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## **DISTURBANCE REJECTION FOPID CONTROL OF ROTOR BY MULTI-OBJECTIVE BB-BC OPTIMIZATION ALGORITHM**

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### **ABSTRACT**

This paper presents a FOPID tuning method for disturbance reject control by using multi-objective BB-BC optimization algorithm. Proposed method allows multi-objective optimization of set-point performance and disturbance rejection performances of FOPID control system. The objective function to be minimized is composed of the weighted sum of MSE for set-point performance and RDR for disturbance rejection improvement. The proposed optimization performs maximization of RDR and minimization of MSE and it can deal with the tradeoff between RDR performance and step-point performance. Application of the method is shown for auto-tuning of FOPID controller that is employed for control of TRMS model. We observed that low-frequency RDR indices can be used to improve disturbance rejection performance in multi-objective controller tuning problems. Particularly, for flight control application, disturbance reject control is very substantial to robust performance of propulsion systems.

### **INTRODUCTION**

FOPID controllers have been suggested by Podlubny in 1999 [1] by substituting integer order derivative and integrator of classical PID with fractional one. These modification lead to two additional order parameters  $\lambda$  and  $\mu$  and these modification can improve frequency response of classical PID controller, which allows to obtain better control performance and better stability compared to PID controllers [2].

Nowadays, FOPID controller begins to find application in practice applications due to its advantages to

classical PID controller, which has turned into a standard for industrial controller class [3]. Many study reported that FOPID controller provides better control performance and stability compared to PID controllers [4]. Due to increasing practical utilization of FOPID controllers, tuning of FOPID control system to obtain a desired control response for specific control mission is becoming more important and many methods have been developed to address the FOPID controller tuning problem [5]. These methods can be mainly classified in two groups, which are methods based on analytical optimization methods [6] and methods based on heuristic methods [7]. Due to high computational complexity of fractional order control system, heuristic optimization methods presents advantages of set and trail search methodology. However, efforts to improve performance of heuristic optimization methods, particularly for controller tuning problems, are needed and increasing. Many methods was suggested or modified to obtain desired control performance such as; SMDO method [8], Tabu search based optimization algorithm [9], Fruit Fly Optimization algorithm [10], Cuckoo search algorithm [11], Adaptive Particle Swarm Optimization algorithm [12].

Another advantage of employment for heuristic optimization methods in control application is that they allow multi-objective optimization. Control tuning problem is indeed resolving tradeoff between many objectives such as low overshoots, fast settling, disturbance rejection, robust stability etc. Considering these objectives in controller tuning allows to obtain a good controller responding to many application constraints. Performance of the optimization process has been increased by combining many objective functions based on average errors (MSE, ITAE, ISE, IAE), rising time, settling time, steady state

error, gain margin, phase margin etc. In the literature, many studies employing multi-objective optimization of these constraints can be found. For instance, FOPID controller design method based on minimization of IAE, absolute steady state error and settling time external optimization algorithm for an automatic voltage regulator system are proposed in [13]. Real coded population based method is proposed for tuning PI and PID controller parameter according to overshoot, steady state error, rise time and settling time multi objective functions [14]. Overshoot, rising time, settling time, steady state error, IAE, integral of the square input, gain margin and phase margin are used as a multi objective function for tuning FOPID controller parameters with particle swarm optimization algorithm for automatic voltage regulator in [15]. Investigates the multi-objective optimal design of distributed order fractional damper with new hybrid method is presented in [16].

In this paper, we present a method for disturbance rejection FOPID design based on multi objective application BB-BC algorithm. The objective function to be minimized is composed of the weighted sum of MSE, which is for improvement of set-point performance, and RDR, which is for improvement of disturbance rejection improvement. Previously, Alagoz et al. defined RDR indices for closed loop PID and FOPID controllers and investigated the bounds of RDR performance of closed loop control system [17, 18]. In current study, we present a scheme to apply RDR index in controller tuning based on multi-objective heuristic optimization approach. The proposed scheme performs maximization of RDR and minimization of MSE and it can deal with the tradeoff between RDR performance and step point performance. Disturbance reject control is particularly very substantial to robust performance of flight control applications. Application of the method is shown for auto-tuning of FOPID controller for the control of TRMS model. Results showed that low-frequency RDR indices can be effective for improvement of disturbance rejection in multi-objective heuristic tuning methods.

## NOMENCLATURE

FOPID	Fractional Order Proportional Integral Derivative.
BB-BC	Big Bang Big Crunch
SMDO	Stochastic Multi parameter Divergence Optimization
MSE	Mean Square Error.
RDR	Reference to Disturbance Ratio
TRMS	Twin Rotor Multi Input Multi Output System
PID	Proportional Integral and Derivative
ITAE	Integral Time Absolute Error
ISE	Integral Square Error
IAE	Integral Absolute Error

## PRELIMINARIES

### Fundamental of Fractional Order System

The Caputo definition of fractional order differentiation was given based on  $\Gamma(\cdot)$ , namely Euler's gamma function, as follows [19],

$${}_a D_t^\alpha = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^n(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad n-1 \leq \alpha < n \quad (1)$$

The equation (2) leads to very useful property on Laplace transform of fractional order derivative which is given as  $L(D^\alpha f(t)) = s^\alpha F(s)$  for zero initial conditions. Fractional order transfer functions are expressed in a general form,

$$T(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{i=0}^m b_i s^{\varphi_i}}{\sum_{i=0}^n a_i s^{\alpha_i}}, \quad (2)$$

where denominator polynomial coefficients,  $a_i$ , and numerator polynomial coefficients,  $b_i$ , are positive real numbers. The fractional orders are  $\alpha_i \in R$ , and  $\varphi_i \in R$ , [19].

Implementation of theoretical fractional order derivative is not practical due to requiring high computational power. In this reason, integer order approximation of fractional order models are used in the practical implementation of fractional order system models. The continued fraction expansion (CFE) method provides satisfactory accuracy for practical application [20, 21]. In this study, we used Valerio's toolbox for practical implementation of fractional order PID controller (FOPID) by CFE approximation method. Transfer function of FOPID controller is written in general form as,

$$C(s) = k_p + \frac{k_i}{s^\lambda} + k_d s^\mu \quad (3)$$

where, parameters  $k_p, k_d$  and  $k_i$  are controller coefficients, and  $\lambda$  and  $\mu$  are fractional order of controller function. It has been reported that additional two parameters  $\lambda$  and  $\mu$  can improve control performance of classical PID controller [22].

### Multi-Objective Controller Tuning For Improved Disturbance Rejection

Two important performance criterions for time response of control system are set-point performance and disturbance rejection performance in practical control system implementations. Set-point performance can be evaluated by mean squared error performance that is expressed as,

$$E_{MSE} = \frac{1}{n} \sum_{i=1}^n (e(t))^2 \quad (4)$$

For disturbance rejection performance, reference to disturbance ratio (RDR) has been proposed to quantitative assessment of

disturbance rejection capacity of closed loop control systems [17, 18]. In this approach, a closed loop linear control system is assumed to be combination of two communication channel models and RDR indicator measures what extent the reference signal transmitted to the system output relative to the disturbance signal transmission [18]. For improved disturbance rejection performance, RDR index should be maximized for control system. RDR spectrum was expressed for closed loop PID and FOPID systems [17] depending on controller parameters.

$$RDR(\omega) = \left( (k_p + k_i \cos(\frac{\pi}{2}\lambda)\omega^{-\lambda} + k_d \cos(\frac{\pi}{2}\mu)\omega^\mu)^2 + \left( (k_d \sin(\frac{\pi}{2}\mu)\omega^\mu - k_i \sin(\frac{\pi}{2}\lambda)\omega^{-\lambda})^2 \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \quad (5)$$

[18]. Since control system mainly works in low frequency ranges, RDR performance for low frequency region can be expressed as,

$$RDR_L = \sum_{\omega_i \in (0, \omega_{max})} RDR(\omega_i) \quad (6)$$

Therefore, we define objective function as weighted sum of squared error performance and RDR performance as,

$$J = \min \{ c_1 E_{MSE} + c_2 RDR_L^{-1} \} \quad (7)$$

where  $c_1$  and  $c_2$  are weight coefficients that give direction to optimization deal with the tradeoff between RDR performance and step-point performance of control systems. Since RDR need maximization to improve disturbance rejection,  $RDR_L^{-1}$  is used to turn it to a minimization problem. Thus, maximization of RDR performance and minimization of set-point performance can be possible by minimizing weighted multi-objective  $J$  function. In the following section, BB-BC optimization algorithm is used to minimize this objective function.

### Big Bang Big Crunch Optimization Algorithm

BB-BC optimization algorithm is proposed by Erol and Eksin [23] as heuristic optimization methods. This algorithm has two phases, which are big bang and big crunch. Algorithm generates random points like swarm or a candidate solution set in big bang phase. But, in big crunch phase, algorithm calculates centre of mass or minimum cost functions values [24]. Optimization steps of the BB-BC optimization algorithm can be summarized as follow [23]:

**Steps of the BB-BC optimization algorithm:**

**Step 1:** Generation of candidate solutions with a random numbers between limits of the search space.

**Step 2:** Fitness function values of each member of candidate solution set are calculated.

**Step 3:** Center of the mass value is generated according to following equation. Best fit individual can be chosen as the center of mass, which is written by Eq. (8)

$$x_c = \frac{\sum_{i=1}^N \frac{1}{f_i} x_i}{\sum_{i=1}^N \frac{1}{f_i}} \quad (8)$$

where  $x_i$  is a point within n dimensional search space generated,  $f_i$  is a fitness function value of this point,  $N$  is the population size of the optimization algorithm

**Step 4:** Calculate new candidates around the center of mass by adding or subtracting random numbers whose value decreases as the iteration steps elapse. This can be formalized as equation  $x_{new} = x_c + lr/k$  where  $x_c$  stands for center of mass that is calculated with (8),  $l$  is the upper limit of the parameter,  $r$  is a normal random number and  $k$  is the iteration step.

**Step 5:** Return to Step 2 until stopping criteria has been satisfied.

### FOPID CONTROLLER DESIGN WITH MULTI OBJECTIVE BB-BC OPTIMIZATION ALGORITHM

In this section, BB-BC optimization algorithm is employed for tuning FOPID controller parameters according to multi objective functions (MO) defined by equation (7), which performs the weighted sum of MSE and RDR performances. The RDR calculates the ratio of reference signal energy to disturbance signal energy at the system output and it is very useful for quantitative evaluation of disturbance rejection capacity of closed loop control systems [18]. MSE is used to evaluate set-point performance for step reference input. Here, to deal with the tradeoff between disturbance rejection and step-point performance, minimization of MSE and maximization of RDR are carried out simultaneously.

Flight control system encounters disturbances during to flight. For this reasons, improvement of RDR performance can improve robust performance of flight control systems. In this study, we tested the proposed disturbance rejection FOPID tuning method in the control of TRMS model. We applied to step input disturbance in this simulation model and observed disturbance rejection performance of methods and controllers. Block diagram of proposed method is shown in Figure 1. According to Figure 1, BB-BC optimization algorithm adjusts FOPID controller parameters according to multi objective function given by equation (7). MSE error is calculated with step response of the main rotor of the TRMS RDR<sub>L</sub> is the calculated according to equation (6) for low frequency ( $\omega$ ) range between [0-10].

In order to extent search range of BB-BC algorithm, the initial values of  $k_p, k_d, k_i, \lambda$  and  $\mu$  is added (+), subtracted(-), divided (/) and multiplied (\*) by a random number in the range of [0,1]. For instance, for  $k_p$ , four new candidate point can be

generated as  $k_p + rand$ ,  $k_p - rand$ ,  $k_p / rand$  and  $k_p * rand$ . This modification of BB-BC algorithm is inspired from Base Optimization method [25], which can increase search range randomly.

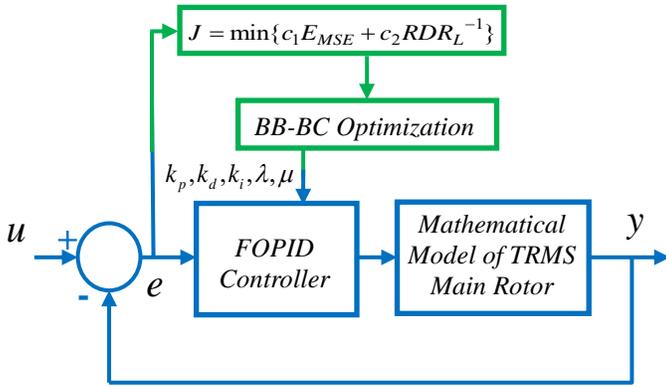


Figure 1: FOPID controller optimization process

**Multi objective BB-BC optimization algorithm steps:**

**Step 1:** Initialize  $k_p, k_d, k_i, \lambda$  and  $\mu$  parameters and expand this set of tem by adding (+), subtracting(-), dividing (/) and multiplying (\*) by a random number.

**Step 2:** Calculate cost values of expanded parameter set by using equation (7).

**Step 3:** Find the center of mass according to Eq. 9 for expanded parameter set. For example, following equation calculates mass center for  $k_p$ .

$$x^{k_p}_c = \frac{\sum_{i=1}^N \frac{1}{f_i} k_{p_i}}{\sum_{i=1}^N \frac{1}{f_i}} \quad (9)$$

**Step 4:** Calculate new candidates around the center of mass by adding or subtracting a random number. New candidates of FOPID controller parameters can be generated with  $x_{new} = x_c \pm lr/k$ , where  $l$  is the upper limit of the parameter,  $r$  is a normal random number and  $k$  is the iteration step. For example, new value of  $k_p$  is calculated with  $k_{p_{new}} = k_{p_c} + lr/k$ . Other parameters new values can be calculated with these corresponding equations.

**Step 5:** Return to Step 2 until stopping criteria has been satisfied.

**SIMULATION RESULTS**

In this section, simulation results are presented and performances of FOPID and PID controller are compared. Mathematical model of TRMS main rotor, which is used in simulation as plant, was obtained in [8]. Simulation model of closed loop model FOPID and PID control system with TRMS

main rotor model is shown in Fig. 2. In this test, we applied step reference at the beginning of simulation and after mid of simulation, we applied a step disturbance to system. Thus, one can see the both step and disturbance responses on the same simulation. As a simulation model of FOPID controller, we used a non-integer toolbox by developed by Valerio for approximate modeling of FOPID controller [26]. In this study, CFE approximation of FOPID controller is configured in Valerio’s toolbox.

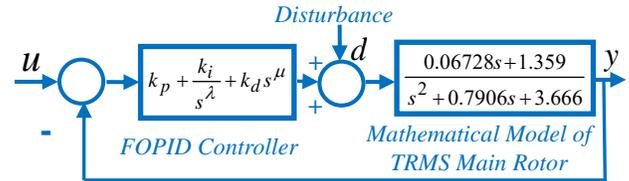


Figure 2: Simulation model of FOPID controller system of TRMS main rotors with input disturbance model.

Figure 3 shows evolution of cost function during BB-BC optimization of FOPID controller. Decrease of cost values indicates improvement of control system response according to multi objective function (equation 7). Figure 4 compares the performance of this multi objective function for two different controllers, as classical PID and FOPID. Both FOPID and PID controllers are tuned with proposed BB-BC optimization algorithm. Values of FOPID and PID controller parameters obtained can be found in Table 1. As seen in the figure, FOPID controller gives better set-point and disturbance rejection performance compared to classical PID controller. FOPID controller can significantly reduce negative effects of disturbance at the system output. Consequently, FOPID can present rather robust performance than PID controllers according to results of our simulations.

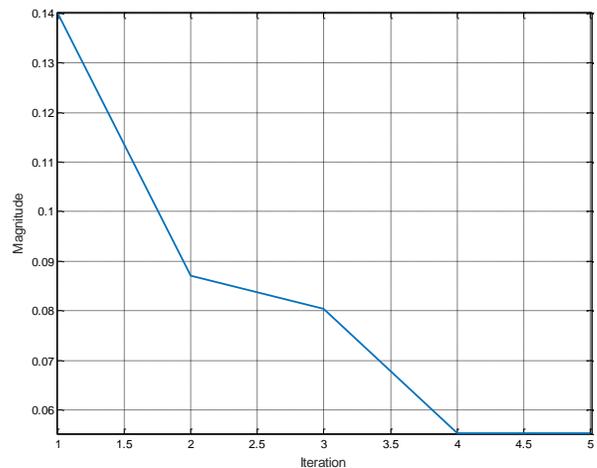
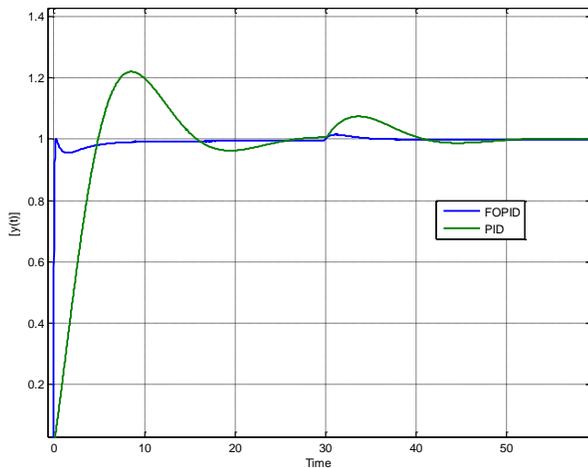
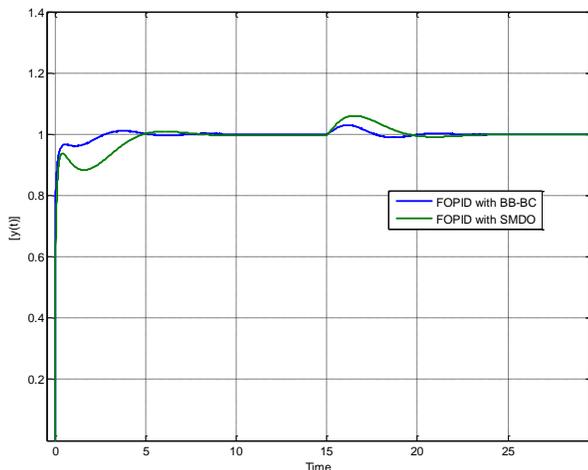


Figure 3: Evolution of cost function during BB-BC optimization of FOPID controller



**Figure 4:** Comparisons of step reference and step disturbance performances for FOPID and classical PID controllers

Figure 5 compares results that are obtained the proposed multi objective function (equation (7)) by two different optimization method, which are SMDO [8] and BB-BC optimization algorithm. Both optimizer yields disturbance reject FOPID controller designs, however results indicates that BB-BC optimization algorithm can yield better results than SMDO optimization in simulations. SMDO method performs search by less number of set and trials and small parameter steps in limited direction and it is effective for real-time fine-tuning of systems. In simulation, BB-BC uses a set of candidate solution spreading wider range of search space and therefore it can find better solutions. FOPID controller designs obtained by SMDO and proposed BB-BC optimization is listed in Table 1.



**Figure 5:** Comparisons of step reference and step disturbance performances for FOPID and classical PID controllers

**Table 1:** Controller parameters values that generated multi objective BB-BC and SMDO

	$k_p$	$k_i$	$k_d$	$\lambda$	$\mu$
<b>FOPID generated by Multi objective BB-BC</b>	27.46	18.04	26.82	0.79	0.89
<b>FOPID generated SMDO [8]</b>	9.90	7.78	11.89	1.0053	1.07
<b>PID generated by Multi objective BB-BC</b>	14.23	11.97	6.93	0	0

## CONCLUSION

The paper presented a disturbance rejection FOPID controller tuning method based on multi objective BB-BC optimization methods. Test results were shown for control problem of TRMS main rotor model. Our results validate that FOPID controller can yield superior disturbance rejection performance compared to classical PID when both controller are tuned by BB-BC optimization method under the same conditions. We also compared results of two different optimization methods, which are the proposed BB-BC optimization and SMDO. Both optimization methods yield disturbance reject FOPID designs, however results of BB-BC is better than SMDO method in simulations, mainly, because of large search ranges and larger set of candidate solutions.

This study also shows that RDR objective is very useful for disturbance reject closed loop control system design. RDR spectrum depends on controller parameter and angular frequency and allows to optimization for disturbance rejection at any frequency range. In this study, we performed optimization for low frequency region RDR performance, which is very substantial for control systems in practice. Proposed objective function improves RDR performance and step-point performance and it is useful to deals with the tradeoff between disturbance rejection performance and step-point performance of closed loop control systems. Future study can be planed to practical validation of proposed multi objective cost function for tuning real FOPID control systems.

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