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Fixed-wing MAV adaptive PD control based on a modified MIT rule with sliding-mode control

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Abstract— **This paper presents an adaptive PD control law by using a modified MIT rule. The adjustment mechanism of the MIT rule has been implemented with three types of sliding-mode control, i.e., classical sliding-mode control, second order sliding-mode (2-SM), and high order sliding-mode control (HOSM). The proposed controllers have been designed for the directional and lateral dynamics of a fixed-wing mini aerial autonomous vehicle (MAV). Several simulations have been carried out in order to analyze the modified MIT rule.**

I. INTRODUCTION

In control theory, there exist several dynamic systems possessing constant uncertain parameters or parameters varying slowly [1]. For example, when we develop an MAV (Mini Aerial Vehicle), we can add or remove sensors or batteries, then we modify the weight and consequently the inertia parameters. In the same way, when an MAV flies in bad weather, it is exposed to changes in the air density which are usually considered as a constant value. In order to solve the aforementioned problems, several control laws could be designed. One of such options is the use of an adaptive controller [2] allowing the MAV performs a stable flight under such conditions. The adaptive control has been applied in areas as the robot manipulators, airplanes, rockets, chemical process, electronic systems, ships, bioengineering, etc. [1]. We can find in the literature some works related to the MIT rule, as in [3], where it has been applied the MIT rule based in model reference adaptive control (MRAC) for the regulation of a second order system; the contribution of [3] is the modification of the MIT rule with the objective to obtain a major amplitude of reference desired, due to the MIT rule tends to be unstable with large reference values, and this is one of the inconvenient in work with the MIT rule and even more, is sensitive to big numerical changes in the adaptation gain, this is, even with decimal changes in this gain, the system tends to be unstable. In [3] only show an example of how to make one gain adaptable of the control law, and the other gain of the control law proposed is not adaptable.

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In [4], the MIT rule has been applied to a distillation process, considering a linearized model, in addition to applying an adaptive feedback control for two parameters of the controller, and by considering as reference a unitary step signal. In [5], they have presented a comparison of the MIT rule with the gradient method and by the Lyapunov method. In [5] has been designed a control law for tracking and regulation for an aspheric tank with selecting a small reference for the input, and they have not presented a modification to the MIT rule. The works mentioned at the top have shown results using the Matlab software (simulation results).

In this work, we have applied the MIT rule with the sliding modes theory, the objective of this union is to obtain an adaptive control with gain scheduling. Thus, the MIT rule with the sliding mode theory has been used as the adjustment mechanism for the gains of a PD control law, this adaptive controller is applied in order to lead the yaw and roll angles to a desired angular position. To the theory of the MIT rule we have added a sliding mode control, and due to that with this union presented the chattering effect, we have added a second order sliding mode in order to reduce this effect, and finally we have appreciated than the effect chattering is presented still with two sliding modes, so we have added a high order sliding mode in order to reduced or almost eliminate the effect chattering in the design of the adaptive control.

In addition to the changes made to the MIT rule, we have obtained a lower sensitivity in the adaptive gain, and it is possible to vary the reference required with bigger values, unlike the aforementioned works. Furthermore, we have proposed a different definition of the MIT rule with sign function and with sign − sign functions for the adaptive mechanism to that shown in [6], [1]. Our proposal is based on the theory of sliding-modes, that is, we have designed a sliding surface with the output of the plant which defines the aerodynamic (directional-lateral) of a fixed-wing aerial vehicle and with the output of the model-reference and considering this, we have designed the adaptive mechanism. We also added a disturbance which acts on the process input [6], with the objective of test the robustness of the adaptive controller in presence of not modeled disturbances in fixed-wing MAVs. Such disturbances are small, but unknown and random.

The paper is organized as follows: section II shows the equations that define the dynamical model of the MAV. The section III illustrates the general theory of the MIT rule; in section IV, it is presented the control law design for the

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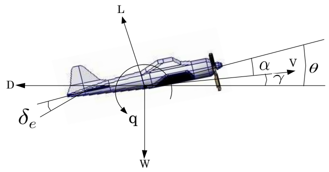

Fig. 1. Pure pitching motion

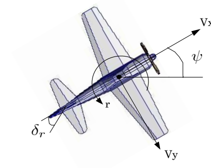

Fig. 2. Pure yawing motion

directional and lateral dynamics. In section V, it is presented the altitude control design. In section VI it is shown the simulation results and an analysis of the error signals and the efforts of the control inputs. Finally, Section VII presents the conclusions of this work.

II. AIRPLANE MODEL

In order to obtain the model equations, by omitting any flexible structure of the MAV, the fixed-wing MAV can then considered as a rigid body. Also, we do not consider the curvature of the earth, it is considered just as a plane because we assume that the fixed-wing MAV will only fly short distances. With the previous considerations, we obtain the model by applying the Newton's laws of motion.

A. Longitudinal dynamics

The used dynamical model to control the altitude of the MAV is given by [14]:

$$\dot{\theta} = q \tag{1}$$
$$\dot{q} = M_q q + M_{\delta_e}\delta_e \tag{2}$$
$$\dot{h} = V\sin(\theta) \tag{3}$$

where V is the magnitude of the airplane speed, θ denotes the pitch angle. q is the pitch angular rate with respect to the y-axis of the aircraft body, h defines the airplane altitude and δ_e represents the elevator deviation [14]. In aerodynamics, M_q and M_{δ_e} are linked with the stability derivatives which are implicit in the pitch motion. We can see these variables in the in Figure 1. The aerodynamic stability derivatives are defined by:

$$M_q = \frac{\rho S V \bar{c}^2}{4I_{yy}} C_{m_q}$$
$$M_{\delta_e} = \frac{\rho V^2 S\bar{c}}{2I_{yy}} C_{m_{\delta_e}}$$

where:

ρ: Air density (1.05 kg/m^3).
S: Wing area (0.09 m^2).
$\bar{c}$: Standard mean chord (0.14 m).
b: Wingspan, (0.914 m).
I_{yy}: Moment of inertia in pitch (0.17 $kg \cdot m^2$).
C_{m_q}: Dimensionless coefficient for longitudinal movement, obtained experimentally (-50).
$C_{m_{\delta_e}}$: Dimensionless coefficient for elevator movement, obtained experimentally (0.25).

B. Directional-lateral dynamic

The lateral dynamics generates the roll motion and, at the same time, induces a yaw motion (and vice versa), then a natural coupling exists between the rotations about the roll and yaw axes [10]. In our case, to solve it, we have considered a decoupling of the yaw and roll angles [2]. Thus, each angle can be controlled independently. Generally, the effects of the engine thrust are also ignored [10]. In the Figure 2, the yaw angle (directional dynamics) is represented, which can be described by the following equations:

$$\dot{\psi} = r \tag{4}$$
$$\dot{r} = N_r r + N_{\delta_r}\delta_r \tag{5}$$

where ψ represents the yaw angle and r denotes the yaw rate with respect to the center of gravity of the MAV. δ_r is the rudder deflection. N_r and N_{δ_r} are the stability derivatives for yaw motion. The aerodynamic stability derivatives for the yaw angle are defined as [10]:

$$N_r = \frac{\rho V S b^2}{4I_{zz}} C_{n_r}$$
$$N_{\delta_r} = \frac{\rho V^2 S b}{2I_{zz}} C_{n_{\delta_r}}$$

where:

I_{zz}: Moment of inertia in yaw (0.02 $kg \cdot m^2$).
C_{n_r}: Dimensionless coefficient for the yaw angle, obtained experimentally (-0.01).
$C_{n_{\delta r}}$: Dimensionless coefficient for the rudder movement, obtained experimentally (0.0005).

The following equations describe the dynamics for the roll angle (lateral dynamics):

$$\dot{\phi} = p \tag{6}$$
$$\dot{p} = L_p p + L_{\delta_a}\delta_a \tag{7}$$

where p denotes the roll rate, ϕ describes the roll angle, and δ_a represents the deviation of the ailerons. L_p and $L_{\delta a}$ represent the stability derivatives of the roll motion [14]. In the Figure 3, they are shown the variables of the roll motion. The aerodynamic stability derivatives for roll angle are defined as [10]:

$$L_p = \frac{\rho V S b^2}{4I_{xx}} C_{l_p}$$
$$L_{\delta_a} = \frac{\rho V^2 S b}{2I_{xx}} C_{l_{\delta_a}}$$

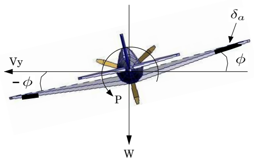

Fig. 3. Pure rolling motion

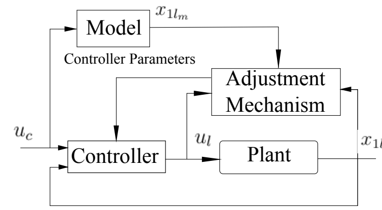

Fig. 4. Block diagram of the MRAS

where:

I_{xx}: Moment of inertia in roll (0.16 $kg \cdot m^2$).
C_{lp}: Dimensionless coefficient for roll angle, obtained experimentally (-0.15).
$C_{l_{\delta_a}}$: Dimensionless coefficient for ailerons movement, obtained experimentally (0.005).

C. Change of variables of the directional-lateral aerodynamic model

In order to design the adaptive control law, we have conducted a change in the variables notation for the directional and lateral dynamics; this is due to the fact that the dynamics are similar. Then, the directional dynamics is represented in the new variables by:

$$\dot{x}_{1\psi} = x_{2\psi} \quad (8)$$
$$\dot{x}_{2\psi} = C_{1\psi} x_{2\psi} + C_{2\psi} u_{\psi} \quad (9)$$

The change of variables for the lateral dynamics is also defined by:

$$\dot{x}_{1\phi} = x_{2\phi} \quad (10)$$
$$\dot{x}_{2\phi} = C_{1\phi} x_{2\phi} + C_{2\phi} u_{\phi} \quad (11)$$

III. MIT RULE

The MIT rule has been the first proposed approach to design adaptive control by reference model. The name is derived from the fact that it was developed at the MIT Laboratory Instrumentation. In order to describe the MIT rule, let us consider a closed-loop system in which the controller has one adjustable parameter θ. The desired closed-loop response is obtained from model-reference whose output is given by y_m. The error e is defined between the output y of the closed-loop system and the output y_m, from the model-reference. One way to solve it is to adjust the parameter in such a way that a loss function (12) is minimized; the loss function is given by:

$$J(\theta) = \frac{1}{2} e^2 \quad (12)$$

To minimize J, we must change the parameters in the direction of the negative gradient of J, thus,

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma e \frac{\partial e}{\partial \theta} \quad (13)$$

The equation (13) represents the called MIT rule. The partial derivative $\partial e/\partial \theta$ is the derivative of the sensitivity of the system and defines how the error is influenced by the adjustable parameter. If it is assumed that the changes of the parameters are slower than the other variables in the system, then the derivative $\partial e/\partial \theta$ can be evaluated under the assumption that θ is constant. There are many alternatives for the loss function (12). For example, if it is chosen:

$$J(\theta) = |e| \quad (14)$$

thus, the gradient method would be given by:

$$\frac{d\theta}{dt} = -\gamma \frac{\partial e}{\partial \theta} \operatorname{sign}(e) \quad (15)$$

The first implemented model reference adaptive system (MRAS) was based on (15). However, there are many other formulations in the literature, for example, in [6] it is presented:

$$\frac{d\theta}{dt} = -\gamma \operatorname{sign}\left(\frac{\partial e}{\partial \theta}\right) \operatorname{sign}(e) \quad (16)$$

which is known as the algorithm $\operatorname{sign} - \operatorname{sign}$.

IV. DESIGN OF THE ADAPTIVE CONTROL

We have designed a PD control law with adaptive gains, thus, the adaptive part of the controller is given by the proportional and the derivative gains. These gains are defined as k_p and k_v respectively. The methodology to design the adaptive control is based on the MRAS, in order to design the adjustment mechanism by the MIT rule. We have modified the MIT rule by inserting the theory of first order sliding-mode, second order (2-SM) and high order sliding-mode (HOSM), with the purpose of obtaining a robust control law that stabilizing the system, and always trying to remove the chattering effect. The block diagram representing the MRAS is shown in Figure 4, where the Plant represents the directional-lateral aerodynamic model of the fixed-wing MAV, and the block called as Model represents the model-reference.

Consider the equations (8)-(11), and let us use the subindex $l = \psi, \phi$ for the directional-lateral dynamics, respectively. Thus, u_l defines the control input. Thereby, The adaptive control is given by:

$$u_l = \hat{k}_{pla} e_l + \hat{k}_{vla} \dot{e}_l \quad (17)$$

where $\hat{k}_{pla}$ and $\hat{k}_{vla}$ are called as the position and velocity gains, respectively, thus, these are the adaptive gains. The error of the directional and lateral dynamics has been defined as $e_l = x_{1l} - x_{1l}^d$. The gains of the PD control have implicit a subscript to indicate the algorithm that has been applied as adjustment mechanism, $a = a_1, a_2, a_3, a_4$ where a_1 corresponds to the MIT rule, a_2 corresponds to the MIT rule with sliding-mode, a_3 uses the MIT rule with 2-sliding-mode, and finally a_4 represents the MIT rule with HOSM. Therefore, for the design of the MIT rule, it is introduced an error given by:

$$e_{l_m} = x_{1l_m} - x_{1l} \tag{18}$$

where x_{1l_m} is the output from the reference model. We have followed the methodology that has been presented in [6] for the MIT rule, taking this into account, the aerodynamic model has been transformed into the representation of a transference function in order to develop the derivatives of sensitivity; these have been obtained by computing partial derivatives with respect to the controller parameters $\hat{k}_{pla}$ and $\hat{k}_{vla}$. Thus, the closed-loop transfer function with the adaptive PD controller has been defined as:

$$x_{1l} = \frac{C_{2l}(\hat{k}_{pl} + \hat{k}_{vl}s)}{s^2 + (C_{1l} + C_{2l}\hat{k}_{vl})s + C_{2l}\hat{k}_{pl}} x_{1l}^d \tag{19}$$

and the model of reference for the directional-lateral dynamics has been defined as:

$$x_{1lm} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} x_{1l}^d \tag{20}$$

where $\zeta = 3.17$ and $\omega = 3.16$. Consider (18)-(20) and calculate the partial derivatives with respect to $\hat{k}_{pla}$ and $\hat{k}_{vla}$, then it is obtained:

$$\frac{\partial e_{l_m}}{\partial \hat{k}_{pl}} = \frac{C_{2l}}{s^2 + (C_{1l} + C_{2l}\hat{k}_{vl})s + C_{2l}\hat{k}_{pl}} (x_{1l} - x_{1l}^d) \tag{21}$$

$$\frac{\partial e_{l_m}}{\partial \hat{k}_{vl}} = \frac{C_{2l}s}{s^2 + (C_{1l} + C_{2l}\hat{k}_{vl})s + C_{2l}\hat{k}_{pl}} (x_{1l} - x_{1l}^d) \tag{22}$$

Generally, the expressions (21) and (22) cannot be used due to the unknown parameters $\hat{k}_{pla}$ and $\hat{k}_{vla}$. Thus, an optimum case has been assumed, it is defined as:

$$s^2 + (C_{1l} + C_{2l}\hat{k}_{vl})s + C_{2l}\hat{k}_{pl} = s^2 + 2\zeta\omega_n s + \omega_n^2 \tag{23}$$

thus, after these approximations, we have obtained the differential equations of the adaptive PD controller.

$$\dot{\hat{k}}_{pla_1} = -\gamma_{1l}\left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(x_{1l} - x_{1l}^d)\right) e_{l_m} \tag{24}$$

$$\dot{\hat{k}}_{vla_1} = -\gamma_{2l}\left(\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}(x_{1l} - x_{1l}^d)\right) e_{l_m} \tag{25}$$

Now, it is proposed an MIT rule with second order sliding-mode; this approach is different than the defined in (15), and then, it is defined a sliding-mode surface as $s_{1l} = \dot{x}_{1lm} - x_{2l} + k_{1l}e_{l_m}$ (we search to increase the stability to the adjustment mechanism), where k_{1l} is a positive gain. Then, the differential equations of the adaptive controller, with the methodology by sliding-mode, are given by:

$$\dot{\hat{k}}_{pla_2} = -\gamma_{1l}\left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(x_{1l} - x_{1l}^d)\right) (\beta_{p1l}\operatorname{sign}(s_{1l})) \tag{26}$$

$$\dot{\hat{k}}_{vla_2} = -\gamma_{2l}\left(\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}(x_{1l} - x_{1l}^d)\right) (\beta_{v1l}\operatorname{sign}(s_{1l})) \tag{27}$$

where β_{p1l} and β_{v1l} are positive values. Due to the chattering effect of the first order sliding-mode, let us design an adjustment mechanism with a second order sliding-mode. These second order sliding-mode includes a robust differentiator of first order [7]. This differentiator is defined by:

$$\begin{aligned} \dot{x}_0 = v_0 &= -\lambda_0|x_0 - s_{1l}|^{1/2}\operatorname{sign}(x_0 - s_{1l}) + x_1 \\ \dot{x}_1 &= -\lambda_1\operatorname{sign}(x_1 - v_0) \end{aligned}$$

where x_0 and x_1 are real-time estimations of s_{1l} and $\dot{s}_{1l}$, respectively. The values of λ_1 and λ_2 are constant positives. Thus, the differential equations of the adaptive PD controller with a second order sliding-mode are defined by:

$$\dot{\hat{k}}_{pla_3} = -\gamma_{1l}\left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(x_{1l} - x_{1l}^d)\right) (\beta_{p1l}\operatorname{sign}(s_{1l}) + \beta_{p2l}\operatorname{sign}(\dot{s}_{1l})) \tag{28}$$

$$\dot{\hat{k}}_{vla_3} = -\gamma_{2l}\left(\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}(x_{1l} - x_{1l}^d)\right) (\beta_{v1l}\operatorname{sign}(s_{1l}) + \beta_{v2l}\operatorname{sign}(\dot{s}_{1l})) \tag{29}$$

where β_{p1l}, β_{p2l}, β_{v1l} and β_{v2l} are positive gains.

In order to reduce or eliminated the chattering effect in the second order sliding-mode, we have designed an adjustment mechanism with HOSM. To design the adjustment mechanism, it is necessary a robust differentiator of second order [7], which is given by:

$$\begin{aligned} \dot{x}_0 = v_0 &= -\lambda_0|x_0 - s_{1l}|^{2/3}\operatorname{sign}(x_0 - s_{1l}) + x_1 \\ \dot{x}_1 = v_1 &= -\lambda_1|x_1 - v_0|^{1/2}\operatorname{sign}(x_1 - v_0) + x_2 \\ \dot{x}_2 &= -\lambda_2\operatorname{sign}|x_2 - v_1| \end{aligned}$$

where x_0, x_1 y x_2 are real-time estimations of s_{1l}, $\dot{s}_{1l}$ and $\ddot{s}_{1l}$. The values of λ_0, λ_1 and λ_2 are defined as positive constants. Finally, the differential equations of the adaptive PD controller with HOSM are defined by:

$$\dot{\hat{k}}_{pla_4} = -\gamma_{1l}\left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(x_{1l} - x_{1l}^d)\right) (\alpha_{pl}[\ddot{s}_{1l} + 2(|\dot{s}_{1l}|^3 + |s_{1l}|^2)^{1/6} \operatorname{sign}(\dot{s}_{1l} + |s_{1l}|^{2/3}\operatorname{sign}(s_{1l}))]) \tag{30}$$

$$\dot{\hat{k}}_{vla_4} = -\gamma_{2l}\left(\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}(x_{1l} - x_{1l}^d)\right) (\alpha_{vl}[\ddot{s}_{1l} + 2(|\dot{s}_{1l}|^3 + |s_{1l}|^2)^{1/6} \operatorname{sign}(\dot{s}_{1l} + |s_{1l}|^{2/3}\operatorname{sign}(s_{1l}))]) \tag{31}$$

where α_{pl} and α_{vl} are positive constant gains.

V. ALTITUDE CONTROL LAW

In order to keep a fixed altitude, we have designed a PD control law by considering the longitudinal dynamics equations (1)-(3). The altitude error has been defined as $\tilde{e}_h = h_d - h$, and it denotes the difference between the desired altitude h_d with respect to the current altitude h, where h is obtained by integrating (3). The desired altitude is achieved by controlling the pitch angle, thus it has been defined an error for this angle, given by $\tilde{e}_\theta = \theta_d - \theta$, where $\theta_d = \arctan(\tilde{e}_h/\varsigma)$; θ_d is the desired pitch angle, ς denotes

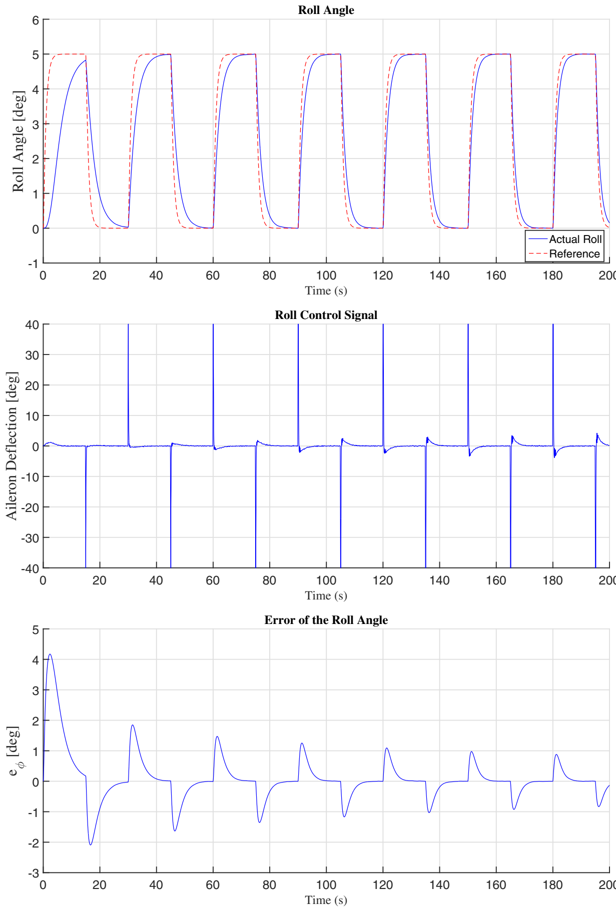

Fig. 5. Adaptive PD controller using the MIT rule in roll angle (with disturbances)

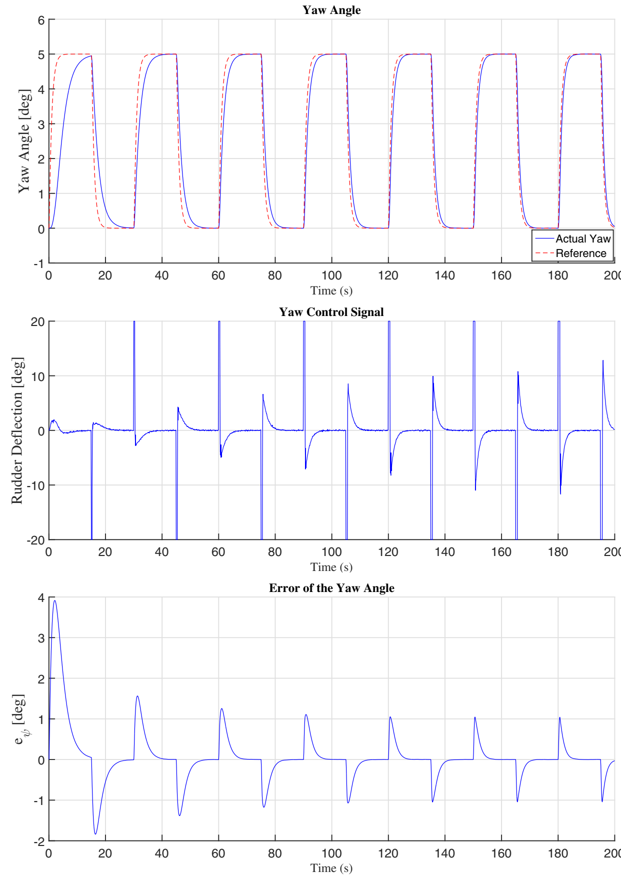

Fig. 6. Adaptive PD controller using the MIT rule in yaw angle (with disturbances)

the longitude from the center of mass of the fixed-wing MAV to the nose of the UAV.

The PD control law, for the altitude motion, is given by:

$$\delta_e = k_{ph}\tilde{e}_l + k_{vh}\dot{\tilde{e}}_l \tag{32}$$

where k_{ph} and k_{vh} are the positive constant gains of the PD controller

VI. Simulation results

A. MIT rule

The Figure 5 shows the obtained results when using the MIT rule in roll angle. The dashed line corresponds to the output signal of the model-reference, and the solid line represents the actual roll angle. In the same Figure, it is shown the control action which has been saturated in $\pm 40°$; the obtained value of the control law is too big for our fixed-wing MAV platform which will be considered for the experimental test in a future work, the value allowed is $\pm 20°$. We can also see, in this Figures, the related error signal between the model-reference and the actual roll angle. We have conducted several simulations in order to reduce the control law signal, but it was not possible to obtain a better response.

For the yaw angle case, the MIT rule has presented a good performance, see the Figure 6, that is, the signal value of the control law is inside of the allowed values for the rudder ($\pm 20°$). In this Figure, we can also observe the results for the yaw angle and its error.

B. MIT rule with sliding-mode

The Figure 7 shows the results of the MIT rule with a first order sliding-mode. It has been observed an improvement with respect to the results that have been obtained with the MIT rule; it can be observed than the actual roll angle achieves the model-reference signal in a lower time in comparison with the MIT rule. We also observe that the control action is saturated in $\pm 20°$ which is the allowed value for the ailerons of our fixed-wing MAV. However, we observe an inconvenient in this algorithm, that is, the presence of the chattering effect in the control signal.

In the Figure 8, it is presented the obtained results when applying the MIT rule with the first order sliding-mode. This adaptive control algorithm also presents a lower error and control effort in comparison with the MIT rule for the yaw angle. In addition, this algorithm reduces the noise that has presented with the MIT rule. Besides, it also generates a low chattering effect at the control signal.

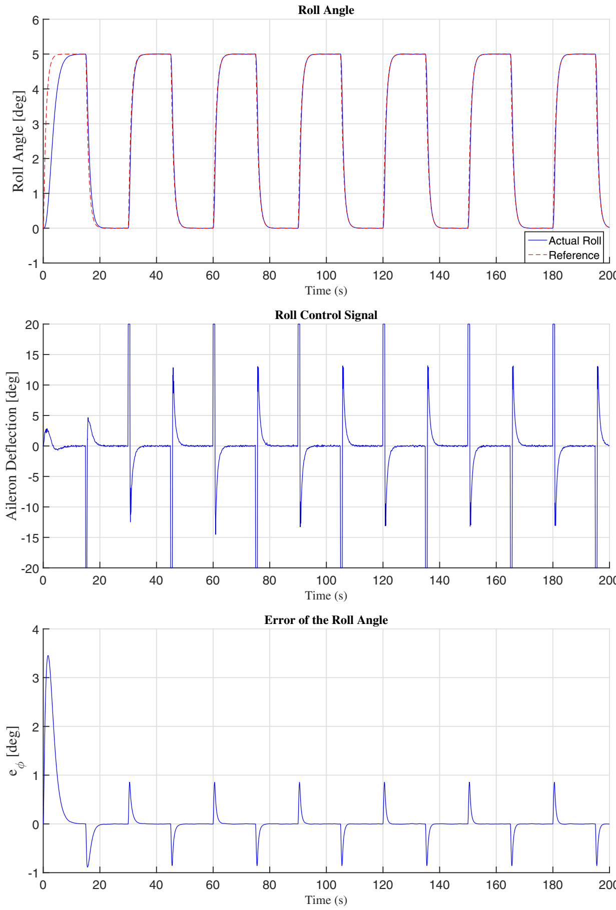

Fig. 7. Adaptive PD controller using the MIT rule with first order sliding-mode in roll angle (with disturbances)

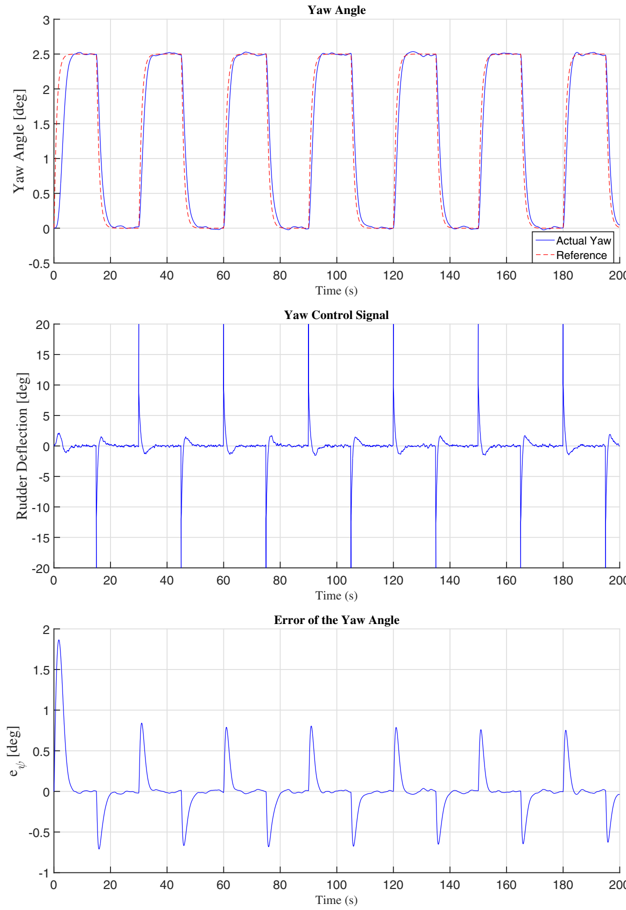

Fig. 8. Adaptive PD controller when using the MIT rule with first order sliding-mode in the yaw angle (with disturbances)

C. MIT rule with second order sliding-mode

In the Figure 9, it is observed the performance of the MIT rule with second order sliding-mode. With this technique, we have reduced the chattering effect in some parts of the control signal, with respect to the MIT rule and the MIT rule with first order sliding-mode.

The MIT rule with second order sliding-mode for the yaw angle has presented a minimal difference in the reduction of chattering effect; due to the minimal chattering effect that has been generated by the MIT rule with first order sliding-mode, but if the model-reference has a higher amplitude, then the chattering effect will also increase.

D. MIT rule with high order sliding-mode

The Figure 11 shows the performance of the MIT rule with high order sliding-mode (HOSM) for the roll angle. The chattering effect has been reduced significantly and the noise generated by the MIT rule has been eliminated.

The Figure 12 shows the behavior of the MIT rule with high order sliding-mode in the yaw angle.

E. Reduction of the chattering effect

The Figures 13 and 15 present the control signals without the disturbances in order to appreciate the chattering reduction. We observe that it was only necessary the use of a second order sliding-mode in order to reduce the chattering effect. Nonetheless, the MIT rule with HOSM is necessary, given that, if the model-reference increase the angle amplitude then the chattering effect increase too, then we propose the use of the HOSM to eliminate such an effect. On the other hand, the Figures 14 and 16 show the control signals with disturbances. Now, we can observe how the sliding mode technique is trying the reduce the disturbances, even if they were not modeled in the controller design.

In the Figure17 is presented the obtained trajectory with the altitude control and the adaptive control for the yaw and roll angles.

F. Analysis of the error signals and the efforts of the control laws

In order to analyze the error signals and the effort of the control laws, we have applied the $\mathscr{L}_2$ norm to the error:

$$\mathscr{L}_2[e_l] = \sqrt{\frac{1}{T-t_0}\int_{t_0}^{T} \|e_l\|^2 dt} \tag{33}$$

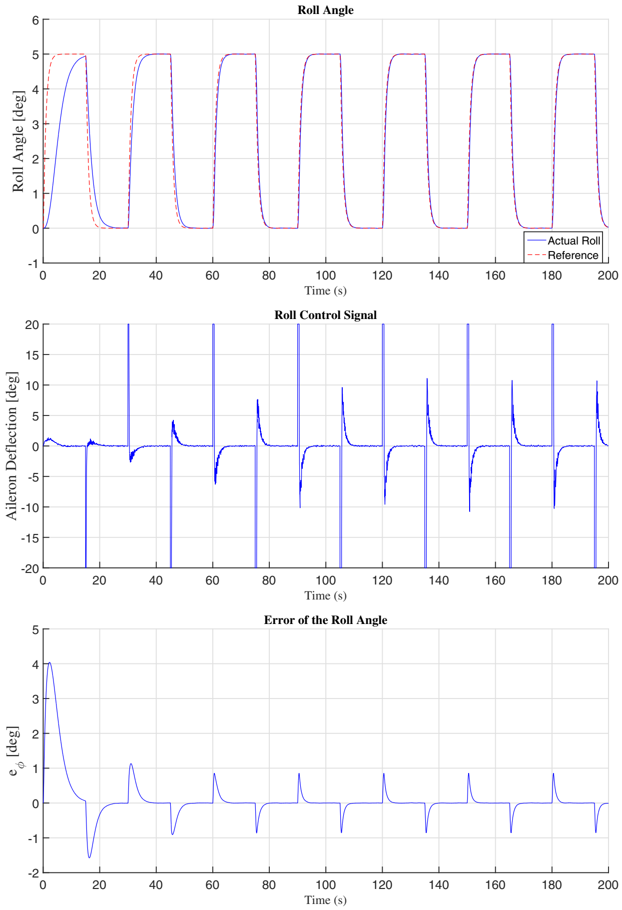

Fig. 9. Adaptive PD controller when using the MIT rule with second order sliding-mode in roll angle (with disturbances)

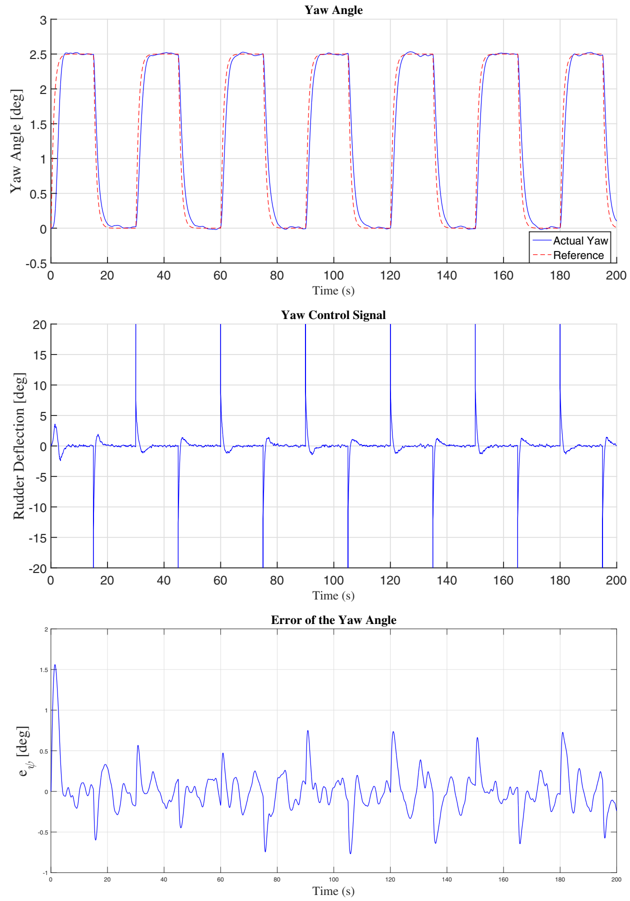

Fig. 10. Adaptive PD controller when using the MIT rule with second order sliding-mode in yaw angle (with disturbances)

and it is also used to the effort of the control law; it has been defined as:

$$\mathscr{L}_2[\delta_s] = \sqrt{\frac{1}{T-t_0}\int_{t_0}^{T} \|\delta_s\|^2 dt} \tag{34}$$

The errors and efforts that have been obtained with the norms (33) and (34) are shown in the tables I and II for the roll and yaw angles, respectively. In Tables I and II, we can appreciate that the MIT rule with a first order sliding-mode has presented lower errors, as well as in the controller efforts for the yaw angle. However, if we observe the Figure 13, it has the presence of the chattering effect in the control signal, thus, if we want to eliminate it, we need to use the MIT rule with HOSM, in spite of its highest error and effort of the controller. With this proposal, we have eliminated the chattering effect and some noise generated by the MIT rule in the control signal.

TABLE I

$\mathscr{L}_2$ NORM FOR THE ERRORS AND THE EFFORTS OF THE CONTROL LAWS ON THE ROLL ANGLE

Roll angle	$e_\phi[deg]$	$u_\phi[deg]$
MIT	0.3587	0.5986
MIT-SM	0.3573	0.6035
MIT-2SM	0.9582	0.6744
MIT-HOSM	4.8928	3.4218

TABLE II

$\mathscr{L}_2$ NORM FOR THE ERRORS AND THE EFFORTS OF THE CONTROL LAWS ON THE YAW ANGLE.

Yaw angle	$e_\psi[deg]$	$u_\psi[deg]$
MIT	0.3587	0.5540
MIT-SM	0.1787	0.2457
MIT-2SM	0.4791	0.2966
MIT-HOSM	2.4464	1.3002

VII. CONCLUSIONS

In this work, the MIT rule was modified by adding sliding-mode techniques in order to obtain a robust mechanism for an adaptive PD controller, and the system has been subjected to disturbances no modeled in the control law. The adaptive control laws have been applied in the yaw and roll angles of a fixed-wing MAV, and a classic PD control has been applied for the altitude motion. The simulation results have shown that the PD adaptive control with the MIT rule was saturated to $\pm 40°$, for roll angle, and this value cannot be allowed by

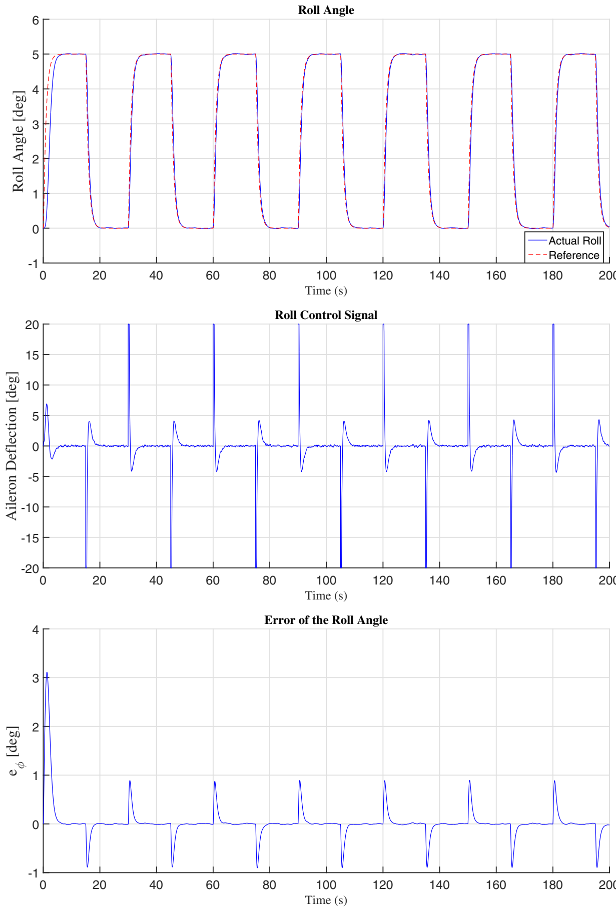

Fig. 11. Adaptive PD controller using the MIT rule with high order sliding-mode in roll angle (with disturbances)

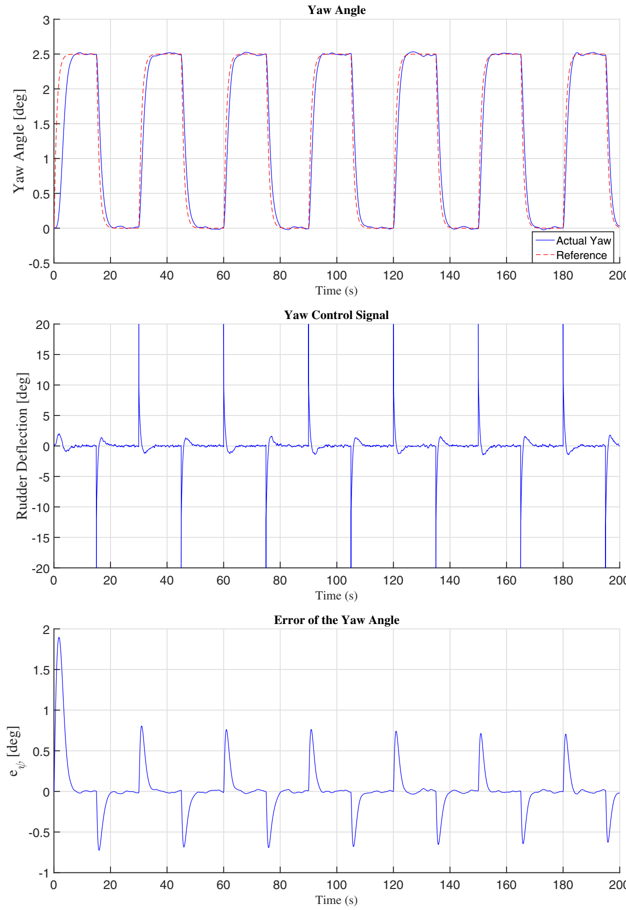

Fig. 12. Adaptive PD controller when using the MIT rule with high order sliding-mode in yaw angle (with disturbances)

our fixed-wing MAV platform, even more the control signal has presented some noise without the disturbances, and also the tuning of the adaptive gains are not easy, that is, with some decimal changes in this gains the system tends to the instability in the yaw and roll angles.

For the MIT rule with first order sliding-mode, it has been obtained the desired tracking with an allowed the control law signal of $\pm 20^\circ$, and even with the linkage of these techniques, it has obtained a lower error and effort of controller in the yaw angle, in comparison with the other presented techniques, however in the control signal for the roll angle, it has presented the undesired chattering effect, that is appreciated better in the results of the control signal without disturbances. The chattering effect, in the control signal, has been reduced by the MIT rule with second order sliding-mode, but it has not eliminated of the control signal for the roll angle. Whereas in yaw angle, the chattering effect has been eliminated, that is, when we consider low amplitude values of the model-reference.

A good behavior in the control signal has been presented by the MIT rule with high order sliding-mode. The chattering effect has been reduced or even eliminated. Furthermore, the control signal is inside of the allowed values for our fixed-wing MAV, that is, in $\pm 20^\circ$ of deflection of the control surfaces. We have also observed that it is easier to tune the adaptive gains with the MIT rule with HOSM, however, we have a bigger error and control effort in comparison with the others proposed techniques.

Finally, the control objective is achieved even if the disturbances are not modeled in the control system. In the future work, these control laws will be implanted in a fixed-wing MAV testbed. By doing some changes, we consider that our proposal could be tested in a VTOL UAV platform.

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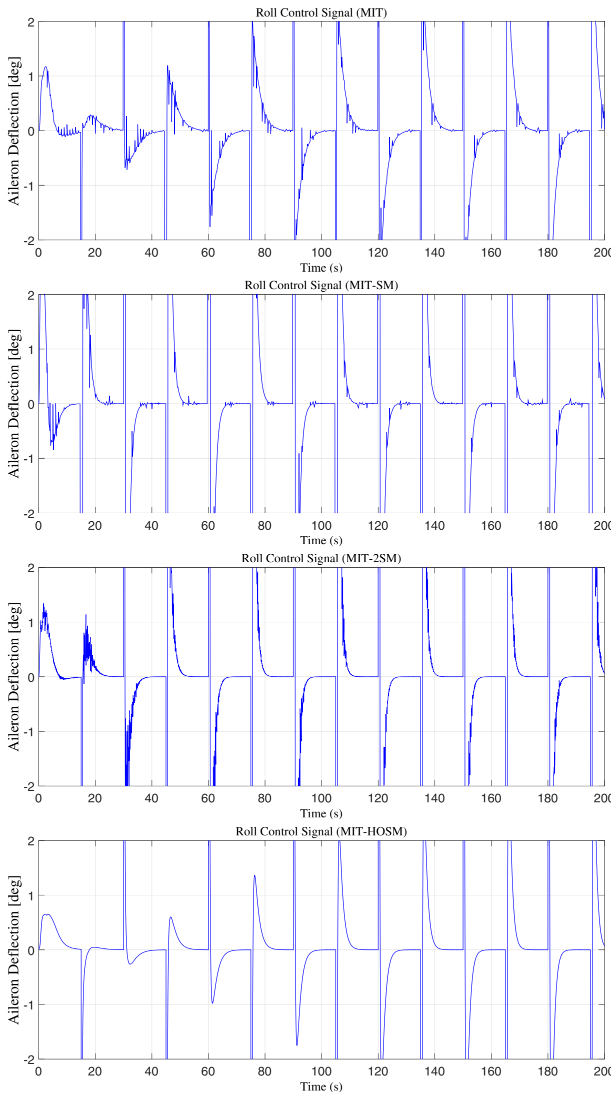

Fig. 13. Zoom of the control actions of the adaptive PD control when using the MIT rule with sliding-mode in the roll angle (without disturbances)

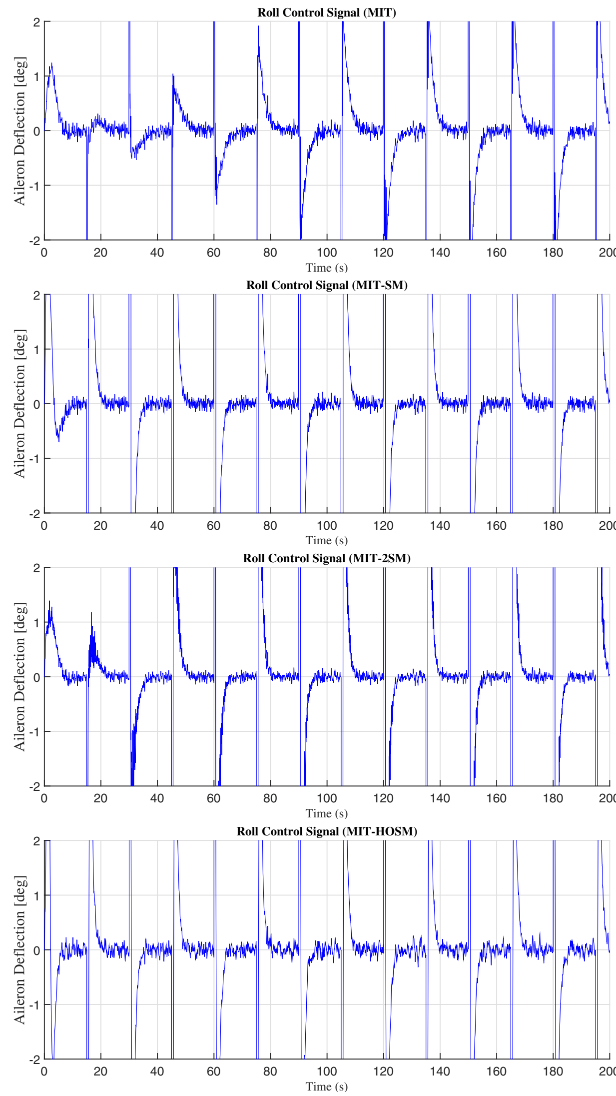

Fig. 14. Zoom of the control actions of the adaptive PD control when using the MIT rule with sliding-mode in the roll angle (with disturbances)

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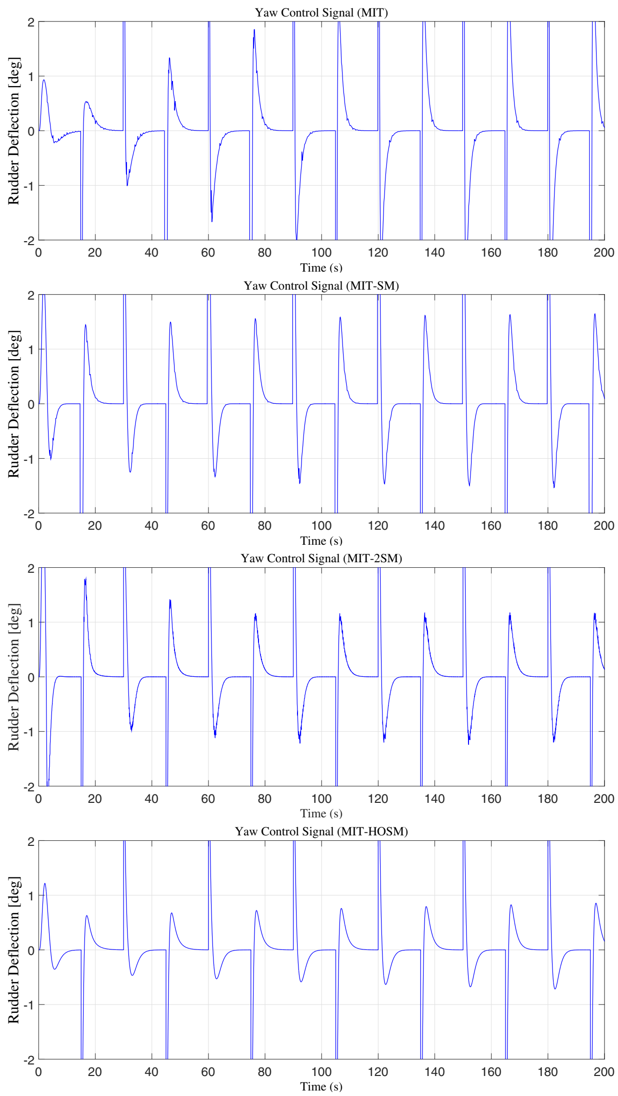

Fig. 15. Zoom of the control actions of adaptive PD control when using the MIT rule with sliding-mode in the yaw angle (without disturbances).

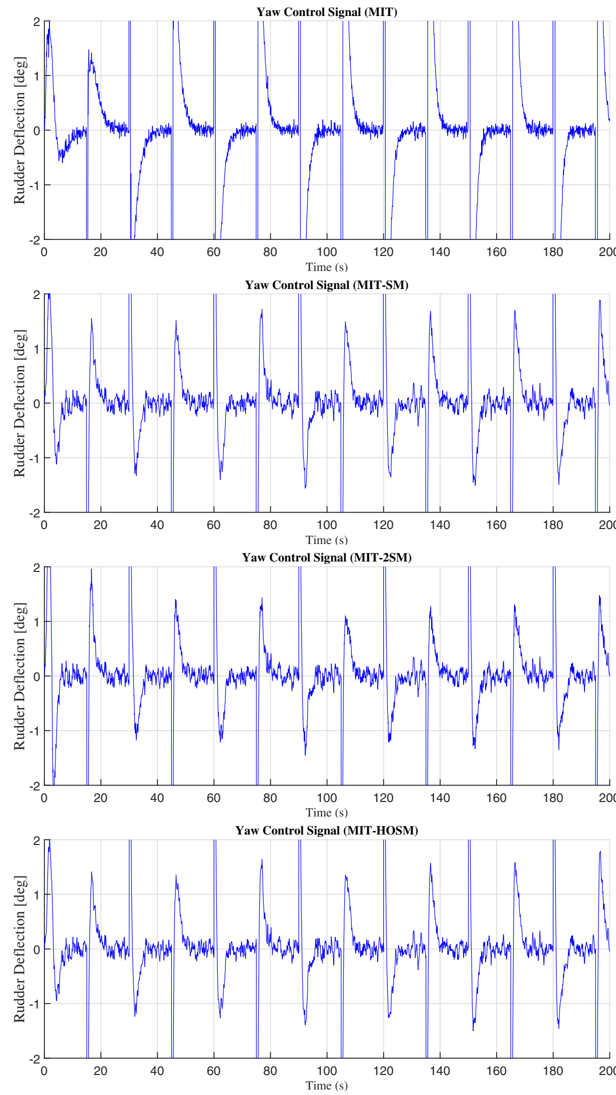

Fig. 16. Zoom of the control actions of adaptive PD control when using the MIT rule with sliding-mode in the yaw angle (with disturbances).

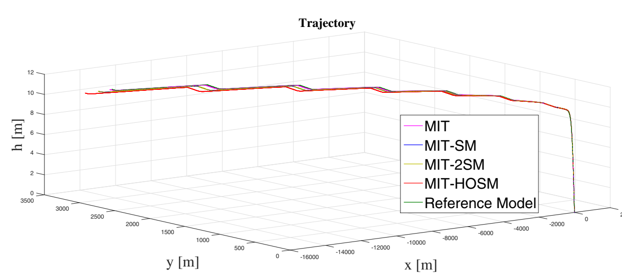

Fig. 17. Obtained trajectory of the fixed-wing MAV for the proposed control laws