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Fractional Order Zero Phase Error Tracking Control for Continuous Time Non-minimum Phase Processes*

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Abstract— **This paper presents a Fractional-Order Zero Phase Error Tracking Controller (FOZPETC) for the non-minimum phase processes. The conventional Zero Phase Error Tracking Controller (ZPETC) results in high bandwidth of the system and improves the tracking performance of the overall system. Meanwhile, the overall system may be quite sensitive to the high frequency disturbance and noise. We apply the fractional-order pole-zero cancellation techniques to the non-minimum phase zeros and use the FOZPETC instead of ZPETC to improve the disturbance rejection capacity. A numerical example is provided to illustrate the proposed fractional-order tracking zero phase controller design method.**

I. INTRODUCTION

In tracking control systems, i.e., servo systems, the objective is to make the actual output track the reference input with minimal error. Researchers have investigated many methods to improve the tracking performance. Tomizuka [1] proposed the perfect tracking control (PTC) method in 1987. In the perfect tracking control system, in order to track the trajectory exactly, the transfer function between the desired output and the actual output should equal unity, which may be achieved by a feedforward controller which acts as the inverse of the closed-loop system. The poles of the closed-loop system can be modified by pole placement method, and all the poles should lie in the left half plane (LHP) of the complex plane such that the closed-loop system is stable, these poles can be canceled easily by inversion. However, the feedforward controller cannot be used to cancel the non-minimum phase zeros, since such a cancellation leads to system instability [2].

The existence of the noncancellable zeros will generate phase error and gain error. To remove the drawbacks, Tomizuka [1] proposed an effective method that is capable to eliminate the phase error introduced by the non-minimum phase zeros. This strategy is called Zero Phase Error Tracking Control (ZPETC). This tracking controller reshapes the reference signal by cancelling all the poles and cancelable zeros of the closed-loop transfer function, meanwhile, it provides a chance for the system actual output to track the desired output with zero phase error for all frequency points. In similarity, Yeh and Hsu [3], Mustafa [4] used ZPETC without factorization of zeros and injected a gain compensation filter. Adnan et al. [5] designed the gain compensation filter using comparing coefficients method and studied the effect of zero locations to tracking performance. Liu et al. [6] applied the ZPETC methodology to fractional-order systems, and validate the effectiveness of the proposed method in hardware-in-the-loop thermal peltier platform.

The ZPETC strategy has broad applications in motion control. It also plays a significant role in process control, for example, when the reference signal of the temperature control system is pre-established by the operator, this preview input information can be utilized by ZPETC to improve the tracking performance. The ZPETC results in a high bandwidth by removing the poles and cancelable zeros, hence improves the tracking performance. However, at the same time, the system robustness is undesirable and the system is sensitive to the internal and external noises.

Fractional-order calculus has attracted increasing attention in human operator behavior analysis [7], [8], energy informatics [9], control domain [10], [11], [12], especially the research of fractional order PID controllers [13], [14], [15]. In [16], Li et al. proposed a new systematic design method for fractional-order proportional and derivative (FOPD) controller, which ensures the phase Bode plot to be flat around the given gain crossover frequency, which ensures the system robustness. Merrikh-Bayat [17] proposed a new feedback control strategy for non-minimum phase processes using the fractional-order unstable pole-zero cancellation method. The non-minimum phase zeros put limitations both in time domain performance and frequency domain performance, such as the undershoot and restricted stability margins [18]. This method partially cancelled the non-minimum phase zeros without leading to internal instability. The author showed that this fractional-order cancelation of the non-minimum zeros considerably increases the phase margin and gain margin of the system.

In order to improve the disturbance rejection capacity of the ZPETC based control systems, the fractional-order pole-zero cancellation technique is firstly implemented to partially cancel the non-minimum phase zeros, and then the tracking controller is modified to track the reference input without phase error.

In this paper, we are trying to mitigate the ZPETC system sensitivity to noise and increase the system disturbance rejection capacity. We apply the fractional pole-zero cancellation methodology to the non-minimum phase system and design a Fractional-order Zero Phase Error Tracking

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Controller (FOZPETC) to track the reference without phase error. The rest of this paper is organized as follows: section II introduces the concept of the ZPETC. The fractional-order pole-zero cancellation methodology is presented in section III. Section IV presents the idea of continuous fractional-order Zero phase error tracking control. A numerical example is illustrated in section V. Conclusions are given in section VI.

II. Concept of Continuous Time Zero Phase Error Tracking Controller

The feedback control method is widely used for its simplicity and disturbance rejection capability, however, one obvious drawback of the negative feedback control is the phase lag. To achieve superior tracking performance, the feedforward control methodology is always operated combined with feedback techniques. Assuming that the feedback controller has already been designed for the plant, the transfer function of the closed-loop system is expressed as

$$G(s) = \frac{B(s)}{A(s)}. \tag{1}$$

The relationship between the reference input signal $R(s)$ and system output signal $Y(s)$ is yield as:

$$Y(s) = G(s)R(s). \tag{2}$$

Consider a feedforward controller which reshapes the reference input in the following form:

$$R(s) = F(s)Y_d(s), \tag{3}$$

where $Y_d(s)$ is the desired output, $F(s)$ is the feedforward controller. The objective is to ensure that the system output $Y(s)$ equals the desired output $Y_d(s)$, which means that the tracking controller provides perfect tracking without any error and distortion as long as all the initial conditions are zero. It can be easily seen that, the tracking controller should be the inverse of the closed-loop transfer function:

$$F(s) = \frac{A(s)}{B(s)}. \tag{4}$$

This is called Perfect Tracking Control (PTC) [1]. It should be noted that this controller can only be implemented when $B(s)$ does not contain any non-minimum phase zero. Theoretically, these non-minimum phase zeros can be canceled by the poles of the controller. However in real processes, the external environment disturbance and the internal parameters perturbation can lead to the zero drift, subsequently the RHP (Right Half Plane) poles of the tracking controller will lead to the internal instability [2]. If any zero lies in RHP or on the imaginary axis, even lies in the LHP but close to the imaginary axis, the output of the system will explode or oscillate [1]. Let us consider the closed-loop system in equation (4), and factorize the numerator $B(s)$ into two parts:

$$B(s) = B_a(s)B_u(s), \tag{5}$$

where $B_a(s)$ includes all the zeros of system which are acceptable poles of the tracking controller, and $B_u(s)$ contains all the unacceptable zeros, as well as the time delay, as the poles of the tracking controller $F(s)$. It should be noted that $B_u(s)$ may include zeros lying in the stability region but very close to stability boundary.

The tracking controller which can cancel all the cancelable zeros and poles can be achieved as:

$$F(s) = \frac{A(s)}{B_a(s)B_u(0)}, \tag{6}$$

where $B_u(0)$ is used to scale the steady state gain of the overall system transfer function to unity. Thus for zero initial state, the overall system output is obtained as

$$Y(s) = \frac{B_u(s)}{B_u(0)}Y_d(s). \tag{7}$$

It is noted that the existence of the non-cancelable zeros in equation (7) create amplitude distortion and phase shift between actual output and desired output, the tracking performance is not perfect. It is necessary to find a way to compensate the drawbacks, such as the phase error.

Suppose $B_u(s)$ has n non-minimum phase zeros at $s = b_k(k = 1, 2, \cdots, n)$, and consider the tracking controller behaviour in frequency domain by $s = jw$:

$$B_u(jw) = (jw - b_1)(jw - b_2)\cdots(jw - b_n). \tag{8}$$

In order to make the tracking controller generate zero phase shift, which means that the frequency domain performance of the $F(s)$ should be a real number without imaginary part, we consider:

$$B_u(-jw) = (-jw - b_1)(-jw - b_2)\cdots(-jw - b_n). \tag{9}$$

The following property is obtained as:

$$\begin{aligned} & B_u(jw)B_u(-jw) \\ &= (jw - b_1)(-jw - b_1)\cdots(jw - b_n)(-jw - b_n) \\ &= (w^2 - b_1^2)\cdots(w^2 - b_n^2). \end{aligned} \tag{10}$$

By multiplying $B_u(jw)$, the polynomial becomes a real number, which means no phase shift will be introduced as in (10), and in low frequency region, the frequency response gain is close to 1. Thus, after applying another scaling factor $B_u(0)$, we can obtain the transfer function of the tracking controller as follows:

$$F(s) = \frac{A(s)B_u(-s)}{B_a(s)B_u^2(0)}, \tag{11}$$

and the transfer function from $Y_d(s)$ to $Y(s)$ is

$$G_{overall}(s) = \frac{B_u(s)B_u(-s)}{B_u^2(0)}. \tag{12}$$

By using the feedforward controller (11), the overall transfer function becomes a real number, and the phase shift between $Y(s)$ and $Y_d(s)$ becomes zero for all frequencies, ((11)) is called the Zero Phase Error tracking Controller (ZPETC).

III. Concept of Fractional Order Pole-zero Cancelation

Let us first study the frequency domain behavior of the fractional-order transfer function [17]

$$P(s) = 1 - (\frac{s}{z})^{\alpha}, \tag{13}$$

where $0 < \alpha < 1$, z is a positive real constant. Replace s with jw. The frequency response of (13) is:

$$\begin{aligned} &P(jw) \\ &= 1 - (\frac{w}{z})^{\alpha}\cos(\frac{\pi\alpha}{2}) - j(\frac{w}{z})^{\alpha}\sin(\frac{\pi\alpha}{2}). \end{aligned} \tag{14}$$

Figure 1 is the Bode plot of $P(s)$ for $z = 1$, $\alpha = 0.1, 0.2, \cdots, 1$. It is observed that the magnitude of $P(jw)$ increases and the phase gets deceased as α increases. When $0 < \alpha < 1$, the magnitude of $P(jw)$ becomes smaller than the value when $\alpha = 1$.

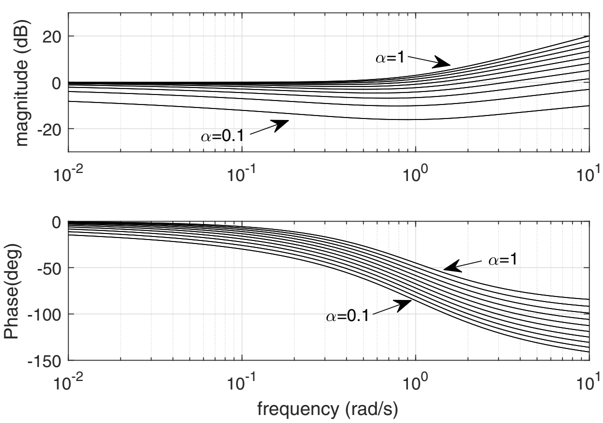

Fig. 1: Bode plot of $1 - (\frac{s}{z})^{\alpha}$

It is known that the non-minimum phase zeros put limitations on the robust stability of the feedback system for two main reasons: First, the non-minimum phase zero increases the Bode magnitude plot of the open loop transfer function, which leads to a higher crossover frequency. Secondly, the injection of negative phase of the non-minimum phase zero pushes the Bode phase plot downward. These two reasons together decrease the phase margin and gain margin of the open-loop transfer function.

For fractional order zero in form of equation (13), it generates magnitude smaller than the open-loop transfer function, which pushes the crossover frequency lower than the integer order zeros. Meanwhile, the lower phase of the fractional order zero pushes the Bode phase plot downward, which may decrease the phase margin. Thus, fractional order zeros bring both positive effect (lower crossover frequency) and bad effect (lower phase) to the system, offering potentially helpful tradeoff.

Now we introduce the fractional-order pole-zero cancelation, which transforms the integer order zeros to fractional-order zeros. Consider a linear time-invariant (LTI) system with transfer function $G(s)$ with a non-minimum phase zero at $s = z$, where z is a positive real number. Such a transfer function can be decomposed as

$$G(s) = (1 - \frac{s}{z})G_0(s), \tag{15}$$

where $G_0(s)$ includes all the minimum phase zeros and poles of the system transfer function. The term $1 - \frac{s}{z}$ can be expanded using fractional power of s as follows:

$$1 - \frac{s}{z} = [1 - (\frac{s}{z})^{1/n}]\sum_{k=1}^{n}(\frac{s}{z})^{(k-1)/n}, \tag{16}$$

where n can be considered as any positive integer. Defining

$$Q_{z,n}(s) \triangleq \sum_{k=1}^{n}(\frac{s}{z})^{(k-1)/n}, \tag{17}$$

and multiplying $1/Q_{z,n}(s)$ to both sides of (15) yield

$$G_f(s) = [1 - (\frac{s}{z})^{1/n}]G_0(s). \tag{18}$$

$G_f(s)$ and $G(s)$ have the same poles and zeros and steady-state gains, except that $G_f(s)$ has a lower order non-minimum phase zero at $s = z$.

The above operation can be described with the control diagram in Fig. 2, where $C(s)$ is the controller, $G(s)$ is the process, $1/Q_{z,n}(s)$ is the fractional-order pole-zero canceller.

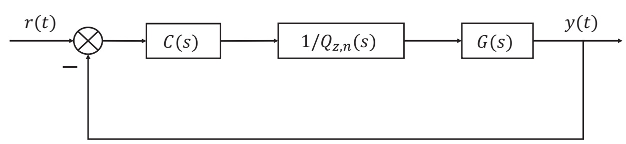

Fig. 2: Control structure of fractional-order cancellation for non-minimum phase processes

When the above control structure is used, it is necessary to discuss the stability of the overall system. Suppose $G(s)$ is stable. The internal stability of the fractional-order zero canceller $1/Q_{z,n}(s)$ dominates the overall system stability. It is clearly known that the integer order zero canceller will lead to instability due to the unstable pole at $s = z$. Applying the fractional pole-zero cancelation by injecting $1/Q_{z,n}(s)$, will not lead to internal instability, which will be explained in the following [17]. Instead of the pole at $s = z$, the canceller $1/Q_{z,n}(s)$ has poles at the roots of $Q_{z,n}(s) = 0$. It is important to point out that the stability of a fractional system cannot be examined simply by investigating the poles in the RHP of the complex s plane. It is noted in [19], [20] that such a fractional transfer function is stable if all roots of $\widetilde{Q}_{z,n}(w)$, defined as

$$\widetilde{Q}_{z,n}(w) = Q_{z,n}(s)|_{s^{1/n}=w}, \tag{19}$$

lie in the angular sector defined by $|\arg(jw)| > \pi/2n$ in w-plane, or equivalently, $Q_{z,n}(s) = 0$ does not have any root in the closed RHP of the first Riemann sheet [21]. Actually, $1/Q_{z,n}(s)$ has $n-1$ poles distributed on a Riemann surface with n Riemann sheets, where the origin is a branch point of order $n-1$ [22]. More precisely, the kth pole is expressed as $s_k = ze^{j2k\pi}$ for $k = 1, 2, \cdots, n-1$, it does not have any pole in the right half plane of the first Riemann sheet. Thus, the zero canceller suggested here does not change the internal stability of the feedback system.

IV. Concept of Continuous Fractional-order Zero Phase Error Tracking controller

In section II, we have introduced the concept of continuous Zero Phase Error Tracking Controller in form (11). It is mentioned in the previous section that the non-minimum phase zero will increase the gain crossover frequency and push the Bode phase plot downward, which leads to a lower phase margin. By applying the additional part $B_u(-s)$, the magnitude of the transfer function will get increased by the corresponding minimum phase zeros at $s_j = -z_j$, $(j = 1, 2, \cdots, q)$. For the above reasons, the crossover frequency of the overall system is pretty high after ZPETC compensated, as well as the system magnitude. The compensated system may have bad robust stability to high frequency disturbance and noise.

In order to improve the disturbance rejection capacity of the ZPETC compensated system, first, we apply the fractional-order pole-zero cancelation technique to the non-minimum phase zeros of the already designed closed loop system.

Considering the closed-loop system (1) which was first compensated with the poles and minimum phase zeros cancelation:

$$H(s) = \frac{B_u(s)}{B_u(0)}. \tag{20}$$

Assuming $H(s)$ has q non-minimum phase zeros at $s_i = z_i$, $(i = 1, 2, ..., q)$, we apply the fractional order pole-zero canceler to each non-minimum phase zero z_i by multiplying $1/Q_{z_i,n_i}(s)$, where

$$Q_{z_i,n_i} = \sum_{k=1}^{n_i} (\frac{s}{z_i})^{(k-1)/n_i}, \tag{21}$$

the overall transfer function is yielded as

$$\begin{aligned} \widetilde{H}(s) &= \frac{\widetilde{B_u}(s)}{B_u(0)} \\ &= \frac{(1-(\frac{s}{z_1})^{\frac{1}{n_1}}(1-(\frac{s}{z_2})^{1/n_2})\cdots(1-(\frac{s}{z_q})^{1/n_q})}{B_u(0)}. \end{aligned}$$

Similar to the conventional ZPETC, this feedforward controller generates tracking error between the desired input and system output. In the following, we will try to remove the phase shift of this feedforward controller. Let us still consider this problem in frequency domain, according to Euler's function, it is obtained:

$$\begin{aligned} 1-(\tfrac{jw}{z_i})^{\frac{1}{n_i}} &= 1-(\tfrac{w}{z_i})^{\frac{1}{n_i}} e^{j(\frac{\pi}{2}\frac{1}{n_i})} \\ &= 1-(\tfrac{w}{z_i})^{\frac{1}{n_i}}(\cos(\tfrac{\pi}{2n_i}) + j\sin(\tfrac{\pi}{2n_i})) \\ &= M(w) - jN(w), \end{aligned} \tag{22}$$

$$\begin{aligned} 1-(\tfrac{-jw}{z_i})^{\frac{1}{n_i}} &= 1-(\tfrac{w}{z_i})^{\frac{1}{n_i}} e^{j(-\frac{\pi}{2}\frac{1}{n_i})} \\ &= 1-(\tfrac{w}{z_i})^{\frac{1}{n_i}}(\cos(\tfrac{\pi}{2n_i}) - j\sin(\tfrac{\pi}{2n_i})) \\ &= M(w) + jN(w). \end{aligned} \tag{23}$$

Based on the above equations, it is obtained:

$$\begin{aligned} &(1-(\frac{jw}{z_i})^{\frac{1}{n_i}})(1-(\frac{-jw}{z_i})^{\frac{1}{n_i}}) \\ &= [M(w) - jN(w)][M(w) + jN(w)] \\ &= M(w)^2 + N(w)^2. \end{aligned} \tag{24}$$

By utilizing the above property, we apply $\widetilde{B_u}(-s)$ in $H(s)$, as well as the DC gain scaler, the transfer function of the overall system is obtained as:

$$\widetilde{H}(s) = \frac{\widetilde{B_u}(s)\widetilde{B_u}(-s)}{B_u^2(0)}, \tag{25}$$

where

$$B_u(-s) = (1-(-\frac{s}{z_1})^{1/n_1})\prod_{i=2}^{q}(1-(-\frac{s}{z_i})^{1/n_i}). \tag{26}$$

It is easily concluded that equation (25) gives a real number, which means the fractional-order tracking controller does not generate phase error between the desired output and the system output. The relationship between the desired output and the system output can be expressed as

$$Y(s) = \frac{\widetilde{B_u}(s)\widetilde{B_u}(-s)}{B_u^2(0)} Y_d(s), \tag{27}$$

and the fractional order tracking controller is

$$\widetilde{F}(s) = \frac{A(s)\widetilde{B_u}(-s)}{Q_{z,n}(s)B_a(s)B_u^2(0)}, \tag{28}$$

where

$$Q_{z,n}(s) = Q_{z_1,n_1}(s)Q_{z_2,n_2}(s)\cdots Q_{z_q,n_q}(s), \tag{29}$$

the control structure of FOZPETC is depicted in Fig. 3.

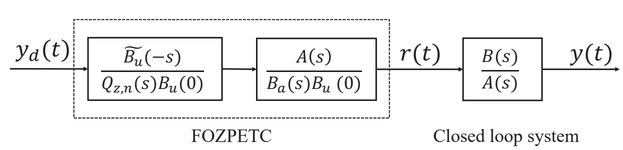

Fig. 3: Control structure of fractional-order cancellation for non-minimum phase processes

It is shown in Fig. 3 that, the non-minimum phase zero is partially cancelled by the $Q_{z,n}(s)$, and the remaining fractional-order zero $\widetilde{B_u}(s)$ has lower magnitude. By multiplying the factor $\widetilde{B_u}(-s)$, the phase shift equals zero, and from equations (22) and (23) it can be easily verified that

$$\begin{aligned}&\left|(1-(\frac{jw}{z_i})^{1/n_i})(1-(-\frac{jw}{z_i})^{1/n_i})\right| \\ &= 1+(\frac{w}{z_i})^{\frac{2}{n_i}}-2(\frac{w}{z_i})^{\frac{1}{n_i}}\cos(\frac{\pi}{2n_i}).\end{aligned} \tag{30}$$

Comparing to the magnitude of $(1-(\frac{jw}{z_i}))(1-(\frac{-jw}{z_i}))$, which equals $1+(\frac{w}{z_i})^2$, we can see that the magnitude of the fractional-order ZPETC compensated system transfer function is smaller than the integer-order case.

V. Illustrative Examples

In this section, we present a numerical example to show that the FOZPETC method can improve the system disturbance rejection capacity effectively comparing with ZPETC strategy. Suppose that the closed-loop system transfer function is:

$$G(s)=\frac{(1-s/10)}{(1+s/8)(1+s/4)}. \tag{31}$$

Notice that the closed-loop transfer function (31) has one non-minimum phase zero at $s=10$, according to equation (11), the conventional ZPETC should be:

$$F(s)=(1+s/8)(1+s/4)(1+s/10), \tag{32}$$

the overall system transfer function between the desired output and real output is

$$H(s)=(1+s/10)(1-s/10), \tag{33}$$

and the Bode plot of the transfer function (33) is depicted in Fig. 4.

By applying the fractional-order pole-zero cancellation strategy, as it is pointed out in [17], in many cases using small values such as $n=2$ or $n=3$ leads to satisfactory results, the order we choose here is $n=2$:

$$\frac{1}{Q_{1,2}(s)}=\frac{1}{(s/10)^{1/2}+1}, \tag{34}$$

according to equation (28), the fractional-order tracking controller is:

$$\widetilde{F}(s)=\frac{(1+s/10)(1+s/4)(1-(-s/10)^{1/2})}{(1+(s/10)^{1/2})}, \tag{35}$$

the transfer function between the desired output and the real output is:

$$\widetilde{H}(s)=(1-(s/10)^{1/2})(1-(-s/10)^{1/2}), \tag{36}$$

the Bode plot of equation (36) is depicted in Fig. 4.

We can see that all the Bode phase plots of the ZPETC and FOZPETC are zero, and the Bode magnitude plots of the FOZPETC are much lower than the conventional ZPETC. As we mentioned above, the Bode magnitude plot of ZPETC equals $1+(w/10)^2$, which is a monotonic-increasing function with respect to w. In Fig. 4, the Bode magnitude plot of ZPETC system equals 3dB when frequency is 6.4229rad/s, which means that the ZPETC system double amplifies

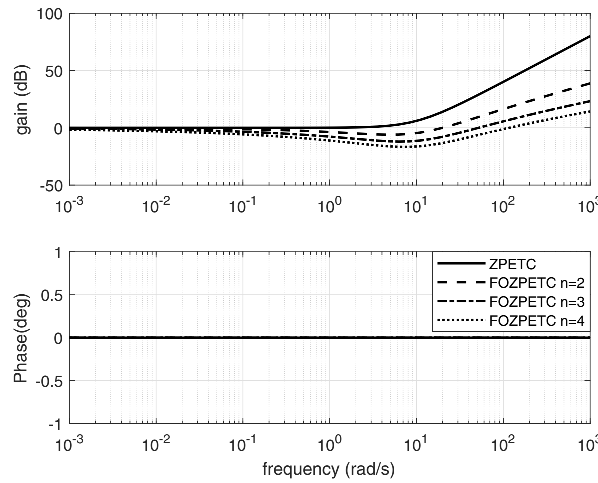

Fig. 4: Bode plot of the ZPETC and FOZPETC based systems

the power of input signals at the certain frequency. Thus when the environmental noise with a frequency higher than 6.4229rad/s, the system will amplify this signal obviously, which may bring suicidally damage to the system. From Eq. (30), the magnitude of FOZPETC is not a monotonic-increasing function with respect to w, it reaches the minimum value at

$$w_{min}=z(\cos\frac{\pi}{2n})^n, \tag{37}$$

and the minimum value is

$$\left|\widetilde{H}(jw)\right|_{w=w_{min}}=\sin\frac{\pi}{2n}, \tag{38}$$

which means the minimum magnitude of the FOZPETC system decreases as n increases. From Fig. 4, we can see that the Bode magnitude plot of FOZPETC decreases below 0 dB, and gets increased afterwards, and the magnitude of the system with $n=3$ is lower than the magnitude of the system with $n=2$. We can get a intuitively conclusion that the FOZPETC strategy will attenuate the system noise with frequency around w_{min}. It is easily obtained that the magnitude of FOZPETC system ($n=2$) equals 3dB at 27.6349rad/s, while which is 73.4931rad/s for FOZPETC system ($n=3$). when $n=4$, the magnitude reaches 3dB at 176.2952rad/s. Thus, the FOZPETC technique will improve the system's ability of noise rejection super significantly.

Figure 5 is the time response for the above four systems, the desired output is $y_d(t)=\sin(0.5t)$, which can be seen as the noise input. From Fig. 5, the phase shift between the desired output and the system output for these four systems are zero, and the magnitude of ZPETC system output is bigger than 1, which means that the ZPETC system will amplify the noise and then the system may be possibly damaged or even broken down. While the magnitude of the FOZPETC system output is smaller than 1, especially for $n=3$ and $n=4$, which means that the FOZPETC system will attenuate the noise. From above analysis, we can get a conclusion that the FOZPETC technique will improve

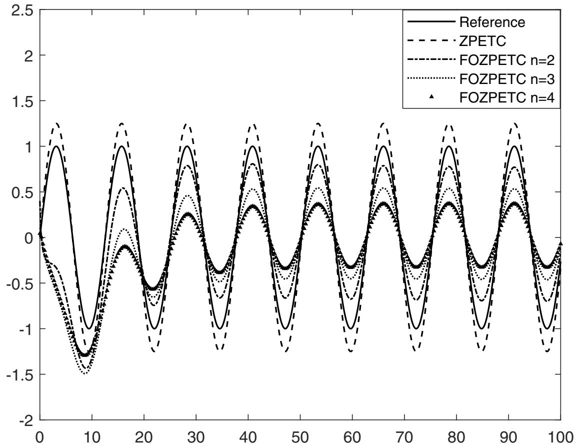

Fig. 5: Time response of the overall system

the system's capacity of disturbance and noise rejection, especially when n increases. As is known that the tradeoff between noise rejection and time response performance is an important issue in the controller design. It is noticed from Fig. 5, the tracking performance for FOZPETC gets inferior when the n increases, which is the reasonable consequence for the lower bandwidth. In practice systems, the order of the proposed FOZPETC controller should be determined according to the design requirements. If the noise rejection is a quite significant consideration, high order of the FOZPETC controller can be chosen, while lower order should be implemented when both of the tracking performance and noise rejection capacity are taken into consideration.

VI. CONCLUSIONS

This paper provides a tradeoff possibility to improve the disturbance rejection capability for the ZPETC compensated system by the Fractional Order Zero Phase Error Tracking Controller. The conventional ZPETC structure exactly improves the tracking performance of the original closed-loop system, but the unsatisfactory disturbance rejection will lead to aggressive system behaviours, and make the system quite sensitive to the environmental disturbance and noise. The fractional-order pole-zero cancellation technique can decrease both magnitude and phase of the system prominently and we apply this technique to the ZPETC structure for non-minimum phase processes, and the additional fractional compensator makes the phase shift of the overall transfer function remain zero. This proposed FOZPETC improves the system disturbance rejection capacity significantly. But one of the drawbacks of this method is that the order of fractional-order pole-zero canceller cannot vary continuously, and the FOZPETC will result in the deterioration to the system tracking performance. This will be our future work to investigate the continuous varying order of fractional-order pole-zero canceller so as to improve the disturbance rejection capacity without any undesirable loss of tracking performance.

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