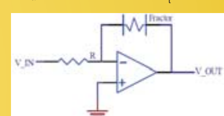


Rule of thumb

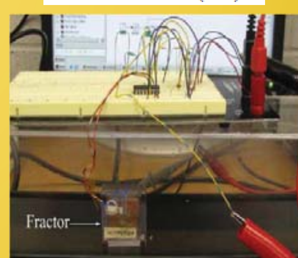
- Power law, Scale-free, scale invariant
- Heavy tailedness, fat tail
- Long range dependence (LRD)
- Porous media, Anomaly
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …

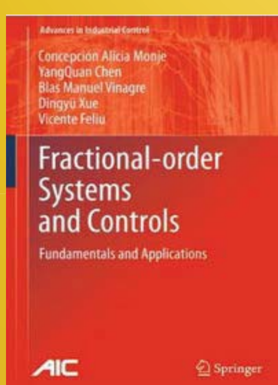
"Go fractional!" – YangQuan Chen

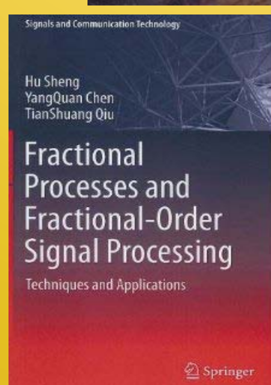
7/13/1865. "Go west, young man." – Horace Greeley (1811-1872)

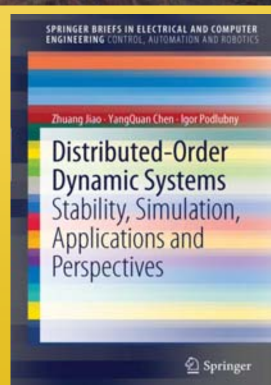


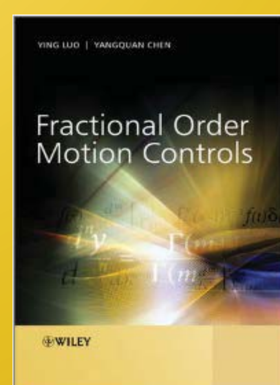
$$G(s) = -\frac{K}{R(s\tau)^{\lambda}}$$











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