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Is Our Universe Expanding Dynamics Fractional Order?

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http://mechatronics.ucmerced.edu/research/applied-fractional-calculus

Mechatronics, Embedded Systems and Automation (MESA) Lab

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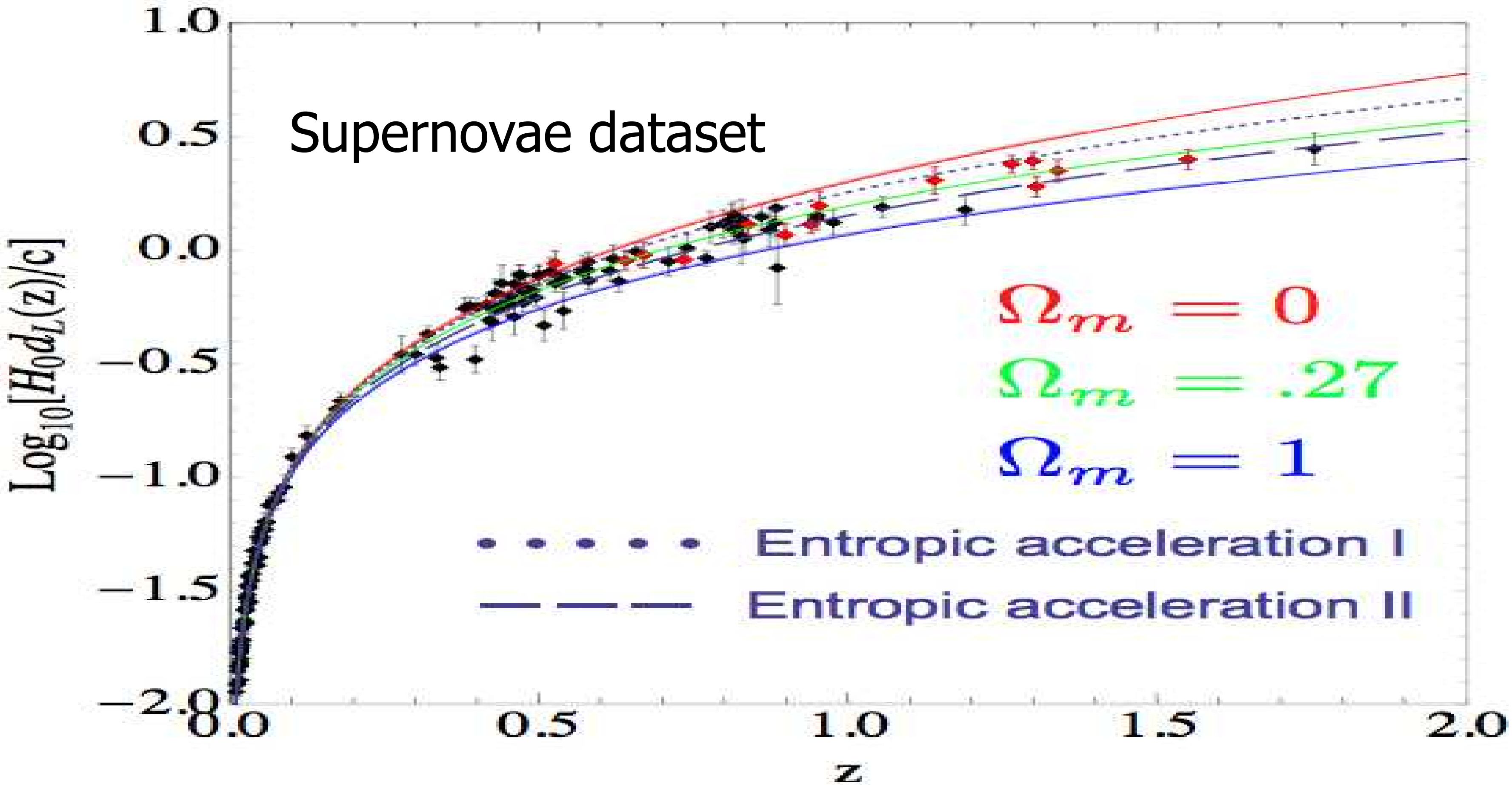
Nov. 6, 2012. Tuesday 11:00-12:00

Castle Research Facility Room #22

[redacted]

Motivation



G. Smoot et al., Phys. Lett. B 696 (2011) 273-277.

Nobel Price in Physics 2006: G. Smoot & J. Mather

Is Our Universe Expanding Dynamics Fractional Order?

- Before the time of Einstein:

  Static (Cosmological Constant)

- Since1929 (Edwin Hubble):

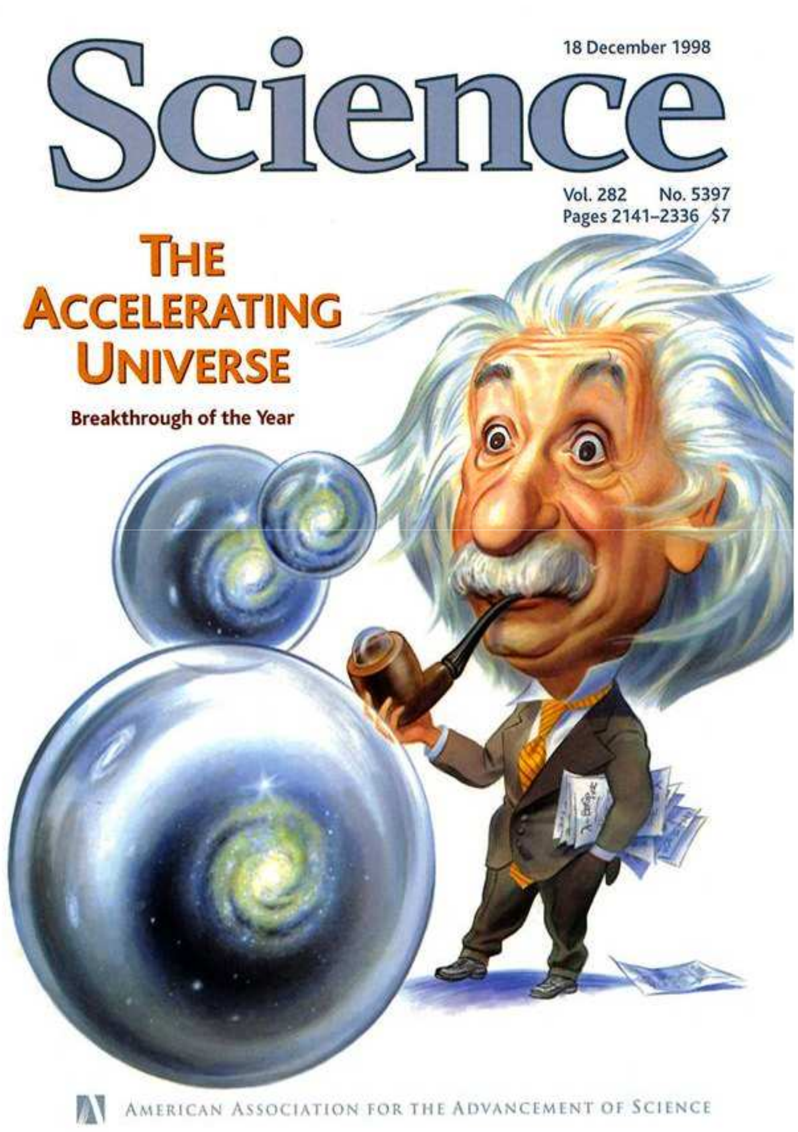
  Expanding

- Since1998 (Riess et al. & Perlmutter et al.):

  Accelerating (Dark Energy)

- Today:

  Accelerating Law---Fractional Calculus



Physics 2011

Saul Perlmutter

Brian P. Schmidt

Adam G. Riess

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How to Measure Our Expanding Universe?

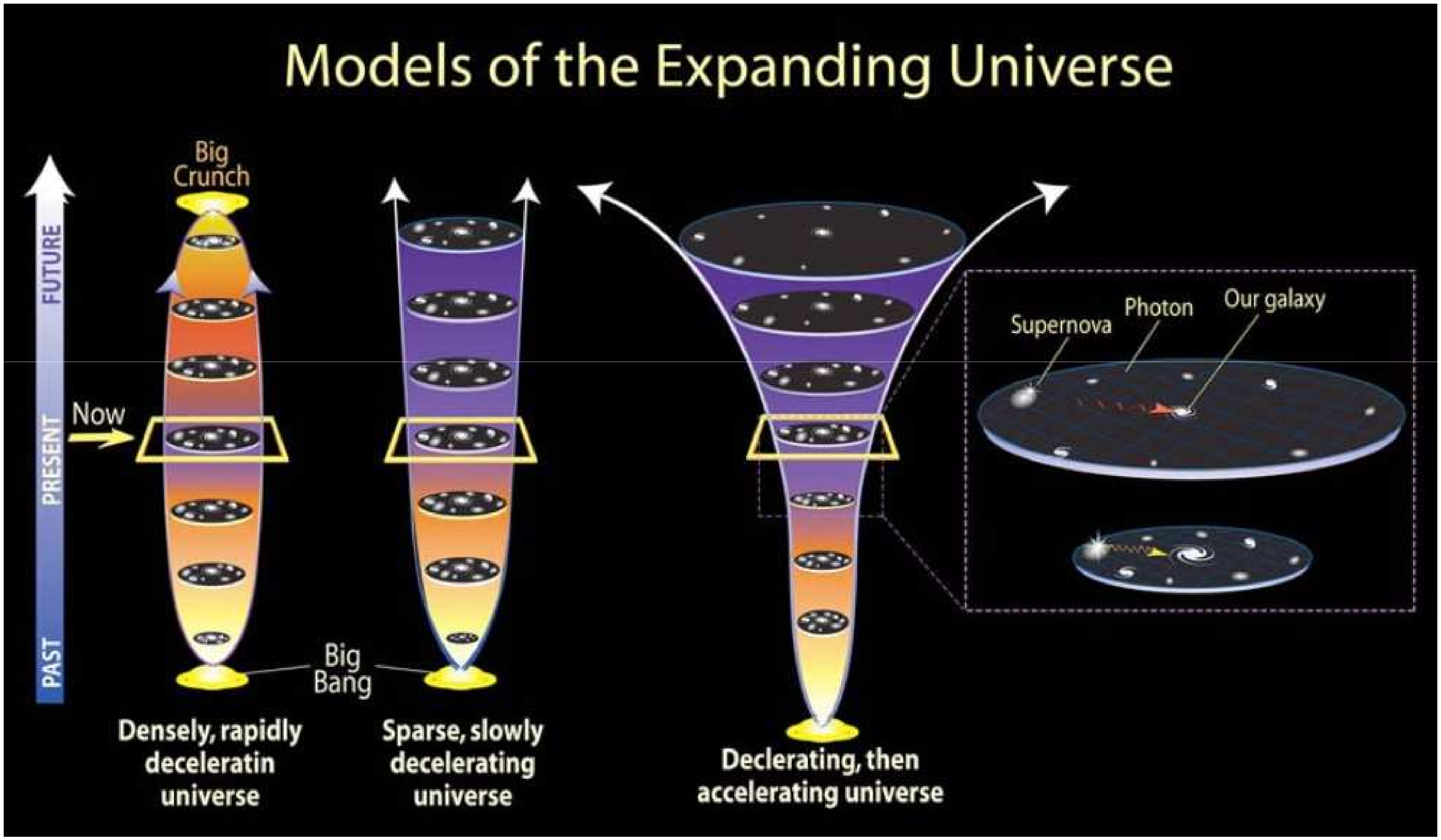
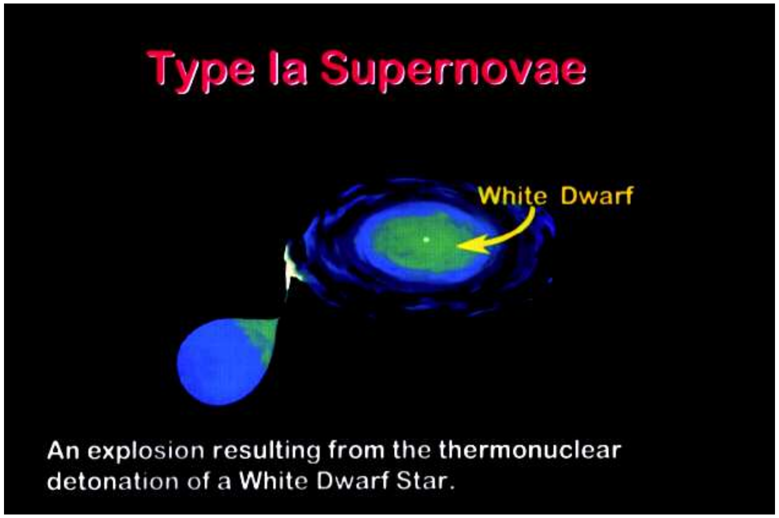


Figure credit: Dr. Adam G. Riess

Type Ia Supenovae (SNIa)--- Standard Candle



Distance measurement :

The relative age of the Universe at the time of the supernova explosion

Redshift measurement:

The growth factor of the Universe at the time a supernova exploded

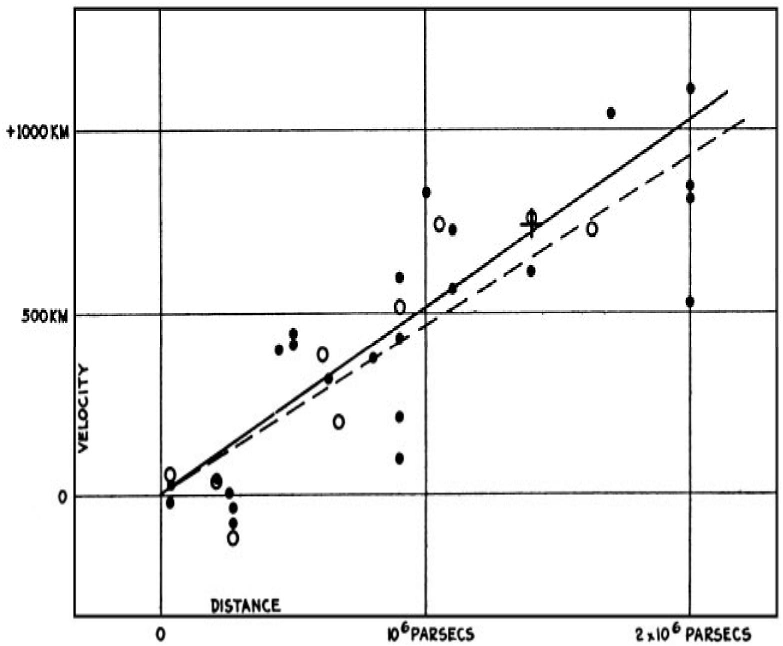
Hubble Diagram

Figure credit: Dr. Adam G. Riess

This relationship between distance modulus and redshift can be considered as a purely kinematic record of the universe’s expansion history.

That is to say, the relative positions of galaxies can tell us whether the universe was ever accelerating or decelerating, regardless of its cause.

Hubble's Data



(E.P. Hubble, PNAS 15 (1929) 168-173.)

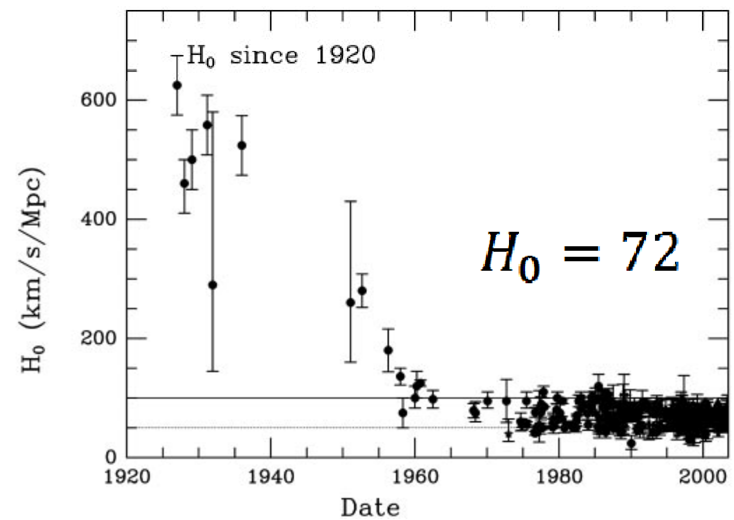
Hubble's Law:

$$c\,z = v = H_0\,d$$

Findings:

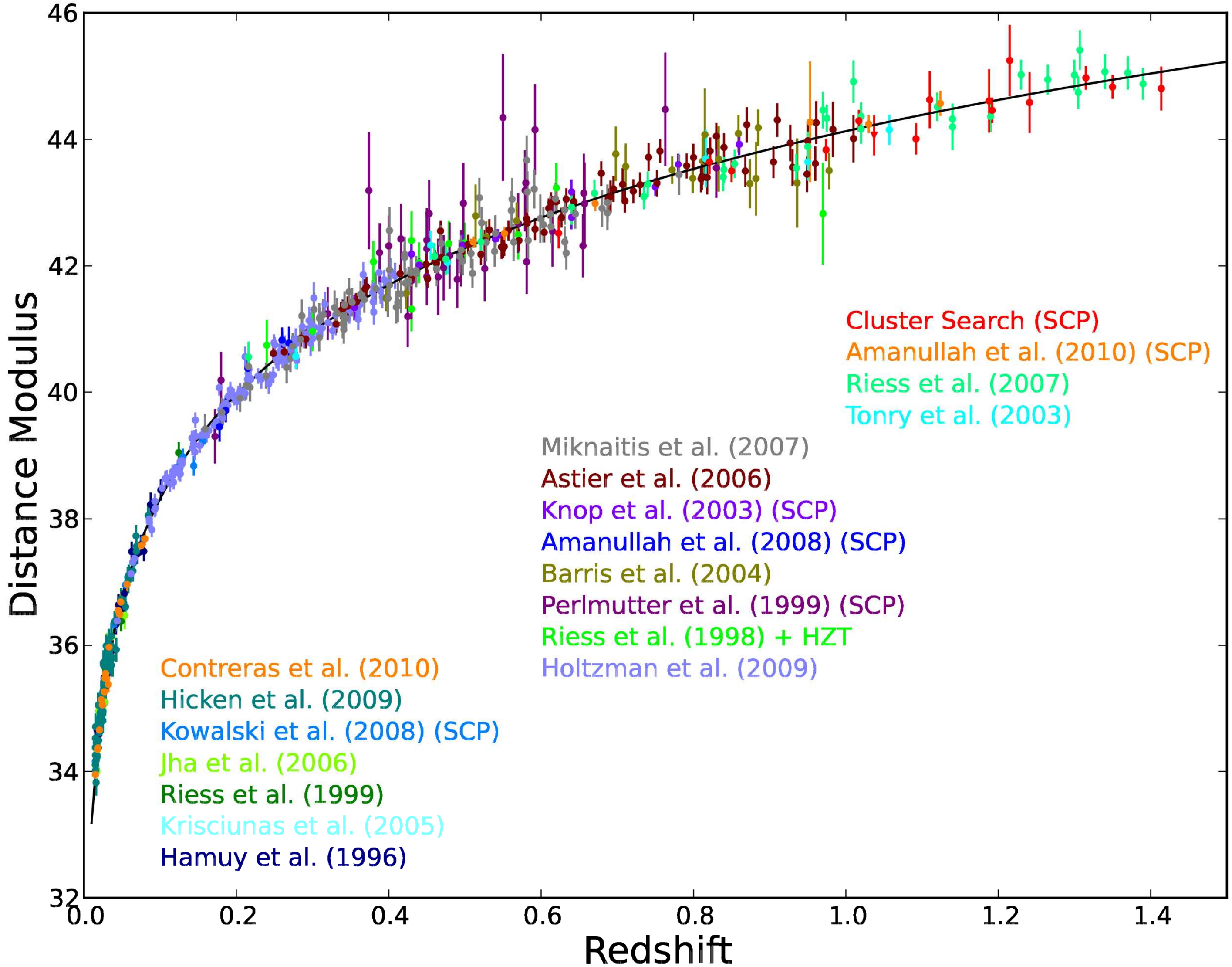
Hubble showed that galaxies recede from us in all directions and more distant ones recede more rapidly in proportion to their distance.

The Universe is expanding!



(R.P. Kirshner, PNAS 101 (2003) 8-13.)

13.7 billion years



Slide credit: Supernova Cosmology Project

"Fractional Order Thinking"

IS OUR UNIVERSE EXPANDING DYNAMICS FRACTIONAL ORDER?

WHAT IS THE BENEFIT BY USING FRACTIONAL CALCULUS?

$$\ldots, \quad \frac{d^{-2}f}{dt^{-2}}, \quad \frac{d^{-1}f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2f}{dt^2}, \quad \ldots$$

$$\Downarrow$$

$${}^{C}D_0^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad (n-1 < \alpha \leq n, \ \ n \in \mathbb{N})$$

$$\mathscr{L}\{{}^{C}D_0^{\alpha} f(t); s\} = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0)$$

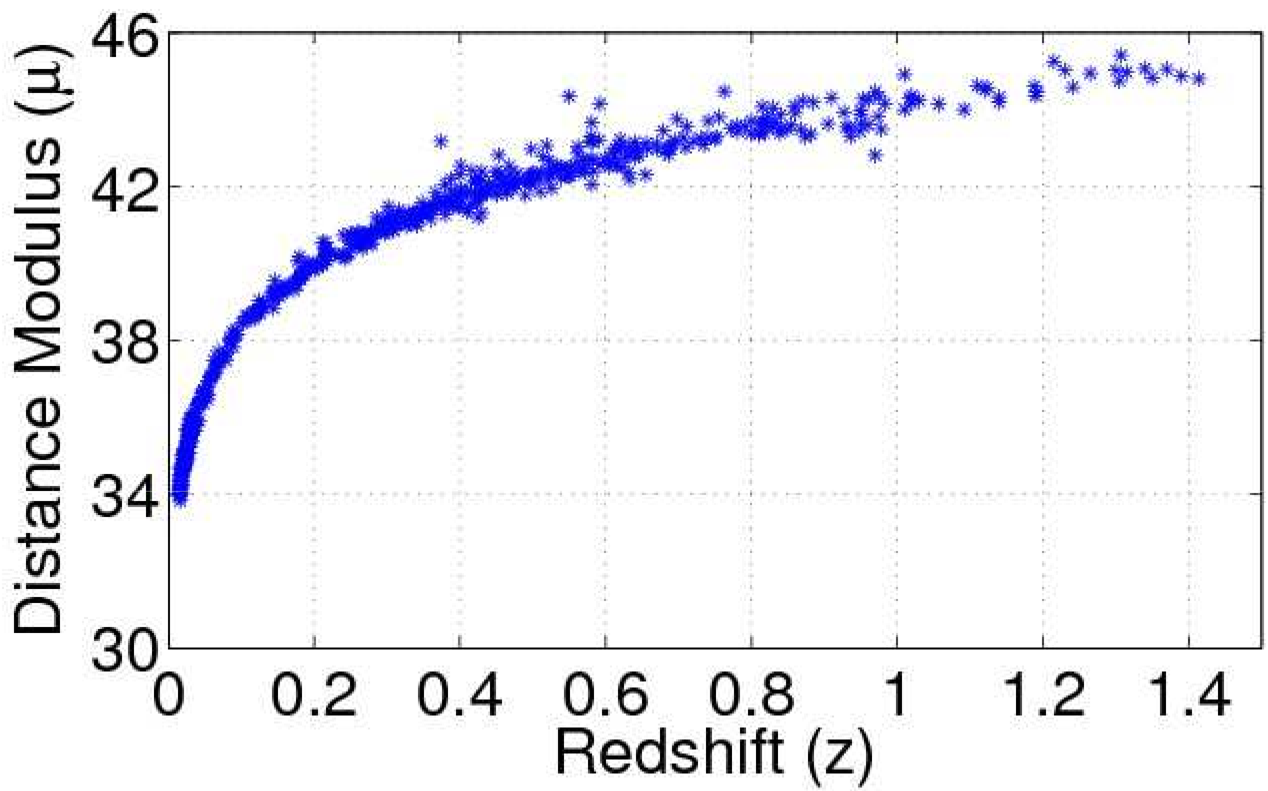
Mittag-Leffer function:

$$E_{\alpha,\beta}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(k * \alpha + \beta)}, \quad \Re(\alpha) > 0, \Re(\beta) > 0$$

$$\mathscr{L}\{t^{\beta-1} E_{\alpha,\beta}(\pm\lambda t^{\alpha}); s\} = \frac{s^{\alpha-\beta}}{s^{\alpha} \mp \lambda}$$

Special cases:

$$E_{0,1}(t) = \frac{1}{1-t}, \quad E_{1,1}(t) = \exp(t), \quad E_{1,2}(t) = \frac{\exp(t) - 1}{t}$$



Data fitting → (Accelerating law) → Mittag-Leffler function

Approach: $f(t) = Kt^{\alpha-1}E_{\alpha,\alpha}(-\lambda t^{\alpha}), \quad (1 < \alpha < 2)$

$\Updownarrow$

$$f(t) = \int_0^t y(\tau)g(t-\tau)d\tau$$

Where $g(t) = \frac{K}{y_1}t^{\alpha-2}E_{1,\alpha-1}\Big(-\frac{y_2}{y_1}t\Big)$ and

$${}^C D_0^{\alpha} y(t) + \lambda y(t) = 0, \quad y(0) = y_1, \quad y'(0) = y_2$$

Dataset: $[z(k), \mu(k)]$

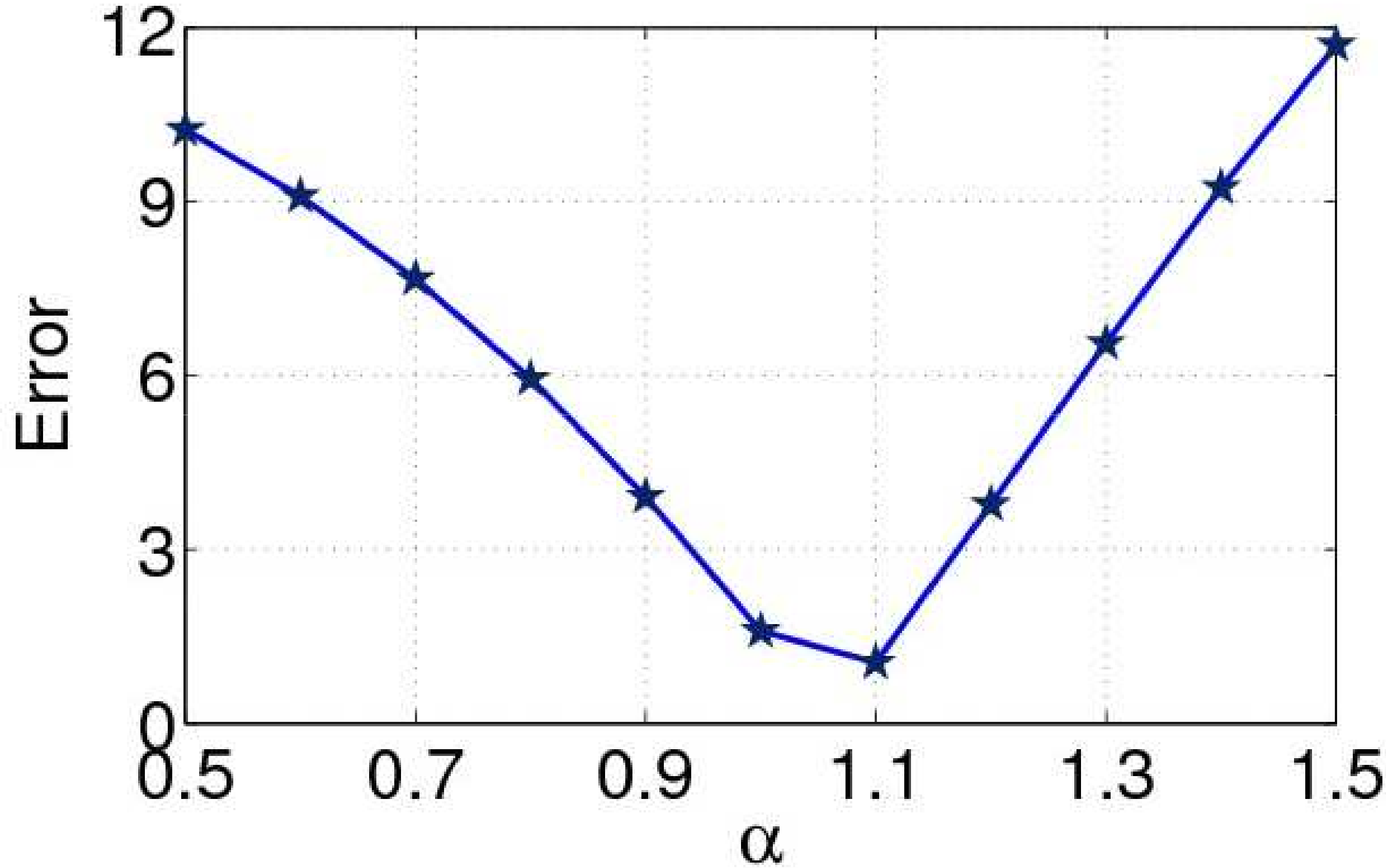
http://supernova.lbl.gov/Union/figures/SCPUnion2.1_mu_vs_z.txt

Fitting function: $[z(k), f(k)]$

$$f(t) = Kt^{\alpha-1}E_{\alpha,\alpha}(-\lambda t^{\alpha}), \quad (1 < \alpha < 2)$$

Aim: $J = \min \sqrt{\frac{\sum_{k=1}^{580} \left(f(k) - \mu(k)\right)^2}{580}}$

580: Number of the type Ia supernovae up to date


12
9
6
3
0
Error
0.5
0.7
0.9
1.1
1.3
1.5
α

Matlab Codes

```
clear all;clc;
options=optimset('TolX',1e-15,'TolFun',1e-15);
z=importdata('z.mat');
mu=importdata('mu.mat');
[x,FVAL,EXITFLAG]=fminsearch(@(x) mlf_scp(x,mu,z),[1,2,1.1],options);
K=x(1);lambda=x(2);alpha=x(3);
y=mlf(alpha,alpha,-lambda*z.^alpha);
y=K*y.*z.^(alpha-1);
figure;
scatter(z,mu)
hold on
plot(z,y,'k','LineWidth',2);
grid on;
xlabel('Redshift z');ylabel('Distance Modulus \mu');
```

```
function [J]=mlf_scp(x,y0,z)
K=x(1);lambda=x(2);alpha=x(3);
N=length(y0);
y=mlf(alpha,alpha,-lambda*z.^alpha);
y=K*y.*z.^(alpha-1);
J=sqrt((y-y0)*(y-y0)'/N);
```

http://www.mathworks.com/matlabcentral/fileexchange/8738

Results

$$K = 42.7638$$

$$\alpha = 1.0616$$

$$J = 0.2649$$

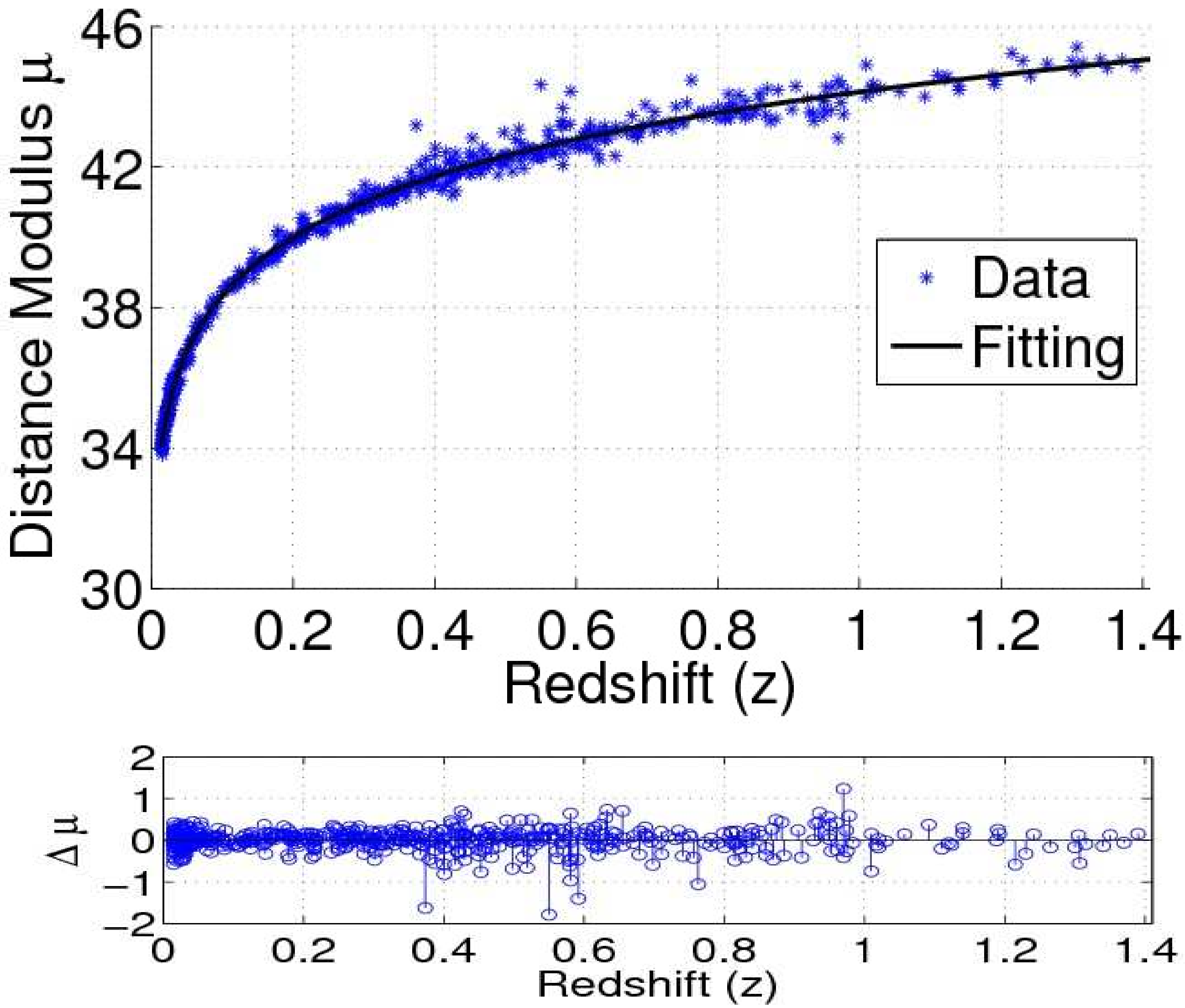
$$\lambda = 9.5671 \times 10^{-4}$$

$$f(t) = 42.7638 t^{0.0616} E_{1.0616,1.0616}(-9.5671 \times 10^{-4} t^{1.0616})$$

$$f(t) = \int_0^t y(\tau) g(t-\tau) d\tau$$

$$g(t) = \frac{42.7638}{y_1 t^{0.9384}} E_{1,0.0616}\left(-\frac{y_2}{y_1} t\right)$$

$${}^{C}D_0^{1.0616} y(t) + 9.5671 \times 10^{-4} y(t) = 0, \quad y(0) = y_1, \quad y'(0) = y_2$$


46
42
38
34
30
Distance Modulus μ
0
0.2
0.4
0.6
0.8
1
1.2
1.4
Redshift (z)
Data
Fitting
2
1
0
−1
−2
Δμ
Redshift (z)

M-L function *VS* Exp functions

Fitting functions		J	N
$f_1(t) = Kt^{\alpha-1}E_{\alpha,\alpha}(-\lambda t^\alpha)$		0.2649	3
$f_2(t) = \sum_{i=1}^{n} K_i \exp(-\lambda_i t)$	n=1	1.5874	2
	n=2	1.0680	4
	n=3	0.5053	6
	n=4	0.9241	8
	n=5	2.7227	10
$f_3(t) = \sum_{i=1}^{n} K_i \exp(-\lambda_i t^\alpha)$	n=1	0.2687	3
	n=2	0.2671	5
	n=3	0.2691	7
	n=4	0.2672	9
	n=5	0.2674	11

J: Square Error, N: Number of Parameters

Benefit by Using Fractional calculus

1. The evolution of expanding dynamics of our universe obeys a Mittag-Leffler function, meaning that the accelerating law can be described by some fractional order differential equation, whose solution is the Mittag-Leffler function.

2. It can help the astronomers to discover new Type Ia supernovae with high redshift according to the proposed Mittag-Leffler function.

3. The fitting model is simper than other models.

$$\mu_0 = m - M = 5\log d_L + 25$$

$$d_L = c(1+z)\int_0^z \frac{du}{H(u)}$$
$$= c(1+z)H_0^{-1}\int_0^z \exp\left\{-\int_0^u [1+q(u)]d\ln(1+u)\right\}du,$$

$$H(z) = \frac{\dot{a}}{a}, \quad q(z) \equiv \frac{-\ddot{a}/a}{H^2(z)} = \frac{dH^{-1}(z)}{dt} - 1.$$

$$a(t) = a_0\left\{1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \frac{1}{3!}j_0H_0^3(t-t_0)^3 + O\left[(t-t_0)^4\right]\right\},$$

$$j(t) = +(\dddot{a}/a)(\dot{a}/a)^{-3}.$$

Discussion

$$f(t) = Kt^{\alpha-1}E_{\alpha,\alpha}(-\lambda t^{\alpha}), \quad (1 < \alpha < 2)$$

$$f(t) = \int_0^t y(\tau)g(t-\tau)d\tau$$

Where $g(t) = \frac{K}{y_1}t^{\alpha-2}E_{1,\alpha-1}\left(-\frac{y_2}{y_1}t\right)$ and

$${}^{C}D_0^{\alpha}y(t) + \lambda y(t) = 0, \quad y(0) = y_1, \quad y'(0) = y_2$$

Discussion

- Variable order:

$$\alpha \to \alpha(t)$$

- Distributed order:

$$D_{(\varphi)} f(t) = \int_{\beta_1}^{\beta_2} \varphi(\alpha)\, D^{\alpha} f(t)\, d\alpha$$

Thank you for your attention!

All Questions are welcome!