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Variable Gain Feedback PD^{α}-Type Iterative Learning Control for Fractional Nonlinear Systems With Time-Delay

JING WANG[1], (Member, IEEE), CHENCHEN YU[1], YI LIU[1], DONG SHEN[1], (Senior Member, IEEE), AND YANGQUAN CHEN[2], (Member, IEEE)

[1]College of Information Science and Technology, Beijing University of Chemical Technology, Beijing 100029, China
[2]Mechatronics, Embedded Systems and Automation Lab, University of California at Merced, Merced, CA 95343, USA

Corresponding author: Jing Wang (jwang@mail.buct.edu.cn)

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ABSTRACT A variable gain feedback PD^{α}-type iterative learning control (ILC) update rate is proposed for the fractional-order nonlinear systems with time delay. The learning update rate combines the open-loop and closed-loop strategy, in which the system's current tracking error and the previous iterative control of the tracking error are simultaneously used to correct the control effect. So, the proposed method could both speed up the convergence rate along the iteration direction and reduce the tracking error along the time direction. Furthermore, the algorithm ensures that the system has good learning efficiency and control performance at the different running time and iterative batches due to the time-iteration-variable learning gain. The sufficient condition for the convergence of the proposed algorithm is analyzed. Finally, the validity of the proposed method is verified on a numerical example and a wind power generation system.

INDEX TERMS Nonlinear systems with time-delay, fractional order system, time-iteration-variable gain, feedback ILC.

I. INTRODUCTION

Iterative learning control (ILC) is one of the important research directions in the field of intelligent control and it also widely used in practice [1]. The advantages of ILC can be reflected in complex systems with nonlinear characteristic and the uncertain structural information. The basic ILC scheme takes full advantage of the previous implementation information. The repeated control trajectory is modified according to the tracking or performance errors obtained from the previous iterations. Then the desired trajectory is ultimately achieved, and the perfect control is completed [2]–[5].

The ILC strategy has developed tremendously in the integer order systems and has been widely used in many fields, such as industrial robot, medical system [6], [7]. However, some real systems are essentially described as the fractional order models in order to better characterize the dynamic process. So we should pay close attention to the fractional order system and its related control problem based on the fractional calculus theory. It includes the iterative learning control algorithm and the initial value problem of fractional linear system [8]–[10]. Fractional calculus originates in the 17th century and has been widely applied to physical and engineering practice now [11]–[17]. The fractional controllers also have been widely used to improve the performance and robustness of control systems [18]. The fractional calculus control theory is studied [19]–[21].

The fractional order iterative learning control (FOILC) is first proposed in 2001, and the ILC strategy is applied to the fractional system has become a new research hotspot [22]–[26]. Many FOILC problems are proposed in order to improve the control performance of linear or nonlinear systems [19]–[21], [27]. A D^{α}-type ILC update rate is first proposed and the detailed analysis was carried out [28], [29]. The PD^{α}-type ILC is studied in the linear time invariant system [30]. The time domain analysis of FOILC is discussed [31], [32].

Many industrial processes have time-lag, so it is necessary to study the stability and convergence of FOILC control systems with time-delay. Here the FOILC for the fractional

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nonlinear systems with time-lag is considered. How to deal with the trajectory tracking control problem for fractional nonlinear system with time-lag? To answer this question, a variable gain feedback PD^α-type ILC update rate is proposed and the sufficient condition of convergence for tracking errors are discussed in this paper. The main innovation is that for fractional nonlinear system with time-delay, the variable gain feedback structure is adopted on the basis of the closed-loop PD^α-type ILC algorithm. The closed-loop strategy aims to introduce the information of the current iteration which will improve the stability and convergence [33]. So the proposed ILC algorithm can realize the good tracking performance and the fast convergence. Another highlight of the proposed control algorithm lies on its variable gain strategy in which the learning gains are varied with the iterations k and time t. The variable gain are adjusted based on the system errors.

The structure of this paper is as follows. Section II introduces some of the basic definitions of the fractional calculus and the vector norms. Section III proposes the variable gain feedback PD^α-type ILC update rate for a class of fractional-order nonlinear systems with time-delay, and the system convergence are proved. Section IV gives two experiments, a numerical example and a wind power generation system, to verify the proposed method. Section V draws the conclusions and puts forward the future work.

II. PRELIMINARIES

First, we give some classical definitions of the fractional calculus and the vector norms [34]–[36], which will be used in the following sections of this paper.

Definition 1: The Caputo fractional-order integral with order $\alpha \in (0, 1)$ is defined as

$$ {}_{t_0}^{C}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, $$

when $t_0 = 0$,

$$ {}_{0}^{C}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau. $$

The Caputo fractional derivatives is defined as

$$ {}_{t_0}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau, \quad (n-1<\alpha<n). $$

The λ-norm of a function $e(t)$ is defined as

$$ \|e(t)\|_\lambda = \sup_{0\le t\le T} \left\{ e^{-\lambda t} \|e(t)\|_\infty \right\}, \quad t \in [0, T], $$

where the maximum norm of $e(t)$ is defined as

$$ \|e(t)\|_\infty = \max_{0\le t\le T} \{|e(t)|\}. $$

III. THE VARIABLE GAIN FEEDBACK PD^α-TYPE ILC

The fractional-order nonlinear system with time-delay can be expressed as

$$ \begin{cases} y_k^{(\alpha)}(t) = f(t, y_k(t), y_k(t-\tau), u_k(t)), & t \in [0, T], \\ y_k(t) = g_k(t), & t \in [-\tau, 0], \end{cases} \tag{1} $$

where $\alpha \in (0, 1)$, $y_k(t) \in R$ and $u_k(t) \in R$, $\cdot^{(\alpha)}$ represents the α_{th}-order Caputo derivative of t, $g_k(t)$ is a continuous function defined on $[-\tau, 0]$, and the function f satisfies

$$ \left\| \frac{\partial f}{\partial y_k(t-\tau)} \right\|_\infty \le a, \quad \left\| \frac{\partial f}{\partial u_k(t)} \right\|_\infty \le b, \quad \left\| \frac{\partial f}{\partial y_k(t)} \right\|_\infty \le c, \tag{2} $$

where a, b, and c are constants greater than zero, respectively. Assume that, for a given trajectory function $y_d(t)$, there exists an unique $u_d(t)$ satisfies

$$ \begin{cases} y_d^{(\alpha)}(t) = f(t, y_d(t), y_d(t-\tau), u_d(t)), & t \in [0, T], \\ y_d(t) = g_d(t), & t \in [-\tau, 0]. \end{cases} \tag{3} $$

Here $u_d(t)$ and $y_d(t)$ denote the desired system control input and reference system output, respectively. The tracking error of the system is $e_k(t) = y_d(t) - y_k(t)$. $t \in [-\tau, 0]$, $\tau > 0$, $g_k(t) = g_d(t)$, $u_k(t) = u_d(t)$, $e_k(t) = 0$; $t \in [0, T]$, $y_k(0) = y_d(0)$.

The variable gain feedback PD^α-type ILC update rate is proposed for fractional nonlinear systems with time-delay. The sufficient condition for the convergence of the proposed algorithm is analyzed and given by introducing the λ-norm. The fractional-order variable gain feedback PD^α-type ILC update rate is as follows,

$$ \begin{aligned} u_{k+1}(t) = u_k(t) &+ K_{p1}(k+1)e_k(t) + K_{d1}(k+1)e_k^{(\alpha)}(t) \\ &+ K_{p2}(t)e_{k+1}(t) + K_{d2}(t)e_{k+1}^{(\alpha)}(t), \end{aligned} \tag{4} $$

where $K_{p1}(k+1)$, $K_{d1}(k+1)$, $K_{p2}(t)$, $K_{d2}(t)$ are the variable gain functions that vary with the number of iterations k and time t, respectively. $K_{p1}(k+1)$, $K_{d1}(k+1)$ are defined as

$$ \begin{aligned} K_{p1}(k+1) &= r(k+1)K_{p1}(0), \\ K_{d1}(k+1) &= r(k+1)K_{d1}(0), \end{aligned} $$

where $K_{p1}(0)$ and $K_{d1}(0)$ are the initial values of the learning gains, and $0 < r(k) < 1$ is a monotonically decreasing function about the number of iterations k $(k = 0, 1, 2, \cdots)$. $K_{p2}(t)$, $K_{d2}(t)$ are defined as

$$ \begin{aligned} K_{p2}(t) &= K_{p2}(0)\left[\sigma\left(1 - \frac{2}{\pi}\arctan|e_{k+1}(t)|\right)\right], \\ K_{d2}(t) &= K_{d2}(0)\left[1 - \sigma\left(1 - \frac{2}{\pi}\arctan|e_{k+1}(t)|\right)\right], \end{aligned} $$

where $K_{p2}(0)$ and $K_{d2}(0)$ are the initial values of the learning gains, $0 < \sigma < 1$.

From a batch perspective, $K_{p1}(k+1)$ and $K_{d1}(k+1)$ are related to k. At the beginning of the iteration, $e_k(t)$ is relatively large and $K_{p1}(k+1)$ is relatively large. The control effect of the learning law is more obvious. When the iteration error $e_k(t)$ is gradually reduced with the iteration k increasing, $K_{p1}(k+1)$ becomes greater. It is found that the control correction effect remains steady as a condition as possible along the iteration direction. This ensures that the iterative process is gradually amended until stable. From the time point of view, $K_{p2}(t)$ and $K_{d2}(t)$ are related to $e_{k+1}(t)$.

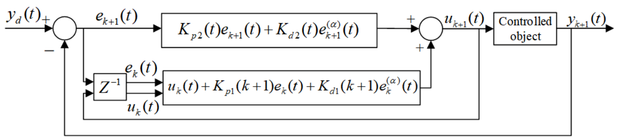


FIGURE 1. **Variable gain feedback PD^{α}-type ILC algorithm.**

At the initial time, $e_{k+1}(t)$ is relatively large and $K_{p2}(t)$ is relatively small. As t tends to T, $K_{p2}(t)$ increases properly, $K_{d2}(t)$ decreases appropriately, y gradually approaches y_d. Therefore, the proposed ILC strategy can adaptively adjust the control law and significantly obtain a fast convergence rate and high tracking precision.

The basic structure of variable gain feedback PD^{α}-type ILC algorithm is shown in Figure 1. The proposed ILC algorithm consists of two parts that simultaneously introduce the information from the current and history iterations. Their learning gains are varying with the number of iterations k and the system time t, respectively. Therefore, these two strategies, close-loop and variable gain, can significantly improve the system convergence, compared to the traditional open-loop iterative learning control.

Lemma 1: For the system (1) and the reference system (3), the tracking error $e_k^{(\alpha)}(t)$ is as follows,

$$\begin{aligned} e_k^{(\alpha)}(t) &= f_d - f_k \\ &= \frac{\partial f}{\partial u_k(t)}\Delta u_k(t) \\ &\quad + \frac{\partial f}{\partial y_k(t)} e_k(t) \\ &\quad + \frac{\partial f}{\partial y_k(t-\tau)} e_k(t-\tau), \end{aligned}$$

where f is a continuous differentiable function,

$$\begin{aligned} f_d &= f(t, y_d(t), y_d(t-\tau), u_d(t)), \\ f_k &= f(t, y_k(t), y_k(t-\tau), u_k(t)), \\ \Delta u_k(t) &= u_d(t) - u_k(t), \\ e_k(t) &= y_d(t) - y_k(t), \\ e_k(t-\tau) &= y_d(t-\tau) - y_k(t-\tau). \end{aligned}$$

Proof: Directly from the system (1) and its reference (3), we have

$$\begin{aligned} f_d - f_k &= f(t, y_d(t), y_d(t-\tau), u_d(t)) \\ &\quad - f(t, y_k(t), y_k(t-\tau), u_k(t)) \\ &= f(t, y_d(t), y_d(t-\tau), u_d(t)) \\ &\quad - f(t, y_k(t), y_d(t-\tau), u_d(t)) \\ &\quad + f(t, y_k(t), y_d(t-\tau), u_d(t)) \\ &\quad - f(t, y_k(t), y_k(t-\tau), u_d(t)) \\ &\quad + f(t, y_k(t), y_k(t-\tau), u_d(t)) \\ &\quad - f(t, y_k(t), y_k(t-\tau), u_k(t)) \\ &= \frac{\partial f}{\partial u_k(t)}\Delta u_k(t) \\ &\quad + \frac{\partial f}{\partial y_k(t)} e_k(t) \\ &\quad + \frac{\partial f}{\partial y_k(t-\tau)} e_k(t-\tau). \end{aligned}$$

□

Lemma 2: For the system (1) and the reference system (3), if satisfied (2) then there exists a sufficient large λ satisfying

$$\|e_k\|_\lambda \le O(\lambda^{-1})\|\Delta u_k\|_\lambda,$$

where $O(\lambda^{-1})$ denotes the same order infinitesimal function about parameter λ.

Proof: Implement the integral operation ${}_0D_t^{-\alpha}$ to both sides of Eq. (3) with the initial condition $y_k(0) = y_d(0)$,

$$\begin{aligned} \|e_k\|_\lambda &= \sup_{0\le t\le T}\left\{e^{-\lambda t}\|e_k\|_\infty\right\} \\ &= \sup_{0\le t\le T}\left\{e^{-\lambda t}\|D^{-\alpha}[f_d - f_k]\|_\infty\right\} \\ &\le \sup_{0\le t\le T}\int_0^t e^{-\lambda t}\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\|[f_d - f_k]\|_\infty ds \\ &\le \sup_{0\le t\le T}\int_0^t e^{-\lambda t}\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}[c\|e_k(s)\|_\infty \\ &\quad + a\|e_k(s-\tau)\|_\infty + b\|\Delta u_k(s)\|_\infty]ds \\ &\le \sup_{0\le t\le T}\int_0^t e^{-\lambda(t-s)}\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\sup_{0\le s\le T} e^{-\lambda s}[c\|e_k\|_\infty \\ &\quad + b\|\Delta u_k(s)\|_\infty]ds \\ &\quad + a\sup_{0\le t\le T}\int_0^t e^{-\lambda t}\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\|e_k(s-\tau)\|_\infty ds \\ &\le [c\|e_k\|_\lambda + b\|\Delta u_k\|_\lambda]\cdot\sup_{0\le t\le T}\int_0^t e^{-\lambda(t-s)}\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}ds \\ &\quad + a\sup_{0\le t\le T}\int_{-\tau}^{t-\tau} e^{-\lambda t}\frac{(t-s'-\tau)^{\alpha-1}}{\Gamma(\alpha)}\|e_k(s')\|_\infty ds' \\ &\le \frac{(1-e^{-\lambda T})T^\alpha}{\lambda\Gamma(\alpha+1)}[c\|e_k\|_\lambda + b\|\Delta u_k\|_\lambda] \\ &\quad + a\sup_{0\le t\le T}\int_{-\tau}^{t-\tau} e^{-\lambda t}\frac{(t-s-\tau)^{\alpha-1}}{\Gamma(\alpha)}\|e_k(s)\|_\infty ds, \end{aligned} \tag{5}$$

where

$$\begin{aligned} &\sup_{0\le t\le T}\int_{-\tau}^{t-\tau} e^{-\lambda t}\frac{(t-s-\tau)^{\alpha-1}}{\Gamma(\alpha)}\|e_k(s)\|_\infty ds \\ &\le \sup_{0\le t\le T}\int_{-\tau}^{0} e^{-\lambda t}\frac{(t-s-\tau)^{\alpha-1}}{\Gamma(\alpha)}\|e_k(s)\|_\infty ds \\ &\quad + \sup_{0\le t\le T}\int_{0}^{t-\tau} e^{-\lambda t}\frac{(t-s-\tau)^{\alpha-1}}{\Gamma(\alpha)}\|e_k(s)\|_\infty ds \\ &\le \sup_{0\le t\le T}\int_{0}^{t-\tau} e^{-\lambda s}\|e_k(s)\|_\infty e^{-\lambda(t-s)}\frac{(t-s-\tau)^{\alpha-1}}{\Gamma(\alpha)}ds \\ &\le \sup_{0\le t\le T}\int_{0}^{t-\tau}\left[\sup_{0\le t\le T} e^{-\lambda t}\|e_k(t)\|_\infty\right] \\ &\quad e^{-\lambda(t-s)}\frac{(t-s-\tau)^{\alpha-1}}{\Gamma(\alpha)}ds \end{aligned}$$

$$\le \sup_{0\le t\le T} e^{-\lambda t}\|e_k(t)\|_\infty \sup_{0\le t\le T}\int_0^{t-\tau} e^{-\lambda(t-s)}\frac{(t-s-\tau)^{\alpha-1}}{\Gamma(\alpha)}ds$$
$$\le \|e_k\|_\lambda \sup_{0\le t\le T}\int_0^{t-\tau} e^{-\lambda(t-s)}ds\int_0^{t-\tau}\frac{(t-s-\tau)^{\alpha-1}}{\Gamma(\alpha)}ds$$
$$\le \|e_k\|_\lambda \frac{(e^{-\lambda\tau}-e^{-\lambda T})(T-\tau)^\alpha}{\lambda\Gamma(\alpha+1)}. \tag{6}$$

Linking Eq. (5) and Eq. (6), we have

$$\|e_k\|_\lambda \le \frac{(1-e^{-\lambda T})T^\alpha}{\lambda\Gamma(\alpha+1)}[c\|e_k\|_\lambda + b\|\Delta u_k\|_\lambda] + \frac{a(e^{-\lambda\tau}-e^{-\lambda T})(T-\tau)^\alpha}{\lambda\Gamma(\alpha+1)}\|e_k\|_\lambda,$$
$$= \frac{(1-e^{-\lambda T})bT^\alpha}{\lambda\Gamma(\alpha+1)-c(1-e^{-\lambda T}T^\alpha)-a(e^{-\lambda\tau}-e^{-\lambda T})(T-\tau)^\alpha}\|\Delta u_k\|_\lambda,$$

where λ is large enough such that

$$\lambda\Gamma(\alpha+1)-c(1-e^{-\lambda T}T^\alpha)-a(e^{-\lambda\tau}-e^{-\lambda T})(T-\tau)^\alpha>0.$$

Define

$$O(\lambda^{-1}) = \frac{b(1-e^{-\lambda T})T^\alpha}{\lambda\Gamma(\alpha+1)-c(1-e^{-\lambda T}T^\alpha)-a(e^{-\lambda\tau}-e^{-\lambda T})(T-\tau)^\alpha}.$$

then

$$\|e_k\|_\lambda \le O(\lambda^{-1})\|\Delta u_k\|_\lambda.$$

□

Lemma 3: For the system(1) and the variable gain feedback PD^α-type ILC scheme (4) and reference system (3), suppose $\Delta u_{k+1} = u_d - u_{k+1}$ ($k = 0, 1, 2, \cdots$), we have

$$\|\Delta u_{k+1}\|_\lambda \le \rho\|\Delta u_k\|_\lambda.$$

Proof: Define $A_k(t) = \frac{\partial f}{\partial y_k(t)}$, $B_k(t) = \frac{\partial f}{\partial u_k(t)}$, $C_k(t) = \frac{\partial f}{\partial y_k(t-\tau)}$,

$$\begin{aligned}
&\Delta u_{k+1}\\
&= u_d - u_{k+1}\\
&= u_d - u_k - K_{p1}(k+1)e_k(t) - K_{d1}(k+1)e_k^{(\alpha)}(t)\\
&\quad - K_{p2}(t)e_{k+1}(t) - K_{d2}(t)e_{k+1}^{(\alpha)}(t)\\
&= \Delta u_k - K_{p1}(k+1)e_k(t) - K_{p2}(t)e_{k+1}(t)\\
&\quad - K_{d1}(k+1)[f_d - f_k] - K_{d2}(t)[f_d - f_{k+1}]\\
&= \Delta u_k - K_{p1}(k+1)e_k(t) - K_{p2}(t)e_{k+1}(t)\\
&\quad - K_{d1}(k+1)\\
&\quad [A_k(t)e_k(t) + B_k(t)\Delta u_k(t) + C_k(t)e_k(t-\tau)]\\
&\quad - K_{d2}(t)\\
&\quad [A_k(t)e_{k+1}(t) + B_k(t)\Delta u_{k+1}(t) + C_k(t)e_{k+1}(t-\tau)]\\
&= \Delta u_k - B_k(t)K_{d1}(k+1)\Delta u_k\\
&\quad - [K_{p1}(k+1) + A_k(t)K_{d1}(k+1)]e_k(t)\\
&\quad - [K_{p2}(t) + A_k(t)K_{d2}(t)]e_{k+1}(t)\\
&\quad - B_k(t)K_{d2}(t)\Delta u_{k+1}(t) - C_k(t)K_{d1}(k+1)e_k(t-\tau)\\
&\quad - C_k(t)K_{d2}(t)e_{k+1}(t-\tau)\\
&= [1+B_k(t)K_{d2}(t)]^{-1}\Delta u_k\\
&\quad - [1+B_k(t)K_{d2}(t)]^{-1}B_k(t)K_{d1}(k+1)\Delta u_k\\
&\quad - [1+B_k(t)K_{d2}(t)]^{-1}\\
&\quad [K_{p1}(k+1) + A_k(t)K_{d1}(k+1)]e_k(t)\\
&\quad - [1+B_k(t)K_{d2}(t)]^{-1}[K_{p2}(t) + A_k(t)K_{d2}(t)]e_{k+1}(t)\\
&\quad - [1+B_k(t)K_{d2}(t)]^{-1}C_k(t)K_{d1}(k+1)e_k(t-\tau)\\
&\quad - [1+B_k(t)K_{d2}(t)]^{-1}C_k(t)K_{d2}(t)e_{k+1}(t-\tau).
\end{aligned} \tag{7}$$

Taking the maximum norm on both sides of (7) yields

$$\begin{aligned}
&\|\Delta u_{k+1}\|_\infty\\
&\le \left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty \|\Delta u_k\|_\infty\\
&\quad + \left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty \|B_k(t)K_{d1}(k+1)\|_\infty\|\Delta u_k\|_\infty\\
&\quad + \left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty\\
&\quad \left\|K_{p1}(k+1) + A_k(t)K_{d1}(k+1)\right\|_\infty\|e_k\|_\infty\\
&\quad + \left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty\\
&\quad \left\|K_{p2}(t) + A_k(t)K_{d2}(t)\right\|_\infty\|e_{k+1}(t)\|_\infty\\
&\quad + \left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty\\
&\quad \|C_k(t)K_{d1}(k+1)\|_\infty\|e_k(t-\tau)\|_\infty\\
&\quad + \left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty\\
&\quad \|C_k(t)K_{d2}(t)\|_\infty\|e_{k+1}(t-\tau)\|_\infty.
\end{aligned} \tag{8}$$

Multiplied the inequality (8) by $e^{-\lambda t}$ and applying λ-norm, it is obvious that

$$\begin{aligned}
\|\Delta u_{k+1}\|_\lambda &= \sup_{0\le t\le T}\left\{e^{-\lambda t}\|\Delta u_{k+1}\|_\infty\right\}\\
&\le \sup_{0\le t\le T}\left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty\\
&\quad [1+\|B_k(t)K_{d1}(k+1)\|_\infty]\|\Delta u_k\|_\lambda\\
&\quad + \sup_{0\le t\le T}\left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty\\
&\quad \left\|K_{p1}(k+1) + A_k(t)K_{d1}(k+1)\right\|_\infty\|e_k(t)\|_\lambda\\
&\quad + \sup_{0\le t\le T}\left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty\\
&\quad \left\|K_{p2}(t) + A_k(t)K_{d2}(t)\right\|_\infty\|e_{k+1}(t)\|_\lambda\\
&\quad + \sup_{0\le t\le T}\left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty\\
&\quad \|C_k(t)K_{d1}(k+1)\|_\infty \sup_{0\le t\le T}\|e_k(t-\tau)\|_\infty e^{-\lambda t}\\
&\quad + \sup_{0\le t\le T}\left\|[1+B_k(t)K_{d2}(t)]^{-1}\right\|_\infty\\
&\quad \|C_k(t)K_{d2}(t)\|_\infty \sup_{0\le t\le T}\|e_{k+1}(t-\tau)\|_\infty e^{-\lambda t}.
\end{aligned} \tag{9}$$

Because $0 < r(k) < 1$, we choose the appropriate $K_{d1}(0)$ such that

$$\begin{aligned}\rho' &= \sup_{0\le t\le T} \|[1+B_k(t)K_{d2}(t)]^{-1}\|_\infty \\ &\quad [1+\|r(k+1)B_k(t)K_{d1}(0)\|_\infty] \\ &< \sup_{0\le t\le T} \|[1+B_k(t)K_{d2}(t)]^{-1}\|_\infty \\ &\quad [1+\|B_k(t)K_{d1}(0)\|_\infty] \\ &= \rho < 1,\end{aligned}$$

where

$$\begin{aligned}\rho' &= \sup_{0\le t\le T} \|[1+B_k(t)K_{d2}(t)]^{-1}\|_\infty \\ &\quad [1+\|r(k+1)B_k(t)K_{d1}(0)\|_\infty], \\ \mu &= \sup_{0\le t\le T} \|[1+B_k(t)K_{d2}(t)]^{-1}\|_\infty \\ &\quad \|[K_{p2}(t)+A_k(t)K_{d2}(t)]\|_\infty, \\ \gamma &= \sup_{0\le t\le T} \|[1+B_k(t)K_{d2}(t)]^{-1}\|_\infty \|C_k(t)K_{d2}(t)\|_\infty, \\ \nu &= \sup_{0\le t\le T} \|[1+B_k(t)K_{d2}(t)]^{-1}\|_\infty \\ &\quad \|K_{p1}(k+1)+A_k(t)K_{d1}(k+1)\|_\infty, \\ \delta &= \sup_{0\le t\le T} \|[1+B_k(t)K_{d2}(t)]^{-1}\|_\infty \\ &\quad \|C_k(t)K_{d1}(k+1)\|_\infty,\end{aligned}$$

$$\begin{aligned}&\sup_{0\le t\le T} e^{-\lambda t}\|e_k(t-\tau)\|_\infty \\ &= \sup_{-\tau\le t-\tau\le T-\tau} e^{-\lambda(t-\tau)}\|e_k(t-\tau)\|_\infty e^{-\lambda\tau} \\ &= \sup_{-\tau\le t'\le T-\tau} e^{-\lambda t'}\|e_k(t')\|_\infty e^{-\lambda\tau} \\ &= \sup_{-\tau\le t\le T-\tau} e^{-\lambda t}\|e_k(t)\|_\infty e^{-\lambda\tau} \\ &\le e^{-\lambda\tau}\|e_k\|_\lambda,\end{aligned}$$

$$\begin{aligned}&\sup_{0\le t\le T} e^{-\lambda t}\|e_{k+1}(t-\tau)\|_\infty \\ &= \sup_{-\tau\le t-\tau\le T-\tau} e^{-\lambda(t-\tau)}\|e_{k+1}(t-\tau)\|_\infty e^{-\lambda\tau} \\ &= \sup_{-\tau\le t'\le T-\tau} e^{-\lambda t'}\|e_{k+1}(t')\|_\infty e^{-\lambda\tau} \\ &= \sup_{-\tau\le t\le T-\tau} e^{-\lambda t}\|e_{k+1}(t)\|_\infty e^{-\lambda\tau} \\ &\le e^{-\lambda\tau}\|e_{k+1}\|_\lambda.\end{aligned}$$

Link it with Eq. (9),

$$\begin{aligned}\|\Delta u_{k+1}\|_\lambda &\le \rho'\|\Delta u_k\|_\lambda + \nu\|e_k\|_\lambda + \mu\|e_{k+1}\|_\lambda \\ &\quad + \delta e^{-\lambda\tau}\|e_k\|_\lambda + \gamma e^{-\lambda\tau}\|e_{k+1}\|_\lambda. \qquad (10)\end{aligned}$$

According to *Lemma 3.2* and the inequality (10), we have

$$\begin{aligned}\|\Delta u_{k+1}\|_\lambda &\le [\rho' + (\nu+\delta e^{-\lambda\tau})O(\lambda^{-1})]\|\Delta u_k\|_\lambda \\ &\quad + (\mu+\gamma e^{-\lambda\tau})O(\lambda^{-1})\|\Delta u_{k+1}\|_\lambda.\end{aligned}$$

Let λ is sufficiently large, $\|\Delta u_{k+1}\|_\lambda \le \rho\|\Delta u_k\|_\lambda$. □

Theorem 1: For the system (1) with the variable gain feedback PD^{α}-type ILC update rate (4) and reference system (3), where $t \in [-\tau, T]$, suppose $f(t, y(t), y(t-\tau), u(t))$ satisfies (2), and

$$\begin{aligned}\rho &= \sup_{0\le t\le T} \|[1+B_k(t)K_{d2}(t)]^{-1}\|_\infty[1+\|B_k(t)K_{d1}(0)\|_\infty] \\ &< 1,\end{aligned}$$

we have $\lim_{k\to\infty} y_k(t) = y_d(t)$, $t \in [-\tau, T]$.

It can be known that the convergence conditions ρ only depend on $K_{d1}(0)$ but not $K_{p1}(0)$ according to Theorem 1. So we just need to determine the initial values of the learning gains $K_{d1}(0)$ to assure $\rho < 1$.

Proof: As long as yield $\rho < 1$, there is a large enough λ to satisfy $\rho' + (\nu + \delta e^{-\lambda\tau})O(\lambda^{-1}) < 1$. So,

$$\lim_{k\to\infty} \|\Delta u_k\|_\lambda = \lim_{k\to\infty} \|u_d - u_k\|_\lambda = 0.$$

In other words, $\lim_{k\to\infty} u_k(t) = u_d(t)$, where $t \in [-\tau, T]$. According to the existence and uniqueness theorem of fractional-order differential equation, it is obtained that

$$\lim_{k\to\infty} y_k(t) = y_d(t).$$

□

IV. SIMULATIONS

Example 1: Consider the fractional-order nonlinear system with time-delay as follows

$$y^{(0.5)}(t) = 0.5y^2(t) + 0.1y(t-0.4) + 0.5u(t), \qquad (11)$$

The closed-loop PD^{α}-type ILC law with fixed gains is

$$u_{k+1}(t) = u_k(t) + e_{k+1}(t) + e_{k+1}^{(0.5)}(t), \qquad (12)$$

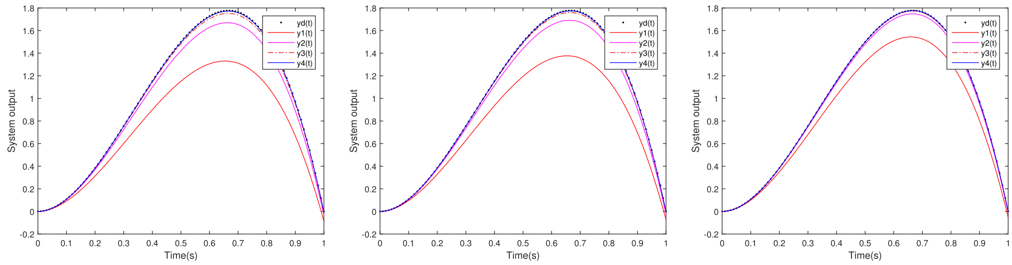
Furthermore, two kinds of variable gain feedback PD^{α}-type ILC law are given,

$$\begin{aligned}u_{k+1}(t) &= u_k(t) + (1/k)e_k(t) + (1/2k)e_k^{(0.5)}(t) \\ &\quad + e_{k+1}(t) + e_{k+1}^{(0.5)}(t), \qquad (13)\end{aligned}$$

and

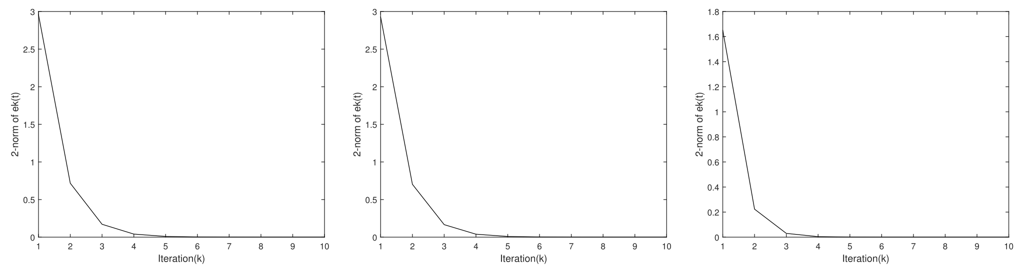
$$\begin{aligned}u_{k+1}(t) &= u_k(t) + (1/k)e_k(t) + (1/2k)e_k^{(0.5)}(t) \\ &\quad + 4\left[0.5\left(1-\frac{2}{\pi}\arctan|e_{k+1}(t)|\right)\right]e_{k+1}(t) \\ &\quad + 4\left[1-0.5\left(1-\nu\frac{2}{\pi}\arctan|e_{k+1}(t)|\right)\right]e_{k+1}^{(0.5)}(t).0 \qquad (14)\end{aligned}$$

In addition, $K_{p1}(0) = 1$, $K_{d1}(0) = 1/2$, the control input initial value is $u_0(t) = 0$, the initial system output is $y(0) = 0$, $T = 1$ and the reference trajectory function is $y_d(t) = 12\ t^2(1-t)$. Under the control of ILC update rate (13), it is easy to get $\rho = |1+1/2|^{-1}|1+1/4| = 5/6 < 1$. Under the control of ILC update rate (14), $K_{d2}(t) \in (2, 4)$, $\rho \in (5/12, 5/8)$, which satisfies all $\rho < 1$. Figure 2 and Figure 3 are the simulation results of the closed-loop PD^{α}-type, the variable gain feedback PD^{α}-type ILC update rate (13) and (14).



(a) ILC law (12) (b) ILC law (13) (c) ILC law (14)

FIGURE 2. System output: System(11), reference(15).



(a) ILC law (12) (b) ILC law (13) (c) ILC law (14)

FIGURE 3. Tracking error: System(11), reference(15).

TABLE 1. Comparison of $\|e\|_2$: System(11), reference(15).

Number of iterations	1	2	3	4	5
learning law(12)	2.9575	0.7171	0.1714	0.0409	0.0099
learning law (13)	2.9274	0.7028	0.1664	0.0393	0.0095
learning law (14)	1.6505	0.2236	0.0301	0.0042	0.0007

TABLE 2. Comparison of $\|e\|_2$: System(11), reference(16).

Number of iterations	1	2	3	4	5
learning law(12)	1.7707	0.4268	0.1058	0.0306	0.0152
learning law(13)	1.7517	0.4177	0.1024	0.0293	0.0146
learning law(14)	0.9915	0.1354	0.0210	0.0075	0.0064

Figure 2 is the system (11) outputs under the action of ILC update rate (12)-(14), respectively. Where iteration $k = 1, 2, \cdots$, the reference trajectory function is $y_d(t) = 12\,t^2(1-t)$ and $T = 1$.

Figure 3 is the two norm of $y_d(t) - y_k(t)$ while fractional-order nonlinear system with time-delay (11) under the action of ILC update rate (12)-(14), respectively. And the reference trajectory function is as follows:

$$y_d(t) = 12t^2(1-t), \tag{15}$$

For fractional-order system with time-delay (11) and the closed-loop PD^{α}-type, the variable gain feedback PD^{α}-type (13) and the variable gain feedback PD^{α}-type (14) ILC update rate, the reference trajectory $y_d(t)$ is a segmentation function

$$y_d(t) = \begin{cases} 0, & 0 < t < 0.1, \\ \dfrac{1}{2}, & 0.1 \le t < 0.6, \\ 1, & 0.6 \le t < 1. \end{cases} \tag{16}$$

Figure 4 and Figure 5 show the corresponding simulation results of system (11), reference (16).

Example 2: Give an example of wind power generation. Because the actual electrical and capacitive electrical properties are essentially fractional facts, the use of fractional descriptions is more receptive to their intrinsic characteristics and engineering values [21], [31], [37], [38].

$$\begin{cases} x^{(0.5)}(t) = Ax(t) + Bu(t), \\ y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(t). \end{cases} \tag{17}$$

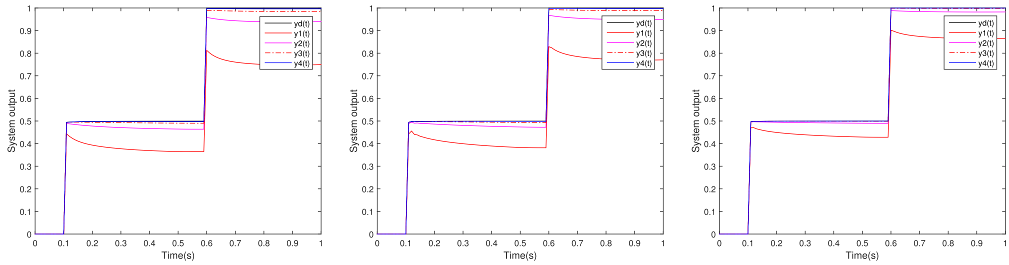
$$A = \begin{bmatrix} a_1 + a_1 R_0 & a_3 + x_{30} & a_3 x_{20} \\ b_1 x_{30} & b_2 + b_3 R_0 & b_1 x_{10} + b_4 \\ 0 & 0 & c_2 v_0 + c_3 x_{30} \end{bmatrix},$$

$$B = \begin{bmatrix} a_2 x_{10} & a_1 x_{30} & 0 \\ b_3 x_{20} & b_2 x_{30} & 0 \\ 0 & 0 & c_1 v_0 + c_3 v_{30} \end{bmatrix}.$$

the expression of each parameter is

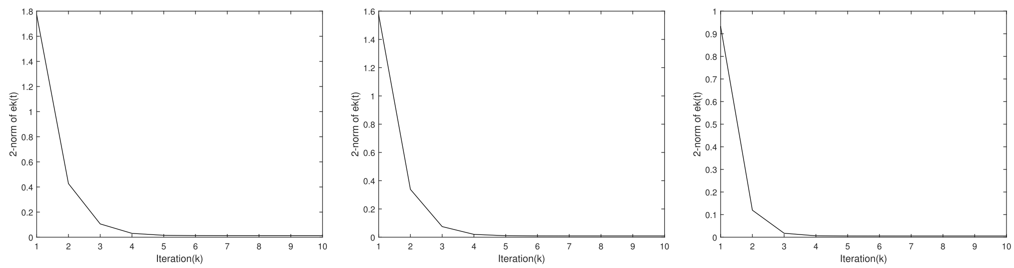
$$a_1 = -\frac{R_s}{L_d + L_s}, \quad a_2 = -\frac{1}{L_d + L_s}, \; a_3 = p\frac{L_q - L_s}{L_d + L_s},$$

$$b_1 = -p\frac{L_d + L_s}{L_q + L_s}, \quad b_2 = -\frac{R_s}{L_q + L_s},$$



(a) ILC update rate (12) (b) ILC update rate (13) (c) ILC update rate (14)

FIGURE 4. **System output: System(11), reference(16).**



(a) ILC update rate (12) (b) ILC update rate (13) (c) ILC update rate (14)

FIGURE 5. **Tracking error: System(11), reference(16).**

$$b_3 = -\frac{1}{L_q + L_s}, \quad b_4 = \frac{p\psi_m}{L_d + L_s},$$
$$c_1 = \frac{2d_1}{J}, \quad c_2 = \frac{d_2}{J}, \ c_3 = \frac{2d_3}{J},$$

where d_1, d_2, d_3 are constants associated with air density and wind turbine radius; x_{10}, x_{20}, x_{30}, and R_0 are the steady-state values of the corresponding variables at the operating point, p is the pole number of the motor. R_1, R_2 are the equivalent resistance of the system and L_s is the equivalent inductance of the system, L_d and L_q represent the inductance of d, q axis, respectively. Ψ_m is the magnetic flux.

The main parameters are as follows: $L_d = L_q = 0.04156\ H$, $R_s = 3.3\Omega$, $p = 1$, $J = 0.5042 kg \cdot m^2$, the correlation constant is $d_1 = 0.002$, $d_2 = 0.002$, $d_3 = 0.002$, $v_0 = 2.1$, $v_{20} = 5.2$, the steady-state values of the variables at the run point are, respectively $x_{10} = -0.2$, $x_{20} = 0.6$, $x_{30} = 120.6$, $R_0 = 1.6$.

So the system matrices are

$$A = \begin{bmatrix} -5.8225 & 119.6988 & -0.5407 \\ -120.6 & -5.8225 & 0.3207 \\ 0 & 0 & 1.4601 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.2377 & -1.344 & 0 \\ -0.713 & -472.5711 & 0 \\ 0 & 0 & 0.0785 \end{bmatrix}.$$

The expected speed is $y_d(t) = \sin(4\pi t)$, select initial control input $u_0(t) = [0\ 0\ 0]^T$, $r(k) = 1/k$. In the variable gain feedback PD^{α}-type ILC update rate, select the learning gain:

$$k_{p1}(0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad k_{d1}(0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}.$$

Under the control of ILC update rate (14), $K_{d2}(t) \in (2, 4)$, $\rho \in (0.79, 0.89)$, which satisfies all $\rho < 1$ and convergence condition of *Theorem 1*.

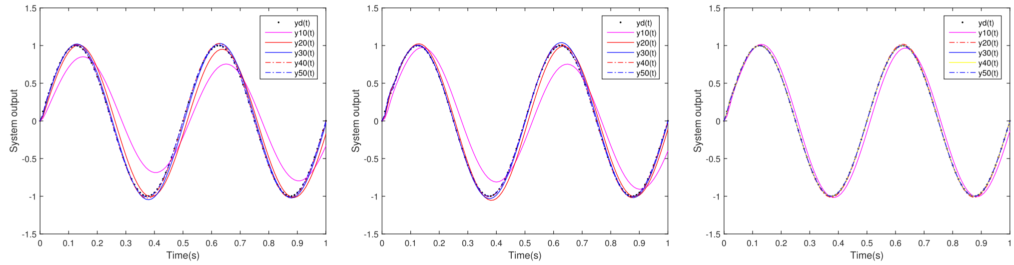
Figure 6 and Figure 7 show the corresponding simulation results of system(17), reference(18).

$$y_d(t) = \sin(4\pi t), \tag{18}$$

It can be seen from Figure 2 and Figure 7 that the iterative number of variable gain feedback PD^{α}-type ILC update rate is smaller than the iterative number of closed-loop PD^{α}-type ILC update rate when the tracking error of fractional nonlinear time-delay system (11) and (17). At the same time, through the data in Table 1, Table 2 and Table 3, it can be seen that the two norms of the tracking error with the variable gain feedback PD^{α}-type ILC update rate are smaller than the two norms of the tracking error with closed-loop PD^{α}-type ILC update rate after each iteration. Therefore, compared with the closed-loop PD^{α}-type ILC update rate, the variable gain

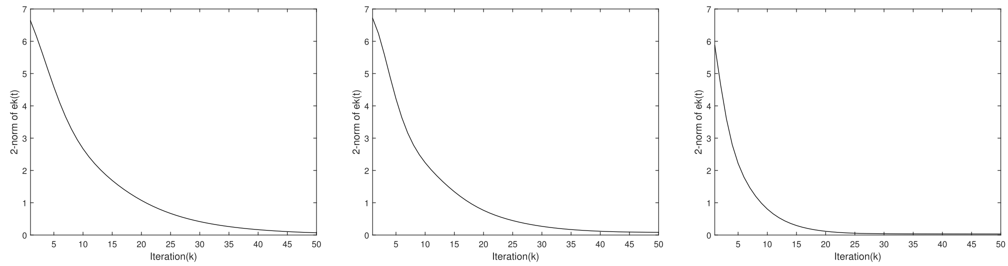
TABLE 3. **Comparison of $\|e\|_2$: System(17), reference(18).**

Number of iterations	1	10	20	30	40
learning law(12)	6.6340	2.6715	1.0735	0.4165	0.1659
learning law(13)	6.6562	2.6346	1.0495	0.4004	0.1589
learning law(14)	5.8918	0.9313	0.1468	0.0329	0.0202



(a) ILC update rate (12) (b) ILC update rate (13) (c) ILC update rate (14)

FIGURE 6. System output: System(17), reference(18).



(a) ILC update rate (12) (b) ILC update rate (13) (c) ILC update rate (14)

FIGURE 7. Tracking error: System(17), reference(18).

feedback PD^{α}-type ILC update rate has a better convergence effect.

V. CONCLUSION

For a class of fractional-order nonlinear systems with time-delay, a variable gain feedback controller is added on the basis of the traditional closed-loop PD^{α}-type ILC algorithm. By introducing the λ-norm, we get the sufficient condition that the system tracking error is bounded convergence. The simulation results show that the variable gain feedback PD^{α}-type ILC system has faster convergence speed and better tracking effect. However, the time-delay is not considered separately when designing the controller in this paper. The time-delay does not affect the stability of the system on the iteration axis when k tends to infinity. But the system convergence along the time axis cannot be guaranteed within one batch due to the effect of time-delay. Then designing the controller and simultaneously considering the time-delay in one batch should be the further research topic. The appropriate iteration learning controller should guarantee the systemąŕs stability both in iteration direction and time direction.

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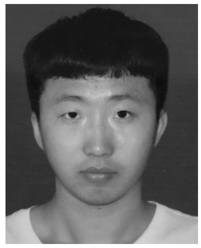
JING WANG was born in Tangshang, Hebei, China, in 1972. She received the B.S. degree in industry automation and the Ph.D. degree in control theory and control engineering from Northeastern University, China, in 1994 and 1998, respectively.

She is currently a Professor with the College of Information Science and Technology, Beijing University of Chemical Technology, China. Her research interests include complex systems, process monitoring, fault detection and diagnosis, and advance control.

CHENCHEN YU received the bachelor's degree in automation from Northeast Forestry University, China, in 2016. He is currently pursuing the M.S. degree in control science and engineering with the School of Information Science and Technology, Beijing University of Chemical Technology.

His research interest includes iterative learning control for fractional-order systems.



YI LIU was born in Jilin City, Jilin, China, in 1996. He received the B.S. degree in automation from the University of Science and Technology Liaoning, China, in 2018. He is currently pursuing the M.S. degree in control science and control engineering with the Beijing University of Chemical Technology.

His research interests include fractional processes and fractional-order signal processing.

DONG SHEN (M'10–SM'17) received the B.S. degree in mathematics from the School of Mathematics, Shandong University, Jinan, China, in 2005, and the Ph.D. degree in mathematics from the Key Laboratory of Systems and Control, Institute of Systems Science, Academy of Mathematics and System Science, Chinese Academy of Sciences (CAS), Beijing, China, in 2010.

From 2010 to 2012, he was a Postdoctoral Fellow with the State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, CAS. From 2016 to 2017, he was a Visiting Scholar with the National University of Singapore (NUS), Singapore. Since 2012, he has been with the College of Information Science and Technology, Beijing University of Chemical Technology (BUCT), Beijing, where he is currently a Professor. He has published more than 80 refereed journal and conference papers. He has (co-)authored *Iterative Learning Control under Iteration-Varying Lengths: Synthesis and Analysis* (Springer, 2019), *Iterative Learning Control With Passive Incomplete Information: Algorithm Design and Convergence Analysis* (Springer, 2018), *Iterative Learning Control for Multi-Agent Systems Coordination* (Wiley, 2017), and *Stochastic Iterative Learning Control* (Science Press, 2016, in Chinese). His current research interests include iterative learning control and stochastic control and optimization.

Dr. Shen received the IEEE CSS Beijing Chapter Young Author Prize, in 2014, and the Wentsun Wu Artificial Intelligence Science and Technology Progress Award, in 2012.

YANGQUAN CHEN received the Ph.D. degree from Nanyang Technological University, Singapore, in 1998. He was a Faculty of Electrical Engineering, Utah State University, from 2000 to 2012. He joined the School of Engineering, University of California at Merced, Merced, in 2012. His research interests include mechatronics for sustainability, cognitive process control, small multi-UAV-based cooperative multi-spectral personal remote sensing, applied fractional calculus in controls, modeling and complex signal processing, and distributed measurement and control of distributed parameter systems with mobile actuator and sensor networks.