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Event-triggered boundary feedback control for networked reaction-subdiffusion processes with input uncertainties

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ABSTRACT

This paper is concerned with the event-triggered boundary feedback control problems for networked reaction-subdiffusion processes governed by time fractional reaction-diffusion systems with unknown time-varying input uncertainties over sensor/actuator networks. The event-triggered boundary state feedback controller is first designed and implemented via backstepping technique. Moreover, we realize that the availability of full-state measurements in many practical applications may be impossible due to the difficulties in measuring. To solve this limitation, we design an extended Luenberger observer that embeds within the networked sensor to estimate the whole states of the systems under consideration. Based on this, the boundary output feedback event-triggered implementation of the studied system in the context of sensor/actuator networks is then proposed. It is shown that both two kinds of event-triggered strategies could significantly asymptotically stabilize the estimation with the Zeno phenomenon being excluded. Two numerical illustrations are finally included.

1. Introduction

In recent years, control systems are often performed over sensor/actuator networks due to the advantages in term of energy saving and structure flexibility [10,19,26,37]. One fundamental issue in such control systems is how to determine the transmission instants in the context of sensor/actuator networks so that the stability of the closed-loop systems under consideration can be guaranteed by designing feedback controllers. As cited in [42,44], two methods are commonly used to solve this problem, namely the time-triggered control method where the passing of time decides when the control inputs should be taken and the event-triggered control method where the control task is performed immediately if the given event-triggered condition is violated. One of advantages of using time-triggered methods is that it can make the control process easy to be implemented and analyzed. However, when the system approaches its steady state, time-triggered control methods would waste both computation and energy resources. Besides, if the sampling period is approaching zero, a great deal of "unnecessary" sampling signals will be sent into the shared communication network with limited bandwidth, which would lead to a higher workload of the network [12,38,44]. Therefore, in these cases, event-triggered control scheme should be considered. Another reason why the use of event-triggered control method is of interest is that it is closer in nature to the

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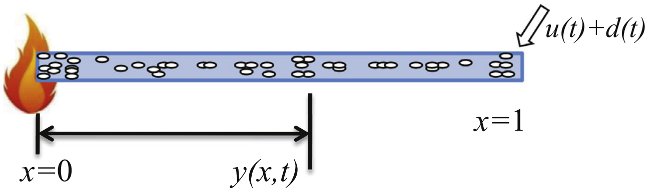


Fig. 1. Schematics for the model of a thin spatially inhomogeneous rod with state $y(x, t)$ and the boundary inputs acting on the right-sided $x = 1$.

way a human behaves as a controller [4]. With these advantages, event-triggered control has become an active research area and has been widely performed to deal with several control and filtering issues such as output feedback control [28], state feedback control [29] and adaptive control [40] for lumped parameter systems governed by ordinary differential equations.

Nowadays, the event-triggered control strategy has been proposed to discuss the synchronization problem for complex networks with diffusion term [11]. Recalling the existing literatures, the studies of networked reaction-subdiffusion processes have drawn increasing attention and time fractional reaction-diffusion systems in the context of sensor/actuator networks have been proved to be powerful tools to describe them (see e.g. [5,16,31]). This is due to the fact that fractional-order derivative is defined as a kind of convolution and good at modeling the dynamics inheriting memory and heritage properties where integer-order derivative approaches appear to fail [2,15,39]. Here time fractional reaction-diffusion systems can be regarded as an extension of the conventional reaction-diffusion systems, where the first-order time derivative is generalized to a fractional derivative of order $\alpha \in (0, 1]$. Thus, the investigations of event-triggered control problem for time fractional reaction-diffusion systems over sensor/actuator networks should be both interesting and challenging so as to explain and improve their rather complex dynamical behaviors.

The focus of this paper is on time fractional reaction-diffusion systems with a Caputo fractional-order derivative ${}_0^CD_t^\alpha$, $\alpha \in (0, 1)$ in one space dimension of the form:

$$\begin{cases} {}_0^CD_t^\alpha y(x,t) = \triangle y(x,t) + b(x)y(x,t) \text{ in } (0,1)\times[0,\infty), \\ y_x(0,t) = 0, \ \ y(1,t) = u(t) + d(t) \text{ in } [0,\infty), \\ y(x,0) = y_0(x) \text{ in } (0,1), \end{cases} \tag{1.1}$$

where $\triangle := \partial^2/\partial x^2$ is the Laplace operator, $b \in C^1(0, 1)$, $y_0 \in L^2(0, 1)$, ${}_0^CD_t^\alpha$ is the Caputo fractional derivative given by [22]

$${}_0^CD_t^\alpha y(\cdot,t) = {}_0I_t^{1-\alpha}\frac{\partial y}{\partial t}(\cdot,t) \text{ and } {}_0I_t^\alpha y(\cdot,t) = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} y(\cdot,s)ds \tag{1.2}$$

represents the Riemann–Liouville fractional integral. Here $L^2(0, 1)$ denotes the usual square integrable function space endowed with the inner product $(\cdot, \cdot)$ and the norm $\|\cdot\|$, $u \in L^2[0, \infty)$ is the boundary control input and $d(t) \in \mathbb{R}$ represents the unknown uncertainties which exist in all practical applications. The boundary conditions claim that the control input and the uncertainties only happen at the right end $(x = 1)$ and the left end $(x = 0)$ is supposed to be insulated.

The authors in [20] claimed that time fractional reaction-diffusion system is usually used to model the transport processes in a spatially inhomogeneous environment. Typical examples include the chemical reaction processes in dispersive transport media [43], reheating processes of the heterogeneous metal slabs [16], or the flow through porous media with a source [36], etc. Then system (1.1) can be viewed as a model of a thin spatially inhomogeneous rod with not only the heat loss on the right-sided $(x = 1)$ but also the heat generation inside the rod (see Fig. 1). For richer information on using time fractional reaction-diffusion systems to model real-life applications, we refer the reader to monographs [21,35,39] and the references cited therein.

To deal with the unknown input uncertainties $d(t)$, several methods have been used for the conventional parabolic distributed parameter systems (DPSs). A sliding mode control strategy has been applied in [8], which shows a good robust performance. The authors in [25] have designed a combined backstepping and sliding mode controller for the one-dimensional unstable parabolic DPSs with boundary disturbances. However, there is a need for further studies on the sliding mode control theory of time fractional reaction-diffusion systems. Note that recently another method called uncertainty and disturbance estimator (UDE) based control has received much attention as shown in [34,45], which has been extended to discussing the parabolic DPSs [9]. The UDE-based method only requires that the Laplace transformation of the uncertain signal exists. Then we here adopt this method to estimate the unknown uncertainties of system (1.1).

Motivated by these above considerations, in this paper, we consider the event-triggered boundary feedback control for the infinite-dimensional time fractional reaction-diffusion system (1.1) in the context of sensor/actuator networks. To the best of our knowledge, no result is available on this topic. Even for the special case of conventional non-fractional reaction-diffusion systems, our proposed approaches are novel and challenging. To realize this, if all components of the state vector for the considered system can be measured by networked sensors, we design an event-triggered boundary state feedback controller based on the measured states via backstepping technique to asymptotically stabilize the closed-loop system. An issue that requires careful consideration in the implementation of event-triggered control strategy is to guarantee a positive lower boundedness of the minimum inter-event time that avoids the occurrence of infinite transmissions in a finite time, i.e., the Zeno phenomenon [7]. It is worth mentioning that the past several decades have witnessed significant developments in using backstepping technique to study the control problem of non-fractional diffusion systems. In [13], the backstepping method has been extended to considering the boundary feedback stabilization problem of a time fractional reaction-diffusion system with both Dirichlet and Neumann boundary conditions. Then we continue to use this technique to prove our main event-triggered boundary control results. Moreover, we realize that the availability of full-state measurements in some practical cases may be impossible for direct measurement due to the difficulties in measuring [41]. To solve this limitation, it is often possible to determine the state by building an observer that uses measurements of the inputs and outputs of the studied system, along with a model of the system dynamics, to estimate the state. This is the event-triggered boundary output feedback control, where the control action is updated based on the obtained estimation of the states when a certain stability threshold is breached. In this paper, we give an extended Luenberger observer that embeds within the networked sensor estimating the whole states to realize this. In particular, if the observer error is equal to zero consistently, then both the event-triggered boundary state feedback controllers and the event-triggered boundary output feedback controllers have same performance.

In our previous work [17], the event-driven boundary state feedback control problem for time fractional reaction-diffusion systems with Dirichlet boundary conditions has been studied. The present paper is an extension of [17], where we propose both the state and the output feedback event-triggered boundary control strategies to stabilize the subdiffusion system at hand. It is shown that both two kinds of event-triggered controllers could significantly asymptotically stabilize the estimation. As a result, the proposed methods lead to a lower network load while guaranteeing the desired levels of performance. This is appealing in many situations of real applications.

The reminder of the paper is organized as follows. Section 2 gives some preliminary results to be used thereafter. The main results on the design and implementation of corresponding event-triggered boundary state feedback controller are presented in Section 3. In section 4, to overcome the limitation of lack of full-state measurements, we design an extended Luenberger observer and then propose a event-triggered boundary output feedback controller to stabilize the system at hand. Two numerical examples are finally included.

2. Preliminary results

To discuss the stabilization problem of system (1.1), we introduce the following special function

$$E_{\alpha,\beta}(t)=\sum_{k=0}^{\infty}\frac{t^k}{\Gamma(\alpha k+\beta)},\ \ \mathbb{R}(\alpha)>0,\ \ t\in\mathbb{C}, \tag{2.1}$$

which is known as the Mittag–Leffler function in two parameter and in particular, we write $E_{\alpha,1}(t)=E_\alpha(t)$ for short when $\beta=1$. If $\alpha=\beta=1$, it reduces to the conventional exponential function. Then Mittag–Leffler function can be regarded as an extension of the exponential function.

Lemma 2.1 ([33]). *Let $\alpha<2$, β be an arbitrary real number and $\frac{\pi\alpha}{2}<\theta<\min\{\pi,\pi\alpha\}$. If $\theta\leqslant|\arg(z)|\leqslant\pi,\ |z|\geq 0$, then*

$$\left|E_{\alpha,\beta}(z)\right|\leqslant\frac{M}{1+|z|} \tag{2.2}$$

holds for some constants $M>0$.

By Lemma 2.1, if $\alpha\in(0,1)$, $\beta=1$, $\lambda\in\mathbb{R}$ and $\lambda>0$, we get that $|\arg(-\lambda t^\alpha)|=\pi\in[\theta,\pi]$ and then

$$E_\alpha(-\lambda t^\alpha)\leqslant\frac{M}{1+\lambda t^\alpha}\leqslant\frac{M}{\lambda}t^{-\alpha},\ \ t>0, \tag{2.3}$$

where λ is used to regulate the convergence speed.

Lemma 2.2. *Given $\lambda>0$, $t\geq 0$, $\alpha\in(0,1)$, it follows that*

$$\int_0^t\frac{E_{\alpha,\alpha}(-\lambda(t-s)^\alpha)}{(t-s)^{1-\alpha}}s^{-\alpha}ds=\Gamma(1-\alpha)E_\alpha(-\lambda t^\alpha). \tag{2.4}$$

Proof. Given $\lambda>0$, one has

$$\begin{aligned}\int_0^t\frac{E_{\alpha,\alpha}(-\lambda(t-s)^\alpha)}{(t-s)^{1-\alpha}}s^{-\alpha}ds&=\sum_{k=0}^{\infty}\frac{(-\lambda)^k}{\Gamma(\alpha k+\alpha)}\int_0^t(t-s)^{\alpha k+\alpha-1}s^{-\alpha}ds\\&=\sum_{k=0}^{\infty}\frac{(-\lambda)^k t^{\alpha k+\alpha-1-\alpha+1}}{\Gamma(\alpha k+\alpha)}\int_0^1(1-\tau)^{\alpha k+\alpha-1}\tau^{-\alpha}d\tau\\&=\sum_{k=0}^{\infty}\frac{(-\lambda)^k}{\Gamma(\alpha k+\alpha)}B(\alpha k+\alpha,1-\alpha)t^{\alpha k}\end{aligned}$$

$$= \Gamma(1-\alpha)\sum_{k=0}^{\infty}\frac{(-\lambda)^k t^{\alpha k}}{\Gamma(\alpha k+1)}$$
$$= \Gamma(1-\alpha)E_\alpha(-\lambda t^\alpha), \tag{2.5}$$

where $B(p,q)=\int_0^1(1-t)^{p-1}t^{q-1}dt=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$ represents the Beta function. This completes the proof. □

Next, we express the solution of system (1.1) via spectral theory of system operator $\triangle$.

For operator $\triangle$ with the boundary conditions $y_x(0)=y(1)=0$, let $\lambda_n=n^2\pi^2$ and $\xi_n(x)=\sqrt{2}\cos((n-1/2)\pi x)$, $n=1,2,\ldots$. We obtain that [32]

1. $-\lambda_n$ is the eigenvalue of $\triangle$;
2. $\xi_n(x)$ is the eigenfunction of $\triangle$ corresponding to $-\lambda_n$ and $\{\xi_n\}_{n\geq 1}$ forms a Riesz basis of $L^2(0,1)$.

By Matignon [30], system (1.1) can not be stable if some eigenvalues of $\triangle+b(x)$ are bigger than zero. Then we conclude that system (1.1) is unstable if $b(x)$ is positive and large enough.

Similar to Ge et al. [14], we deduce the dual relationship between the following two systems:

$$\begin{cases}\triangle\varphi(x)=0 \text{ in } (0,1),\\ \varphi_x(0)=h_1,\ \varphi(1)=h_2\end{cases} \text{ and } \begin{cases}\triangle\psi(x)=f(x) \text{ in } (0,1),\\ \psi_x(0)=\psi(1)=0,\end{cases} \tag{2.6}$$

which plays a key role in obtaining our results.

Lemma 2.3. *Given $f\in L^2(0,1)$. Then*

$$\int_0^1\varphi(x)f(x)dx=h_2\psi_x(1)+h_1\psi(0), \tag{2.7}$$

where φ and ψ denote the solutions of (2.6), respectively. In particular, if $f=-\lambda_n\xi_n$, we have

$$-\lambda_n(\varphi,\xi_n)=h_2\frac{d\xi_n}{dx}(1)+h_1\xi_n(0). \tag{2.8}$$

Proof. Consider (2.6), based on the following one dimensional Green's identity

$$\int_0^1\psi(x)\triangle\varphi(x)-\varphi(x)\triangle\psi(x)dx=\left(\psi(x)\varphi_x(x)-\varphi(x)\psi_x(x)\right)|_0^1, \tag{2.9}$$

it yields that

$$0=\int_0^1\psi(x)\triangle\varphi(x)dx=\int_0^1\varphi(x)\triangle\psi(x)dx-\varphi(1)\psi_x(1)-\varphi_x(0)\psi(0)$$
$$=\int_0^1\varphi(x)f(x)dx-h_2\psi_x(1)-h_1\psi(0). \tag{2.10}$$

If $f=-\lambda_n\xi_n$, one has $\psi(x)=\xi_n(x)$ and $\triangle\xi_n=-\lambda_n\xi_n$. Then

$$\int_0^1\varphi(x)\triangle\xi_n(x)dx=-\lambda_n\int_0^1\varphi(x)\xi_n(x)dx=h_2\tfrac{d\xi_n}{dx}(1)+h_1\xi_n(0). \tag{2.11}$$

The proof is complete. □

Lemma 2.4 ([14]). *Given $\theta_1\in L^2((0,1)\times[0,\infty))$, $\theta_2\in L^2[0,\infty)$, a function ψ is said to be the unique mild solution of system*

$$\begin{cases}{}_0^CD_t^\alpha\psi(x,t)=\triangle\psi(x,t)+\theta_1(x,t) \text{ in } (0,1)\times[0,\infty),\\ \psi_x(0,t)=\theta_2(t),\ \psi(1,t)=\theta_3(t) \text{ in } [0,\infty),\\ \psi(x,0)=\psi_0(x) \text{ in } (0,1)\end{cases} \tag{2.12}$$

if it satisfies

$$\psi(x,t)=\sum_{n=1}^{\infty}E_\alpha(-\lambda_n t^\alpha)(\psi_0,\xi_n)\xi_n(x)+\sum_{n=1}^{\infty}\int_0^t\frac{E_{\alpha,\alpha}(-\lambda_n(t-\tau)^\alpha)}{(t-\tau)^{1-\alpha}}(\xi_n,\theta_1(\cdot,\tau))d\tau\,\xi_n(x)$$
$$+\sum_{n=1}^{\infty}\int_0^t\frac{E_{\alpha,\alpha}(-\lambda_n(t-\tau)^\alpha)}{(t-\tau)^{1-\alpha}}\left(\xi_n(0)\theta_2(\tau)+\frac{\partial\xi_n}{\partial x}(1)\theta_3(\tau)\right)d\tau\,\xi_n(x). \tag{2.13}$$

For the proof of Lemma 2.4, we refer the readers to [14], where the solution expressions of studied systems with control inputs emerging in the differential equation as distributed inputs and as boundary inputs in the boundary conditions are discussed. Here we investigate a similar case, which is not exactly the same but the proof is based on similar arguments. Then we omit the detailed proof.

3. Event-triggered boundary state feedback control

Consider system (1.1) over sensor/actuator networks, for simplicity, we write $A := \triangle + b(x)$ with

$$\mathrm{Dom}(A) = \{\phi \in L^2(0,1) : \phi, \phi' \text{ are absolutely continuous and } \phi_x(0) = \phi(1) = 0\}. \tag{3.1}$$

If the control input u is assumed to be not continuously implemented, but is updated at certain instants $\{t_k\}_{k\geq 1}$, for each time interval $[t_k, t_{k+1})$, it can be reformulated as follows

$$\begin{cases} {}_0^C D_t^\alpha y(x,t) = Ay(x,t) \text{ in } (0,1)\times[t_k, t_{k+1}), \\ y_x(0,t) = 0,\ y(1,t) = u(t_k) + d(t) \text{ in } [t_k, t_{k+1}), \\ y(x,0) = y_0(x) \ \text{ in } (0,1). \end{cases} \tag{3.2}$$

3.1. Equivalent transform via backstepping

Similar to the argument in [13], the integral transformation

$$\omega(x,t) = y(x,t) - \int_0^x g(x,\xi)y(\xi,t)d\xi \tag{3.3}$$

with $\omega_0(x) = y_0(x) - \int_0^x g(x,\xi)y_0(\xi)d\xi$ is adopted to convert (3.2) into a target system to be specified later. Then we get the following proposition, whose proof can be found in Appendix A.1.

Proposition 3.1. *Suppose that the kernel g(x, ξ) is chosen satisfying*

$$\begin{cases} g_{xx}(x,\xi) - g_{\xi\xi}(x,\xi) = (b(\xi)-\lambda)g(x,\xi),\ \ 0<\xi<x<1, \\ 2g(x,x) = \lambda x - \int_0^x b(s)ds,\ \ 0<x<1, \\ g_\xi(x,0) = 0,\ \ 0<\xi<1. \end{cases} \tag{3.4}$$

Then the integral transformation (3.3) can equivalently convert the dynamic (3.2) into

$$\begin{cases} {}_0^C D_t^\alpha \omega(x,t) = \triangle\omega(x,t) - \lambda\omega(x,t) \text{ in } (0,1)\times[t_k, t_{k+1}), \\ \omega_x(0,t) = 0 \text{ in } [t_k, t_{k+1}), \\ \omega(1,t) = u(t_k) + d(t) - \int_0^1 g(1,\xi)y(\xi,t)d\xi \text{ in } [t_k, t_{k+1}), \\ \omega(x,0) = \omega_0(x) \text{ in } (0,1) \end{cases} \tag{3.5}$$

for some constant $\lambda > 0$.

3.2. Event-triggered controller design

Before stating our main results in this section, we give an estimation of the unknown time-varying input uncertainties following the ideas of Zhong and Rees [45] and Ren et al. [34].

To estimate the unknown time-varying uncertainties, we design a low-pass filter F as follows:

$$\hat{d}(t) = \mathcal{L}^{-1}\{F(s)\}(t) * d(t), \tag{3.6}$$

where $\hat{d}$ denotes the estimation of d, $\mathcal{L}^{-1}$ is the inverse Laplace operator, $*$ represents the convolution operator and F satisfies

$$\lim_{s\to 0} s(1-F(s)) = 0. \tag{3.7}$$

Let $\tilde{d}(t) = d(t) - \hat{d}(t)$. By $\lim_{s\to 0} s\mathcal{L}\{\tilde{d}\}(s) = \lim_{s\to 0} s\mathcal{L}\left\{d-\hat{d}\right\}(s) = \lim_{s\to 0} s(1-F(s))\mathcal{L}\{d\}(s) = 0$. If $d(t)$ is bounded, the final value theorem yields that $\lim_{t\to\infty}\tilde{d}(t) = 0$.

Design the event-driven boundary controller as

$$u(t_k) = \int_0^1 g(1,\xi)y(\xi,t_k)d\xi - \hat{d}(t_k), \tag{3.8}$$

we get that

$$\begin{aligned} u(t_k) &= \int_0^1 g(1,\xi)y(\xi,t_k)d\xi - \hat{d}(t_k) + \hat{d}(t) - \hat{d}(t) \\ &= \int_0^1 g(1,\xi)y(\xi,t_k)d\xi - \hat{d}(t_k) + \hat{d}(t) - \mathcal{L}^{-1}\{F(s)\}(t) * (y(1,t) - u(t_k)) \end{aligned}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{1-F(s)}\right\}(t) * \int_0^1 g(1,\xi)y(\xi,t_k)d\xi - \mathcal{L}^{-1}\left\{\frac{1}{1-F(s)}\right\}(t) * \left(\hat{d}(t_k) - \hat{d}(t)\right)$$
$$-\mathcal{L}^{-1}\left\{\frac{F(s)}{1-F(s)}\right\}(t) * y(1,t). \tag{3.9}$$

Assume that the first event happens at $t_0 = 0$, we design that the next instant t_{k+1} is determined by

$$t_{k+1} = \min\left\{t > t_k : \|y(\cdot,t) - y(\cdot,t_k)\| + \left|\hat{d}(t) - \hat{d}(t_k)\right| \geqslant \breve{e}E_\alpha(-\mu t^\alpha)\right\},\ k \in \mathbb{N}, \tag{3.10}$$

where $\breve{e}$ is the event threshold, $\mu > 0$ is a given constant and $\{t_k\}_{k\in\mathbb{N}}$ represents the event-driven instants to show when the actuator signal is updated. Different from a time-triggered scheme, the event generator (3.10) only supervises the difference between L^2-norm of the states and the estimation of input uncertainties sampled in discrete instants having no interest in what happens in between updates. Moreover, we see in the end of this section that the release instants $\{t_k\}_{k\in\mathbb{N}}$ constructed satisfies $\min_{k\in\mathbb{N}}\{t_{k+1} - t_k\} > 0$. This could exclude the occurrence of Zeno phenomenon.

Remark 3.1. It is worth mentioning that the sampled state satisfying the inequality (3.10) will be sent out to the controller side. With this, compared with some available control strategics (see e.g. [6,13]), the burden of the network communication is significantly reduced so as to save the communication bandwidth in the network and the transmission energy.

Note that system (3.2) can be equivalently converted to (3.5) via the inevitable integral transformation (3.3) if the boundary controller is chosen as (3.8). From Proposition 3.1, there exists a positive constant ν such that

$$\|y(\cdot,t)\| \leqslant \nu\|\omega(\cdot,t)\| \text{ and } \|\omega_0\| \leqslant \nu\|y_0\|. \tag{3.11}$$

In what follows, we therefore, focus on studying the stability of system (3.5) with

$$\omega(1,t) = \tilde{d}(t) + \int_0^1 g(1,\xi)(y(\xi,t_k) - y(\xi,t))d\xi - \hat{d}(t_k) + \hat{d}(t) \tag{3.12}$$

under the event-triggered rule (3.10). We then get the following result.

Theorem 3.1. *Suppose that (3.7) and all conditions of Proposition 3.1 are satisfied. Let $u(t)$ be designed as (3.9). If both $\omega_x(1, t)$ and $d(t)$ are bounded, the Laplace transforms of $\omega^2(\cdot, t)$, $\omega_x(\cdot, t)$ and d exit. Then the closed-loop system (1.1) under the event-triggered rule (3.10) is asymptotically stable.*

The following lemma plays a central role in the proof of Theorem 3.1.

Lemma 3.1 ([1,3]). *Let ϕ: $[0, \infty) \to \mathbf{R}$ be a differentiable function. Then, for any given $t \geq 0$,*

$$\frac{1}{2}{}_0^CD_t^\alpha\phi^2(t) \leqslant \phi(t){}_0^CD_t^\alpha\phi(t),\ \alpha \in (0,1). \tag{3.13}$$

Proof of Theorem 3.1. Observing that $\omega(\cdot, t)$ is a differentiable function with respect to t, let

$$W(t) = \frac{1}{2}\int_0^1 \omega(x,t)^2dx. \tag{3.14}$$

Lemma 3.1 and Eq. (3.12) yield that

$$\begin{aligned}{}_0^CD_t^\alpha W(t) &= \frac{1}{2}\int_0^1 {}_0^CD_t^\alpha\omega(x,t)^2dx \leqslant \int_0^1 \omega(x,t){}_0^CD_t^\alpha\omega(x,t)dx \\ &= \int_0^1 \omega(x,t)\omega_{xx}(x,t)dx - \lambda\int_0^1 \omega(x,t)^2dx \\ &= \omega_x(1,t)\tilde{d}(t) + \omega_x(1,t)\left[\hat{d}(t) - \hat{d}(t_k)\right] - \int_0^1 \omega_x(x,t)^2dx \\ &\quad +\omega_x(1,t)\int_0^1 g(1,\xi)(y(\xi,t_k) - y(\xi,t))d\xi - \lambda\int_0^1 \omega(x,t)^2dx \\ &\leqslant \omega_x(1,t)\left(\tilde{d}(t) + \delta_1(t)\right) - 2\lambda W(t),\end{aligned} \tag{3.15}$$

where $\delta_1(t) = \hat{d}(t) - \hat{d}(t_k) + \int_0^1 g(1,\xi)(y(\xi,t_k) - y(\xi,t))d\xi$. Set

$$Q(t) = \omega_x(1,t)(\tilde{d}(t) + \delta_1(t)) - 2\lambda W(t) - {}_0^CD_t^\alpha W(t) \geqslant 0. \tag{3.16}$$

Taking Laplace transform on both sides of (3.16) gives $\mathcal{L}(W)(s) = \frac{s^{\alpha-1}W(0)+\mathcal{L}(\omega_x(1,\cdot)(\tilde{d}+\delta_1))(s)-\mathcal{L}(Q)(s)}{s^\alpha+2\lambda}$, where $W(0) = \frac{1}{2}\int_0^1 \omega(x,0)^2dx \geqslant 0$. It then follows from the uniqueness, existence theorem [33] that the unique solution of (3.16) satisfies

$$W(t) = E_\alpha(-2\lambda t^\alpha)W(0) + \left[\omega_x(1,t)(\tilde{d}(t) + \delta_1(t))\right] * \left[t^{\alpha-1}E_{\alpha,\alpha}(-2\lambda t^\alpha)\right] - Q(t) * \left[t^{\alpha-1}E_{\alpha,\alpha}(-2\lambda t^\alpha)\right]. \tag{3.17}$$

Since $Q(t) \geq 0$ and $t^{\alpha-1}E_{\alpha,\alpha}(-2\lambda t^{\alpha}) > 0$ for all $t > 0$,

$$W(t) \leqslant E_{\alpha}(-2\lambda t^{\alpha})W(0) + \left[\omega_x(1,t)(\tilde{d}(t) + \delta_1(t))\right] * \tfrac{E_{\alpha,\alpha}(-2\lambda t^{\alpha})}{t^{1-\alpha}}, \ t > 0. \tag{3.18}$$

The boundedness of $\omega_x(1, t)$ implies that there exits a constant $M_{\omega} > 0$ such that $|\omega_x(1, t)| \leq M_{\omega}$. From (3.7), we obtain that $\tilde{d}(t) \to 0$ as $t \to \infty$. Taking into account

$$\lim_{t\to\infty} \int_0^t \tfrac{E_{\alpha,\alpha}(-2\lambda\tau^{\alpha})}{\tau^{1-\alpha}} d\tau = \lim_{t\to\infty} \sum_{k=0}^{\infty} \int_0^t \tfrac{(-2\lambda)^k \tau^{\alpha k+\alpha-1}}{\Gamma(\alpha k+\alpha)} d\tau = \lim_{t\to\infty} \tfrac{1-E_{\alpha}(-2\lambda t^{\alpha})}{2\lambda} = \tfrac{1}{2\lambda}, \tag{3.19}$$

one has $t^{\alpha-1}E_{\alpha,\alpha}(-2\lambda\tau^{\alpha}) \in L^1[0,\infty)$ and then

$$\tilde{d}(t) * \left[\frac{E_{\alpha,\alpha}(-2\lambda t^{\alpha})}{t^{1-\alpha}}\right] \to 0 \text{ as } t \to \infty. \tag{3.20}$$

This is true following from the fact that the convolution of an L^1 function with a function tending to zero does, itself, tend to zero [6]. Moreover, the event-triggered rule (3.10) implies that

$$\delta_1(t) \leqslant \breve{e}\max\{C_g, 1\}E_{\alpha}(-\mu t^{\alpha}) \text{ when } \ t \in [t_k, t_{k+1}), \tag{3.21}$$

where $C_g = \max |g(x, \xi)|$ is a constant defined as in Appendix A.1. Then Lemma 2.2 yields that

$$\begin{aligned} \delta_1(t) * \frac{E_{\alpha,\alpha}(-2\lambda t^{\alpha})}{t^{1-\alpha}} &\leqslant \breve{e}\max\{C_g, 1\} \int_0^t \frac{E_{\alpha,\alpha}(-2\lambda(t-s)^{\alpha})}{(t-s)^{1-\alpha}} E_{\alpha}(-\mu s^{\alpha}) ds \\ &\leqslant \frac{M\breve{e}\max\{C_g, 1\}}{\mu} \int_0^t \frac{E_{\alpha,\alpha}(-2\lambda(t-s)^{\alpha})}{(t-s)^{1-\alpha}} s^{-\alpha} ds \\ &\leqslant \frac{M\breve{e}\max\{C_g, 1\}\Gamma(1-\alpha)}{\mu} E_{\alpha}(-2\lambda t^{\alpha}). \end{aligned} \tag{3.22}$$

Furthermore, since $E_{\alpha}(-2\lambda t^{\alpha}) \to 0$ as $t \to \infty$, we conclude that system (1.1) is asymptotically stable under the event-driven rule (3.10) and the proof is finished. □

3.3. Minimum inter-event time

To avoid the Zeno phenomenon, a positive lower bounded minimum inter-event time should be guaranteed [7].

Theorem 3.2. *Suppose that all conditions of Theorem 3.1 hold. Then the minimum inter-event time $T_{\min}$ given by*

$$T_{\min} = \min_{k=0,1,2,\dots} \{t_{k+1} - t_k\} \tag{3.23}$$

is lower bounded provided that t_k is defined as (3.10).

Proof. For any $k = 0, 1, 2, \dots,$ let $\bar{e}(x,t) = y(x,t) - y(x,t_k), \ t \in [t_k, t_{k+1})$. Then the definition of Caputo fractional derivative leads to

$${}_0^C D_t^{\alpha} \bar{e}(x,t) = {}_0^C D_t^{\alpha} y(x,t) = A\bar{e}(x,t) + Ay(x,t_k) \tag{3.24}$$

with boundary conditions $\bar{e}_x(0,t) = 0, \ \bar{e}(1,t) = d(t) - d(t_k)$, initial condition $\bar{e}_0(x) = 0$ and $\bar{e}(x,t_k) = 0$.

By Lemma 2.4, together with $\bar{e}(x,t_k) = 0$, the solution of system (3.24) can be given using the spectral theory of operator A, namely,

$$\begin{aligned} \bar{e}(x,t) = &\sum_{n=1}^{\infty} \int_{t_k}^t \frac{E_{\alpha,\alpha}((b(x)-\lambda_n)\tau^{\alpha})}{\tau^{1-\alpha}} d\tau (Ay(\cdot,t_k), \xi_n)\xi_n(x) \\ &+ \sum_{n=1}^{\infty} \int_0^t \frac{E_{\alpha,\alpha}((b(x)-\lambda_n)(t-\tau)^{\alpha})}{(t-\tau)^{1-\alpha}} (d(\tau) - d(t_k)) d\tau \frac{\partial \xi_n}{\partial x}(1)\xi_n(x) \\ &- \sum_{n=1}^{\infty} \int_0^{t_k} \frac{E_{\alpha,\alpha}((b(x)-\lambda_n)(t_k-\tau)^{\alpha})}{(t_k-\tau)^{1-\alpha}} (d(\tau) - d(t_k)) d\tau \frac{\partial \xi_n}{\partial x}(1)\xi_n(x), \ t \in [t_k, t_{k+1}). \end{aligned} \tag{3.25}$$

Since b is positive and large enough, without loss of generality, suppose that $b^* = \max_{x\in[0,1]} b(x) > \pi^2$, one has $b(x) - \lambda_n \leqslant b^* - \pi^2$. Notice that $E_{\alpha,\alpha}(t^{\alpha})$ is an increasing function [18], for any $t > \tau \geq 0$, one has $E_{\alpha,\alpha}((b(x)-\lambda_n)\tau^{\alpha}) \leqslant E_{\alpha,\alpha}((b^*-\pi^2)t^{\alpha})$. Besides, it follows from the boundedness of $d(t)$ that $|d(t) - d(t_k)| \leqslant C_1$ for some constant $C_1 > 0$, and then

$$\|\bar{e}(\cdot,t)\| \leqslant \tfrac{\|Ay(\cdot,t_k)\| E_{\alpha,\alpha}((b^*-\pi^2)t^{\alpha})}{\alpha} \left(t^{\alpha} - t_k^{\alpha}\right) + \sqrt{2} C_1 \pi c^* \delta_f(t, t_k), \ t \in [t_k, t_{k+1}), \tag{3.26}$$

where $c^*=\left(\sum_{n=1}^{\infty}\frac{\left(n-\frac{1}{2}\right)^2}{\left(n^2\pi^2-b^*\right)^2}\right)^{\frac{1}{2}}$, $\delta_f(t,t_k)=3\left\{\sum_{n=1}^{\infty}\left(f_n(t)-f_n(t_k)\right)^2\right\}^{1/2}+\left\{\sum_{n=1}^{\infty}f_n^2(t-t_k)\right\}^{1/2}$ and $f_n(t)=E_\alpha((b^*-n^2\pi^2)t^\alpha)$. Moreover, (3.10) yields that when $\|\bar{e}(\cdot,t)\|+\left|\hat{d}(t)-\hat{d}(t_k)\right|\geqslant\breve{e}E_\alpha(-\mu t^\alpha)$, a new event will occur. Therefore, a lower bound time $t_*>t_k$ can be found such that

$$\frac{\|Ay(\cdot,t_k)\|E_{\alpha,\alpha}((b^*-\pi^2)t_*^\alpha)}{\alpha E_\alpha(-\mu t_*{}^\alpha)}\left(t_*^\alpha-t_k^\alpha\right)+\frac{\sqrt{2}C_1\pi c^*\delta_f(t_*,t_k)}{E_\alpha(-\mu t_*{}^\alpha)}+\frac{\left|\hat{d}(t_*)-\hat{d}(t_k)\right|}{E_\alpha(-\mu t_*{}^\alpha)}=\breve{e}.$$

This implies that the minimum inter-event time $T_{\min}:=t_*-t_k$ is lower bounded and the proof is finished. □

4. Event-triggered boundary output feedback control

4.1. Observer design

In order to consider the event-triggered boundary output feedback stabilization problem of system (1.1), it is supposed that the measurement is given by a pointwise sensor to observe the value of $y(x, t)$ at $x=0$ with $y(0,t)\not\equiv 0$:

$$z(t)=Cy(x,t):=y(0,t). \tag{4.1}$$

To overcome the problems caused by the lack of full-state measurements, we design an extended Luenberger observer that embeds within the networked sensor estimating the whole states of system (1.1) as follows:

$$\begin{cases} {}_0^CD_t^\alpha\hat{y}(x,t)=A\hat{y}(x,t)+K_0(x)(z(t)-\hat{z}(t)) \text{ in } (0,1)\times[0,\infty),\\ \hat{y}_x(0,t)=K_1(z(t)-\hat{z}(t)),\ \hat{y}(1,t)=u(t)+\hat{d}(t) \text{ in } [0,\infty),\\ \hat{y}(x,0)=\hat{y}_0(x) \text{ in } (0,1),\end{cases} \tag{4.2}$$

where $\hat{z}(t)=\hat{y}(0,t)$. Then the observer estimation error $e(x,t):=y(x,t)-\hat{y}(x,t)$ with $e_0(x)=y_0(x)-\hat{y}_0(x)$ satisfies

$$\begin{cases} {}_0^CD_t^\alpha e(x,t)=Ae(x,t)-K_0(x)e(0,t) \text{ in } (0,1)\times[0,\infty),\\ e_x(0,t)=-K_1e(0,t),\ e(1,t)=\tilde{d}(t) \text{ in } [0,\infty),\\ e(x,0)=e_0(x) \text{ in } (0,1).\end{cases} \tag{4.3}$$

4.2. Equivalent transforms via backstepping

i) For the error dynamic system (4.3), consider the integral transformation

$$e(x,t)=\theta(x,t)-\int_0^x m(x,\xi)\theta(\xi,t)d\xi \tag{4.4}$$

with $\theta_0(x)=e_0(x)-\int_0^x m(x,\xi)e_0(\xi)d\xi$, we obtain the following result, whose proof can be found in Appendix A.2.

Proposition 4.1. *Suppose that the observer gain $K_0(x)$ and K_1 are designed as*

$$K_0(x)=m_\xi(x,0) \text{ and } K_1=m(0,0). \tag{4.5}$$

Then if the kernel $m(x,\xi)$ is chosen satisfying

$$\begin{cases} m_{xx}(x,\xi)-m_{\xi\xi}(x,\xi)=-(b(\xi)+\lambda)m(x,\xi), 0<\xi<x<1,\\ 2\frac{d}{dx}m(x,x)=\lambda+b(x),\ \ 0<x<1,\\ m(1,\xi)=0,\ \ 0<\xi<1,\end{cases} \tag{4.6}$$

the integral transformation (4.4) can equivalently convert the error dynamic (4.3) into

$$\begin{cases} {}_0^CD_t^\alpha\theta(x,t)=(\triangle-\lambda)\theta(x,t) \text{ in } (0,1)\times[0,\infty),\\ \theta_x(0,t)=0,\ \theta(1,t)=\tilde{d}(t) \text{ in } [0,\infty),\\ \theta(x,0)=\theta_0(x) \text{ in } (0,1)\end{cases} \tag{4.7}$$

for some constant $\lambda>0$.

ii) For the observer system (4.2) with event-triggered control strategy, suppose that $K_0(x)$ is still governed by (4.5), we rewrite system (4.2) as follows

$$\begin{cases} {}_0^CD_t^\alpha\hat{y}(x,t)=A\hat{y}(x,t)+K_0(x)e(0,t) \text{ in } (0,1)\times[t_k,t_{k+1}),\\ \hat{y}_x(0,t)=K_1e(0,t),\ \hat{y}(1,t)=u(t_k)+\hat{d}(t) \text{ in } [t_k,t_{k+1}),\\ \hat{y}(x,0)=\hat{y}_0(x) \text{ in } (0,1),\end{cases} \tag{4.8}$$

where $\{t_k\}_{k\in\mathbb{N}}$, representing the time when the control input u is updated, is a sequence of strictly increasing instants to be specified later.

Similarly, by the integral transformation

$$\rho(x,t)=\hat{y}(x,t)-\int_0^x h(x,\xi)\hat{y}(\xi,t)d\xi \tag{4.9}$$

with $\rho_0(x) = \hat{y}_0(x) - \int_0^x h(x,\xi)\hat{y}_0(\xi)d\xi$. The following proposition holds, whose proof is given in Appendix A.3.

Proposition 4.2. *Suppose that $K_0(x)$, K_1 still satisfy (4.5) and h solves (3.4). Then the observer (4.8) is equivalent to*

$$\begin{cases} {}_0^C D_t^\alpha \rho(x,t) = (\triangle - \lambda)\rho(x,t) + H(x)e(0,t) \text{ in } (0,1)\times[t_k, t_{k+1}), \\ \rho_x(0,t) = K_1 e(0,t) \text{ in } [t_k, t_{k+1}), \\ \rho(1,t) = u(t_k) + \hat{d}(t) - \int_0^1 h(1,\xi)\hat{y}(\xi,t)d\xi \text{ in } [t_k, t_{k+1}), \\ \rho(x,0) = \rho_0(x) \text{ in } (0,1) \end{cases} \tag{4.10}$$

for some constant $\lambda > 0$, where $H(x) = h(x,0)K_1 - \int_0^x h(x,\xi)K_0(\xi)d\xi + K_0(x)$.

4.3. Stabilization of system (1.1)

For the observer system (4.8), suppose that the controller is

$$u(t_k) = \int_0^1 h(1,\xi)\hat{y}(\xi,t_k)d\xi - \hat{d}(t_k) \tag{4.11}$$

and the event-triggered control strategy is designed as follows:

$$t_{k+1} = \min\left\{t > t_k : \left\|\hat{y}(\cdot,t) - \hat{y}(\cdot,t_k)\right\| + \left|\hat{d}(t) - \hat{d}(t_k)\right| \geqslant \breve{e}E_\alpha(-\mu t^\alpha)\right\},\ k \in \mathbb{N}, \tag{4.12}$$

where $t_0 = 0$, $\breve{e}$ is the event threshold, $\mu > 0$ is a given constant and $\{t_k\}_{k\in\mathbb{N}}$ denotes the event-triggered instants. In this case, the event generator supervises the difference between L^2-norm of the observer estimation states and the estimation of input uncertainties sampled in discrete instants. Similarly, the sampled observer estimation state satisfying the inequality (4.12) will be sent out to the controller side. Now we obtain the following main result.

Theorem 4.1. *Assume that (3.7), all conditions of Propositions 4.1 and 4.2 hold. Let $u(t)$ be designed as (4.11). Then if $d(t)$, $\theta_x(1, t)$ and $\rho_x(1, t)$ are bounded, the closed-loop system (1.1) under the event-triggered rule (4.12) is asymptotically stable.*

Proof. By Propositions 4.1 and 4.2, a constant $\nu_1 > 0$ can be found such that

$$\|e(\cdot,t)\| \leqslant \nu_1\|\theta(\cdot,t)\|,\ \|\theta_0\| \leqslant \nu_1\|e_0\| \text{ and } \|\hat{y}(\cdot,t)\| \leqslant \nu_1\|\rho(\cdot,t)\|,\ \|\rho_0\| \leqslant \nu_1\|\hat{y}_0\|. \tag{4.13}$$

Similarly, (3.7) implies that $\lim_{s\to 0} s\mathcal{L}\{\tilde{d}\}(s) = \lim_{s\to 0} s(1-F(s))\mathcal{L}\{d\}(s) = 0$. We have $\lim_{t\to\infty}\tilde{d}(t) = 0$ based on the final value theorem.

Choose the weighted Lyapunov functional candidate as follows

$$J(t) = \frac{s_1}{2}\int_0^1 \theta^2(x,t)dx + \frac{s_2}{2}\int_0^1 \rho^2(x,t)dx, \tag{4.14}$$

where $s_i \geq 0$, $i = 1, 2$ are two constants to be specified later. By Lemma 3.1, one has

$$\begin{aligned} {}_0^C D_t^\alpha J(t) &\leqslant s_1\int_0^1 \theta(x,t){}_0^C D_t^\alpha\theta(x,t)dx + s_2\int_0^1 \rho(x,t){}_0^C D_t^\alpha\rho(x,t)dx \\ &= -2\lambda J(t) + s_1\theta_x(1,t)\tilde{d}(t) - s_1\int_0^1 \theta_x^2(x,t)dx \\ &\quad + s_2\rho_x(1,t)\rho(1,t) - s_2\int_0^1 \rho_x^2(x,t)dx \\ &\quad + s_2\left[\int_0^1 \rho(x,t)H(x)dx - K_1\rho(0,t)\right]\theta(0,t). \end{aligned} \tag{4.15}$$

Applying Young inequality and the following inequality

$$\theta^2(0,t) \leqslant \theta^2(1,t) + \int_0^1 \theta_x^2(x,t)dx = \tilde{d}^2(t) + \int_0^1 \theta_x^2(x,t)dx, \tag{4.16}$$

there is

$$\begin{aligned} & s_2\left[\int_0^1 \rho(x,t)H(x)dx - K_1\rho(0,t)\right]\theta(0,t) \\ & \leqslant \frac{1}{4}\theta^2(0,t) + s_2^2\left[\int_0^1 \rho(x,t)H(x)dx - K_1\rho(0,t)\right]^2 \end{aligned}$$

$$\leqslant \frac{1}{4}\tilde{d}^2(t) + \frac{1}{4}\int_0^1 \theta_x^2(x,t)dx + s_2^2 M_H \|\rho(\cdot,t)\|^2 + s_2^2 K_1^2 \rho^2(1,t) + s_2^2 K_1^2 \int_0^1 \rho_x^2(x,t)dx$$
$$\leqslant \frac{1}{4}\tilde{d}^2(t) + s_2^2 K_1^2 \rho^2(1,t) + 2s_2 M_H J(t) + \frac{1}{4}\int_0^1 \theta_x^2(x,t)dx + s_2^2 K_1^2 \int_0^1 \rho_x^2(x,t)dx, \tag{4.17}$$

where $M_H = \max\limits_{x\in[0,1]} H(x)$. This, together with the boundedness of $\rho_x(1, t)$ and $\theta_x(1, t)$, shows that

$$\begin{aligned} {}_0^C D_t^\alpha J(t) \leqslant & -2(\lambda - s_2 M_H)J(t) - \left(s_1 - \frac{1}{4}\right)\int_0^1 \theta_x^2(x,t)dx \\ & - s_2\left(1 - s_2K_1^2\right)\int_0^1 \rho_x^2(x,t)dx + s_1|\theta_x(1,t)|\tilde{d}(t) \\ & + \frac{1}{4}\tilde{d}^2(t) + s_2|\rho_x(1,t)|\rho(1,t) + s_2^2K_1^2\rho^2(1,t). \end{aligned} \tag{4.18}$$

Moreover, if $s_1 > 1/4$ and $s_2 < \min\left\{1/K_1^2, \lambda/M_H\right\}$, it yields that

$${}_0^C D_t^\alpha J(t) \leqslant -2(\lambda - s_2 M_H)J(t) + \delta_2(t), \tag{4.19}$$

where $\delta_2(t) = s_1|\theta_x(1,t)|\tilde{d}(t) + \frac{1}{4}\tilde{d}^2(t) + s_2|\rho_x(1,t)|\rho(1,t) + s_2^2K_1^2\rho^2(1,t)$. Define $Q_2(t) = \delta_2(t) - 2(\lambda - s_2M_H)J(t) - {}_0^C D_t^\alpha J(t) \geqslant 0$. We get that

$$\mathcal{L}(J)(s) = \frac{s^{\alpha-1}J(0) + \mathcal{L}(\delta_2)(s) - \mathcal{L}(Q_2)(s)}{s^\alpha + 2(\lambda - s_2M_H)}, \tag{4.20}$$

where $J(0) = \frac{s_1}{2}\int_0^1 \theta_0^2(x)dx + \frac{s_2}{2}\int_0^1 \rho_0^2(x)dx \geqslant 0$. It then follows from the uniqueness, existence theorem in [33] that

$$\begin{aligned} J(t) = & E_\alpha(-2(\lambda - s_2M_H)t^\alpha)J(0) + \delta_2(t) * \left[t^{\alpha-1}E_{\alpha,\alpha}(-2(\lambda - s_2M_H)t^\alpha)\right] \\ & - Q_2(t) * \left[t^{\alpha-1}E_{\alpha,\alpha}(-2(\lambda - s_2M_H)t^\alpha)\right] \\ \leqslant & E_\alpha(-2(\lambda - s_2M_H)t^\alpha)J(0) + \delta_2(t) * \left[\frac{E_{\alpha,\alpha}(-2(\lambda - s_2M_H)t^\alpha)}{t^{1-\alpha}}\right]. \end{aligned} \tag{4.21}$$

Similar to the statements in the proof of Theorem 3.1, based on the boundedness of $\theta_x(1, t)$, we obtain that

$$s_1|\theta_x(1,t)|\tilde{d}(t) + \frac{1}{4}\tilde{d}^2(t) \to 0 \text{ as } t \to \infty. \tag{4.22}$$

Moreover, since $E_\alpha(-\mu t^\alpha) \in (0, 1]$ and $E_\alpha^2(-\mu t^\alpha) \leqslant E_\alpha(-\mu t^\alpha)$ for all $\mu > 0$, $t \geq 0$,

$$\left(s_2|\rho_x(1,t)|\rho(1,t) + s_2^2K_1^2\rho^2(1,t)\right) * \frac{E_{\alpha,\alpha}(-2(\lambda - s_2M_H)t^\alpha)}{t^{1-\alpha}} \leqslant C_2\Gamma(1-\alpha)E_\alpha(-2(\lambda - s_2M_H)t^\alpha),$$

where $C_2 = \left(s_2|\rho_x(1,t)| + s_2^2K_1^2\right)M\breve{e}\max\{C_h, 1\}/\mu$ and $C_h = \max|h(x,\xi)|$. Then system (1.1) is asymptotically stable under the event-triggered rule (4.12) and the proof is complete. □

4.4. Minimum inter-event time

To avoid the Zeno phenomenon, we obtain the following result.

Theorem 4.2. *If all conditions of Theorem 4.1 hold. Then the minimum inter-event time $T_{\min}$ given by*

$$T_{\min} = \min_{k=0,1,2,\ldots}\{t_{k+1} - t_k\} \tag{4.23}$$

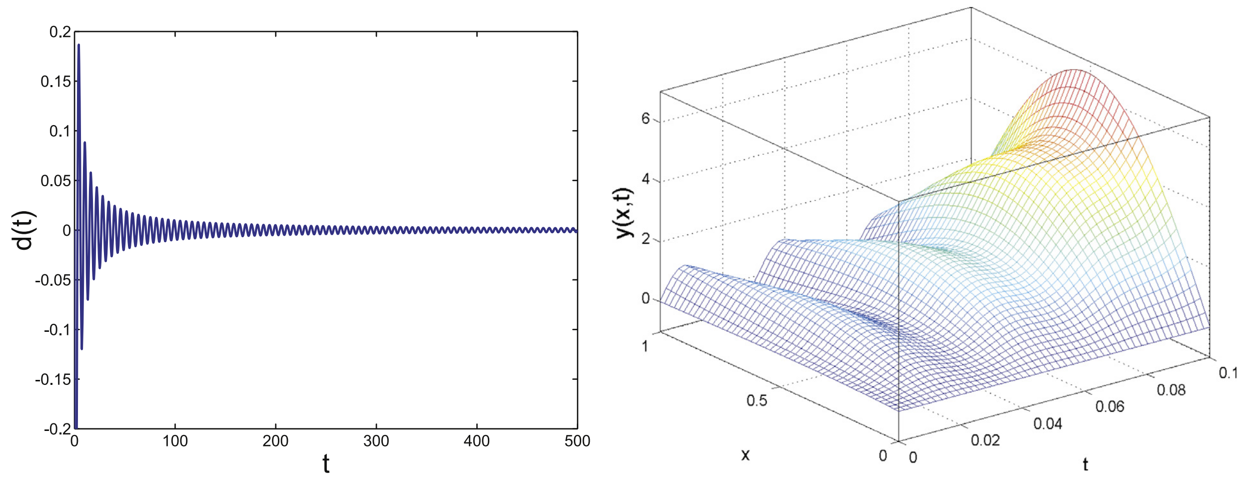
is lower bounded.

Proof. Let $\hat{e}(x,t) = \hat{y}(x,t) - \hat{y}(x,t_k)$, $t \in [t_k, t_{k+1})$. Consider system (4.8), we have $\hat{e}(x,t_k) = 0$ and

$$\begin{aligned} {}_0^C D_t^\alpha \hat{e}(x,t) &= {}_0^C D_t^\alpha \hat{y}(x,t) = A\hat{y}(x,t) + K_0(x)e(0,t) \\ &= A\hat{e}(x,t) + A\hat{y}(x,t_k) + K_0(x)e(0,t) \end{aligned} \tag{4.24}$$

with the initial condition $\hat{e}_0(x) = \hat{e}(x,0) = 0$ and the boundary conditions

$$\hat{e}_x(0,t) = K_1(e(0,t) - e(0,t_k)),\ \hat{e}(1,t) = \hat{d}(t) - \hat{d}(t_k). \tag{4.25}$$



(i) Uncertainties $d(t)$. (ii) Open loop $y(x,t)$ with $u \equiv 0$.

Fig. 2. Plot of the open loop system (1.1) with $u \equiv 0$.

Since $\hat{e}(x, t_k) = 0$, by Lemma 2.4, it yields that

$$\begin{aligned}\hat{e}(x,t) = &\sum_{n=1}^{\infty}\int_{t_k}^{t}\frac{E_{\alpha,\alpha}((b(x)-\lambda_n)\tau^{\alpha})}{\tau^{1-\alpha}}d\tau(A\hat{y}(\cdot,t_k),\xi_n)\xi_n(x)\\ &+\sum_{n=1}^{\infty}\int_{0}^{t}\frac{E_{\alpha,\alpha}((b(x)-\lambda_n)(t-\tau)^{\alpha})}{(t-\tau)^{1-\alpha}}\delta_3(\tau)d\tau\xi_n(x)\\ &-\sum_{n=1}^{\infty}\int_{0}^{t_k}\frac{E_{\alpha,\alpha}((b(x)-\lambda_n)(t_k-\tau)^{\alpha})}{(t_k-\tau)^{1-\alpha}}\delta_3(\tau)d\tau\xi_n(x),\ \ t\in[t_k,t_{k+1}),\end{aligned} \tag{4.26}$$

where $\delta_3(t) = (K_0, \xi_n)e(0,t) + \xi_n(0)\hat{e}_x(0,t) + \frac{\partial \xi_n}{\partial x}(1)\hat{e}(1,t)$. By (4.13) and the conclusion of Theorem 4.1, a constant $C_3 > 0$ can be found such that

$$C_3 \geqslant \max\left\{|e(0,t)|, |\hat{e}_x(0,t)|, |\hat{d}(t)-\hat{d}(t_k)|\right\}. \tag{4.27}$$

Similarly, suppose that $b^* = \max_{x\in[0,1]} b(x) > \pi^2$, one has

$$\begin{aligned}\|\hat{e}(\cdot,t)\| \leqslant &\frac{\|A\hat{y}(\cdot,t_k)\|E_{\alpha,\alpha}((b^*-\pi^2)t^{\alpha})}{\alpha}\left(t^{\alpha}-t_k^{\alpha}\right)+\frac{C_3\left(\|K_0\|+\sqrt{2}\right)E_{\alpha,\alpha}((b^*-\pi^2)t^{\alpha})}{\alpha}\left(t^{\alpha}-t_k^{\alpha}\right)\\ &+\sqrt{2}C_3\pi c^*\delta_f(t,t_k),\ \ t\in[t_k,t_{k+1}).\end{aligned} \tag{4.28}$$

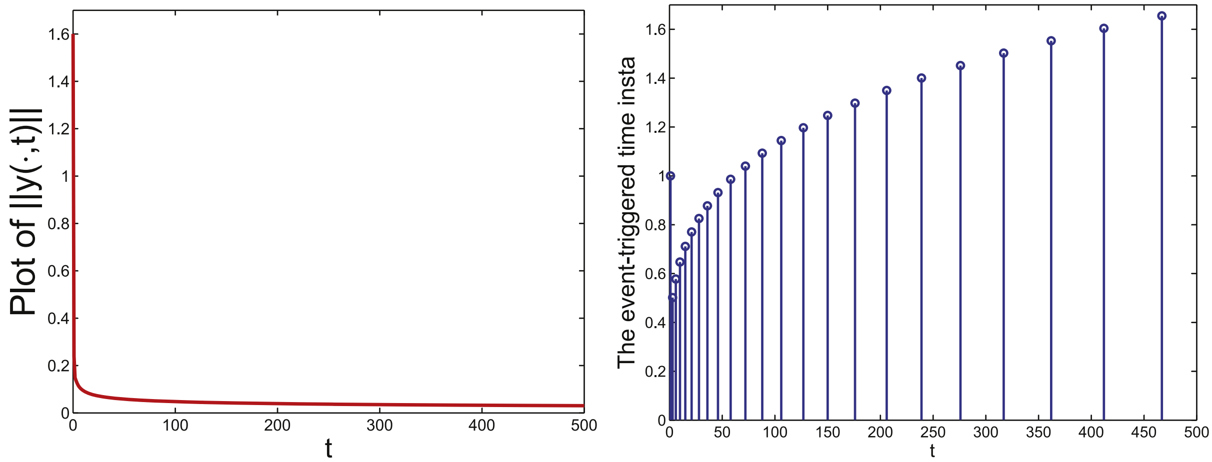
where c^* and $\delta_f(t,\, t_k)$ are defined as in the proof of Theorem 3.2. Observing that a new event will occur when $\left\|\hat{e}(\cdot,t)\right\| + \left|\hat{d}(t)-\hat{d}(t_k)\right| \geqslant \breve{e}E_{\alpha}(-\mu t^{\alpha})$. Therefore, a lower bound time $t_\flat > t_k$ can be found such that

$$\frac{\left\{\|A\hat{y}(\cdot,t_k)\|+C_3\left(\|K_0\|+\sqrt{2}\right)\right\}E_{\alpha,\alpha}((b^*-\pi^2)t_\flat^{\alpha})}{\alpha E_{\alpha}(-\mu t_\flat^{\alpha})}\left(t_\flat^{\alpha}-t_k^{\alpha}\right)+\frac{\sqrt{2}C_3\pi c^*\delta_f(t_\flat,t_k)}{E_{\alpha}(-\mu t_\flat^{\alpha})}+\frac{\left|\hat{d}(t_\flat)-\hat{d}(t_k)\right|}{E_{\alpha}(-\mu t_\flat^{\alpha})}=\breve{e}.$$

This implies that the minimum inter-event time $T_{\min} := t_\flat - t_k$ is lower bounded and the proof is finished. □

5. Numerical examples

In this section, we work out two simulate examples to test our results. To obtain the simulation results, the high-order approximations to Caputo fractional derivative introduced by Li and Zeng [24] is used. Choose the parameters in system (1.1) as $\alpha = 0.5$, $b = 15$ and $y_0(x) = \frac{x(1-x)}{2}e^{-x}$. Assume that $d(t) = \frac{\sin(200t)}{1+t}$, which is shown in (i) of Fig. 2. Then system (1.1) with $u \equiv 0$ is unstable as presented in Fig. 2 (ii) by observing that $15 \geq \pi^2$.



(1) Closed-loop $\|y(\cdot,t)\|$. (2) Event-triggered instants.

Fig. 3. Evolution of system (1.1) with event-triggered boundary state feedback controller.

Case 1: Event-triggered boundary state feedback control

To this end, let $\breve{e} = 0.05$ in (3.10) and let $\lambda = 1$ in all target systems. By Chapter 4 of [23], we get that the solution of hyperbolic PDE (3.4) is

$$g(x,\xi) = -14x\frac{I_1\left(\sqrt{14(x^2-\xi^2)}\right)}{\sqrt{14(x^2-\xi^2)}}, \tag{5.1}$$

where I_1 represents the modified Bessel functions of order one. Let the low-pass filter F be

$$F(s) = \frac{1}{s^\alpha + 1}. \tag{5.2}$$

Then (3.7) holds. By $\mathcal{L}\{t^{\beta-1}E_{\alpha,\beta}(\pm\lambda t^\alpha)\} = \frac{s^{\alpha-\beta}}{s^\alpha \mp \lambda}, \quad s \geqslant |\lambda|^{1/\alpha}$, we obtain that

$$\hat{d}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^\alpha+1}\right\}(t) * d(t) = \frac{E_{\alpha,\alpha}(-t^\alpha)}{t^{1-\alpha}} * d(t). \tag{5.3}$$

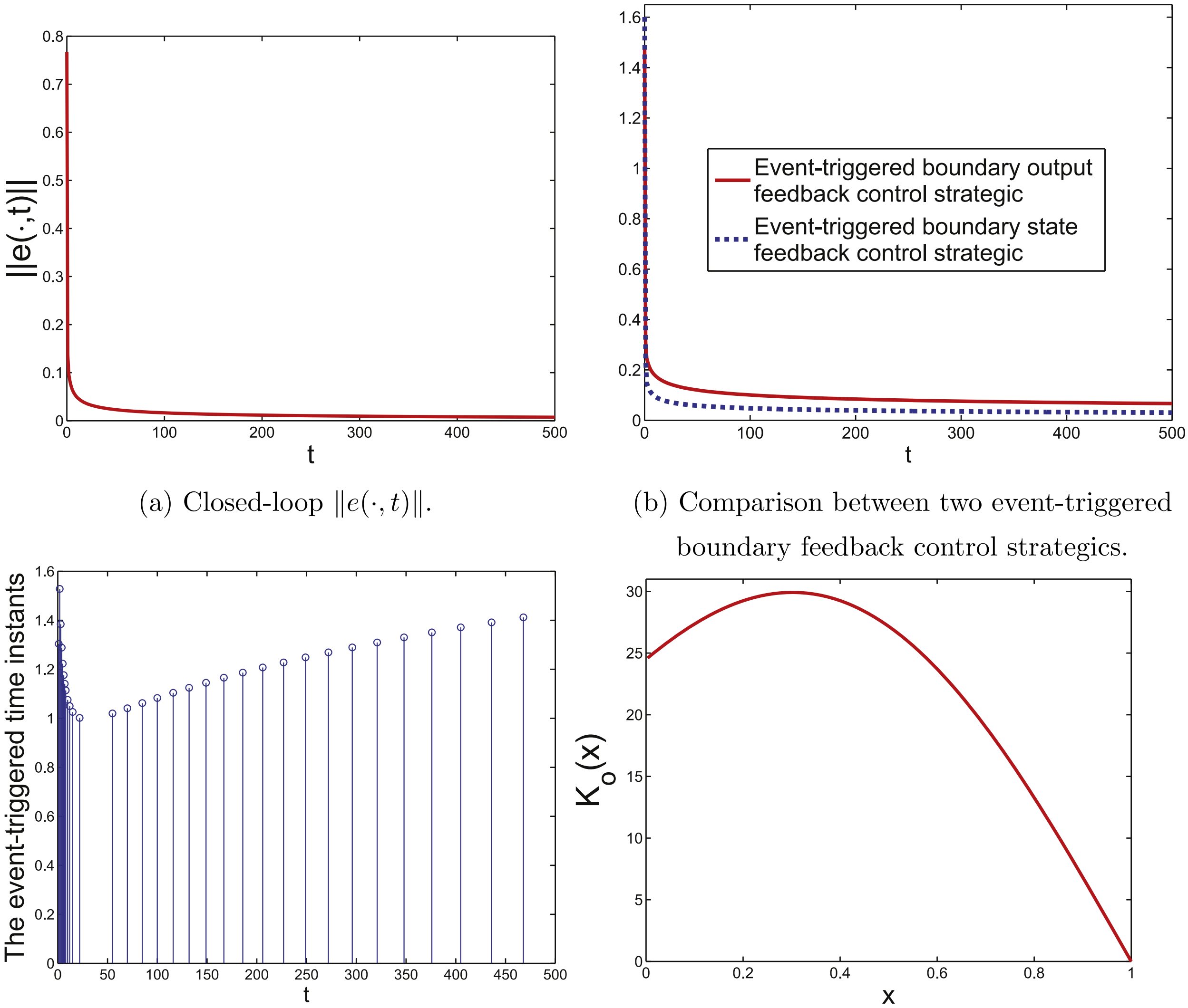
With this, we plot the spatial L^2-norm of the solution $y(x, t)$ to system (1.1) with event-triggered boundary state feedback controller (3.10) in (1) of Fig. 3. This shows that the event-triggered boundary state feedback controller can significantly asymptotically stabilize the system under consideration. Here the event generator is designed by calculating the difference between L^2-norm of the states and the estimation of input uncertainties sampled in discrete instants. Moreover, (2) of Fig. 3 shows the event-triggered instants when the control input is updated. In this simulate process, we choose the whole time interval as [0,500], which is divided into 500 parts. Then the number of the time-triggered instants is 500. Therefore, we conclude that the event-triggered strategy could significantly reduce the number of release time so as to relieve the burden of the network communication and then save the transmission energy while guaranteeing the desired levels of performance.

Case 2: Event-triggered boundary output feedback control

In this case, we continuous to consider the same system as in the case 1 and choose $F(s)$ as in (5.2) too. Besides, choose $\hat{y}_0(x) = \frac{x(1-x)}{4}e^{-x}$. Then $e_0(x) = \frac{x(1-x)}{4}e^{-x}$. It follows from the statements in Chapter 5 of [23] that $h(x,\xi) = g(x,\xi)$ and

$$m(x,\xi) = -14(1-x)\frac{I_1\left(\sqrt{14(2-x-\xi)(x-\xi)}\right)}{\sqrt{14(2-x-\xi)(x-\xi)}}. \tag{5.4}$$

The results shown in Fig. 4(a) and (b) implies that the unstable system can be asymptotically stabilized by the event-triggered boundary output feedback controller. Here the event generator is designed by supervising the difference between L^2-norm of the observer estimation states and the estimation of input uncertainties sampled in discrete instants. With this, comparison between two event-triggered boundary feedback control strategics is presented in Fig. 4(b). The corresponding instants are shown in (c) of Fig. 4. Similarly, we claim that the event-triggered boundary output feedback control strategy



(a) Closed-loop $\|e(\cdot,t)\|$.

(b) Comparison between two event-triggered boundary feedback control strategics.

(c) Event-triggered instants.

(d) The observer gain $K_0(x)$.

Fig. 4. Plots of system (1.1) with event-triggered boundary output feedback controller.

could also significantly reduce the number of release time while guaranteeing the desired performance. Besides, we obtain that $K_1 = -7$ and Fig. 4(d) shows the evolution of observer gain $K_0(x)$.

6. Conclusion

In this paper, both the event-triggered boundary state feedback controllers and event-triggered boundary output feedback controllers are designed via backstepping to asymptotically stabilize the time fractional reaction-diffusion systems with unknown input uncertainties in the context of sensor/actuator networks. It is shown that both two kinds of event-triggered controllers could significantly converge the estimation and guarantee the desirable control performance. This is appealing in many situations of real-world. Moreover, event-triggered control problems for more complex networked anomalous diffusion processes as well as optimal control problems are of great interest.

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Appendix A

A1. Proof of Proposition 3.1

Denote $g_\xi(x,x)=\frac{\partial}{\partial\xi}g(x,\xi)|_{\xi=x},\ g_x(x,x)=\frac{\partial}{\partial x}g(x,\xi)|_{\xi=x}$ and $\frac{d}{dx}g(x,x)=g_x(x,x)+g_\xi(x,x)$. Differentiating (3.3) with respect to x, we obtain that

$$\omega_{xx}(x,t)=y_{xx}(x,t)-\frac{d}{dx}g(x,x)y(x,t)-g(x,x)y_x(x,t)-g_x(x,x)y(x,t)-\int_0^x g_{xx}(x,\xi)y(\xi,t)d\xi. \tag{A.5}$$

By ${}_0^CD_t^\alpha y(x,t)=Ay(x,t),\ y_x(0,t)=0$ and

$$\begin{aligned}\int_0^x g(x,\xi)y_{\xi\xi}(\xi,t)d\xi \ &= \int_0^x g_{\xi\xi}(x,\xi)y(\xi,t)d\xi+g(x,x)y_x(x,t)-g(x,0)y_x(0,t)\\ &\quad -g_\xi(x,x)y(x,t)+g_\xi(x,0)y(0,t),\end{aligned} \tag{A.6}$$

it follows that

$$\begin{aligned}0&={}_0^CD_t^\alpha\omega(x,t)-\omega_{xx}(x,t)-\lambda\omega(x,t)\\
&={}_0^CD_t^\alpha y(x,t)-\int_0^x g(x,\xi){}_0^CD_t^\alpha y(\xi,t)d\xi-y_{xx}(x,t)+\left[\int_0^x g(x,\xi)y(\xi,t)d\xi\right]_{xx}\\
&\quad-\lambda y(x,t)+\lambda\int_0^x g(x,\xi)y(\xi,t)d\xi\\
&={}_0^CD_t^\alpha y(x,t)-y_{xx}(x,t)-\int_0^x g(x,\xi)y_{\xi\xi}(\xi,t)d\xi-\int_0^x g(x,\xi)b(\xi)y(\xi,t)d\xi\\
&\quad+\left[\int_0^x g(x,\xi)y(\xi,t)d\xi\right]_{xx}-\lambda y(x,t)+\lambda\int_0^x g(x,\xi)y(\xi,t)d\xi\\
&={}_0^CD_t^\alpha y(x,t)-y_{xx}(x,t)-\int_0^x g_{\xi\xi}(x,\xi)y(\xi,t)d\xi-g(x,x)y_x(x,t)+g(x,0)y_x(0,t)\\
&\quad+g_\xi(x,x)y(x,t)-g_\xi(x,0)y(0,t)-\int_0^x g(x,\xi)b(\xi)y(\xi,t)d\xi+\frac{d}{dx}g(x,x)y(x,t)\\
&\quad+g(x,x)y_x(x,t)+g_x(x,x)y(x,t)+\int_0^x g_{xx}(x,\xi)y(\xi,t)d\xi-\lambda y(x,t)+\lambda\int_0^x g(x,\xi)y(\xi,t)d\xi\\
&=\int_0^x y(\xi,t)\big\{g_{xx}(x,\xi)-g_{\xi\xi}(x,\xi)+(\lambda-b(\xi))g(x,\xi)\big\}d\xi+y(x,t)\left(b(x)-\lambda+2\frac{d}{dx}g(x,x)\right)\\
&\quad-g_\xi(x,0)y(0,t).\end{aligned} \tag{A.7}$$

Integrating $b(x)-\lambda+2\frac{d}{dx}g(x,x)=0$ with respect to x yields that $g(x,x)=\int_0^x\frac{\lambda-b(s)}{2}ds+g(0,0)$. Here $g(0,0)$ can be obtained using the boundary condition $y_x(0,t)=\omega_x(0,t)=0$, which shows that $g(0,0)=0$. Besides, the right side boundary condition gives

$$\omega(1,t)=y(1,t)-\int_0^1 g(1,\xi)y(\xi,t)d\xi. \tag{A.8}$$

Thus, if g is chosen satisfying (3.4), we obtain system (3.5).

Next, we show both the integral transformation (3.3) and its inverse are bounded.

By Krstić and Smyshlyaev [23] and Liu [27], the following existence results on the kernel system (3.4) holds.

Lemma A.1 ([23,27]). *If $b\in C^1[0,1]$, then system (3.4) has a unique bounded solution which is twice continuously differentiable in $0<\xi<x<1$.*

Define G: $L^2(0,1)\to L^2(0,1)$ as

$$\omega(x,\cdot)=(Gy)(x,\cdot):=y(x,\cdot)-\int_0^x g(x,\xi)y(\xi,\cdot)d\xi, \tag{A.9}$$

where g is the solution of (3.4). By Lemma Appendix A.1, since g is bounded, denote $C_g=\max|g(x,\xi)|$, it is not difficult to get that G is a bounded operator.

Set $\varphi(x,t)=\int_0^x g(x,\xi)y(\xi,t)d\xi$. Then $\omega(x,t)=y(x,t)-\varphi(x,t)$ and

$$\varphi(x,t)=\int_0^x g(x,\xi)\omega(\xi,t)d\xi+\int_0^x g(x,\xi)\varphi(\xi,t)d\xi. \tag{A.10}$$

Let $\varphi_0(x,t)=\int_0^x g(x,\xi)\omega(\xi,t)d\xi$ and $\varphi_n(x,t)=\int_0^x g(x,\xi)\varphi_{n-1}(\xi,t)d\xi$. We obtain that

$$|\varphi_0(x,t)|\leqslant C_g\|\omega(\cdot,t)\|,\ \ |\varphi_1(x,t)|\leqslant C_g^2\|\omega(\cdot,t)\|x, \tag{A.11}$$

$$|\varphi_2(x,t)| \leqslant \frac{C_g^3\|\omega(\cdot,t)\|}{2!}x^2, \ \ |\varphi_n(x,t)| \leqslant \frac{C_g^{n+1}\|\omega(\cdot,t)\|}{n!}x^n. \tag{A.12}$$

Therefore, the series $\varphi(x,t) = \sum_{n=0}^{\infty} \varphi_n(x,t)$ is absolutely and uniformly convergent and that forms the solution of (A.10). This means that G^{-1} exists and is bounded. The proof is completed.

A2. Proof of Proposition 4.1

Similar to the proof of Proposition 3.1, differentiating the transformation (4.4), by $\theta_x(0,t) = 0,$ we see

$$e_{xx}(x,t) = \theta_{xx}(x,t) - \frac{d}{dx}m(x,x)\theta(x,t) - m(x,x)\theta_x(x,t) - m_x(x,x)\theta(x,t) - \int_0^x m_{xx}(x,\xi)\theta(\xi,t)d\xi \tag{A.13}$$

and

$$\begin{aligned}
{}_0^C D_t^\alpha e(x,t) &= {}_0^C D_t^\alpha \theta(x,t) - \int_0^x m(x,\xi) {}_0^C D_t^\alpha \theta(\xi,t)d\xi \\
&= {}_0^C D_t^\alpha \theta(x,t) - \int_0^x m_{\xi\xi}(x,\xi)\theta(\xi,t)d\xi - m(x,x)\theta_x(x,t) + m_\xi(x,x)\theta(x,t) \\
&\quad - m_\xi(x,0)\theta(0,t) + \lambda\int_0^x m(x,\xi)\theta(\xi,t)d\xi.
\end{aligned} \tag{A.14}$$

It then follows from (4.3) and (A.13),(A.14) that

$$\begin{aligned}
0 &= {}_0^C D_t^\alpha \theta(x,t) - \int_0^x m_{\xi\xi}(x,\xi)\theta(\xi,t)d\xi - m(x,x)\theta_x(x,t) + m_\xi(x,x)\theta(x,t) \\
&\quad - m_\xi(x,0)\theta(0,t) + \lambda\int_0^x m(x,\xi)\theta(\xi,t)d\xi - \theta_{xx}(x,t) + \frac{d}{dx}m(x,x)\theta(x,t) \\
&\quad + m(x,x)\theta_x(x,t) + m_x(x,x)\theta(x,t) + \int_0^x m_{xx}(x,\xi)\theta(\xi,t)d\xi - b(x)\theta(x,t) \\
&\quad + b(x)\int_0^x m(x,\xi)\theta(\xi,t)d\xi + K_0(x)\theta(0,t) \\
&= \left[-b(x) - \lambda + 2\frac{d}{dx}m(x,x)\right]\theta(x,t) + \int_0^x \theta(\xi,t)\begin{Bmatrix} m_{xx}(x,\xi) - m_{\xi\xi}(x,\xi) \\ +(\lambda + b(x))m(x,\xi)\end{Bmatrix}d\xi \\
&\quad + (K_0(x) - m_\xi(x,0))\theta(0,t).
\end{aligned} \tag{A.15}$$

The boundary conditions of (4.3) and (4.7) yield respectively,

$$K_1 = m(0,0) \text{ and } m(1,\xi) = 0. \tag{A.16}$$

This implies that if m is chosen satisfying (4.6), system (4.3) can be transformed into (3.5).

Moreover, similar to the arguments in Part A.1, by Krstić and Smyshlyaev [23], it is not difficult to get that both the integral transformation (4.4) and its inverse are bounded. Then the proof of Proposition 4.1 is completed.

A3. Proof of Proposition 4.2

Similarly, for the transform (4.9), we have

$$\begin{aligned}
&{}_0^C D_t^\alpha \rho(x,t) - \rho_{xx}(x,t) - \lambda\rho(x,t) \\
&= {}_0^C D_t^\alpha \hat{y}(x,t) - \hat{y}_{xx}(x,t) - \int_0^x h(x,\xi)\hat{y}_{\xi\xi}(\xi,t)d\xi - \int_0^x h(x,\xi)b(\xi)\hat{y}(\xi,t)d\xi \\
&\quad - \int_0^x h(x,\xi)K_0(\xi)d\xi e(0,t) + \left[\int_0^x h(x,\xi)\hat{y}(\xi,t)d\xi\right]_{xx} - \lambda\hat{y}(x,t) \\
&\quad + \lambda\int_0^x h(x,\xi)\hat{y}(\xi,t)d\xi \\
&= b(x)\hat{y}(x,t) + K_0(x)e(0,t) - \int_0^x h_{\xi\xi}(x,\xi)\hat{y}(\xi,t)d\xi - h(x,x)\hat{y}_x(x,t) \\
&\quad + h(x,0)\hat{y}_x(0,t) + h_\xi(x,x)\hat{y}(x,t) - h_\xi(x,0)\hat{y}(0,t) - \int_0^x h(x,\xi)b(\xi)\hat{y}(\xi,t)d\xi
\end{aligned}$$

$$
\begin{aligned}
&-\int_0^x h(x,\xi)K_0(\xi)d\xi e(0,t) + \frac{d}{dx}h(x,x)\hat{y}(x,t) + h(x,x)\hat{y}_x(x,t) + h_x(x,x)\hat{y}(x,t) \\
&-\lambda\hat{y}(x,t) + \int_0^x h_{xx}(x,\xi)\hat{y}(\xi,t)d\xi + \lambda\int_0^x h(x,\xi)\hat{y}(\xi,t)d\xi \\
=&\int_0^x \hat{y}(\xi,t)\big\{h_{xx}(x,\xi) - h_{\xi\xi}(x,\xi) + (\lambda - b(\xi))h(x,\xi)\big\}d\xi - h_\xi(x,0)\hat{y}(0,t) \\
&+\hat{y}(x,t)\left(b(x) - \lambda + 2\frac{d}{dx}h(x,x)\right) + e(0,t)\left[h(x,0)K_1 - \int_0^x h(x,\xi)K_0(\xi)d\xi + K_0(x)\right].
\end{aligned}
\tag{A.17}
$$

Let $x=0$ and $x=1$ in (4.9). It follows respectively, that $\rho_x(0,t)=\hat{y}_x(0,t)=K_1e(0,t)$ if $h(0,0)=0$ and

$$
\rho(1,t) = u(t_k) + \tilde{d}(t_k) - \int_0^1 h(1,\xi)\hat{y}(\xi,t)d\xi. \tag{A.18}
$$

Then system (4.8) is equivalent to (4.10) provided that h is the solution of system (3.4).

Let H_2: $L^2(0,1)\to L^2(0,1)$ be $\omega^*(x,\cdot) = (H_2\hat{y})(x,\cdot) := \hat{y}(x,\cdot) - \int_0^x h(x,\xi)\hat{y}(\xi,\cdot)d\xi$. By Krstić and Smyshlyaev [23], since h is bounded, denote $C_h = \max|h(x,\xi)|$, we get that H_2 is a bounded operator. Similar to the proof of Proposition 3.1, H_2^{-1} exists and is bounded. The proof is finished.

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