An Improved Cooperative Team Spraying Control of a Diffusion Process With a Moving or Static Pollution Source

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***Abstract*—This paper is concerned with a control problem of a diffusion process with the help of static mesh sensor networks in a certain region of interest and a team of networked mobile actuators carrying chemical neutralizers. The major contribution of this paper can be divided into three parts: the first is the construction of a cyber-physical system framework based on centroidal Voronoi tessellations (CVTs), the second is the convergence analysis of the actuators location, and the last is a novel proportional integral (PI) control method for actuator motion planning and neutralizing control (e.g., spraying) of a diffusion process with a moving or static pollution source, which is more effective than a proportional (P) control method. An optimal spraying control cost function is constructed. Then, the minimization problem of the spraying amount is addressed. Moreover, a new CVT algorithm based on the novel PI control method, henceforth called PI-CVT algorithm, is introduced together with the convergence analysis of the actuators location via a PI control law. Finally, a modified simulation platform called diffusion-mobile-actuators-sensors-2-dimension-proportional integral derivative (Diff-MAS2D-PID) is illustrated. In addition, a numerical simulation example for the diffusion process is presented to verify the effectiveness of our proposed controllers.**

***Index Terms*—Centroidal Voronoi tessellations (CVTs), diffusion processes, mobile actuator-sensor networks (MAS-Net), PI control.**

I. Introduction

IN reality, diffusion can be regarded as the net movement of molecules or atoms from a region of high concentration to a region of low concentration. It has lots of applications in chemistry [1], physics [2], biology [3], economics [4] and engineering [5]. Many diffusion processes can be modeled as distributed parameter systems (DPSs), see [6]–[8] for more knowledge.

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Earlier works on controlling the diffusion process with a static pollution source were motivated by the applications of centroidal Voronoi tessellations (CVTs) [9]–[11] and mobile actuator-sensor networks (MAS-Net) [12]. Notable pioneering work on CVTs was provided by Du *et al*. [13]. As we know, the CVT algorithm is a kind of non-model method which can solve the problem of coverage control [14]–[17]. Motivated by the work [11], [18] that the CVT algorithm was used for the sensor placement problem in coverage control and the sensor location problem in feedback control of partial differential equation (PDE) systems, CVTs were extended to the work on diffusion control for the actuator deployment problem with the static sensor networks in [19]. Here the CVT algorithm is also applicable to our work in this paper. Actually, the monitoring and control of a diffusion process can be regarded as the optimal sensor/actuator placement or motion planning problem [20], [21]. More recently, there have been emerged related work on the use of the CVT algorithm for diffusion process control based on MAS-Net [6], [21]–[24]. And other work [25]–[27] on boundary control for PDEs based on the backstepping technique has also arisen.

To the best of our knowledge, the diffusion system in the previous research work is mostly discussed together with proportional (P) controllers and a static pollution source. In fact, a moving pollution source may be more practical than a static one. For instance, smart enemy target sources can be viewed as moving pollution sources in military warfare, which are expected to be eliminated under a group of mobile actuators (robots). Additionally, in some engineering applications, a static or steady state error can be induced by the P control method, which can be taken as a drawback of this method. Owing to this drawback, proportional integral (PI) controllers with the advantage of eliminating steady error and better flexibility (more accurate control) are used in practice instead of P controllers. However, very few results are provided on the control of a diffusion process with a moving or static pollution source based on PI controllers and the CVT algorithm currently.

Motivated by the fact a novel PI method has a better performance than a P control method (i.e., P controller for actuators motion control and spraying control) on controlling a diffusion process [19], [28], we use this PI control method to address the control problem for the diffusion process. Note that, the novelty of the PI method is a PI controller for actuator motion control together with a PI controller with time-delay (for more details, see the last part in Section II) for actuator spraying control. Moreover, a modified simulation platform (diffusion-mobile-actuators-sensors-2-dimension-proportional integral derivative (Diff-MAS2D-PID)) is built for the diffusion system together with PI, PD and PID controllers and moving or static pollution sources. In numerical simulations, we choose the appropriate parameter to minimize the difference of total pollution amount and total spraying amount, which makes the area of interest not overdosed. In addition, this paper addresses a diffusion control problem for an integer-order system by integer-order PI controllers, and the anomalous diffusion control problem in work [29] is realized by fractional-order controllers. From the theoretical point of view, here the convergence analysis of the mobile actuators location by the Lyapunov method in this paper could provide some insights into the control theory of diffusion processes.

The remainder of this paper is organized as follows. The problem statement is introduced in Section II briefly. In Section III, the control problem of a diffusion process is discussed. Section IV gives the convergence analysis of the actuators location. In Section V, a modified simulation platform (Diff-MAS2D-PID) is described. Then, numerical simulations are given to test the effectiveness of our proposed method. Finally, conclusions and some future work are provided in Section VI.

II. Problem Statement

The problem of actuator motion control and spraying control can be framed in a cyber-physical system [21]. In this section, we specifically illustrate the diffusion problem.

A diffusion process could occur in an area Ψ, which represents a convex polytope in $\mathcal{R}^2$. A group of n mobile actuators can move freely to a certain position in this region. Denote the set $P=\{p_1,p_2,\ldots,p_n\}$ as the position coordinate set of mobile actuators, where p_i represents the position coordinate of the ith actuator. $\phi(x,y,t)\colon \Psi\rightarrow\mathbb{R}_+$ can be described as the pollution concentration in the area Ψ. In order to simplify the presentation of the dynamic process, the pollution concentration $\phi(x,y,t)$ is governed by the following PDE

$$\frac{\partial\phi}{\partial t}=a\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+f_d(x,y,t)+f_c(\tilde{\phi},x,y,t) \tag{1}$$

where $a>0$ denotes a diffusion coefficient, $f_d(x,y,t)$ is a moving or static pollution source, $\tilde{\phi}$ represents the measured data of ϕ from the sensors, and $f_c(\tilde{\phi},x,y,t)$ is a control input whose exact representation is determined by designers.

We suppose that every actuator at its position will receive information from static sensors, move to the high pollution concentration and then release chemical neutralization by the controller. In this paper, our control objectives are introduced as follows:

1) Control the diffusion process rationally and decrease the amount of total pollutants.

2) Minimize the polluted area which is heavily affected.

3) Improve the spraying speed and neutralize the pollution as quickly as possible without making the area of interest overdosed.

Ψ can be divided into a collection of n polytopes $\Theta=\{V_1,\ldots,V_n\}$. Suppose that $\forall i\in\{1,\ldots,n\}$, there exists

$$V_i=\{s\in\Psi\mid |s-p_i|<|s-p_j|,\, j\neq i\},\quad \forall j\in\{1,\ldots,n\} \tag{2}$$

where $|\cdot|$ represents the Euclidean distance, and s is an arbitrary point in the area Ψ. It is obvious that $V_i\cap V_j=\emptyset$ for $i\neq j$ and $\cup_{i=1}^{n}\bar{V}_i=\bar{\Psi}$, where $\bar{V}_i=V_i\cup\partial V_i$, $\bar{\Psi}=\Psi\cup\partial\Psi$, ∂V_i and $\partial\Psi$ represent boundaries of V_i and Ψ respectively.

The set of regions $(V_i)_{i=1}^{n}$ is defined as the Voronoi tessellation which is generated by the set of points $(p_i)_{i=1}^{n}$. When V_i is adjacent to V_j, the point p_i can be viewed as a neighbor of the point p_j. In particular, if the generators $(p_i)_{i=1}^{n}$ of the Voronoi cells are also themselves centroids (centers of masses) of these cells, such a Voronoi tessellation is called a CVT.

Fig. 1(a) shows that a CVT can be constructed by 10 random points in the area $(0,1)\times(0,1)$ with a density function $\phi(x,y)=e^{-8(x+0.2)^2-8(y+0.2)^2}$. In fact, Voronoi cells are always generated by trajectories of mobile actuators at a certain range of time rather than some random points, which could be caused by the movement of actuators, as shown in Fig. 1(b). Suppose that actuators move a short distance and keep safe distance with each other in a certain time while Voronoi cells change along with moving trajectories of mobile actuators. Moreover, due to distance between any two points in the pollution area and moving trajectories of actuators, pollution area can be redistricted. Hence, moving trajectories of actuators are involved in their divisiory regions.

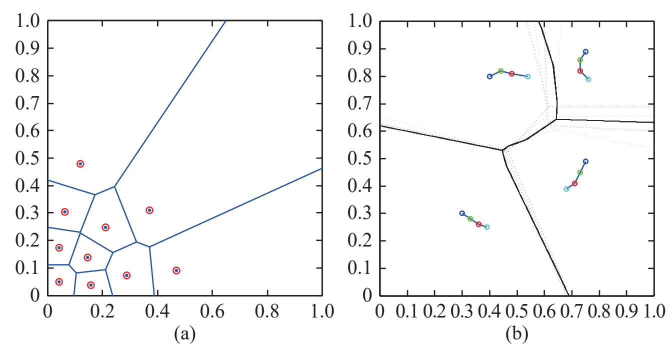

Fig. 1. A CVT constructed by 10 random points and trajectories of 4 mobile actuators.

Approximately, the variation of actuators space can be neglected in a certain condition and converted into time-delay, which can be expressed as δt. It will be used for actuator neutralizing control in Section IV-C. In other words, the meaning of δt is to reduce the impact of the measurement error of the actuator at one position on control of the actuator at another position.

III. Control of Diffusion Process

In this section, we introduce the objective optimization problem of the diffusion process for actuator motion control and spraying control.

A. Actuator Position Control Problem

As we know, the CVT algorithm is used to solve the time-invariant environment problem whose density function is time-independent. Based on the arguments in [19, Section V], it still can keep the validation of solving our problem in time-varying environment when the evolution of diffusion process is slow, compared with the convergence rate of the Lloyd's method. This method can be used as a deterministic algorithm to generate a CVT, for which more details can be found in [10], [19], [24]. In order to apply the CVT algorithm for actuator motion control, the time varying could be negligible, and then the pollutant concentration function could be taken as a time-independent function. Due to the absence of specific form of the PDE system (1), we choose the CVT algorithm for the actuator location control since the CVT algorithm is a non-model algorithm, i.e., it does not need to know the exact mathematical form of the model. The proposed control strategy may be uncommon and may have some limitations for the uncertain dynamic system.

Apparently, to control the diffusion process and minimize the heavily affected area, the mobile actuators should get close to the affected area with high pollution concentration and be far from the lightly affected area. However, it seems unreasonable to put all the mobile actuators close to the pollution source since the diffused pollutants far away from the pollution source also need to be eliminated timely. Given above control requirements, the cost function that needs to be minimized is introduced [11], [19]

$$J_1(P,\Theta) = \sum_{i=1}^{n} \int_{V_i} \phi(s)|s-p_i|^2 ds, \quad s \in \Psi$$
$$\text{s.t.} \quad |\dot{p}_i| < k_{\text{vol}}, |\ddot{p}_i| < k_{\text{acc}} \tag{3}$$

where $\phi(s)$ denotes concentration at the point s in the area Ψ, k_{vol} and k_{acc} are the upper bounds of the velocity and the accelerated velocity of mobile actuators, respectively. We will give more details on the optimization result of the cost function (3) later.

In what follows, we shall write

$$J_{1,P} = J_1(P,\Theta). \tag{4}$$

In addition, some smoothness properties of $J_{1,P}$ can be deduced, i.e., the function $J_{1,P}$ is at least continuously differentiable on $\mathbb{R}^n \setminus \{P \in \mathbb{R}^n | p_i = p_j \text{ for some } i, j \in \{1,\dots,n\}, i \neq j\}$ due to the dependence of Voronoi cells $(V_i)_{i=1}^n$ on $P = (p_i,\dots,p_n)$, for all $p_i \neq p_j$.

Similarly to the arguments in [11], given a region $V \subset \mathbb{R}^2$, the density function $\rho(s) \geq 0$ is defined on it. Let us give some basic definitions, including definitions of mass and center of mass [11], [19]

$$M_V := \int_V \rho(s) ds, \quad C_V := \frac{1}{M_V} \int_V s\rho(s) ds. \tag{5}$$

Next, let us consider the actuators location optimization problem (3) again. In order to illustrate this specific problem clearly, here, suppose the density function $\rho(s) = \phi(s)$. Obviously, for each Voronoi cell V_i, the mass and the center of mass are as follows:

$$M_{V_i} := \int_{V_i} \phi(s) ds$$
$$C_{V_i} = \bar{p}_i := \frac{1}{M_{V_i}} \int_{V_i} s\phi(s) ds. \tag{6}$$

Considering the parallel axis theorem, we obtain the partial derivative of the cost function $J_{1,P}$ as follows:

$$\frac{\partial J_{1,P}}{\partial p_i} = 2M_{V_i}(p_i - \bar{p}_i). \tag{7}$$

A necessary condition to minimize the $J_{1,P}$ is that $\{p_i, V_i\}_{i=1}^n$ can be a CVT [6], [10] of Ψ. The local minimum points for the above cost function $J_{1,P}$ are centroids for their Voronoi cells, which can be described as below:

$$\bar{p}_i = \arg\min_{p_i} J_{1,P}. \tag{8}$$

Note that we denote the actuators' configuration as a centroidal Voronoi configuration, in which each actuator is located at the centroid of its Voronoi cell. The actuator motion control problem is essentially an optimization coverage control problem. Its purpose is to make the actuators reach a centroidal Voronoi configuration, i.e., each actuator's location can converge to the centroid of its Voronoi cell.

For the purpose of computing the location of mobile actuators by CVTs, the Lloyd's method is utilized to determine CVTs here. In many practical applications, the robot (actuator or sensor) has only limited communication capabilities. We assume that mobile actuators can communicate with other actuators and sensors in a certain adjustable range. A modified distribution algorithm [19] is introduced, which has the advantage in reducing the computation amount.

B. Actuator Neutralizing Control Problem

Moreover, the pollution should be neutralized as soon as possible without making the area of interest overdosed. However, spraying chemicals amount may not be consistent with the amount of pollutant. In order to reduce the disparities of spraying amount and pollutant amount, we construct a cost function needed to be minimized, which can be described by the following constraint optimization problem

$$J_2(U_s(t),\Theta) = \left(\sum_{i=1}^{n} \int_{V_i} \phi(s) ds - \sum_{i=1}^{n} \int_0^t u_{s_i}(\tau) d\tau\right)^2, \quad s \in \Psi$$
$$\text{s.t.} \quad \sum_{i=1}^{n} \int_0^t u_{s_i}(\tau) d\tau \leq k_s \tag{9}$$

where $U_s(t) = \{u_{s_1}(t) \cdots u_{s_n}(t)\}$, $u_{s_i}(t)$ can be regarded as spraying chemical amount released by the ith actuator, and k_s is an adjustable upper bound. Based on the above control objective, i.e., eliminating the pollution as much as possible without making the area overdosed, the value of k_s needs to be equal to or less than the total pollution amount (i.e., $k_s \leq \sum_{i=1}^n \int_{V_i} \phi(s) ds$). However, due to sensors' measure capability and measure error, the total pollution amount may be hard to obtain accurately, then the fact the value of k_s is completely equal to $\sum_{i=1}^n \int_{V_i} \phi(s) ds$ may be hard to realize. As is illustrated above, in order to avoid spraying overdosed, $k_s < \sum_{i=1}^{n} \int_{V_i} \phi(s)ds$ here.

In the above constrained optimization problem, it is aimed to find an optimal value $U_s^*(t) = \{u_{s_i}^*(t), i = 1, \ldots, n\}$. This optimization result will be addressed in the below theorem. In this situation, the spraying control amount takes the optimal value so that the difference between total spraying chemicals amount and total pollutant amount will be minimized.

First, we will give more details on Karush-Kuhn-Tucker conditions (KKT conditions) [30], [31] as below, which are crucial to the below main theorem.

Definition 1 [30]: Let u^* be a point which satisfies the below constraints

$$M(u^*) = 0, \quad N(u^*) \le 0 \tag{10}$$

and let I be the set of indices i for which $n_i(u^*) = 0$, where $M = (m_1, m_2, \ldots, m_s)$, $N = (n_1, n_2, \ldots, n_t)$, $M, N \in C^1$, and $u^* \in \Omega \subset E^q$ $(s \le q)$ is a set constraint. Thus, if the gradient vectors $\nabla m_j(u^*)$, $\nabla n_i(u^*)$, $1 \le j \le s$, $i \in I$ are linearly independent, then u^* is viewed as a regular point of the constraints (10).

Lemma 1 (KKT conditions) [30] : Let u^* be a minimum point for the problem

$$\begin{aligned} &\text{minimize} \quad f(u) \\ &\text{s.t.} \quad M(u) = 0, \quad N(u) \le 0 \end{aligned} \tag{11}$$

and assume that u^* is a regular point for the constraints (see Definition 1). Then there exist $\alpha \in E^s$ and $\beta \in E^t$ with $\beta \ge 0$ such that

$$\nabla f(u^*) + \alpha^T \nabla M(u^*) + \beta^T \nabla N(u^*) = 0 \tag{12}$$

$$\beta^T N(u^*) = 0. \tag{13}$$

Now, we are ready to state our main result for the optimal value $U_s^*(t)$ according to KKT conditions (Lemma 1), which are necessary conditions of optimality.

Theorem 1: For the above constrained optimization problem, the cost function (9) can approach to optimization, if the ith actuator's spraying amount $u_{s_i}^*(t)$ satisfies the condition given by

$$\sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) d\tau = k_s \tag{14}$$

where $k_s = \sum_{i=1}^{n} \int_{V_i} \phi(s)ds - \lambda/2$, λ is a nonnegative constant.

Proof: By Lemma 1 (KKT conditions), we have

$$\begin{cases} \dfrac{\partial J_2(U_s^*(t))}{\partial u_{s_i}^*(t)} + \lambda \dfrac{\partial \left(\sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) d\tau - k_s \right)}{\partial u_{s_i}^*(t)} = 0 \\ \lambda \ge 0 \\ \lambda \left(\sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) d\tau - k_s \right) = 0. \end{cases} \tag{15}$$

To find an optimal solution, we can try to set none or one constraint active. Then, we will discuss it in below two cases.

Case 1: In the first case, assume that the constraint is not active (i.e, $\lambda = 0$), we obtain

$$\begin{cases} \dfrac{\partial J_2(U_s^*(t))}{\partial u_{s_i}^*(t)} + \lambda \dfrac{\partial \left(\sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) \mathrm{d}\tau - k_s \right)}{\partial u_{s_i}^*(t)} = 0 \\ \lambda = 0 \\ \sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) d\tau < k_s. \end{cases} \tag{16}$$

Using (9) for (16), one can readily get that

$$\sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) \mathrm{d}\tau = \sum_{i=1}^{n} \int_{V_i} \phi(s)ds < k_s. \tag{17}$$

However, it contradicts the fact $k_s < \sum_{i=1}^{n} \int_{V_i} \phi(s)ds$ given in above analysis. Hence, this case will be abandoned.

Case 2: In the second case, suppose that the constraint is active (i.e, $\lambda \ne 0$), which yields

$$\begin{cases} \dfrac{\partial J_2(U_s^*(t))}{\partial u_{s_i}^*(t)} + \lambda \dfrac{\partial \left(\sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) d\tau - k_s \right)}{\partial u_{s_i}^*(t)} = 0 \\ \lambda > 0 \\ \sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) d\tau - k_s = 0. \end{cases} \tag{18}$$

Then, substituting (9) into (18), we can obtain

$$\sum_{i=1}^{n} \int_{V_i} \phi(s)ds - \sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) d\tau - \frac{\lambda}{2} = 0 \tag{19}$$

$$\sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) d\tau = k_s. \tag{20}$$

This has the solution

$$\sum_{i=1}^{n} \int_0^t u_{s_i}^*(\tau) d\tau = \sum_{i=1}^{n} \int_{V_i} \phi(s)ds - \frac{\lambda}{2} = k_s \tag{21}$$

where $k_s = \sum_{i=1}^{n} \int_{V_i} \phi(s)ds - \dfrac{\lambda}{2}$, $\lambda > 0$. ■

As illustrated above, if we choose the optimal value $u_{s_i}^*(t)$ given by (21), the above constrained optimization problem can be addressed.

Remark 1: Given the above analysis, total spraying chemical amount is near to the total pollutant amount as the value of λ becomes smaller. In other words, the tinier discrepancy (between total spraying amount and total pollutant amount) represents the better spraying control effect, so that we can conclude the actuators' spraying amount takes $U_s^*(t) = \arg\min J_2(U_s(t), \Theta)$ for optimal spraying control. The result in above theorem illustrates the existence of the optimal value for spraying control amount.

IV. PI Controller for Diffusion Control

In this section, we will introduce PI controllers for actuator motion planning control and actuator neutralizing control. Meanwhile, we will illustrate the convergence analysis of the actuators location and a new PI-CVT algorithm.

A. PI Controller for Actuator Motion Control

Here, the mobile actuators are treated as virtual particles and their locations meet the following second-order dynamical equation

$$\ddot{p}_i = u_{p_i} \tag{22}$$

where u_{p_i} is the control input for actuator motion path.

The right hand side of the above equation can be described by

$$u_{p_i} = f_i - k_v \dot{p}_i \tag{23}$$

where f_i represents the force input to control the motion of the actuator. f_i is given by a PI controller

$$f_i = k_p(\bar{p}_i - p_i) + k_i \int_0^t (\bar{p}_i - p_i) ds \tag{24}$$

where $\bar{p}_i$ represents the centroid of the current Voronoi cell.

Substituting (24) into (23), we can get

$$u_{p_i} = k_p(\bar{p}_i - p_i) + k_i \int_0^t (\bar{p}_i - p_i) ds - k_v \dot{p}_i. \tag{25}$$

The second term of (23) on the right hand side is the viscous friction introduced in [32]. k_v is the proportion of viscosity coefficient to actuator mass and $\dot{p}_i$ denotes the velocity of the ith actuator. This term is used to eliminate the oscillation behavior of actuators [33] when the actuator is close to the destination. The viscous term assures that the actuator will come to a standstill eventually without external force.

Remark 2: If k_i is equal to 0, the PI controller (24) for control input can be reduced to the P controller [19] for the optimization problem of the actuators location. The PI control method for the actuator location problem seems a more generalized alternative to the P control method.

B. Convergence Analysis of the Actuators Location

Next, we consider the extension of the control design to nonlinear passive-dynamics system [11], [34]. In this paper, the second order system described by a motion equation $\ddot{p}_i = u_{p_i}$ has been investigated. Specifically, suppose that the system's dynamics are passive with control input u_{p_i} and output $\dot{p}_i$. And assume the input with the zero-dynamics manifold $\{\dot{p}_i = 0\}$ is $u_{p_i} = 0$. For such a system, we use the below PI controller for control input

$$\begin{aligned} u_{p_i} = & -k_{\text{pro}} M_{V_i}(p_i - \bar{p}_i) \\ & - k_{\text{int}} M_{V_i} \int_0^t (p_i - \bar{p}_i) ds - k_v \dot{p}_i \end{aligned} \tag{26}$$

where k_{pro}, k_{int} and k_v are positive constants, $M_{V_i} = \int_{V_i} \phi(s) ds$ represents the mass of current Voronoi cell, and $\bar{p}_i$ is the centroid of current Voronoi tessellation.

The convergence analysis on the actuators location (system state) of the closed-loop system (22) induced by (26), can be addressed by the following assumption.

Assumption 1: It is assumed that above positive constants, henceforth called scale positive gains, satisfy $k_{\text{pro}} > k_{\text{int}} > 0$, and $k_v \geq 1$.

We use the Lyapunov stability theory to analyze the dynamic system (22) of the mobile actuators location. By the control input (26) whose gains satisfy Assumption 1, we can obtain the below main result.

Theorem 2: For the passive-dynamics system (22), asymptotic convergence of the actuators location to the set of centroidal Voronoi configurations (i.e., the set of centroids of Voronoi cells) can be achieved by the proposed control input (26) with the PI controller. If this set is finite, the actuators location will converge to a centroidal Voronoi configuration in which each actuator is located at its Voronoi cell's centroid.

Proof: Consider the Lyapunov functional

$$\begin{aligned} \Xi(t) = & \frac{1}{2}(k_{\text{pro}} k_v - k_{\text{int}} k_v) \mathrm{J}_{1,P} \\ & + \frac{1}{2}\sum_{i=1}^{n} \Big(\sqrt{k_{\text{int}} k_v M_{V_i}} (p_i - \bar{p}_i) \\ & + \sqrt{k_{\text{int}} k_v M_{V_i}} \int_0^t (p_i - \bar{p}_i) ds \Big)^2 \\ & + \frac{1}{2}\sum_{i=1}^{n} \Big(\sqrt{k_v} \dot{p}_i + k_v \sqrt{k_v} (p_i - \bar{p}_i) \Big)^2. \end{aligned} \tag{27}$$

Based on the analysis of the CVT algorithm in Section III-A, we have that M_{V_i} and $\bar{p}_i$ are time-invariant, therefore their derivatives are equal to zero. Then, the derivative of the proposed Lyapunov function (27) is given by

$$\begin{aligned} \frac{d\Xi(t)}{dt} = & \frac{1}{2}\sum_{i=1}^{n} \Big(k_{\text{pro}} k_v - k_{\text{int}} k_v \Big) \frac{\partial J_{1,P}}{\partial p_i} \dot{p}_i \\ & + \sum_{i=1}^{n} \Big(\sqrt{k_{\text{int}} k_v M_{V_i}} (p_i - \bar{p}_i) \\ & + \sqrt{k_{\text{int}} k_v M_{V_i}} \int_0^t (p_i - \bar{p}_i) ds \Big) \\ & \times \Big(\sqrt{k_{\text{int}} k_v M_{V_i}} \dot{p}_i + \sqrt{k_{\text{int}} k_v M_{V_i}} (p_i - \bar{p}_i) \Big) \\ & + \sum_{i=1}^{n} (\sqrt{k_v} \dot{p}_i + k_v \sqrt{k_v} (p_i - \bar{p}_i)) \\ & \times \Big(\sqrt{k_v} \ddot{p}_i + k_v \sqrt{k_v} \dot{p}_i \Big). \end{aligned} \tag{28}$$

Choosing the formulation $\dfrac{\partial J_{1,P}}{\partial p_i}$ described by (7), and $\ddot{p}_i$ given by (22), (26), then we obtain

$$\begin{aligned}\frac{d\Xi(t)}{dt} &= \sum_{i=1}^{n}(k_{\text{pro}}k_v - k_{\text{int}}k_v)M_{V_i}\dot{p}_i(p_i-\bar{p}_i)\\&\quad+\sum_{i=1}^{n}k_{\text{int}}k_vM_{V_i}\dot{p}_i(p_i-\bar{p}_i)+\sum_{i=1}^{n}k_{\text{int}}k_vM_{V_i}(p_i-\bar{p}_i)^2\\&\quad+\sum_{i=1}^{n}k_{\text{int}}k_vM_{V_i}\dot{p}_i\int_0^t(p_i-\bar{p}_i)ds\\&\quad+\sum_{i=1}^{n}k_{\text{int}}k_vM_{V_i}(p_i-\bar{p}_i)\int_0^t(p_i-\bar{p}_i)ds\\&\quad-\sum_{i=1}^{n}k_{\text{pro}}k_vM_{V_i}\dot{p}_i(p_i-\bar{p}_i)\\&\quad-\sum_{i=1}^{n}k_{\text{int}}k_vM_{V_i}\dot{p}_i\int_0^t(p_i-\bar{p}_i)ds-\sum_{i=1}^{n}k_v^2k_{\text{pro}}M_{V_i}(p_i-\bar{p}_i)^2\\&\quad-\sum_{i=1}^{n}k_{\text{int}}k_v^2M_{V_i}(p_i-\bar{p}_i)\int_0^t(p_i-\bar{p}_i)ds\\&=\sum_{i=1}^{n}(k_{\text{pro}}k_v-k_{\text{int}}k_v+k_{\text{int}}k_v-k_{\text{pro}}k_v)\\&\quad\times M_{V_i}\dot{p}_i(p_i-\bar{p}_i)+\sum_{i=1}^{n}(k_{\text{int}}k_v-k_{\text{pro}}k_v^2)\\&\quad\times M_{V_i}(p_i-\bar{p}_i)^2+\sum_{i=1}^{n}(k_{\text{int}}k_v-k_{\text{int}}k_v^2)\\&\quad\times M_{V_i}(p_i-\bar{p}_i)\int_0^t(p_i-\bar{p}_i)ds.\end{aligned}\tag{29}$$

Using Assumption 1, we have

$$k_{\text{int}}k_v-k_{\text{pro}}k_v^2<0,\quad k_{\text{int}}k_v-k_{\text{int}}k_v^2\le 0.\tag{30}$$

Note that here $(p_i-\bar{p}_i)\int_0^t(p_i-\bar{p}_i)ds\ge 0$. Combining with above inequality, we further get

$$\begin{aligned}\frac{d\Xi(t)}{dt}&=\sum_{i=1}^{n}(k_{\text{int}}k_v-k_{\text{pro}}k_v^2)M_{V_i}(p_i-\bar{p}_i)^2\\&\quad+\sum_{i=1}^{n}(k_{\text{int}}k_v-k_{\text{int}}k_v^2)M_{V_i}(p_i-\bar{p}_i)\\&\quad\times\int_0^t(p_i-\bar{p}_i)ds\\&\le 0.\end{aligned}\tag{31}$$

The remainder is similar to the argument in [11, proof of Proposition 3.1 and proof of Proposition 5.1]. Based on the LaSalle's principle [11], [35], the actuators location can asymptotically converge to the largest invariant set contained in $\{p_i=\bar{p}_i\}$ which is the set of centroidal Voronoi configurations. Consider the assumption of zero dynamics on the above passive system, we obtain that this set is the solution set of the closed-loop system (22) and is also invariant for (26). Thus, the largest invariant set corresponds to the set of centroidal Voronoi configurations, then the actuators location converges to this set. Moreover, when it is finite, the actuators will reach a specific centroidal Voronoi configuration where each actuator is positioned at its Voronoi cell's centroid. ■

Remark 3: Above convergence analysis of the mobile actuators location can be viewed as evolution of coverage control of mobile sensors network based on the continuous-time Lloyd descent algorithm. We refer to [11] for more details on coverage control of mobile sensors networks. In addition, if we consider the CVT algorithm in time-varying environment, the convergence problem of the mobile actuators location will be complicated. A deeper analysis on it will be investigated in future work with the help of the corresponding arguments in [36].

C. PI Controller for Actuator Neutralizing Control

Motivated by [21, page 175], in this paper, we design the PI controller like the below mathematical expression for neutralizing pollutants in a diffusion process. The relationship of the chemicals amount of each robot released and the average pollutant concentration in the Voronoi cell belonging to that actuator can be described by

$$u_{s_i}(t)=-k_{pr}\bar{\phi}_i(x,y,t)-k_{ir}\int_{t-\delta t}^{t}\bar{\phi}_i(x,y,\tau)d\tau\tag{32}$$

where

$$\bar{\phi}_i(x,y,t)=\frac{\int_{\tilde{V}_i}\phi(x,y,t)dV}{\int_{\tilde{V}_i}dV}\tag{33}$$

represents the average pollutant concentration, $0<\delta t<t$ (see Section II on δt for more details), $i=1,\ldots,n$, k_{pr} and k_{ir} are positive constants.

Let us consider $\tilde{V}_i$ as follows:

$$\tilde{V}_i=V_i\cap C_i\tag{34}$$

where $C_i=\{s\mid |s-p_i|<r_i\}$, r_i represents the sensing range of the ith actuator, and V_i is the Voronoi cell of the ith actuator.

Remark 4: If k_{ir} is equal to 0, the PI control method can be reduced to the P control method [19] for spraying, which takes an important effect in a diffusion process. In addition, for above PI controller (32), we could choose the appropriate coefficient (i.e., k_{pr}, k_{ir}) to minimize the pollution amount and the difference between total pollution amount and total spraying amount, which will be illustrated in the below numerical simulations (see Section V-B for more details).

D. PI-CVT Algorithm

Based on the above analysis, the PI-CVT algorithm is proposed for controlling a diffusion process and will be presented as follows. Algorithm 1 contains two critical parts: actuator motion control and actuator spraying control.

Algorithm 1: PI-CVT algorithm

---

1. Set initial: actuator location $p_i\in\{p_1,\ldots,p_n\}$, pollution source f_d, response time $t=0$.
2. Compute Voronoi cell V_i, $i=1,\ldots,n$ belong to each actuator.
3. Gather the data from sensor within Voronoi cell V_i; compute its centroid $\bar{p}_i$ according to (6) and the average concentration of pollutant $\bar{\phi}_i$ based on (33) in range r_i.
4. Control actuator motion and spraying: compute motion control input u_{p_i} according to (25), compute spray amount u_{s_i} based on (32).
5. Repeat Steps 2–4 until there exists no pollution, then stop.

V. Numerical Simulation

The performance of the novel PI control method for the diffusion process can be demonstrated by comparing the performance between novel PI and P control methods in three aspects: 1) the amount of total pollution; 2) the difference between total pollution amount and total spraying amount; 3) the evolution of state L_2 norm.

In this section, a simulation platform Diff-MAS2D-PID for motion and spraying control of mobile actuators is introduced later. It is also used as the simulation platform to realize two different cases given below:

1) A diffusion process with a moving pollution source.

2) A diffusion process with a static pollution source.

In addition, this novel PI control strategy is based on the CVT algorithm and different from other PI control strategies used for a diffusion process. Therefore, it may be not available to implement the comparison with other PI control strategies which are not based on the CVT algorithm. However, the comparison of control effect between the novel PI control method and the P control method for a diffusion process with a static pollution source could be realized. Note that these two control strategies are both based on the CVT algorithm. Since the control performance of the novel PI control strategy comparing with the P control strategy for a moving disturbance is similar to the counterparts of a static disturbance case, here we only presented the comparison results of a moving pollution source.

A. Diff-MAS2D-PID Simulation Platform

As the technological improvement, a simulation platform called diffusion-mobile-actuators-sensors-2-dimension (Diff-MAS2D) [37], [38] is being used for the control problem of a diffusion process. As the extension of Diff-MAS2D, new simulation platforms called fractional-order-diffusion-mobile-actuators-sensors-2-dimension (FO-Diff-MAS2D) and fractional-order-diffusion-mobile-actuator-sensor-2-dimension fractional-order-proportional-integral (FO-Diff-MAS2D-FOPI) have been introduced to the fractional order anomalous diffusion problem for optimal spraying control [28], [29], [39]. We refer [40] for more basic knowledge about above two simulation platforms.

Motivated by Diff-MAS2D [37], [38], here we develop a modified simulation platform Diff-MAS2D-PID for two-dimensional diffusion control by introducing PID controllers into it. The area concerned is given in uniformization by $\Psi = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1\}$. $M \times N$ sensors evenly distributed in Ψ form a network over the area, which contains n mobile robots releasing the neutralizing chemicals. In the following simulations, the system parameters are set as $M = N = 29$, $n = 4$.

The spatial domain of a diffusion system is discretized by the finite-difference method based on Diff-MAS2D-PID, then the time domain integration is left to MATLAB/Simulink simulation platform. Specifically, Diff-MAS2D-PID can be used to solve the diffusion control problem for actuator location and spraying control via PID controllers. Some main features of Diff-MAS2D-PID are shown below:

1) Sensors and actuators can be collocated or non-collocated.

2) Pollution source can be moving or static.

3) The dynamics equation of actuators location can be modeled as the first order or second order while the diffusion system can be open-loop or closed-loop.

4) The controllers for actuators motion planning and spraying control can be P controllers, PI controllers, PD controllers or PID controllers, which can be applied in $f_c(\tilde{\phi}, x, y, t)$ arbitrarily.

B. Moving Pollution Source

For the case of the moving pollution source, the 4 actuators try to move near the pollution source and release chemistry to neutralize the diffusion process. And the diffusion process is depicted as

$$\frac{\partial \phi}{\partial t} = 0.01\left(\frac{\partial \phi^2}{\partial x^2} + \frac{\partial \phi^2}{\partial y^2}\right) + f_{md}(x,y,t) + f_{mc}(\tilde{\phi}, x, y, t) \tag{35}$$

with the initial condition

$$\phi(x,y,0) = 0 \tag{36}$$

and Neumann boundary conditions

$$\frac{\partial \phi}{\partial \bar{n}} = 0 \tag{37}$$

where f_{md} and f_{mc} represent the moving pollution source and control input, respectively, for the moving issue, $\bar{n}$ is the outward direction normal to the boundary.

The moving pollution source is modeled as a point disturbance f_{md} to the diffusion equation. Moving trajectory for this pollution source could be chosen as arbitrary functions. Without loss of generality, we choose the following form

$$\begin{cases} x = 0.5 + 0.3\cos\dfrac{2\pi t}{50} \\ y = 0.5 + 0.3\sin\dfrac{2\pi t}{50}. \end{cases} \tag{38}$$

And the disturbance equation can be described as follows:

$$f_{md}(t) = 20e^{-t}\big|_{(x=0.5+0.3\cos\frac{2\pi t}{50},\, y=0.5+0.3\sin\frac{2\pi t}{50})}. \tag{39}$$

The moving pollution source begins to diffuse at $t = 0$ to the area Ψ with 4 mobile actuators layout on initial positions at $(0.33, 0.33)$, $(0.33, 0.66)$, $(0.66, 0.33)$, $(0.66, 0.66)$, respectively. In order to show the performance of mobile actuators on controlling the diffusion of the pollutants, control force is added to a diffusion process at $t = 1.0$ s. The time step is set to $\Delta t = 0.004$ s, then the mobile actuators recompute their desired positions every 0.2 s. The total simulation time is $t = 6$ s.

For the actuator motion control, the viscous coefficient is given by $k_v = 1$. To obtain the PI controller for actuator motion control, we will illustrate the tuning steps below. First, choose the optimal value of k_p in a certain range, which makes the total pollution amount lowest. Then, fix the optimal value of k_p, let the value of k_i vary in a certain range, and obtain the optimal value of k_i which also makes the total pollution amount lowest. We will show details in the following figure.

Specifically, Fig. 2(a) shows that the pollution amount gets to the lowest at the end of simulation time when k_p is 9. It is simple to find that the PI control method has a decided advantage over the P control method on the total pollution amount as the value of k_i changes from 0 to 3, as shown in Fig. 2(b). Meanwhile, the PI controller has a serious disadvantage against the P controller as $k_i \geq 6$ in Fig. 2(c). Based on above analysis, we know that the PI control method can present better control performance than the P control method in the range of $0 < k_i < 3$. Without loss of generality, we let $k_i = 1.5$, then we can get the PI controller. Consider this controller, we can obtain the below actuator motion control input

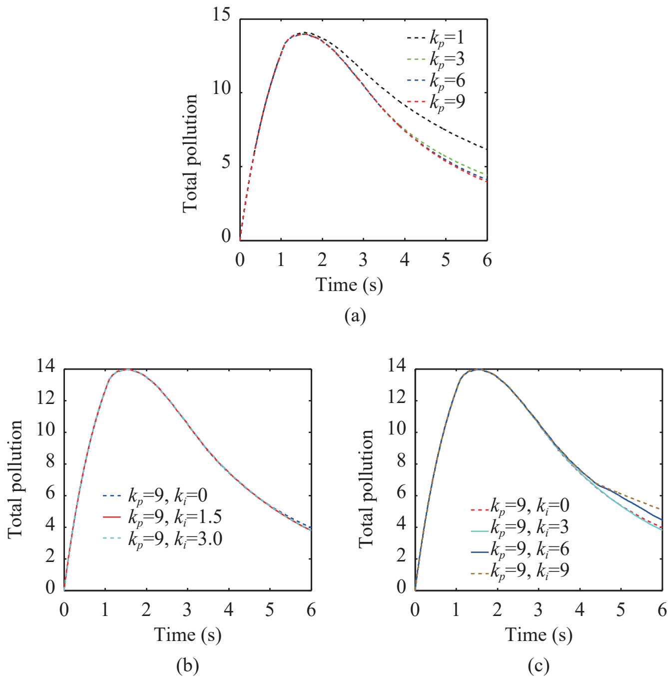

Fig. 2. Evolution of total pollutant amount with different values of k_p and k_i of a static pollution source.

$$u_{p_i} = 9(\bar{p}_i - p_i) + 1.5\int_0^t (\bar{p}_i - p_i)ds - \dot{p}_i. \tag{40}$$

In spraying process, the proportional coefficient is given by $k_{pr} = 10$. To obtain the PI controller for actuator spraying control, we will also illustrate the tuning procedure in below steps. First, give two certain ranges for k_{ir} and δt respectively. Let k_{ir} and δt vary in their ranges. Then, we can obtain the optimal values of k_{ir} and δt, which make the total pollution amount and discrepancy between pollution amount and spraying amount lowest. More details can be found in following figures. For the optimal spraying control strategy, k_{ir} changes from 0 to 30 with the step 10 and δt changes from 0 to 2 s with the step 0.4 s, which is displayed in Fig. 3.

It can be concluded from Fig. 3 that the optimal spraying control can be realized if $\{k_{ir} = 10, \delta t = 2\text{ s}\}$, $\{k_{ir} = 20, \delta t = 1.2\text{ s}\}$, and $\{k_{ir} = 30, \delta t = 1.2\text{ s}\}$ respectively. It can be also seen that the optimal control can be obtained at the end of simulation time as $k_{ir} = 30$ and $\delta t = 1.2$ s in Fig. 4.

Furthermore, the control input could be formulated as

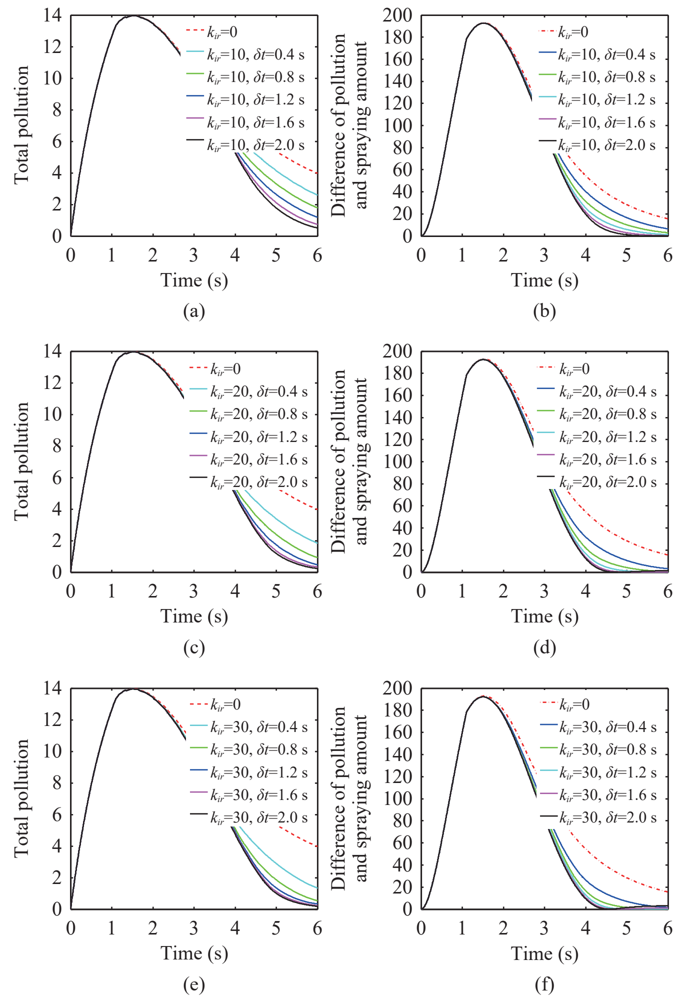

Fig. 3. Spraying control with k_{ir} = 10, 20, 30 and δt = 0.4 s, 0.8 s, 1.2 s, 1.6 s, 2.0 s, of a moving pollution source.

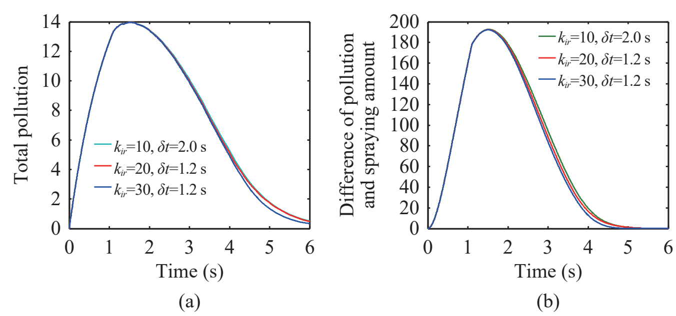

Fig. 4. Spraying control with k_{ir} = 10, 20, 30 and δt = 2.0 s, 1.2 s, 1.2 s of a moving pollution source.

$$u_{s_i}(t) = -10\bar{\phi}_i(x,y,t) - 30\int_{t-1.2}^t \bar{\phi}_i(x,y,\tau)d\tau. \tag{41}$$

For maximizing spraying amount without overdosed, the effectiveness of the PI control method is verified by comparing with the P control method, as shown in Fig. 5. Moreover, from this figure especially Fig. 5 (b), we see the PI control method shows better performance on precision of control.

Fig. 6 presents a comparison of mobile actuators trajectories under the PI control method and the P control method, along

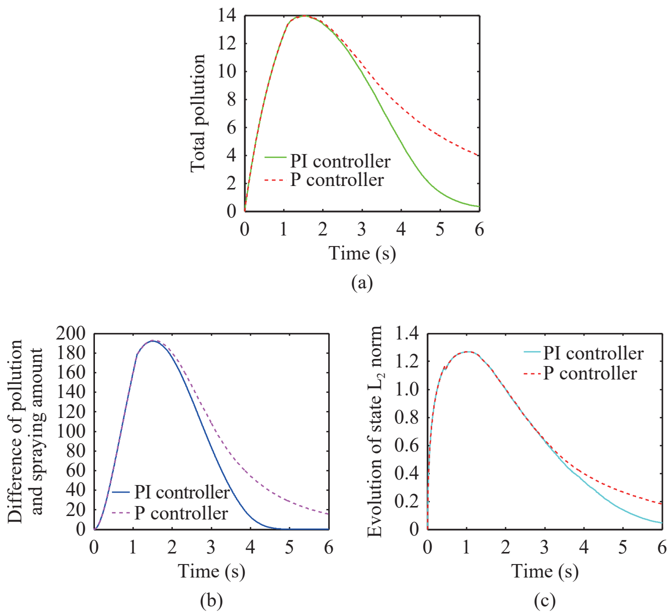

Fig. 5. (a) Evolution of total pollution amount under PI and P controllers of a moving pollution source. (b) difference of total pollution amount and total spraying amount under PI and P controllers of a moving pollution source. (c) evolution of state L_2 norm under PI and P controllers of a moving pollution source.

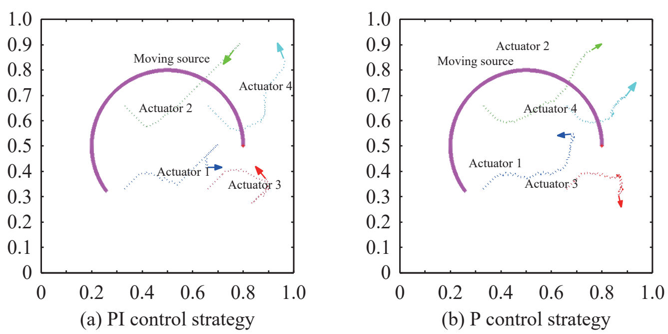

Fig. 6. Trajectories of actuators under PI and P control strategies of a moving pollution source.

with the trajectory of a moving pollution source.

To check the validity of our proposed PI control strategy for neutralizing control, a comparison on spraying amount and spraying speed of each actuator under PI and P control methods has been shown in Fig. 7. It follows that the proposed PI control strategy outperforms the P control strategy in aspects of spraying amount and spraying speed.

Remark 5: If the type of disturbance is different from the exponential decay function in this paper, for example the other type decay function like $\frac{k}{p+t^2}$ $(k, p > 0, t \geq 0)$, then the control effect of the PI control method for the diffusion process is not as good as the one with the exponential decay function. In addition, types of moving trajectory have little effect on performance of the PI control method for the diffusion process. Due to space limitation, deeper analysis on control effect with uncertainties and more types of disturbances will be investigated in our future work.

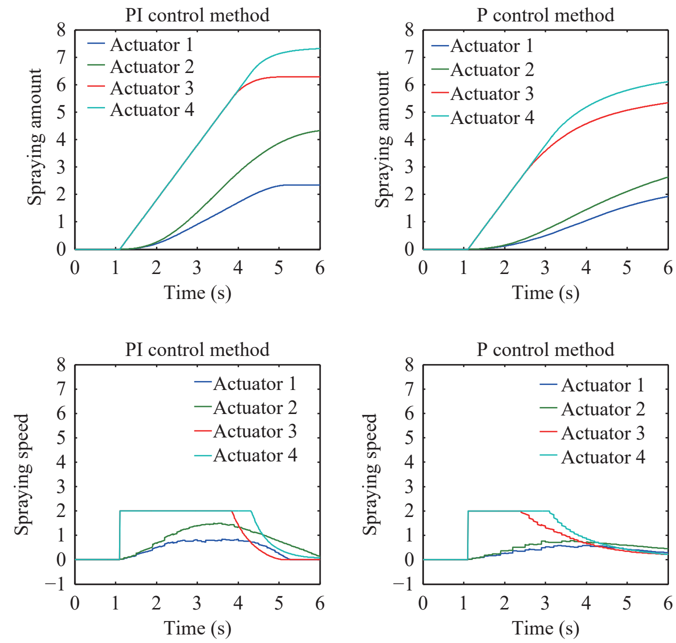

Fig. 7. Evolution of spraying amount and spraying speed under PI and P control methods of a moving pollution source.

VI. Conclusions

In this paper, the control problem of a diffusion process with a moving or static pollution source has been investigated. As the diffusion process evolves slowly enough comparing with the control efforts, CVT applied in time-invariant environment is also available to the time-varying environment. And the convergence problem of the actuators location has been specifically discussed. In addition, with the help of a modified simulation platform (Diff-MAS2D-PID) and the PI-CVT algorithm, we have demonstrated that the novel PI control method is more effective than the P control method for actuator motion and neutralizing control. In the future, the results with the single pollution source can be extended to a diffusion system with multiple pollution sources. Moreover, the proposed method can also be suitable for the diffusion control problem based on the weight cost function of actuator motion control and spraying control.

References

[1] E. V. Dogadkina, M. G. Dontsov, and V. I. Parfenyuk, "The effect of diffusion processes on surface smoothing upon chemical polishing of titanium," *Protection of Metals and Physical Chemistry of Surfaces*, vol. 52, no. 5, pp. 947–953, Sep. 2016.

[2] S. O. Akintunde and P. A. Selyshchev, "The influence of radiationinduced vacancy on the formation of thin-film of compound layer during a reactive diffusion process," *J. Physics and Chemistry of Solids*, vol. 92, pp. 64–69, May 2016.

[3] M. Caputo and C. Cametti, "Fractional derivatives in the diffusion process in heterogeneous systems: the case of transdermal patches," *Mathematical Biosciences*, vol. 291, pp. 38–45, Sep. 2017.

[4] P. Stoneman, "The diffusion of innovations: some reflections," *Int. J. Economics of Business*, vol. 25, no. 1, pp. 85–95, Feb. 2018.

[5] D. Vališ, O. Nováček, K. Hasilováa, and J. Leuchter, "Modelling of degradation and a soft failure moment during the operation of a supercapacitor applying selected diffusion processes," *Engineering Failure Analysis*, vol. 82, pp. 566–582, Dec. 2017.

[6] Y. Q. Chen, Z. M. Wang, and J. S. Liang, "Actuation scheduling in mobile actuator networks for spatial-temporal feedback control of a diffusion process with dynamic obstacle avoidance," in *Proc. IEEE Int. Conf. Mechatronics and Autom.*, vol. 2, pp. 752–757, Jul. 2005.

[7] F. D. Ge, Y. Q. Chen, and C. H. Kou, "Cyber-physical systems as general distributed parameter systems: three types of fractional order models and emerging research opportunities," *IEEE/CAA J. Autom. Sinica*, vol. 2, no. 4, pp. 353–357, Oct. 2015.

[8] M. J. Balas and S. A. Frost, "Robust adaptive model tracking for distributed parameter control of linear infinite-dimensional systems in Hilbert space," *IEEE/CAA J. Autom. Sinica*, vol. 1, no. 3, pp. 294–301, Jul. 2014.

[9] Q. Du, V. Faber, and M. Gunzburger, "Centroidal voronoi tessellations: applications and algorithms," *SIAM Review*, vol. 41, no. 4, pp. 637–676, Dec. 1999.

[10] L. L. Ju, Q. Du, and M. Gunzburger, "Probabilistic methods for centroidal voronoi tessellations and their parallel implementations," *Parallel Computing*, vol. 28, no. 10, pp. 1477–1500, Oct. 2002.

[11] J. Cortés, S. Martínez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Trans. Robotics and Autom.*, vol. 20, no. 2, pp. 243–255, Apr. 2004.

[12] Y. Q. Chen, K. L. Moore, and Z. Song, "Diffusion boundary determination and zone control via mobile actuator-sensor networks (MASnet): challenges and opportunities," in *Proc. SPIE Conf. Intelligent Computing: Theory and Applications II*, vol. 5421, pp. 102–113, Apr. 2004.

[13] Q. Du, M. Gunzburger, and L. L. Ju, "Advances in studies and applications of centroidal voronoi tessellations," *Numerical Mathematics*: *Theory*, *Methods and Applications*, vol. 3, no. 2, pp. 119–142, May 2010.

[14] M. Santos, Y. Diaz-Mercado, and M. Egerstedt, "Coverage control for multirobot teams with heterogeneous sensing capabilities," *IEEE Robotics and Autom. Letters*, vol. 3, no. 2, pp. 919–925, Apr. 2018.

[15] S. Cheng and F. Yuan, "Coverage control for mobile sensor networks with limited communication ranges on a circle," *Automatica*, vol. 92, pp. 155–161, Jun. 2018.

[16] M. Schwager, M. P. Vitus, S. Powers, D. Rus, and C. J. Tomlin, "Robust adaptive coverage control for robotic sensor networks," *IEEE Trans. Control Network Systems*, vol. 4, no. 3, pp. 462–476, Sep. 2017.

[17] J. Habibi, H. Mahboubi, and A. G. Aghdam, "Distributed coverage control of mobile sensor networks subject to measurement error," *IEEE Trans. Autom. Control*, vol. 61, no. 11, pp. 3330–3343, Nov. 2016.

[18] A. L. Faulds and B. B. King, "Sensor location in feedback control of partial differential equation systems," in *Proc. IEEE Int. Conf. on Control Applications*, pp. 536–541, Sep. 2000.

[19] Y. Q. Chen, Z. M. Wang, and J. S. Liang, "Optimal dynamic actuator location in distributed feedback control of a diffusion process," in *Proc. 44th IEEE Conf. Decision and Control, and the European Control Conf.*, pp. 5562–5567, Dec. 2005.

[20] D. Uciński, *Optimal Measurement Methods for Distributed Parameter System Identification*. Boca Raton: CRC Press, 2004.

[21] H. Y. Chao and Y. Q. Chen, *Remote Sensing and Actuation Using Unmanned Vehicles*. Hoboken, NJ, USA: John Wiley & Sons, Inc., 2012.

[22] Q. Y. Chen, Z. M. Wang, and K. L. Moore, "Optimal spraying control of a diffusion process using mobile actuator networks with fractional potential field based dynamic obstacle avoidance," in *Proc. IEEE Int. Conf. Networking, Sensing and Control (ICNSC)*, pp. 107–112, Apr. 2006.

[23] H. Y. Chao, Y. Q. Chen, and W. Ren, "A study of grouping effect on mobile actuator sensor networks for distributed feedback control of diffusion process using central Voronoi tessellations," in *Proc. IEEE Int. Conf. Mechatronics and Automation*, pp. 769–774, Jun. 2006.

[24] H. Y. Chao, Y. Q. Chen, and W. Ren, "Consensus of information in distributed control of a diffusion process using centroidal voronoi tessellations," in *Proc. IEEE Conf. Decision and Control*, pp. 1441–1446, Dec. 2007.

[25] Y. Liu, F. Guo, X. Y. He, and Q. Hui, "Boundary control for an axially moving system with input restriction based on disturbance observers," *IEEE Trans. Systems*, *Man*, *and Cybernetics Systems*, vol. 49, no. 11, pp. 2242–2253, Nov. 2019.

[26] Y. Liu, Z. J. Zhao, and W. He, "Boundary control of an axially moving accelerated/decelerated belt system," *Int. J. Robust and Nonlinear Control*, vol. 26, pp. 3849–3866, Mar. 2016.

[27] Y. Liu, Z. J. Zhao, and W. He, "Stabilization of an axially moving accelerated/decelerated system via an adaptive boundary control," *ISA Trans.*, vol. 64, pp. 394–404, Sep. 2016.

[28] F. D. Ge, J. Chen, Y. Q. Chen, and C. H. Kou, "Optimal spraying control of a tempered time fractional order anomalous diffusion system," in *Proc. Int. Symposium on Fractional Signals and Systems*, pp. 43–48, Oct. 2015.

[29] J. Chen, B. Zhuang, Y. Q. Chen, and B. T. Cui, "Diffusion control for a tempered anomalous diffusion system using fractional-order PI controllers," *ISA Trans.*, vol. 82, pp. 94–106, Nov. 2018.

[30] D. G. Luenberger and Y. Y. Ye, *Linear and Nonlinear Programming*, 3rd ed. Boston: Springer, 2008.

[31] D. P. Bertsekas, *Constrained Optimization and Lagrange Multiplier Methods*. New York: Academic Press, 1982.

[32] A. Howard, M. J. Matarić, and G. S. Sukhatme, "Mobile sensor network deployment using potential fields: a distributed, scalable solution to the area coverage problem," in *Proc. 6th Int. Symposium on Distributed Autonomous Robotic Systems (DARS02)*, vol. 5, pp. 299–308, Jun. 2002.

[33] N. Heo and P. K. Varshney, "Energy-efficient deployment of intelligent mobile sensor networks," *IEEE Trans. Systems*, *Man*, *and Cybernetics–Part A*: *Systems and Humans*, vol. 35, no. 1, pp. 78–92, Jan. 2005.

[34] A. van der Schaft. *L_2-gain and Passivity Techniques in Nonlinear Control*, 2nd ed. New York: Springer-Verlag, 1999.

[35] J. P. Lasalle, "Complete stability of a nonlinear control system," in *Proc. National Academy of Sciences of the United States of America*,

vol. 48, no. 4, pp. 600–603, Apr. 1962.

[36] J. Cortés, S. Martínez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks: variations on a theme," in *Proc. 10th Mediterranean Conf. Control and Automation-MED*, Jul. 2002.

[37] J. S. Liang and Y. Q. Chen, "Diff-MAS2D (version 0.9) user's manual: a simulation platform for controlling distributed parameter systems (diffusion) with networked movable actuators and sensors (MAS) in 2D domain," *Techical Report, Center for Self-Organizing and Intelligent Systems (CSOIS), Department of Electrical and Computer Engineering, College of Engineering, Utah State University, USA*, 2004.

[38] J. S. Liang and Y. Q. Chen, "Diff/Wave-MAS2D: a simulation platform for measurement and actuation scheduling in distributed parameter systems with mobile actuators and sensors," in *Proc. IEEE Int. Conf. Mechatronics and Autom.*, pp. 2228–2233, Jul. 2005.

[39] J. X. Cao, Y. Q. Chen, and C. P. Li, "Multi-UAV-based optimal cropdusting of anomalously diffusing infestation of crops," in *Proc. American Control Conf.*, pp. 1278–1283, Jul. 2015.

[40] D. Y. Xue and Y. Q. Chen, *System Simulation Techniques with MATLAB and Simulink*. New Jersey, USA: Wiley & Sons, 2013.

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