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Crowds involving individuals with disabilities: Modeling heterogeneity using Fractional Order Potential Fields and the Social Force Model

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HIGHLIGHTS

• Fractional Order Potential Fields can be used to respond to heterogeneity in pedestrian crowds.

• Individuals with visual or stamina impairments are overtaken by other pedestrians at high rates.

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ABSTRACT

There is a need to model individuals with physical disabilities in a crowd. This requires both understanding how such individuals impact a crowd and how the crowd impacts them. Previous efforts have adjusted velocity to represent individuals with disabilities in a crowd. The various disabilities that are represented in a crowd of individuals show much more complex and varying interactions that require an improved form of modeling. Individuals with disabilities are in part composed of those using mechanical and electric wheelchairs, those who have impaired vision, and those with diverse mobility-related disabilities. In our research we conducted a large-scale crowd experiment with heterogeneous compositions of crowds involving individuals with disabilities. The initial outcomes demonstrated that each group differed in velocity as well as in the composition of their environment. Additional results showed that interaction within a crowd varied as well. The purpose of this paper is to provide some initial differences found in the study of individuals with disabilities within a crowd and how those differences change pedestrian interaction. Using the nature of Fractional Order Potential Fields (FOPF), this paper provides a method for how pedestrian interaction can be adjusted to respond to the differences created by those with various disabilities in each group. Finally, how a hybrid model between FOPF and the Social Force Model can be used to capture experimental results in simulation.

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1. Introduction

This research evaluates how crowd evacuations affect the movement of individuals with and without disabilities. The purpose of this study is to use the information to develop more effective evacuation policies and procedures, as well as

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Fig. 1. Crowd Study Circuit, 12.2×18.9 m.

to construct built environments that respond more appropriately to the various needs of individuals with disabilities. We demonstrated in initial research that individuals with disabilities were not characterized well and in some circumstances only represented by a change in velocity [1–4]. Shi et al. [5] completed a comprehensive review of the literature and found a great deal of research has been done to collect and observe pedestrian behavior. While these studies provide valuable knowledge on pedestrian behavior, few of these studies addressed vulnerable pedestrians, such as the elderly, children, or individuals with disabilities [6]. Christensen et al. [7] conducted a review of the behavioral measurements of individuals with disabilities in navigating the built environment, finding few studies in this area. Most studies overlooked the heterogeneity of abilities in pedestrian compositions [8–11]. Therefore, there are numerous questions regarding to what extent the movement behavior of individuals with disabilities is affected by heterogeneous conditions expected in crowds.

The authors conducted numerous experiments evaluating the specific movement and interaction of individuals in a crowd, performed to gain a better understanding of heterogeneous interactions. Various factors were important to evaluate and understand in the empirical studies conducted. These included developing an understanding of velocities for each specific group of individuals with disabilities and the spacing in movement between interactions. It also required developing a knowledge of the density and flow information for each heterogeneous group of individuals and finding out how each group responds to the differences in the built environment. Finally, we looked at how the directional movement of individuals are affected as different groups of individual encounter each other. For this paper we separate out individuals with physical disabilities into groups of individuals using motorized wheelchairs and non-motorized wheelchairs, and those who are visually impaired and mobility impaired. We did some initial exploration into the various combinations of disability groups [12,13], but determined that empirical study in the form of crowd experiments were required. This paper focuses on the interaction of overtaking individuals and how this impacts crowd movement for each group of disability. A model is presented that attempts to capture some aspects of the heterogeneous interaction that exist and vary for each group. The following sections will be structured as follows: Section 2 will discuss the Social Force Model, Fractional Order Potential Fields (FOPFs), and the hybrid model. Section 3 discusses the simulation framework that is used. Section 4 discusses preliminary results using data from the experiment in a very simple Social Force Model. Section 5 then shows the results using the hybrid FOPF model. Section 6 concludes with problems encountered so far in the modeling.

These experiments were conducted in 2012 using a variety of individuals with disabilities within a crowd in multiple combinations. An overview as well as some results can be found in [14–17]. Video tracking technology was used to track pedestrians wearing hats around a circuit. A detailed write-up found in [18] describes how this tracking system was used and implemented. Based on these experiments, we determined some differences between each group of disability and their interactions in a crowd. In this paper some of those differences are described, and we create a model to capture aspects of those differences.

A facility at Utah State University was used to conduct the experiments. A circuit was constructed to replicate common built environment features including facility structures such as a doorway, bottleneck corridors, corners, 45-degree corner, and varying hallways widths. All structures were built according to Americans with Disabilities Act Accessibility Guidelines (ADAAG). Fig. 1 shows the size and details of the circuit with the various features of the built environment. A display of the crowd movement in the circuit while wearing tracking markers is also shown in Fig. 1. During each experiment, pedestrians were density within the circuit. Individuals with physical disabilities were included in this injection to match compositions realistic to crowd movement. To fill each circuit to reasonable capacity levels 60–100 participants were required. Both unidirectional and combinations of bi-directional flow were conducted. Each experiment ran for 10 min to allow for both participant rest and system recording time limitations. Data such as interaction behavior, flow, velocities, density, and trajectory information was collected from each experiment.

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Average speed of each disability group through the entire circuit.

Visually impaired	0.7375 m/s
Individual using non-motorized ambulatory device	0.7325 m/s
Individual using motorized wheelchair	0.6425 m/s
Individual without disability	0.81 m/s

Table 2 Mean desired velocity of each crowd type.	
Visually impaired	1.19 m/s
Individual using non-motorized ambulatory device	1.14 m/s
Individual using motorized wheelchair	1.11 m/s
Individual using cane or stamina impaired	0.9 m/s
Individual without disability	1.3 m/s

Table 3

Mean overtakes per pedestrian of each disability type.

Visually impaired	26.92 overtakes/ped
Individual using non-motorized ambulatory device	19.5 overtakes/ped
Individual using motorized wheelchair	20.7 overtakes/ped
Roller walkers	14.83 overtakes/ped
Cane or stamina impaired	44.4 overtakes/ped

2. Empirical results

Walking speeds were gathered and separated out for each group with disabilities. As there were not enough nonmotorized participants, their velocities were combined with those of roller walkers and served as the group of individuals using non-motorized ambulatory devices. The mean travel speed for each group, over various parts of the circuit, can be found in Table 1. In this study, vision impaired and individuals using non-motorized ambulatory devices composed a 12% difference from individuals with no disability. The slowest group, motorized wheelchairs, had a 26% difference in speed.

For the purposes of future modeling, the desired speed for each group of individuals with disabilities was also gathered. This is defined as mean speed found when the individual speed was gathered when there was zero local density representing the speed that the individual travels when not impacted by those around them. This data was gathered through the sessions involving only individuals with disabilities. A table of those values can be found in Table 2. Here it is shown that those using non-motorized wheelchair and those using roller walkers travel at the same mean speed from our gathered experiments. All individuals with disabilities travel at slower rates than those without disabilities.

Given the differences in speed, individuals without disability will move faster than those with a disability. Therefore overtaking behavior dictates flow of a crowd as individuals decide to move past or stay behind a slower moving individual with a disability. To study other potential differences between groups with physical disabilities, the average pedestrian overtake per person is looked at. An overtake can be described as an event where agent *j* is following agent *i*, both in the same direction, and agent *j* moves to a leader position in front of *i*. For the purposes of ruling out possible proxemics issues where people may travel in groups, an overtake for analysis only occurs when agent *j* remains a leader for a significant time within each study. In our experiment, we looked at sessions with combinations of individuals with disabilities than those without. The expectation is that there are more overtakes experienced by the slower individuals with disabilities than those without. The results indicate that individuals without disabilities experience an average of 6 overtakes in a ten-minute session, while individuals with disabilities experience an average per pedestrian of 25 overtakes under similar conditions. The results found in Table 3, show the mean overtakes per individual in each type group. As shown, the mean number of overtakes found per pedestrian during a ten-minute session varied greatly per disability group. The occurrence of overtakes is also examined according to various portions of the circuit representing different common built environment conditions.

A GUI was created to analyze all the data gathered from our large-scale crowd experiments. Two variables in particular, velocity and overtaking, were analyzed. From these results, it was learned that individuals with disabilities shared varying and different walking speeds in crowds as well as in desired walking speeds. Desired walking speeds are the speed an individual walks when there is a zero capacity around them, or they effectively are walking alone. Also, it was shown that crowds overtake each group of individuals with disabilities differently. As pedestrians without disabilities move faster than pedestrians with disabilities their walking speed is only changed during congestion or when they do not move past the slower moving pedestrians. The reasons for why a faster moving individual may not move past a slower one can be broken down into physical congestion, or some form of perception that makes them decide not to move past. This paper will discuss a hybrid microscopic model that will attempt to capture some of the behavior of overtaking in our overall crowd experiment.

There are three levels of modeling in crowd behavior. The first and most broad is macroscopic modeling. In the macroscopic level, all agents are treated homogeneously and concepts of movement are characterized in flow, density, and average velocities. The next level of modeling is mesoscopic modeling. Here the agents are looked at as groups or

combinations that represent distributions of velocity or flow. The final level is microscopic modeling. In microscopic modeling each pedestrian is treated as an individual agent in the crowd. Their interactions with each other and with the environment are accounted for. For this paper, we are looking at microscopic modeling.

3. Fractional Order Potential Fields and the Social Force Model

The most interesting microscopic model, for the purposes of our research, is the Social Force Model developed by Helbing and Molnar [19], and further proven and developed in Helbing et al. [20]. The Social Force or social behavior model was originally draw from a Boltzmann-like gas kinetic model. Further developments lead to a model that is based on the summation of three forces. The Social Force Model is based on the idea that a pedestrian moves as if they were acted upon by three outside forces, the desire to get to their goal, interactions with others, and interactions with their environment.

3.1. Social force

The Social Force Model is structured as an individual point α with a given velocity that changes over time, $\delta \alpha(t)/\delta t = v(\alpha)$. The resultant acceleration from this formula is then affected by a Social Force called *F*. This Force represents the many variables included in the model and which individuals would experience in a crowd such as the environment and other pedestrians. The force *F* on a particular agent *i*, is

$$F = Fi + Fij + Fi_W$$

The desired direction or self-driven force, F_i is a desired velocity to a goal, based on error of velocity to that desired goal. F_{ij} is the interaction of individual *i* with *j*. The social interaction, due to the perceived "personal space" and other physical interactions, described as a repulsive force decreasing over distance. F_{iW} represents interactions of agent *i*, with a wall *W*. The presented model developing the idea that pedestrians move with a set of strategies based on interaction and experience. The best route, which is efficient to the individual, becomes the route they take and it can be modeled. The first force F_i , is the desired force to a goal location *p*. This is based on the position of the agent $r_i(t)$ and its current velocity described as $\frac{\delta r_j(t)}{\delta t} = v_i^{\rightarrow}(t)$. Assuming an individual starts out with the initial goal in mind, the desired velocity direction $v_i^{\rightarrow 0}$ can be found in the equation

$$v_i^{\rightarrow 0}(t) = v_i^o(t)e_i^{\rightarrow}(t)$$

This formula is comprised of the anticipated velocity $v_i^o(t)$, and the desired direction of motion e_i^{\rightarrow} .

Acceleration forces describe those forces acting on a pedestrian. The variable τ_{α} illustrates the relaxation time or in other words the duration of time required for an individual to return to their desired speed after an event took them from their normal speed. The desired force F_i is then described as

$$F_i = \frac{1}{\tau_{\alpha}} v_i^{\rightarrow 0}(t) e_i^{\rightarrow}(t) - v_i^{\rightarrow}(t)$$

The interaction between a pedestrian *i* and *j*, is a complex combination of personal space objectives and physical friction force. In the original introduction, this force was left as a repulsive exponential potential field, for the purposes of simulation [19]. That means that any interaction between pedestrians takes into account forces from every agent within a prescribed region, regardless of being in front of or behind pedestrian *i* and its motion of travel. In Helbing et al. [20], options to introduce line of sight, angle of vision, and other concepts making the force more realistic were presented. For the purposes of this paper, pedestrian interactions will remain basic, with a note that more complex interaction forces will be introduced in the future. The basic interaction force is based on the direction difference between agent *i* and *j* as r_{ij}^{\rightarrow} . The potential field U_{rep} is

$$U_{rep} = \exp(-\|r_{ii}^{\rightarrow}\|),$$

And the subsequent force F_{ij} is

$$F_{ij} = \frac{r_{ij}^{\rightarrow}}{\|r_{ij}^{\rightarrow}\|\exp\left(\|r_{ij}^{\rightarrow}\|\right)}$$

The direction difference r_{iW}^{\rightarrow} illustrates the ending force between agent *i* and a wall object *W*. The force F_{iW} is also represented as an exponential potential force in the equation:

$$F_{iW} = \frac{r_{iW}^{\rightarrow}}{\|r_{iW}^{\rightarrow}\| \exp\left(\|r_{iW}^{\rightarrow}\|\right)}.$$

The forces accumulated together illustrate the collective Social Force acting on a pedestrian as delineated in $F = F_i + F_{ij} + F_{iW}$. With a basic understanding of Social Force, we then describe Fractional Order Potential Fields as a method to characterize the heterogeneity of various physical disability types.

3.2. Fractional Order Potential Fields

Potential fields, [21], are often used in the path planning of robots as found in Ge and Cui [22]. Such fields are used as with attraction fields to drive an agent towards something or repulsive fields as to drive an agent away from something. By assigning repulsive potential fields to objects or other robots and attraction fields to desired goals, a 'safe' path can be calculated for a robot to travel. An example of an attractive potential field between points *i* and *j* with a gain *k* is

$$U(x) = \frac{1}{2}k(x_i - x_j)^2$$

where the force applied to the agent is considered the negative gradient of the potential field as

$$F(x) = -\nabla U(x).$$

Agent *i* is drawn to point *j* as the distance between *i* and *j* increases and a larger value is created in the potential field. For a repulsive field the value increases as the distance decreases. The shape of the field and distances can be varied depending on the form of the equation. One way to vary the shape of the potential field is through the use of Fractional Order Potential Fields (FOPF) [23]. Using a FOPF allows for a larger variety of potential shapes that can be customized to fit a modeling situation. FOPF have been used to describe aerial robotic path planning in Jensen and Chen [24]. For our research, the varying shape will be used to describe how pedestrians and individuals with disabilities interact with each other. In some instances, an individual without a disability may attempt to pass an individual with a disability who is moving slower. However, various circumstances may inhibit the individual's ability to pass and cause them to choose not to pass due to some hesitation, due the size, motion, or something else. While it is not our goal to determine why individuals may overtake at different rates, we intend to create a model to capture the overall response. Through FOPFs, both these extremes and everywhere in between, can be described.

A repulsive FOPF starts from the definition of the Coulombian electric field E(r) is

$$E(r) = \frac{q}{4\pi\epsilon_0 r^2}$$

Integrating this field once produces a punctual charge $V_1(r)$,

$$V_1(r) = \frac{q}{4\pi\epsilon_0 r}$$

twice produces the Coulombian potential field $V_2(r)$

$$V_2(r) = \frac{q \ln r}{4\pi\epsilon_0}.$$

This is a potential field generated by the uniformly distributed charge along a straight line. In analyzing the normalized potential fields of both equations, there is a difference in field strength between the two when distance is varied. In fact, $V_2(r)$ has a stronger force at a larger distance than $V_1(r)$. With successive integrations of the Coulombian electric field equation, the force increases at larger distances. Using fractional calculus any *n*th iteration can be found including fractional order integrals. To do this, the Weyl fractional integral, [25], is of the form:

$$V_n(r) = W^r E(r) = \frac{q}{4\pi\epsilon_0\Gamma(n)} \int_r^\infty \frac{(0-r)^{n-1}}{r^2} d0,$$

where $V_n(r)$ is the *n*th integral of E(r). In the Weyl fractional integral equation, *n* must be greater than 0 and $\Gamma(n)$ is the gamma function,

$$\Gamma(n) = \int_0^1 [\ln(\frac{1}{t})]^{n-1} dt.$$

After manipulation, the equation can be written as

$$V_n(r) = rac{q}{4\pi\epsilon_0} rac{\Gamma(2-n)}{r^{2-n}} orall n \in (0,2)(2,\infty).$$

Normalized between 0 to 1, over a distance range (r_{min} , r_{max}), the potential field is 1 at r_{min} and 0 at r_{max} . The simplification leads to the form

$$U_{drep}(r) = \frac{V_n(r) - V_n(r_{max})}{V_n(r_{min}) - V_n(r_{max})} = \frac{r^{n-2} - r_{max}^{n-2}}{r_{min}^{n-2} - r_{max}^{n-2}}.$$

Placing the minimum distance r_{min} at 1 and a maximum distance of r_{max} at 10, and $1 \le n \le 5$, the result of equation is shown in Fig. 2.

The equation allows for large heavy strong fields at higher n and smaller fields with minimum impact at small n. This means the force at varying distances can be changed by varying the order n and also by modifying the minimum and



Fig. 2. Repulsive Fractional Order Potential Field, $1 \le n \le 5$.

maximum impact distances of r_{min} and r_{max} . This is beneficial in creating maximum force output, as if it is a physical object, and different radii of distance representing perhaps the size of the object at the minimum distance. Also, allowing for the lack of potential field impact beyond a certain maximum distance. One problem with the equation is when n = 2, as the equation becomes a singularity. For that reason, the normalized version of $V_2(r)$ is used in its place. The normalized field in that case is

$$U_{drep(n=2)} = \frac{\ln(r/r_{max})}{\ln(r_{min}/r_{max})}$$

4. Simulation framework

This paper has already determined that groups of individuals with various disabilities have different interactions with each other and the overall crowd. These differences can include many different social behaviors, physical attributes, and knowledge-base. Social behaviors of individuals can be explained in terms of physical attributes, knowledge-base, and aspects of psychology [26,27]. Physical attributes are the space an agent takes up, the speed they are traveling, and other physical interactions with pedestrians and environment. In the case of our analysis, individual overtakes may vary due to the size of the agent being overtaken. Since there may be less room to pass a wheelchair in a congested environment over an individual using a cane. Knowledge-base has to do with an individual's experience with the environment, movement in a crowd, etc. In the case of overtaking a type of disability, this may represent the overtaking individuals experience in overtaking an individual, or if they feel it alright to do in one case versus another. Similarly, the final aspect psychology, involves concepts of an individual's purpose, decision-making, or other behavior attitude interactions with others. While the reasoning to why pedestrian overtakes vary between disability groups, modeling these changes would allow for capturing those events where the lack of a faster moving individual overtaking a slower individual, causes an impact in the overall flow of the crowd.

4.1. PEDSIM simulation

We reviewed many different libraries and programs to find a simulation framework for modeling Social Force. The goal was to find a system that can simulate Social Force, the environment of our conducted experiment, the number of participants used, and the general running times we performed. The library PEDSIM, by Christian Gloor provided for the best framework for testing pedestrian models [28]. The PEDSIM library allows for the simulation of a large number of agents, far over that of the experiment, in real-time conditions with very little lag. The library comes with a basic graphical user interface, which we tailored to our needs. The circuit was coded to the same dimensions as the pedestrian experiment. To facilitate a heterogeneous crowd mixture, an additional agent type was added that can be changed in velocity and would include the FOPF hybrid model with the ability to change the parameters of the model. Fig. 3 illustrates how the PEDSIM framework has been incorporated into the graphical user interface.

With the hybrid FOPF and Social Force Model in place we can adjust the parameters to get desired interaction changes. The order n can be increased to change the number of pedestrians that can overtake an individual. The minimum size r_{min}



Fig. 3. PEDSIM library by Christian Gloor.

can also be changed to meet the spacing difference that may be present between the different disability groups. Taking the negative gradient of the force F_{d_i} represents the interaction between a pedestrian *j* and agent *i* with a disability as

$$F_{d_{ij}} = -\frac{r_{ij}^{\rightarrow} (n-2) \left\| r_{ij}^{\rightarrow} \right\|^{n-3}}{r_{ij}^{\rightarrow} (r_{max}^{n-2} - r_{min}^{n-2})}$$

In the simulation, only the disabled type will have this interaction added to its total forces since it represents the forces imparted on all other individuals within the crowd. Combining this new additional fractional order potential force with the Social Force Model, the new hybrid model for a disabled agent is

$$F = A_{Fi}F_i + B_{Fij}F_{ij} + C_{FiW}F_{iW} + D_{dij}F_{dij}.$$

In this equation, *A_{Fi}*, *B_{Fij}*, *C_{FiW}*, and *D_{dij}* represent gains that can be used to further adjust the overall strength of each force within the combined forces of agent *i*.

This new model now allows for the same standard interactions and properties found in Social Force, but with the additional ability to vary the shape of an additional potential field to modify the way the crowd interacts with the agent. An initial discussion of this model can be found in Stuart et al. [29].

4.2. Preliminary results

To explore the initial abilities of our hybrid model the Social Force Model portion was kept to its simplest form. In this form, only the desired velocity differences were used to describe varying differences between pedestrians or individuals with disabilities. The physical space differences between individuals with and without disabilities were not described in the Social Force Model portion. Then for the FOPF portion of the model, only the parameters of r_{min} , r_{max} , n were varied, in part to vary the effective space differences between individuals with and without disabilities. Simulations were run on the PEDSIM library with one agent containing the disability model, and another agent, released a few seconds later, traveling in the same



Agent overtaking, FOPF with varying n

Fig. 4. Diverse overtaking behavior when varying order n, one to five.

direction. To create similar velocity conditions between an individual with disability vs. one without, the velocity of the disabled individual was set to one m/s while the overtaking individual was set at two m/s. In this scenario, the faster agent should overtake the slower agent, unless prevented by the FOPF potential field. As stated previously, the initial models for both Social Force and FOPF do not include any limiting factors such as direction of travel and angle of vision. The first result, shown in Fig. 4, demonstrates a disabled agent traveling on the dotted line and an overtaking individual without disability on the dashed line. The minimum and maximum field distance were set to one and ten respectively. The order *n* is varied from one to five, with several results included. A time snapshot of position is shown at every three seconds through a 21 s period. With a relatively small *n*, a passing individual has no problem overtaking. With increase of *n* the overtaking distance goes out to two feet and then to three feet in distance within the circuit. At an order n = 4 the force is so great as to keep the faster agent from almost overtaking completely. If the corridor was any smaller, the agent would be forced to follow. The minimum and maximum ranges of the FOPF field influence can be changed too, although caution should be taken as this will also change the influence of the value *n*.

Next an input file was setup for the simulation to match the circumstances found similar to the crowd experiments. A model of the circuit used in our experiments is placed in the simulation and pedestrians were injected every five seconds into the circuit to represent the ramp up of density as done within our crowd experiment. For the simulations, the composition is composed of the average found in our experiments which is 71 individuals without disabilities and nine individuals with disabilities. The agents without disabilities can take in variables of desired velocity and entrance time. The agents with disability use the hybrid model addition and can take in variables of desired velocity, entrance time, r_{min} , r_{max} , and n. For the purposes of all simulations within this simulation only n was focused on and r_{min} was left at one meter for individuals with and without disabilities and r_{max} set to ten meters. Fig. 5, shows the simulation with at low traffic density and the agents spread throughout the circuit. Fig. 6 the simulation at medium traffic levels and Fig. 7 at heavy traffic densities.

5. Standard model results

Since the hybrid model only used a simple form of the Social Force Model, the Social Force Model portion was used first to compare the results. A simplified standard model of the Social Force Model is used because it offers the worst case model results with respect to overtaking. In a very simplified model, faster pedestrians will move past slower pedestrians unless physically blocked from doing so only. The very simple Social Force Model includes no perception of overtaking that would limit faster pedestrians overtaking slower ones. The individuals with disabilities were set from the hybrid model to the standard model that only relies on desired velocity. The results of these simulations will be unrealistic from the point of crowd overall interaction, but the focus of this research is only in demonstrating the ability of the overtake portion of this



Fig. 5. PEDSIM hybrid model low traffic flow.

Standard Social Force Model simulation overtake results.			
Visually impaired	99.02 overtakes/ped		
Individual using non-motorized ambulatory device	160.2 overtakes/ped		
Individual using motorized wheelchair	144.24 overtakes/ped		
Cane or stamina impaired	226.5 overtakes/ped		

model. Note that the original version of Social Force Model involve some flaws. It has been shown that numerical values of certain parameters in the model may produce un-realistic results when the model is applied to an isolated pedestrian or a small number of pedestrians [30,31]. The flaws lead to unrealistic behavior in the form of backwards moving pedestrians (i.e. negative speeds for empirical desired walking speeds), and overlapping of pedestrians. However, such was not observed during this study likely due to the large number of participants involved.

Desired velocity, Table 2, were used as a baseline to compare overtake results to. Within our large-scale crowd experiment, individuals with disabilities were composed of varying groups for each session run. For the simulations, the composition in the crowd were kept only to one group of disability at a time. This further removes the results from reality, but serves as a baseline for our overall analysis of the model. Processing all five groups for overall overtakes, it is shown in Table 4 that the amount of overtaking is far higher than what is experimentally found. This is no surprise since the simulated faster agents are just passing the slower agents as they move around the circuit.

6. Hybrid model results

For the rest of the simulations, the hybrid model is used for all nine individuals with disabilities groups that are simulated within the crowd and used within our previous crowd experiment. The field range values of the FOPF remain fixed and *n* is varied each iteration and then processed to determine what the overall overtake value outcome is for the circuit. This process is done for each of the five disability groups over a range of *n* until the overtake outcome matches or gets close to the value



Fig. 6. PEDSIM hybrid model medium traffic flow.

determined from the experiment. The results found for the vision impaired simulation where the model is set to 1.19 m/s and the individuals without disability are set to 1.3 m/s is found in Fig. 8. The result for individuals with motorized wheelchairs at 1.11 m/s is found in Fig. 9. The results for individuals with canes or stamina impairments at using roller walkers at 0.9 m/s is found in Fig. 10. As both individuals using non-motorized wheelchairs and individuals using roller walkers have the same mean desired velocity of 1.14 m/s, the initial results can be found together on Fig. 11. The first attempt to find an order value that would match experimentation lead to difficulty as found in Fig. 11. It was discovered at this point that the specific time of when each agent with disability was injected into the circuit can matter in the case of simulation. At 1.14 m/s of movement, the agents would end up back at the injection point around the time another agent with disability is to be injected. This meant that the agents with disability are grouped closely together. The large shape created by the hybrid addition of the FOPF combined with the small required number of overtakes to match meant that the agents would occasionally reach jam conditions and get stuck in the circuit. These jam conditions are unrealistic, but a phenomenon of the current design of the simulation. To fix this, the injection times of the simulated individuals with disabilities were changed slightly to allow for a more even spread of agents throughout the circuit. This resulted in the ability to achieve overtake numbers similar to the experiment for both groups with disabilities. The results of this modified simulation can be found in Fig. 12. The jam issue is discussed in the next section. The hybrid model has the ability to match similar outcomes to those found within our empirical experiments. A summary of the results can be found in Table 5. To see where the hybrid model and simulation meet our actual crowd experiment, and were they currently do not, we chose a uni-directional session to simulate. This session, named 1241 (for the time it occurred), is composed of 79 individuals without disabilities and seven individuals with disabilities. The group with disabilities is composed of five members with a visual impairment, one member with a roller walker, and one member with a motorized wheelchair. The results of the simulation were compared with the results of the experimental session for velocity over level of service ranges. Fig. 13 shows those results also in comparison with a simulation of all vision impaired and one of all stamina impaired. Although all three simulations reduce in velocity with increase in density, the results are still not favorable to matching all aspects of a crowd with individuals with disabilities. However, the initial motivation is to present the hybrid model and do some initial matching to the overall overtake outcomes in separated form. Our hybrid model shows promise in being able to capture one heterogeneous characteristic not previously analyzed.



Fig. 7. PEDSIM hybrid model heavy traffic flow.



Fig. 8. Results of hybrid model of vision impaired while varying order.

7. Problems encountered

The addition of a FOPF to the Social Force Model only represents the beginnings of modeling the complex behavior of individuals with disabilities. Subsequently, more analysis of the crowd experiments will be required to continue. As previously stated, more recent renditions of the Social Force Model do not contain any aspects of the example of the



Fig. 9. Results of hybrid model of motorized wheelchair while varying order.



Hybrid FOPF Social Force Analysis with Cane/Stanima at 0.9 m/s

Fig. 10. Results of hybrid model of cane/stamina while varying order.



Hybrid FOPF Social Force Analysis with Roller Walker and Non-motorized Wheelchair at 1.14 m/s

Fig. 11. Results of hybrid model non-motorized wheelchair and roller walker, first results while varying order.

Model included in this study. Newer versions of the Model show that the forces within an individual's range of vision have more influence to that individual's movement than the impact of pedestrians following from behind. Consequently,



Fig. 12. Results of hybrid model non-motorized wheelchair and roller walker, adjusted simulation results while varying order.

Table 5	
Hybrid Social Force Model simulation overtake results	5.

Туре	Hybrid results	Experiment results
Visually impaired	33.68 overtakes/ped	33.45 overtakes/ped
Individual using non-motorized wheelchair	17.75	19.0
Individual using motorized wheelchair	20.84	23.54
Roller walkers	13.6	14.75
Cane or stamina impaired	45.14	42.5



Fig. 13. Session 1241 experimental results versus various simulation results.

the addition of angle of sight modifications to the impact of the FOPF as well as a more current Social Force Model will improve future iterations of this Model. This will hopefully create less of an impact for individuals with disabilities following other pedestrians despite the design of the environment. Future improvements to the Model would optimize the Model's interactions to what was discovered from experimental analysis due to the fact that order *n* can be any order or fractional order. The changeability of *n* allows for the calibration of models and further types of interaction as they were discovered and understood. As the initial simulations include a very simple Social Force Model, future research needs to occur to understand how the hybrid model works with a more complex form of the Social Force Model that can better capture impacts of velocity over density increase and the movement and flows through an environment. From Fig. 13, the velocity outcomes are still much higher than the experimentation results. This is most likely due to the simplification of the interaction behavior of the standard Social Force Model. This is due to the overtake orders being higher than necessary causing inaccuracies in how the simulation currently handles the crowd. The end result is pedestrians not allowing enough agents to pass. It may in fact be that although the overall overtake outcomes were matched, the actual overtake behavior is only partially matched. Further



Fig. 14. Hybrid model jam conditions for doorway and corners.

work to create a better way to study overtake should be explored. Examples found in other forms of transportation may serve well in creating a better definition. This is only the beginning of our exploration. There are many other issues we had with our current simulation framework that need improvement. Currently analysis of each simulation is not integrated into the simulation itself. As analysis is done separately it is more time consuming. Also the current path goals for pedestrians are very simple and in the form of waypoints. More advanced building navigation, which already exists in the field, will improve our results using the hybrid model. Finally, specific jam conditions were observed in the simulated hybrid model. The agents get stuck in the simulation at times. A visual depiction of these conditions is found in Fig. 14. While a work around was determined, these conditions need to be resolved to better explore the hybrid model.

Further, it is somewhat problematic to simultaneously examine individuals with different types of disabilities. Although the intent of this study was to examine a heterogeneous pedestrian stream, as would be expect in real-world conditions such as the actual experiment, future study may wish to examine specific types of disabilities to better understand the behaviors by type of disability.

8. Conclusion

In this paper we presented a hybrid model based on the popular Social Force Model and Fractional Order Potential Fields. This model was introduced as a way to describe heterogeneous differences found in various groups of individuals with disabilities. Through our previously conducted experiments, we have determined preliminary differences between groups in the form of velocity and overtaking interactions within a crowd. Using a simulation platform, we use our hybrid model to explore how FOPFs can be used to try to capture similar outcome results to those found in our large-scale crowd experiment. Those initial results were described and we discussed problems that need to be resolved or explored in future development of our model.

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