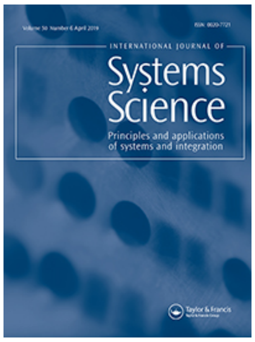

International Journal of Systems Science

ISSN: 0020-7721 (Print) 1464-5319 (Online) Journal homepage: https://www.tandfonline.com/loi/tsys20

Robust passivity and feedback passification of a class of uncertain fractional-order linear systems

Liping Chen, Tingting Li, YangQuan Chen, Ranchao Wu & Suoliang Ge

To cite this article: Liping Chen, Tingting Li, YangQuan Chen, Ranchao Wu & Suoliang Ge (2019) Robust passivity and feedback passification of a class of uncertain fractional-order linear systems, International Journal of Systems Science, 50:6, 1149-1162, DOI: 10.1080/00207721.2019.1597940

To link to this article: https://doi.org/10.1080/00207721.2019.1597940

Published online: 30 Mar 2019.

INTERNATIONAL JOURNAL OF SYSTEMS SCIENCE
2019, VOL. 50, NO. 6, 1149–1162
https://doi.org/10.1080/00207721.2019.1597940

Taylor & Francis
Taylor & Francis Group

Robust passivity and feedback passification of a class of uncertain fractional-order linear systems

Liping Chen[a], Tingting Li[a], YangQuan Chen [b], Ranchao Wu[c] and Suoliang Ge[a]

[a]School of Electrical Engineering and Automation, Hefei University of Technology, Hefei, People's Republic of China; [b]Mechatronics, Embedded Systems and Automation Lab, University of California, Merced, CA, USA; [c]School of Mathematics, Anhui University, Hefei, People's Republic of China

ABSTRACT
Theoretical results on robust passivity and feedback passification of a class of uncertain fractional-order (FO) linear systems are presented in the paper. The system under consideration is subject to time-varying norm-bounded parameter uncertainties in both the state and controlled output matrices. Firstly, some suitable notions of passivity and dissipativity for FO systems are proposed, and the relationship between passivity and stability is obtained. Then, a sufficient condition in the form of linear matrix inequality (LMI) for such system to be robustly passive is given. Based on this condition, the design method of state feedback controller is proposed when the states are available. Moreover, by using matrix singular value decomposition and LMI techniques, the existing condition and method of designing a robust observer-based passive controller for such systems are derived. Numerical simulations demonstrate the effectiveness of the theoretical formulation.

ARTICLE HISTORY
Received 23 February 2018
Accepted 17 March 2019

KEYWORDS
Fractional-order systems; passification; passivity; uncertain

1. Introduction

More and more FO phenomena are found in the field of natural and engineering, such as quantum mechanics, stochastic diffusion, molecular spectroscopy, viscoelastic dynamics, robotics, etc. (Hilfer, 2000; Laskin, 2018; Podlubny, 1999; West, 2016), the modelling, dynamics analysis and control of these phenomena, has become a new research focus of control science and engineering (Ge, Chen, & Kou, 2018; Luo & Chen, 2012; Tepljakov, 2017; Xue, 2017). After several decades of developments, control theory of FO system has made considerable development, particularly stability and stabilisation analysis (Chen, Chai, Wu, & Yang, 2012; Dadras, Dadras, Malek, & Chen, 2017; Dadras & Momeni, 2014; Kaminski, Shorten, & Zeheb, 2015; Kuntanapreeda, 2016; Lu & Chen, 2010), controllability and observability (Ge, Chen, & Kou, 2016, 2017; Sabatier, Farges, Merveillaut, & Feneteau, 2012; Surendra & Nagarajan, 2013), optimal control (Ding, Wang, & Ye, 2012; Wang & Zhou, 2011), system identification (Hartley & Lorenzo, 2003; Jacyntho et al., 2015). However, due to fractional calculus are non-local and have weakly singular kernels, it is far less richness than that of integer order systems, there are still many difficulties and challenges that need to be explored both in the theory and applications, such as passivity, dissipativity theory and passivity-based control (Rakhshan, Gupta, & Goodwine, 2017).

Passivity, and more generally dissipativity exist extensively in physics, applied mathematics and mechanics, etc., have originally arisen from the electrical circuit theory and physical systems (Ortega, 1991). Roughly speaking, a system is passive if the amount of the energy stored by the system is less than which is supplied to the system, that is to say, the system dissipates some energy, but it does not generate energy internally its own (Christopher, Byrnes, & Willems, 1991; Lozano, Brogliato, & Landau, 1992). A beneficial property of passivity is that it is a compositional property for parallel and feedback interconnections (Xia, Antsaklis, Gupta, & Zhu, 2017). It has been shown that it provides a powerful tool for analysis and synthesis of linear and nonlinear system in the form of input and output from the energy point of view, and has been used in different area, for example, chemical reactors systems (Garcia-Sandoval, Gonzalez-Alvarez, & Calderon, 2015) multi-agent systems (Chopra & Spong, 1999), cyber-physical systems (Antsaklis et al., 2013), power electronics system (Zeng, Zhang, & Qiao, 2014), neural networks (Li, Gao, & Shi, 2010), networked control systems (Kottenstette, Hall, Koutsoukos, Sztipanovits, & Antsaklis, 2013) and so

CONTACT Liping Chen lip_chenhut@126.com School of Electrical Engineering and Automation, Hefei University of Technology, 230009 Hefei, People's Republic of China

on. One the other hand, the storage functions is an essential tool to study the issue of passivity and passification, which it can be also considered as a natural candidate for the Lyapunov function under certain condition. For FO system, it is very difficult to construct a Lyapunov function and calculate its fractional derivative. Therefore, to address the issue of passivity and passification of FO systems provides a new approach to consider stability and stabilisation of FO system.

The aim of this paper is to generalise the concept of passivity and dissipativity to FO systems. The main contributions of this paper are summarised as follows: (1) Some notions of passivity and dissipativity for FO systems are shown. (2) Relationship between passivity and stability of FO systems is illustrated as well. (3) Taking a class of FO uncertain linear system as an objective, robust passivity of such system is analysed, state feedback passive controller and observer-based state feedback passive controller are proposed to guarantee the corresponding closed-loop system is robustly passive, respectively.

The rest of this paper is organised as follows. Section 2 describes some basic fractional calculus definitions, proposes passivity and dissipativity notions for FO systems, and the relationship between the stability and passive for FO systems, introduces the problem to be addressed and some necessary lemmas. Section 3 presents the main results and discusses the most relevant details. Section 4 demonstrates the effectiveness of the results for three numerical examples. Finally, some future research directions and conclusions are presented.

Notation: Standard symbols and notation are used throughout the paper. The following symbols stand for: $AC[a,b]$ the space of function f which are absolutely continuous on $[a,b]$. I identity matrix of appropriate order, and $*$ the elements below the main diagonal of a symmetric block matrix. The superscript T the transpose, respectively, $diag\{\cdot\}$ the diagonal matrix. $L_2[0,\infty)$ the space of square summable infinite vector sequences. $X > 0 (< 0)$ a symmetric positive definite (negative definite) matrix. X matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Preliminaries and model description

In this section, the model is formulated, and some definitions, properties and lemmas to be used later are presented.

Definition 2.1 (Podlubny, 1999): The fractional integral with non-integer order $\alpha > 0$ of function $x(t)$ is defined as follows:

$$I^{\alpha}_{t_0,t}x(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}x(\tau)\mathrm{d}\tau,$$

where $\Gamma(\cdot)$ is the Gamma function, $\Gamma(s) = \int_0^{\infty} t^{s-1} \mathrm{e}^{-t}\mathrm{d}t$.

Definition 2.2 (Podlubny, 1999): The Caputo derivative of FO α of function $x(t)$ is defined as follows:

$$\begin{aligned} {}_{C}D^{\alpha}_{t_0,t}x(t) &= I^{n-\alpha}_{t_0,t}\frac{d^n}{\mathrm{d}t^n}x(t) \\ &= \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(t-\tau)^{n-\alpha-1}x^{(n)}(\tau)\mathrm{d}\tau, \end{aligned}$$

where $n-1 < \alpha < n \in Z^{+}$.

It follows from the Definitions 2.1 and 2.2 that the fractional derivative is related to all the history information of a function, while the integer one is only related to its nearby points. That is, the next state of a system not only depends upon its current state but also upon its historical states starting from the initial time. In the following, the notation D^{α} is chosen as the Caputo derivative $D^{\alpha}_{0,t}$.

Consider the following FO system

$$\begin{aligned} D^{\alpha}x(t) &= f(x(t),u(t),t), \\ y(t) &= h(x(t),u(t),t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the input, $y(t) \in R^m$ is the output, $\alpha \in (0,1)$, function f is continuous and locally Lipschitz, and satisfies $f(0,0,t) = 0$, $h(0) = 0$. Obviously, $x=0$ is an equilibrium point for $u(t) = 0$.

Definition 2.3: Function ω is referred to as supply rate of system (1), if function ω from $R^m \times R^m$ to R^m, which is locally integrable along the solutions of (1), that is, for any fixed initial condition $x(0) = x_0 \in R^n$ and any admissible input $u : R \to R^n$

$$\int_0^t |\omega(u(s),y(s))|\mathrm{d}s < +\infty. \quad \forall\, t \geq 0.$$

Definition 2.4: If there exists a nonnegative real-valued function V, called a storage function, such that for all admissible input u, all initial conditions x_0, and all $t \geq 0$, the solution $x(t)$ exists for each $t \geq 0$ and satisfies

$$\begin{aligned} V(x(t)) - V(x_0) &\leq I^{\alpha}_{0,t}w(t) \\ &= \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}w(u(s),y(s))\mathrm{d}s, \end{aligned} \tag{2}$$

then system (1) is called as dissipative with respect to the supply rate ω. Moreover, if there exist a positive definite function S such that

$$V(x(t)) - V(x_0) \le I_{0,t}^{\alpha}\omega(t) - I_{0,t}^{\alpha}S(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\omega(u(s),y(s))\mathrm{d}s - \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}S(s)\mathrm{d}s, \quad (3)$$

then the system is said to be strictly dissipative.

Definition 2.5: If there exist a storage function V satisfying $V(0) = 0$ and (2) or (3), and the supply rate

$$\omega = y^T u,$$

then system (1) is passive or strictly passive, respectively.

Definition 2.6: System (1) is input feed-forward output feedback passive, if there exist $\mu, \nu \in R$ such that system is dissipative with respect to the following supply rete

$$\omega = u^T y - \mu u^T u - \nu y^T y.$$

Property 2.1: *If $x(t) \in C^n[a,b]$, then*

$$I^{\alpha}D^{\alpha}x(t) = x(t) - \sum_{k=0}^{n-1}\frac{x^{(k)}(a)}{k!}(x-a)^k,$$

$$D^{\alpha}I^{\alpha}x(t) = x(t).$$

Remark 2.1: It follows from the Definitions 2.4, 2.5 and Property 2.1 that if

$$D^{\alpha}V(t) \le w(t),$$

then (2) is satisfied. However, the converse is not necessarily true.

Lemma 2.1 (Li, Chen, & Podlubny, 2009): *Let $x=0$ be an equilibrium point for system (2) and $D \in R^n$ be a domain containing the origin. Let $V(t,x(t)) : [0,\infty) \times D \longrightarrow R$ be a continuously differentiable function and locally Lipschitz with respect to x such that*

$$\alpha_1\|x\|^a \le V(t,x(t)) \le \alpha_2\|x\|^{ab},$$
$$D^{\alpha}V(t,x(t)) \le -\alpha_3\|x\|^{ab}, \quad (4)$$

where $t \ge 0$, $x \in D$, $\beta \in (0,1)$, $\alpha_1, \alpha_2, \alpha_3, a$ and b are arbitrary positive constants. Then $x=0$ is Mittag–Leffler stable (asymptotically stable). If the assumptions hold globally on R^n, then $x=0$ is globally Mittag–Leffler stable (asymptotically stable).

Property 2.2: *Assume that condition (4) in Lemma 2.1 is modified to be*

$$D^{\alpha}V(t,x(t)) \le 0, \quad (5)$$

and $V(0) = 0$, the argument still stands, i.e. $x=0$ is globally Mittag–Leffler stable (asymptotically stable).

Based on Property 2.2, one has the following relationship between passivity and Lyapunov stability.

Property 2.3: *If system (1) is passive with a positive semi-definite storage function $V = V(x)$, then the system is stable in the sense of Lyapunov with origin as the equilibrium point.*

Proof: By setting $u=0$ and using Definition 2.5, $D^{\alpha}V \le \omega(0,y) = 0$. It follows from Property 2.2 and Lemma 2.1 that system (1) is Mittag–Leffler stable (asymptotically stable).

In the paper, consider the following FO uncertain systems

$$\begin{aligned} D^{\alpha}x(t) &= (A_0 + \Delta A)x(t) + (B_0 + \Delta B)u(t) \\ &\quad + (B_{\sigma 0} + \Delta B_{\sigma})\sigma(t), \\ y(t) &= (C_0 + \Delta C)x(t) + (D_0 + \Delta D)u(t) \\ &\quad + (D_{\sigma 0} + \Delta D_{\sigma})\sigma(t), \\ z(t) &= C_1 x(t), \end{aligned} \quad (6)$$

where $x(t) = (x_1(t), \ldots, x_n(t))^{\mathrm{T}} \in R^n$ represents the state vector of the system, $\sigma(t) \in R^q$ is the external disturbance which is assumed to belong to $L_2[0,\infty]$ (Sun, Mou, Qiu, Wang, & Gao, 2018; Wang, Qiu, & Feng, 2018), $y(t)$ denote the controlled output, $u(t) \in R^p$ is the control input, $z(t) \in R^l$ is the measured output, the FO α belongs to the interval $(0,1)$. A_0, B_0, C_0, D_0, C_1 are some nominal constant matrices with appropriate dimensions. The uncertain matrices $\Delta A, \Delta B, \Delta C, \Delta D$ are time-varying uncertain matrices with appropriate dimensions subject to the following form:

$$\begin{bmatrix} \Delta A & \Delta B & \Delta B_{\sigma} \\ \Delta C & \Delta D & \Delta D_{\sigma} \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} H(t) \begin{bmatrix} F_1 & F_2 & F_3 \end{bmatrix},$$

where E_1, E_2, and F_1, F_2, F_3 are known constant real matrices with appropriate dimensions, and $H(t)$ are unknown time-varying matrices satisfying

$$H^T(t)H(t) \le I.$$

This paper is concerned with the problems of passivity and passification for system (6), passivity conditions are obtained in the form of LMIs. Based on these conditions, two procedures for designing passification controllers are proposed. For this end, some standard lemmas are presented, which are involved in the following. ■

Lemma 2.2 (Liang, Wu, & Chen, 2015): *Let $x(t) \in R^n$ be a differentiable vector-value function. Then, for any time instant $t \geq t_0$*

$$D^\alpha(x^T(t)Px(t)) \leq (x^T(t)P)D^\alpha x(t) + (D^\alpha x(t))^T Px(t),$$

where $P \in R^{n\times n}$ is a symmetric positive definite matrix, $\alpha \in (0, 1)$.

Lemma 2.3 (Xie, 1996): *Given matrices $Q = Q^T$, H,E and $R = R^T > 0$ of appropriate dimension,*

$$Q + HFE + E^T F^T H^T < 0,$$

for all F satisfying $F^T F \leq R$, if and only if there exist some $\lambda > 0$ such that

$$Q + \lambda HH^T + \lambda^{-1}E^T RE < 0.$$

Lemma 2.4 (Boyd, Ghaoui, Feron, & Balakrishnan, 1994): (*Schur complement*): *For a real matrix $\Sigma = \Sigma^T$, the following assertions are equivalent*

$$\Sigma := \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix} < 0,$$

$$\Sigma_{11} < 0, \Sigma_{22} - \Sigma_{12}^T \Sigma_{11}^{-1} \Sigma_{12} < 0,$$

$$\Sigma_{22} < 0, \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{12}^T < 0.$$

Lemma 2.5 (MacDuffee, 2004): *For any matrix $\Pi \in R^{q\times n}$ with $q < n$ and full row rank* (rank$(\Pi = q)$), *there exists an SVD of Π as follows*

$$\Pi = U\begin{bmatrix} S & 0 \end{bmatrix} V^T,$$

where $S \in R^{q\times q}$ is a diagonal matrix with non-negative diagonal elements in decreasing order, $U \in R^{q\times q}$, $V \in R^{nn}$ are the unitary matrices.

Lemma 2.6 (MacDuffee, 2004): *Given matrix $\Pi \in R^{q\times n}$ with $q < n$ and rank$(\Pi) = q$, assume that $X \in R^{n\times n}$ is a symmetric matrix, then there exists a matrix $X \in R^{q\times q}$ satisfying $\Pi X = X\Pi$ if and only if X can be described as*

$$X = V\begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix} V^T,$$

where $X_{11} \in R^{q\times q}, X_{22} \in R^{(n-q)\times(n-q)}$ and $V \in R^{n\times n}$ is the unitary matrix of SVD of Π.

3. Main results

As stated in the previous section, our aims are to address passivity analysis and passification for system (6), which will be accomplished in three subsection in what follows.

3.1. Robust passivity analysis

In this subsection, robust passivity for uncertain systems (6) with $u(t) = 0$ will be considered, a sufficient condition for the system to be robustly passive is presented.

Theorem 3.1: *If there exist symmetric positive-definite matrices $\tilde{P}$, constant positive scalar λ such that the following LMI holds*

$$\begin{bmatrix} A_0\tilde{P} + \tilde{P}A_0^T & B_{\sigma 0} - \frac{1}{2}\tilde{P}C_0^T & \tilde{P}F_1^T & \lambda E_1 \\ * & -D_{\sigma 0} & F_3^T & -\frac{1}{2}\lambda E_2 \\ * & * & -\lambda I & 0 \\ * & * & * & -\lambda I \end{bmatrix} < 0, \tag{7}$$

then system (6) with $u(t) = 0$ is robustly passive.

Proof: When $u(t) = 0$, system (6) can be rewritten as

$$\begin{aligned} D^\alpha x(t) &= (A_0 + \Delta A)x(t) + (B_{\sigma 0} + \Delta B_\sigma)\sigma(t), \\ y(t) &= (C_0 + \Delta C)x(t) + (D_{\sigma 0} + \Delta D_\sigma)\sigma(t). \end{aligned} \tag{8}$$

Let us select a storage function for system (8) as

$$V(x(t)) = x^T(t)Px(t).$$

Then, based on Lemma 2.2, take the FO derivative along system (8) giving

$$D^\alpha V(x(t)) \leq (x^T(t)P)D^\alpha x(t) + (D^\alpha x(t))^T Px(t).$$

Recalling the passivity Definition 2.5, one has

$$\begin{aligned} &D^\alpha V(x(t)) - y^T(t)\sigma(t) \\ &\quad\leq (x^T(t)P)D^\alpha x(t) + (D^\alpha x(t))^T Px(t) - y^T(t)\sigma(t) \\ &\quad= x^T(t)(PA_0 + A_0^T P + P\Delta A + \Delta A^T P)x(t) \\ &\qquad + x^T(t)PB_{\sigma 0}\sigma(t) + \sigma^T(t)B_{\sigma 0}^T Px(t) \\ &\qquad + x^T(t)P\Delta B_\sigma\sigma(t) + \sigma^T(t)\Delta B_\sigma^T Px(t) \\ &\qquad - x^T(t)C_0^T\sigma(t) - x^T(t)\Delta C^T\sigma(t) \\ &\qquad - \sigma^T(t)D_{\sigma 0}^T\sigma(t) - \sigma^T(t)\Delta D_\sigma^T\sigma(t) \\ &\quad= \eta^T(t)\Pi\eta(t), \end{aligned}$$

where $\eta(t) = [x^T(t), \sigma^T(t)]^T$,

$$\Pi = \begin{bmatrix} \begin{matrix} PA_0 + A_0^T P \\ +P\Delta A + \Delta A^T P \end{matrix} & PB_{\sigma 0} + P\Delta B_\sigma - \frac{1}{2}C_0^T - \frac{1}{2}\Delta C^T \\ * & -D_{\sigma 0} - \frac{1}{2}\Delta D_\sigma - \frac{1}{2}\Delta D_\sigma^T \end{bmatrix}$$

$$= \begin{bmatrix} \Pi_{11} & PB_{\sigma 0} + PE_1H(t)F_3 - \frac{1}{2}C_0^T - \frac{1}{2}(E_2H(t)F_1)^{\mathrm{T}} \\ * & -D_{\sigma 0} - \frac{1}{2}E_2H(t)F_3 - \frac{1}{2}(E_2H(t)F_3)_\sigma^T \end{bmatrix},$$

where $\Pi_{11} = PA_0 + A_0^T P + PE_1H(t)F_1 + (E_1H(t)F_1)^T P$.

Thus, if $\Pi < 0$, the system is passive according to Definition 2.5. In fact, one can observer that $\Pi < 0$ is equivalent to the following condition:

$$\Pi = \begin{bmatrix} PA_0 + A_0^T P & PB_{\sigma 0} - \frac{1}{2}C_0^T \\ * & -D_{\sigma 0} \end{bmatrix} + \begin{bmatrix} PE_1 \\ -\frac{1}{2}E_2 \end{bmatrix} H(t) \begin{bmatrix} F_1 & F_3 \end{bmatrix} + \begin{bmatrix} F_1^T \\ F_3^T \end{bmatrix} H(t) \begin{bmatrix} E_1^T P & -\frac{1}{2}E_2^T \end{bmatrix} < 0. \tag{9}$$

Let $H = [\begin{smallmatrix} PE_1 \\ -\frac{1}{2}E_2 \end{smallmatrix}]$, $E = [F_1 \quad F_3]$ and $F = H(t)$ in Lemma 2.3 and applying Lemma 2.3 to (9), there exist a positive number $\lambda > 0$ such that

$$\begin{bmatrix} PA_0 + A_0^T P & PB_{\sigma 0} - \frac{1}{2}C_0^T \\ * & -D_{\sigma 0} \end{bmatrix} + \lambda \begin{bmatrix} PE_1 \\ -\frac{1}{2}E_2 \end{bmatrix} \begin{bmatrix} E_1^T P & -\frac{1}{2}E_2^T \end{bmatrix} + \lambda^{-1} \begin{bmatrix} F_1^T \\ F_3^T \end{bmatrix} \begin{bmatrix} F_1 & F_3 \end{bmatrix} < 0,$$

which can be rearranged as

$$\begin{bmatrix} \Pi_1 & PB_{\sigma 0} - \frac{1}{2}C_0^T - \frac{1}{2}\lambda PE_1E_2^T + \lambda^{-1}F_1^T F_3 \\ * & -D_{\sigma 0} + \frac{1}{4}\lambda E_2E_2^T + \lambda^{-1}F_3^T F_3 \end{bmatrix} < 0, \tag{10}$$

where $\Pi_1 = PA_0 + A_0^T P + \lambda PE_1E_1^T P + \lambda^{-1}F_1^T F_1$.

Using the well-known Schur complements (Lemma 2.4), (10) can be written as

$$\begin{bmatrix} PA_0 + A_0^T P & PB_{\sigma 0} - \frac{1}{2}C_0^T & F_1^T & \lambda PE_1 \\ * & -D_{\sigma 0} & F_3^T & -\frac{1}{2}\lambda E_2 \\ * & * & -\lambda I & 0 \\ * & * & * & -\lambda I \end{bmatrix} < 0. \tag{11}$$

Multiply by $\mathrm{diag}(P^{-1}, 0, 0, 0)$ on both sides of (11), it yields

$$\begin{bmatrix} A_0P^{-1} + P^{-1}A_0^T & B_{\sigma 0} - \frac{1}{2}P^{-1}C_0^T & P^{-1}F_1^T & \lambda E_1 \\ * & -D_{\sigma 0} & F_3^T & -\frac{1}{2}\lambda E_2 \\ * & * & -\lambda I & 0 \\ * & * & * & -\lambda I \end{bmatrix} < 0. \tag{12}$$

Denote $P^{-1} = \tilde{P}$ in (12), (7) can be obtained directly from (12). This completes the proof. ∎

Remark 3.1: Based on Property 2.3, Theorem 3.1 can be serve as a sufficient condition to guarantee that system (6) is Mittag–Leffer stable.

3.2. State feedback passive control

This subsection is devoted to address the passification problem of system (6). If the states are measurable, the following linear feedback controller will be designed

$$u(t) = Kx(t), \tag{13}$$

where $K \in R^{p\times n}$, such that system (6) with controller (13) is passive.

Theorem 3.2: *If there exist a matrix X with appropriate dimensions, a symmetric positive-definite matrix $\tilde{P}$ and constant positive scalar λ such that the following LMI holds*

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & X^T F_2^T + \tilde{P}F_1^T & \lambda E_1 \\ * & -D_{\sigma 0} & F_3^T & -\frac{1}{2}\lambda E_2 \\ * & * & -\lambda I & 0 \\ * & * & * & -\lambda I \end{bmatrix} < 0, \tag{14}$$

where

$$\Sigma_{11} = A_0\tilde{P} + B_0X + \tilde{P}A_0^T + X^T B_0^T,$$
$$\Sigma_{12} = B_{\sigma 0} - \frac{1}{2}\tilde{P}C_0^T + X^T D_0,$$

then system (6) with the controller (13) is robustly passive. Moreover, a stabilising state-feedback gain matrix is given by

$$K = X\tilde{P}^{-1}. \tag{15}$$

Proof: Substituting the controller (13) into (6) leads to the closed-loop system as follow

$$\begin{aligned} D^\alpha x(t) &= (A_0 + B_0K + \Delta A + \Delta BK)x(t) \\ &\quad + (B_{\sigma 0} + \Delta B_\sigma)\sigma(t), \\ y(t) &= (C_0 + D_0K + \Delta C + \Delta DK)x(t) + (D_{\sigma 0} \\ &\quad + \Delta D_\sigma)\sigma(t). \end{aligned} \tag{16}$$

Since $\Delta A + \Delta BK = E_1H(F_1 + F_2K)$, $\Delta C + \Delta DK = E_2H(F_1 + F_2K)$. By replacing A_0 with $A_0 + B_0K$, C_0 with

$C_0 + D_0K$ and F_1 with $F_1 + F_2K$ in (7), one has

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \tilde{P}(K^TF_2^T + F_1^T) & \lambda E_1 \\ * & -D_{\sigma 0} & F_3^T & -\frac{1}{2}\lambda E_1 \\ * & * & -\lambda I & 0 \\ * & * & * & -\lambda I \end{bmatrix} < 0, \tag{17}$$

where

$$\Omega_{11} = (A_0 + B_0K)\tilde{P} + \tilde{P}(A_0^T + K^TB_0^T),$$
$$\Omega_{12} = B_{\sigma 0} - \frac{1}{2}\tilde{P}(C_0^T + K^TD_0).$$

Defining $K\tilde{P} = X$ in (17), Theorem 3.2 is straightforward. ■

3.3. Observer-based output passive control

The aim of this subsection is to design an observer-based output feedback controller such that systems (6) is robustly passive.

Without loss of generality, assume that the FO system (6) is controllable and observable. Consider a state observer and feedback controller described by

$$\begin{aligned} D^\alpha \hat{x}(t) &= \hat{A}\hat{x}(t) + B_0u(t) + L(z(t) - \hat{z}(t)), \\ \hat{z}(t) &= C_1\hat{x}(t), \\ u(t) &= K_1\hat{x}(t), \end{aligned} \tag{18}$$

where $\hat{x}(t) = (\hat{x}_1(t), \ldots, \hat{x}_n(t))^{\mathrm{T}} \in R^n$ represents the estimated state, $\hat{z}(t)$ is the estimate output vector. $\hat{A}$, K_1 and L are stabilising matrix, state feedback gain matrix and observer gain matrix to be determined with appropriate dimensions, respectively.

Let us denote the estimation error as $e(t) = x(t) - \hat{x}(t)$. In view of (6) and (18), the closed-loop system can be given in the form as follows:

$$\begin{aligned} D^\alpha \hat{x}(t) &= (\hat{A} + B_0K_1)\hat{x}(t) + LC_1e(t), \\ D^\alpha e(t) &= (A_0 + \Delta A - \hat{A} + \Delta BK_1)\hat{x}(t) \\ &\quad + (A_0 + \Delta A - LC_1)e(t) + (B_{\sigma 0} + \Delta B_\sigma)\sigma(t), \\ y(t) &= (C_0 + \Delta C)x(t) + (D_{\sigma 0} + \Delta D_\sigma)\sigma(t), \end{aligned}$$

which can be rewritten as

$$D^\alpha \bar{x}(t) = A_{cl}\bar{x}(t) + (\bar{B}_{\sigma 0} + \Delta\bar{B}_\sigma)\bar{\sigma}(t), \tag{1}$$
$$y(t) = (\bar{C}_0 + \bar{\Delta}C)\bar{x}(t) + (\bar{D}_{\sigma 0} + \Delta\bar{D}_\sigma)\bar{\sigma}(t), \tag{19}$$

where

$$A_{cl} = \begin{bmatrix} \hat{A} + B_0K_1 & LC_1 \\ A_0 + \Delta A - \hat{A} + \Delta BK_1 & A_0 + \Delta A - LC_1 \end{bmatrix},$$
$$\Delta\bar{B}_\sigma = \begin{bmatrix} 0 \\ \Delta B_\sigma \end{bmatrix},$$
$$\bar{x}(t) = \begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix}, \bar{B}_{\sigma 0} = \begin{bmatrix} 0 \\ B_{\sigma 0} \end{bmatrix}, \bar{C}_0 = \begin{bmatrix} C_0 & C_0 \end{bmatrix},$$
$$\Delta\bar{C}_0 = \begin{bmatrix} \Delta C_0 & \Delta C_0 \end{bmatrix},$$
$$\bar{\sigma}(t) = \begin{bmatrix} 0 \\ \sigma(t) \end{bmatrix}, \Delta\bar{D}_{\sigma 0} = \begin{bmatrix} 0 & \Delta D_{\sigma 0} \end{bmatrix},$$
$$\bar{D}_{\sigma 0} = \begin{bmatrix} 0 & D_{\sigma 0} \end{bmatrix}.$$

Now, the design problem can be transformed to look for suitable stabilising matrices $\hat{A}$, state feedback gain K_1 and observer gain matrix L, such that uncertain FO linear system (19) is robustly passive.

Remark 3.2: FO linear observer (18) contains a unknown system matrix, $\hat{A}$, gives an opportunity to better adjust the dynamical properties of the observer-base FO control system, which is difference with the existing FO Luenberger-type observer (24) (Ibrir & Bettayeb, 2015; Lan & Zhou, 2011; Liu, Nie, Wu, & She, 2018).

Theorem 3.3: *If there exist symmetrical matrices $\tilde{P}$ together with matrices $\hat{A}, K_1, L$ of appropriate dimensions and real scalars λ_1 such that*

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & B_{\sigma 0} - \frac{1}{2}\tilde{P}C_0^T & \tilde{P}F_1^T & \lambda_1E_1 \\ * & \Psi_{22} & -\frac{1}{2}\tilde{P}C_0^T - \frac{1}{2}\tilde{P}K_1^TD_0^T & \tilde{P}F_1^T + \tilde{P}K_1^TF_2 & 0 \\ * & * & -D_{\sigma 0} & F_3^T & -\frac{1}{2}\lambda_1E_2 \\ * & * & * & -\lambda_1I & 0 \\ * & * & * & * & -\lambda_1I \end{bmatrix} < 0, \tag{20}$$

where

$$\Psi_{11} = (A_0\tilde{P} - LC_1\tilde{P}) + (\tilde{P}A_0^T - \tilde{P}C_1^TL^T),$$
$$\Psi_{12} = (A_0 - \hat{A} + LC_1)\tilde{P},$$
$$\Psi_{22} = (\hat{A}\tilde{P} + B_0K_1\tilde{P}) + (\tilde{P}\hat{A}^T + \tilde{P}K_1^TB_0^T)$$

then the controlled closed-loop system (19) is robustly passive.

Proof: Choosing storage function for system (19) as

$$V(\bar{x}(t)) = \bar{x}^T(t)\bar{P}\bar{x}(t),$$

where $\bar{P} = \mathrm{diag}(P, P)$.

Taking the FO time derivative of $V(\bar{x}(t))$ along the trajectory of system (19) and employing the passivity definition yields

$$
\begin{aligned}
&D^{\alpha}V(\bar{x}(t)) - \bar{y}^T(t)\bar{\sigma}(t) \\
&\quad \le \hat{x}^T(t)\Big(P(\hat{A}+B_0K_1) + (\hat{A}^T + K_1^TB_0^T)P\Big)\hat{x}(t) \\
&\quad + 2e^T(t)P\Big(A_0 + \Delta A - \hat{A} + \Delta BK_1\Big)\hat{x}(t) \\
&\quad + 2e^T(t)PLC_1\hat{x}(t) + e^T(t)\Big(P(A_0 + \Delta A - LC_1) \\
&\quad + (A_0^T + \Delta A^T - C_1^TL^T)P\Big)e(t) \\
&\quad + 2e^T(t)P(B_{\sigma 0} + \Delta B_{\sigma})\sigma^T(t) \\
&\quad - \hat{x}^T(t)(C_0^T + \Delta C^T)\sigma(t) \\
&\quad - e^T(t)(C_0^T + \Delta C^T)\sigma(t) \\
&\quad - \sigma^T(t)(D_{\sigma 0} + \Delta D_{\sigma})\sigma(t) \\
&\quad - \hat{x}^T(t)(K_1^TD_0^T + K_1^T\Delta D^T)\sigma(t) \\
&\quad = \eta^T(t)\Xi\eta(t),
\end{aligned}
$$

where $\eta(t) = [e^T(t), \hat{x}^T(t), \sigma^T(t)]^T$,

$$
\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & P(B_{\sigma 0} + \Delta B_{\sigma}) - \frac{1}{2}C_0^T - \frac{1}{2}\Delta C^T \\ * & \Xi_{22} & \Xi_{23} \\ * & * & -D_{\sigma 0} - \Delta D_{\sigma} \end{bmatrix},
$$

where

$$
\begin{aligned}
\Xi_{11} &= P(A_0 + \Delta A - LC_1) + (A_0^T + \Delta A^T - C_1^TL^T)P, \\
\Xi_{22} &= P(\hat{A} + B_0K_1) + (\hat{A}^T + K_1^TB_0^T)P, \\
\Xi_{12} &= P(A_0 + \Delta A - \hat{A} + \Delta BK_1 + LC_1), \\
\Xi_{23} &= -\frac{1}{2}C_0^T - \frac{1}{2}\Delta C^T - \frac{1}{2}K_1^TD_0^T - \frac{1}{2}K_1^T\Delta D^T.
\end{aligned}
$$

Matrix Ξ can be written as

$$
\begin{aligned}
\Xi = \Theta = &\begin{bmatrix} \Theta_{11} & P(A_0 - \hat{A} + LC_1) & PB_{\sigma 0} - \frac{1}{2}C_0^T \\ * & P(\hat{A} + B_0K_1) + (\hat{A}^T + K_1^TB_0)P & -\frac{1}{2}C_0^T - \frac{1}{2}K_1^TD_0^T \\ * & * & -D_{\sigma 0} \end{bmatrix} \\
&+ \begin{bmatrix} PE_1 \\ 0 \\ -\frac{1}{2}E_2 \end{bmatrix} H(t) \begin{bmatrix} F_1 & F_1 + F_2K_1 & F_3 \end{bmatrix} \\
&+ \begin{bmatrix} F_1^T \\ F_1^T + K_1^TF_2^T \\ F_3^T \end{bmatrix} H^T(t) \begin{bmatrix} E_1^TP & 0 & -\frac{1}{2}E_2^T \end{bmatrix},
\end{aligned}
$$

where $\Theta_{11} = P(A_0 - LC_1) + (A_0^T - C_1^TL^T)P$.

Taking into account Definition 2.5, the closed-loop controlled system (19) is robustly passive if $\Theta < 0$, which is equivalent to that there exist a constant λ_1 such the following inequality holds by Lemma 2.3,

$$
\begin{aligned}
&\begin{bmatrix} \Theta_{11} & P(A_0 - \hat{A} + LC_1) & PB_{\sigma 0} - \frac{1}{2}C_0^T \\ * & P(\hat{A} + B_0K_1) + (\hat{A}^T + K_1^TB_0)P & -\frac{1}{2}C_0^T - \frac{1}{2}K_1^TD_0^T \\ * & * & -D_{\sigma 0} \end{bmatrix} \\
&+ \lambda_1 \begin{bmatrix} PE_1 \\ 0 \\ -\frac{1}{2}E_2 \end{bmatrix} \begin{bmatrix} E_1^TP & 0 & -\frac{1}{2}E_2^T \end{bmatrix} \\
&+ \lambda_1^{-1} \begin{bmatrix} F_1^T \\ F_1^T + K_1^TF_2^T \\ F_3^T \end{bmatrix} \begin{bmatrix} F_1 & F_1 + F_2K_1 & F_3 \end{bmatrix} < 0,
\end{aligned}
$$

which can be rearranged as

$$
\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & PB_{\sigma 0} - \frac{1}{2}C_0^T - \frac{1}{2}\lambda_1 PE_1E_2^T + \lambda_1^{-1}F_1^TF_3 \\ * & \Phi_{22} & -\frac{1}{2}C_0^T - \frac{1}{2}K_1^TD_0^T + \lambda_1^{-1}(F_1^T + K_1^TF_2^T)F_3 \\ * & * & -D_{\sigma 0} + \lambda_1\frac{1}{4}E_2E_2^T + \lambda_1^{-1}F_3^TF_3 \end{bmatrix} < 0, \tag{21}
$$

where

$$
\begin{aligned}
\Phi_{11} &= P(A_0 - LC_1) + (A_0^T - C_1^TL^T)P \\
&\quad + \lambda_1 PE_1E_1^TP + \lambda_1^{-1}F_1^TF_1. \\
\Phi_{22} &= P(\hat{A} + B_0K_1) + (\hat{A}^T + K_1^TB_0^T)P \\
&\quad + \lambda_1^{-1}(F_1^T + K_1^TF_2^T)(F_1 + F_2K_1), \\
\Phi_{12} &= P(A_0 - \hat{A} + LC_1) + \lambda_1^{-1}F_1^T(F_1 + F_2K_1).
\end{aligned}
$$

By virtue of Lemma 2.4, the fact that (21) is equivalent to

$$
\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & PB_{\sigma 0} - \frac{1}{2}C_0^T & F_1^T & \lambda_1 PE_1 \\ * & \Gamma_{22} & -\frac{1}{2}C_0^T - \frac{1}{2}K_1^TD_0^T & F_1^T + K_1^TF_2^T & 0 \\ * & * & -D_{\sigma 0} & F_3^T & -\frac{1}{2}\lambda_1 E_2 \\ * & * & * & -\lambda_1 I & 0 \\ * & * & * & * & -\lambda_1 I \end{bmatrix} < 0, \tag{22}
$$

where

$$
\begin{aligned}
\Gamma_{11} &= P(A_0 - LC_1) + (A_0^T - C_1^TL^T)P, \\
\Gamma_{12} &= P(A_0 - \hat{A} + LC_1), \\
\Gamma_{22} &= P(\hat{A} + \hat{B}_0K_1) + (\hat{A}^T + K_1^T\hat{B}_0)P.
\end{aligned}
$$

Multiplying both side of the above inequality (22) by the matrix $\mathrm{diag}(P^{-1}, P^{-1}, I, I)$, one has

$$
\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & B_{\sigma 0} - \frac{1}{2}P^{-1}C_0^T & P^{-1}F_1^T & \lambda_1 E_1 \\ * & \Psi_{22} & -\frac{1}{2}P^{-1}C_0^T - \frac{1}{2}P^{-1}K_1^TD_0^T & P^{-1}F_1^T + P^{-1}K_1^TF_2 & 0 \\ * & * & -D_{\sigma 0} & F_3^T & -\frac{1}{2}\lambda_1 E_2 \\ * & * & * & -\lambda_1 I & 0 \\ * & * & * & * & -\lambda_1 I \end{bmatrix}
$$

$$
< 0,
$$

where

$$\Psi_{11} = (A_0P^{-1} - LC_1P^{-1}) + (P^{-1}A_0^T - P^{-1}C_1^TL^T),$$
$$\Psi_{12} = (A_0 - \hat{A} + LC_1)P^{-1},$$
$$\Psi_{22} = (\hat{A}P^{-1} + B_0K_1P^{-1}) + (P^{-1}\hat{A}^T + P^{-1}K_1^TB_0^T).$$

Now, defining $\tilde{P} = P^{-1}$, one obtains inequality (20). This completes the proof.

It is obvious that the matrix inequality (20) in Theorem 3.3 is not an LMI because some crosses of these determined parameters are appearing in (20) in non-linear fashion, such as $LC_1\tilde{P}, \tilde{P}K_1^TB_0^T$, However, it can be transformed into an LMI by employing Lemmas 2.5 and 2.6, which will be shown below. ■

Theorem 3.4: *Assume that the SV D of the output matrix C_1 with full row rank is $C_1 = U\,[\,S\ 0\,]\,V^T$. Then, the closed-loop controlled system (19) is robustly passive if there exist symmetrical matrices $\tilde{P} > 0$, $\tilde{P}_1 > 0, \tilde{P}_2 > 0$, matrices X_1, X_2 and X_3 together with a real scalar $\lambda_1 > 0$ such that*

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & B_{\sigma 0} - \frac{1}{2}\tilde{P}C_0^T & \tilde{P}F_1^T & \lambda_1 E_1 \\ * & \Psi_{22} & -\frac{1}{2}\tilde{P}C_0^T - \frac{1}{2}X_2^TD_0^T & \tilde{P}F_1^T + X_2^TF_2 & 0 \\ * & * & -D_{\sigma 0} & F_3^T & -\frac{1}{2}\lambda_1 E_2 \\ * & * & * & -\lambda_1 I & 0 \\ * & * & * & * & -\lambda_1 I \end{bmatrix} < 0, \tag{23}$$

where

$$\Psi_{11} = (A_0\tilde{P} - X_3C_1) + (\tilde{P}A_0^T - C_1^TX_3^T),$$
$$\Psi_{12} = (A_0\tilde{P} - X_1 + X_3C_1),$$
$$\Psi_{22} = (X_1 + B_0X_2) + (X_1^T + X_2^TB_0^T).,$$
$$\tilde{P} = V\begin{bmatrix} \tilde{P}_1 & 0 \\ 0 & \tilde{P}_2 \end{bmatrix}V^T.$$

Moreover, the robust stabilising matrix, the feedback controller gain and the observer gain are given by

$$\hat{A} = X_1\tilde{P}^{-1}, K_1 = X_2\tilde{P}^{-1}, L = X_3US\tilde{P}_1^{-1}S^{-1}U^{-1}.$$

Proof: Since

$$\tilde{P} = V\begin{bmatrix} \tilde{P}_1 & 0 \\ 0 & \tilde{P}_2 \end{bmatrix}V^T.$$

It follow from Lemma 2.5 that there exists $\bar{\tilde{P}} = US\tilde{P}_1S^{-1}U^{-1}$ such that $C_1\tilde{P} = \bar{\tilde{P}}C_1$, where $\bar{\tilde{P}}^{-1} = US\tilde{P}_1^{-1}S^{-1}U^{-1}$. Denote $X_1 = \hat{A}\tilde{P}, X_2 = K_1\tilde{P}, X_3 = L\bar{\tilde{P}}_1^{-1}$, inequality (23) is equivalent to (20). The proof is completed.

Particularly, if let $\hat{A} = A_0$ in observer (18), ones can obtain the following classical observer-based robust controller for system (6)

$$\begin{aligned} D^\alpha \hat{x}(t) &= A_0\hat{x}(t) + B_0u(t) + L(z(t) - \hat{z}(t)), \\ \hat{z}(t) &= C_1\hat{x}(t), \\ u(t) &= K_1\hat{x}(t). \end{aligned} \tag{24}$$

In this case, the stabilising the feedback controller gain K and the observer gain L can be solved by the following corollary. ■

Corollary 3.1: *Assume that the SV D of the output matrix C_1 with full row rank is $C_1 = U\,[\,S\ 0\,]\,V^T$. Then, the system (19) with FO linear observer (24) is robust passive if there exist symmetrical matrices $\tilde{P} > 0$, $\tilde{P}_1 > 0, \tilde{P}_2 > 0$, matrices X_1 and X_2 together with a real scalar $\lambda_1 > 0$ such that*

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & B_{\sigma 0} - \frac{1}{2}\tilde{P}C_0^T & \tilde{P}F_1^T & \lambda_1 E_1 \\ * & \Psi_{22} & -\frac{1}{2}\tilde{P}C_0^T - \frac{1}{2}X_1^TD_0^T & \tilde{P}F_1^T + X_1^TF_2 & 0 \\ * & * & -D_{\sigma 0} & F_3^T & -\frac{1}{2}\lambda_1 E_2 \\ * & * & * & -\lambda_1 I & 0 \\ * & * & * & * & -\lambda_1 I \end{bmatrix} < 0, \tag{25}$$

where

$$\Psi_{11} = (A_0\tilde{P} - X_2C_1) + (\tilde{P}A_0^T - C_1^TX_2^T),$$
$$\Psi_{12} = X_2C_1,$$
$$\Psi_{22} = (A_0\tilde{P} + B_0X_1) + (\tilde{P}A_0^T + X_1^TB_0^T),$$
$$\tilde{P} = V\begin{bmatrix} \tilde{P}_1 & 0 \\ 0 & \tilde{P}_2 \end{bmatrix}V^T.$$

Moreover, the feedback controller gain and the observer gain are given by

$$K_1 = X_1\tilde{P}^{-1}, L = X_2US\tilde{P}_1^{-1}S^{-1}U^{-1}.$$

Remark 3.3: Note that if the closed-loop controlled system in (16) and (19) are passive according to Theorems 3.2, 3.3, 3.4, respectively, the asymptotic stability of (16) and (19) with $\sigma(t) = 0$ is also guaranteed. That is to say, Theorems 3.2 – 3.4 provide conditions for the existence of state-feedback stabilisation controller for system (6).

Remark 3.4: Sara and Reza (2013) discussed the problem of designing a passivity-based FO integral sliding mode controller for uncertain FO nonlinear systems by using passivity definition of integer-order systems. However, this definition may be not applied directly to FO systems. On the one hand, the dissipative inequality in the definition of dissipativity of integer order systems can not characterise the memory property of fractional energy dissipation of FO systems. On the other hand, fractional systems usually have polynomial convergence speed, rather than the exponential convergence speed that integer order systems generally have, so output of FO systems may diverge. Here, a new passivity definition of FO systems is proposed.

Remark 3.5: A new definition of passivity for FO systems is introduced, relationship between passivity and asymptotic stability is revealed. Obviously, this definition can also be applied to FO nonlinear systems (Chen, He, Chai, & Wu, 2014), FO delayed systems (Chen, Wu, Cao, & Liu, 2015) and FO T-S fuzzy systems (Wag, Qiu, Chadli, & Wang, 2016; Wang et al., 2018), and some passivity conditions of these systems can be derived like this paper.

Remark 3.6: Lu and Chen (2010) and Ma, Lu, and Chen (2014) addressed stability and state feedback control design for the following FO uncertain systems

$$\begin{aligned} D^{\alpha}x(t) &= (A+\Delta A(t))x(t)+(B+\Delta B(t))u(t),\\ y(t) &= Cx(t). \end{aligned} \tag{26}$$

It is obvious that system (26) is a special case of system (6). Theorems 3.1–3.4 can be also applied to system (26). However, these results in Lu and Chen (2010) and Ma et al. (2014) cannot be used to determine the passivity and feedback passification of the system (6).

4. Numerical examples

In this section, the following examples are provided to verify the effectiveness of the proposed theoretical results with computer simulations.

Example 4.1: The boost converter, sometimes called a step-up/down power stage, is an inverting power stage topology. Schematic diagram of a DC–DC boost converter is shown in Figure 1. When S_T OFF and S_D ON, the expression of the FO mathematical model proposed in Chen, Chen, Zhang, and Qiu (2017)) is described by

$$\begin{aligned} D^{\alpha}i_L &= \frac{1}{L}U_{in}-\frac{1}{L}v_C,\\ D^{\alpha}v_C &= \frac{1}{C}i_L-\frac{v_C}{RC}. \end{aligned} \tag{27}$$

Taking i_L and v_C as state variable and selecting the input voltage $U_{in}=12V$, the load resistance $R=$

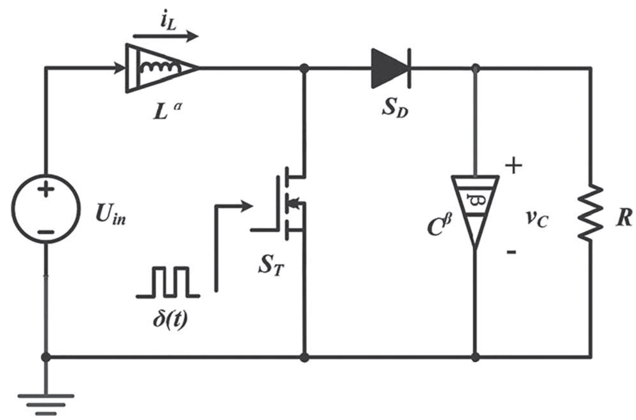

Figure 1. Fractional-order DC–DC boost converter.

40Ω, the $L=477\mu H$, $C=10\mu F/(s)^{1-\alpha}$. Note that Theorem 3.1 is also still valid for nominal model (27). By using LMI Matlab toolbox, one could see that the LMI (7) in Theorem 3.1 with $D_{\sigma}=1$ is feasible. The feasible solution is given by

$$\bar{P}=\begin{bmatrix}172.4275 & 140.6495\\ 140.6495 & 623.7084\end{bmatrix}, \lambda=2.$$

Therefore, it follow from Theorem 3.1 that FO DC–DC converter (27) is passive.

Example 4.2: Consider the uncertain FO linear system (6) with parameters given by

$$A_0=\begin{bmatrix}2 & -1\\ -4 & 0\end{bmatrix}, B_0=\begin{bmatrix}1.5\\ -2.5\end{bmatrix}, B_{\sigma 0}=\begin{bmatrix}0.8\\ -2\end{bmatrix},$$
$$C_0=\begin{bmatrix}1 & -1.2\end{bmatrix}, D_0=2,$$
$$E_1=\begin{bmatrix}0.1\\ 0.1\end{bmatrix}, E_2=0.1, D_{\sigma 0}=1.2, F_1=\begin{bmatrix}0.2 & -0.3\end{bmatrix},$$
$$F_2=0.2, F_3=-0.1, \alpha=0.9.$$

Using MATLAB LMI toolbox, one obtains the feasible solution as follows:

$$\tilde{P}=\begin{bmatrix}0.1237 & 0.0107\\ 0.0107 & 2.0025\end{bmatrix}, \quad X=\begin{bmatrix}-0.4323 & 0.3365\end{bmatrix},$$
$$\lambda=1.4144.$$

The controller gain matrix

$$K=X\tilde{P}^{-1}=\begin{bmatrix}-3.5110 & 0.1868\end{bmatrix}.$$

Hence, the above results show that all the conditions stated in Theorem 3.2 have been satisfied and the controlled system is passive.

In the simulation, an improved predictor-corrector algorithm (Deng, 2007) for FO differential equations is

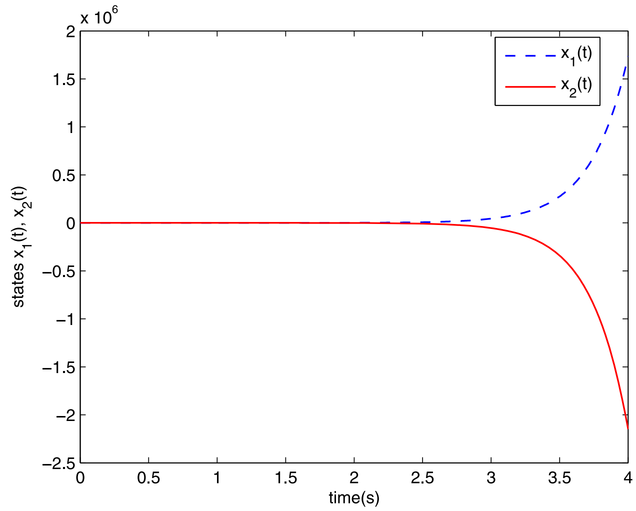

Figure 2. Time response of the selected systems without the control input.

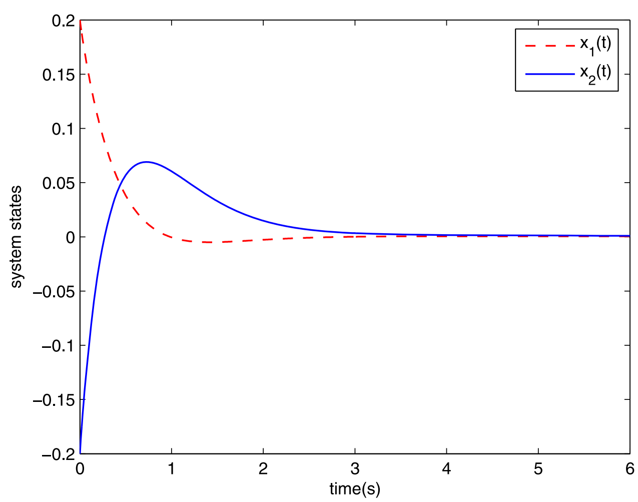

Figure 3. Time response of the selected systems with the control input.

used. Chose $h(t) = 0.2\sin t$, the initial state $x_0 = [0.2\ -0.2]^{\mathrm{T}}$. When the external disturbance $\sigma(t) = 0$, time response of the selected systems without and with the control input are shown in Figures 2 and 3, respectively. Figure 4 shows control input. One can see that the open-loop system is divergent fast and closed-loop controlled system is asymptotically stable under control input.

Example 4.3: Consider a 3-dimensional uncertain FO linear system (6) with the following parameters:

$$A_0 = \begin{bmatrix} -2 & -1 & 0.5 \\ 2 & -0.5 & 1 \\ 1 & 0 & -1.2 \end{bmatrix}, B_{\sigma 0} = \begin{bmatrix} -1 & 3 \\ -2 & 4 \\ 1 & 0 \end{bmatrix},$$

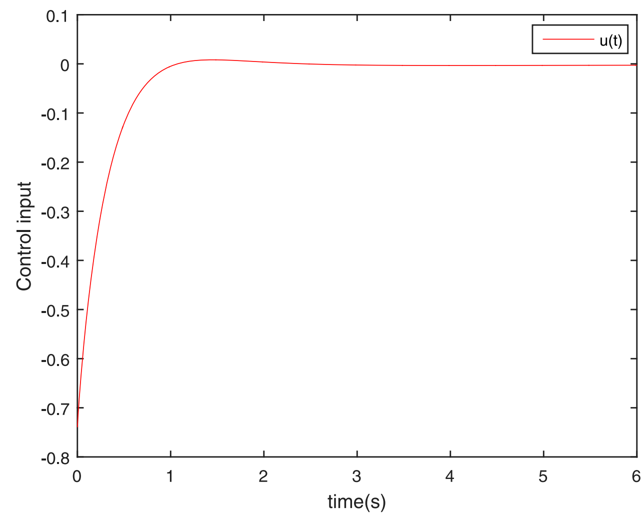

Figure 4. Control input of system in Example 4.2.

$$B_0 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix},$$

$$C_0 = \begin{bmatrix} 4 & -2 & 0 \\ 2 & 3 & -1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix},$$

$$D_{\sigma 0} = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix},$$

$$D_0 = \begin{bmatrix} -2 \\ -3 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 \\ -0.1 \\ 0.2 \end{bmatrix}, E_2 = \begin{bmatrix} 0.3 \\ -0.2 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.2 & -0.3 & 0.1 \end{bmatrix}, F_2 = 0.2,$$

$$F_3 = \begin{bmatrix} -0.1 & 0.2 \end{bmatrix}, \alpha = 0.9.$$

Applying matrixs SVD for C_1, one can obtain

$$U = 1, S = 1.4142, V = \begin{bmatrix} 0.7071 & 0.7071 & 0 \\ 0.7071 & -0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Assume that the state variables are not measurable, one can design observer expressed as (24), by utilising packages Yalmip in Matlab, one finds the LMI (24) in the Corollary 1 is feasible, and feasible solution with the parameters:

$$\tilde{P}_2 = \begin{bmatrix} 0.5830 & -0.1291 \\ -0.1291 & 0.5078 \end{bmatrix},$$

$$X_1 = \begin{bmatrix} 0.6639 & 0.6340 & -0.1025 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 0.4455 & 3.0709 & 0.5151 \end{bmatrix}^T,$$

$$\tilde{P}_1 = 1.1765, \lambda_1 = 2.4216.$$

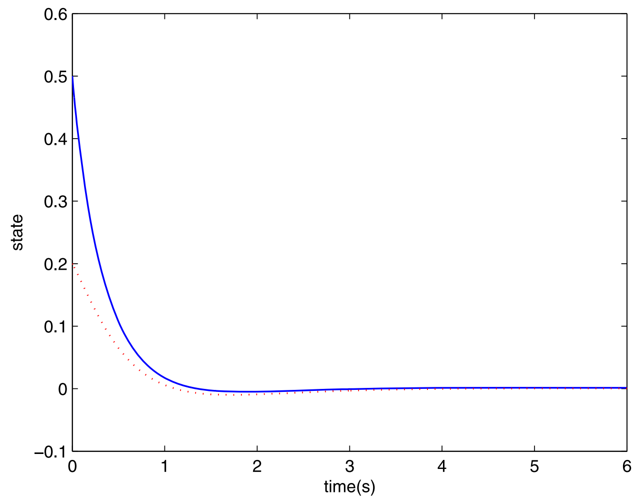

Figure 5. The convergence dynamics of state $x_1(t)$ of the chosen system in Example 2 (solid line: real value, dotted line: estimated value).

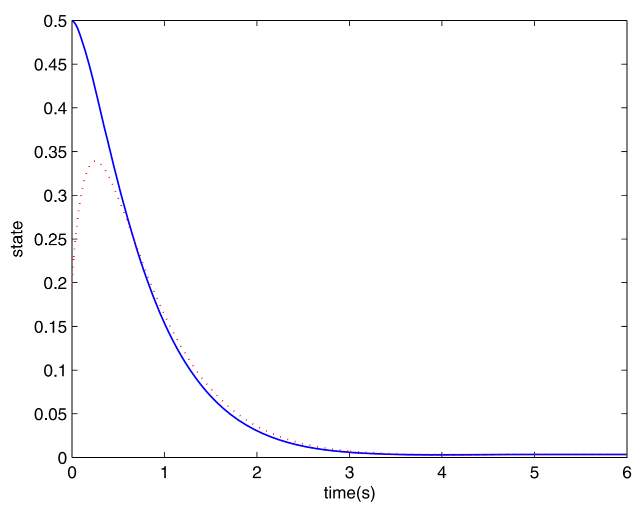

Figure 6. The convergence dynamics of state $x_2(t)$ of the chosen system in Example 2 (solid line: real value, dotted line: estimated value).

Furthermore, the feedback controller gain and the observer gain are given by

$$K_1 = \begin{bmatrix} 0.5453 & 0.5579 & -0.2041 \end{bmatrix},$$
$$L = \begin{bmatrix} 0.3087 & 2.6102 & 0.4378 \end{bmatrix}^T.$$

Therefore, it follows from Corollary 3.1 that system with observer (24) in Example 4.3 is robustly passive.

If one would like to design FO observer (18) to ensure that the controlled system (19) is passive, by employing Theorem (3.4), stabilising matrix, state feedback gain matrix and observer gain matrix can be obtained as

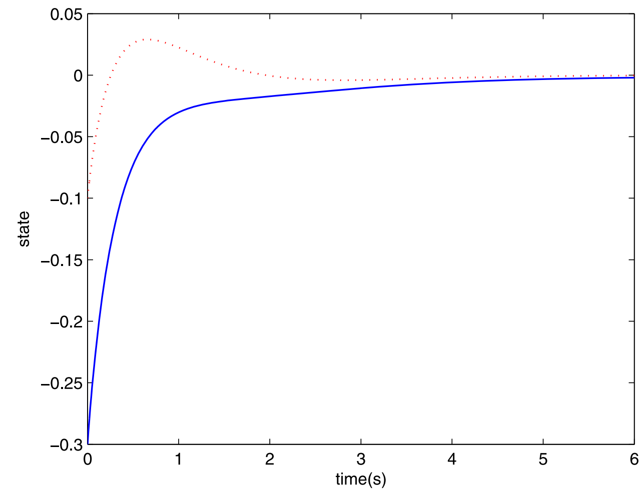

Figure 7. The convergence dynamics of state $x_3(t)$ of the chosen system in Example 2 (solid line: real value, dotted line: estimated value).

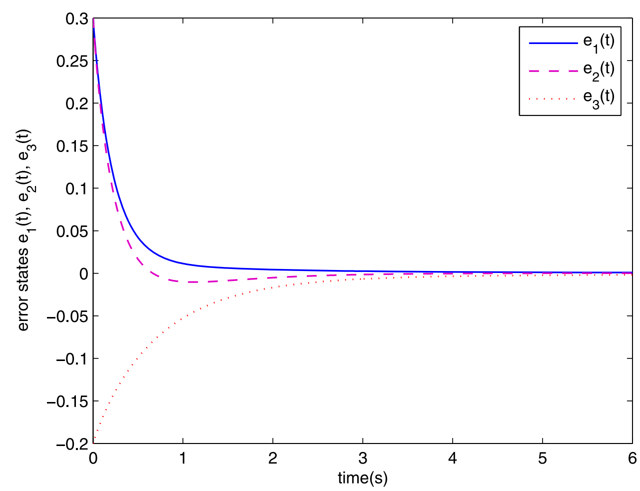

Figure 8. The convergence dynamics of error states using FO observer (24).

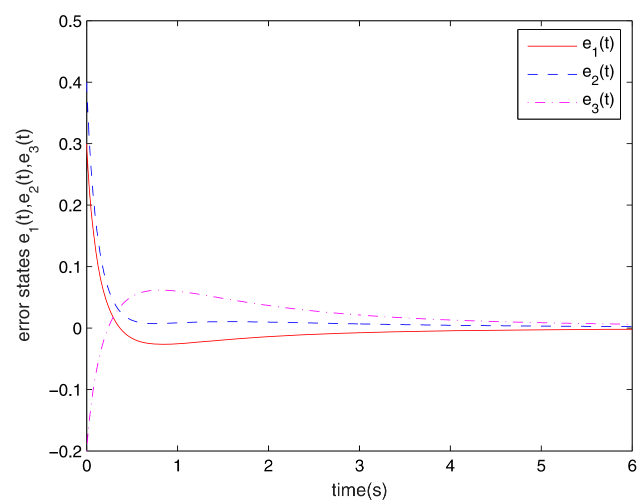

Figure 9. The convergence dynamics of error states using FO observer (18).

follows

$$\hat{A} = \begin{bmatrix} -7.2341 & 3.5473 & -2.9874 \\ -0.5792 & 1.9331 & -1.3529 \\ -0.0009 & -0.0881 & -1.3071 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -1.2216 & 2.0881 & -1.1576 \end{bmatrix},$$
$$L = \begin{bmatrix} 1.9392 & 1.6009 & -0.7635 \end{bmatrix}^T.$$

For simulation, select $h(t) = 0.1\cos(0.2t)$, the real and estimated value initial states are chosen as $x_0 = [0.5\ 0.6\ -0.3]^{\mathrm{T}}$ and $\hat{x}_0 = [0.2\ 0.2\ -0.1]^{\mathrm{T}}$. The trajectories of the real and estimated value states are shown in Figures 5–7. Figure 8 describes dynamic curves of the estimation errors using FO observer (24). Figure 9 shows dynamic curves of the estimation errors using FO observer (18). With the simulation results, it can be seen that the state estimation errors rapidly tend to zero asymptotically as expected and FO observers (18) and (24) are all effective.

5. Conclusion

Robust passivity analysis and passive control for uncertain FO linear systems with time-varying norm-bounded parameter uncertainties has been studied. Some definition of passivity and dissipativity for fractional-order systems were presented. Relationship between passivity and stability of fractional order systems has been built, it is shown that passive fractional-order systems is Mittag–Leffer stable. Furthermore, an LMI sufficient condition for such a system to be robustly passive is given. Sufficient criteria for passive control of the closed loop system are also analysed based on whether the states are available or not. Two illustrative examples are provided to show the usefulness and effectiveness of the presented results. Main features of the paper are summarised as follows, (i) more appropriate passivity and dissipativity definition for FO systems are given, (ii) stability and stabilisation problem of many FO systems can be converted into considering passivity and passivation of FO systems, (iii) passivity of uncertain FO systems can be guaranteed. It is well known that the time-delay phenomenon often appears in many dynamic systems, to discuss passivity and dissipativity issue of fractional-order delayed systems is future work.

Acknowledgments

The authors would like to express their deep gratitude to the Editors and the anonymous referees for their helpful comments and suggestions, which have greatly improved the paper.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the National Natural Science Funds of China [No. 61403115, No. 11571016, No. 51577046], the Fundamental Research Funds for the Central Universities [Nos. JZ2016HGTB0718; JZ2016HGXJ0022], the State Key Program of National Natural Science Foundation of China [No. 51637004] and the National Key Research and Development plan important scientific instruments and equipment development [No. 2016YFF0102200].

Notes on contributors

Liping Chen received the B.S. degree in Applied Mathematics from Anhui Normal University, Wuhu, China, the M.S. degree in Basic Mathematics from Anhui University, Hefei, China, and the Ph.D. degree in School of Automation, Chongqing University, Chongqing, China, in 2007, 2010 and 2013, respectively. He is now an associate professor at Hefei University of Technology. His current research interests include the areas of neural networks, fractional-order systems and nonlinear dynamical systems.

Tingting Li received the B.S. degree in Automation from Anhui University of Technology, Maanshan, China. She is currently pursuing the M.S. degree at the Hefei University of Technology, Hefei, China. His research interests include intelligent optimization algorithm, fractional-order control and power electronics controls.

YangQuan Chen earned his Ph.D. degree in advanced control and instrumentation from Nanyang Technological University, Singapore, in 1998. Dr. Chen was on the faculty of Electrical and Computer Engineering at Utah State University before he joined the School of Engineering, University of California, Merced in 2012 where he teaches “Mechatronics” and “Unmanned Aircraft Systems” for juniors and “Fractional Order Mechanics” and “Nonlinear Control” for graduates. His current research interests include mechatronics for sustainability, MIMO cognitive process control, multi-UAV based cooperative multi- spectral “personal remote sensing” and applications, applied fractional calculus in controls, signal processing and energy informatics; distributed measurement and distributed control of distributed parameter systems using mobile actuator and sensor networks.

Ranchao Wu received the B.S. degree in Applied Mathematics from Anhui Normal University, Wuhu, China, in 1994, and Ph.D. degree in Applied Mathematics from Nanjing University, Nanjing, China, in 2006. He is currently a professor and a doctor adviser at Anhui University. His current research interests include nonlinear dynamic systems, bifurcation and chaos, neural networks, control theory.

Suoliang Ge received the B.S. degree and the M.S. degree in Automation from Hefei University of Technology, Hefei, China, in 1984 and 1989, respectively. He is now an associate professor at Hefei University of Technology. His current research interests include complex systems, nonlinear dynamics, industrial control systems.

ORCID

YangQuan Chen http://orcid.org/0000-0002-7422-5988

References

Antsaklis, P., Goodwine, B., Gupta, V., Mcourt, M., Wang, Y., Wu, P., & Zhu, F. (2013). Control of cyberphysical systems using passivity and dissipativity based methods. *European Journal of Control*, *19*(5), 379–388.
Boyd, S., Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory*. Philadelphia: SIAM Studies in Applied Mathematics.
Chen, L., Chai, Y., Wu, R., & Yang, J. (2012). Stability and stabilization of a class of nonlinear fractional-order systems with caputo derivative. *IEEE Transactions on Circuits and Systems II-Express Briefs*, *59*(9), 602–606.
Chen, X., Chen, Y., Zhang, B., & Qiu, D. (2017). A modeling and analysis method for fractional-order dc–dc converters. *IEEE Transactions on Power Electronics*, *32*(9), 7034–7044.
Chen, L., He, Y., Chai, Y., & Wu, R. (2014). New results on stability and stabilization of a class of nonlinear fractional-order systems. *Nonlinear Dynamics*, *75*(4), 633–641.
Chen, L., Wu, R., Cao, J., & Liu, J.-B. (2015). Stability and synchronization of memristor-based fractional-order delayed neural networks. *Neural Networks*, *71*, 37–44.
Chopra, N., & Spong, M. (1999). *Passivity-based control of multi-agent systems in Advances in robot control*. Berlin: Springer.
Christopher, I., Byrnes, A., & Willems, J. (1991). Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems. *IEEE Transactions on Automatic control*, *36*(11), 1228–1240.
Dadras, S., Dadras, S., Malek, H., & Chen, Y. (2017). A note on the Lyapunov stability of fractional-order nonlinear systems. In *ASME 2017 International design engineering technical conferences and computers and information in engineering conference* (pp. V009T07A033–V009T07A033). American Society of Mechanical Engineers.
Dadras, S., & Momeni, H. R. (2014). Fractional-order dynamic output feedback sliding mode control design for robust stabilization of uncertain fractional-order nonlinear systems. *Asian Journal of Control*, *16*(2), 489–497.
Deng, W. (2007). Numerical algorithm for the time fractional Fokker–Planck equation. *Journal of Computational Physics*, *227*(2), 1510–1522.
Ding, Y., Wang, Z., & Ye, H. (2012). Optimal control of a fractional-order HIV-immune system with memory. *IEEE Transactions on control Systems Technology*, *20*(3), 763–769.
Garcia-Sandoval, J., Gonzalez-Alvarez, V., & Calderon, C. (2015). Stability analysis and passivity properties for a class of chemical reactors: Internal entropy production approach. *Computers & Chenmical Enginering*, *75*, 184–195.
Ge, F., Chen, Y., & Kou, C. (2016). On the regional gradient observability of time fractional diffusion processes. *Automatica*, *74*, 1–9.
Ge, F., Chen, Y., & Kou, C. (2017). Regional controllability analysis of fractional diffusion equations with Riemann–Liouville time fractional derivatives. *Automatica*, *76*, 193–199.
Ge, F., Chen, Y., & Kou, C. (2018). *Regional analysis of time-fractional diffusion processes*. Berlin: Springer.
Hartley, T., & Lorenzo, C. (2003). Fractional-order system identification based on continuous order-distributions. *Signal Processing*, *83*(11), 2287–2300.
Hilfer, R. (2000). *Applications of fractional calculus in physics*. Singapore: World Scientific.
Ibrir, S., & Bettayeb, M. (2015). New sufficient conditions for observer-based control of fractional-order uncertain systems. *Automatica*, *59*, 216–223.
Jacyntho, L., Teixeira, M., Assuncao, E., Cardim, R., Galvao, R., & Hadjiloucas, S. (2015). Identification of fractional-order transfer functions using a step excitation. *IEEE Transactions on Circuits and Systems II-Express Briefs*, *62*(9), 896–900.
Kaminski, J., Shorten, R., & Zeheb, E. (2015). Exact stability test and stabilization for fractional systems. *Systems & Control Letters*, *85*(6), 95–99.
Kottenstette, N., Hall, J., Koutsoukos, X., Sztipanovits, J., & Antsaklis, P. (2013). Design of networked control systems using passivity. *IEEE Transactions on Control Systems Technology*, *21*(3), 649–665.
Kuntanapreeda, S. (2016). Adaptive control of fractional-order unified chaotic systems using a passivity-based control approach. *Nonlinear Dynamics*, *84*(4), 2505–2515.
Lan, Y., & Zhou, Y. (2011). *LMI*-based robust control of fractional-order uncertain linear systems. *Computers and Mathematics with Applications*, *62*, 1460–1471.
Laskin, N. (2018). *Fractional quantum mechanics*. Singapore: World Scientific.
Li, Y., Chen, Y., & Podlubny, I. (2009). Mittag–Leffler stability of fractional order nonlinear dynamic systems. *Automatica*, *45*(8), 1965–1969.
Li, H., Gao, H., & Shi, P. (2010). New passivity analysis for neural networks with discrete and distributed delays. *IEEE Transactions on Neural Networks*, *21*(11), 1842–1847.
Liang, S., Wu, R., & Chen, L. (2015). Comparison principles and stability of nonlinear fractional-order cellular neural networks with multiple time delays. *Neurocomputing*, *168*, 618–625.
Liu, R., Nie, Z., Wu, M., & She, J. (2018). Robust disturbance rejection for uncertain fractional-order systems. *Applied Mathematics and Computation*, *322*, 79–88.
Lozano, R., Brogliato, B., & Landau, I. (1992). Passivity and global stabilization of cascaded nonlinear systems. *IEEE Transactions on Automatic control*, *37*(9), 1386–1388.
Lu, J., & Chen, Y. (Eds.) (2010). Robust stability and stabilization of fractional-order interval systems with the fractional order α: The $0 < \alpha < 1$ case. *IEEE Transactions On Automatic Control*, *55*(1), 152–158.
Luo, Y., & Chen, Y. (2012). *Fractional order motion controls*. United States: Wiley.
Ma, Y., Lu, J., & Chen, W. (2014). Robust stability and stabilization of fractional order linear systems with positive real uncertainty. *ISA Transactions*, *53*(2), 199–209.
MacDuffee, C. (2004). *The theory of matrices*. New York, NY: Dover.
Ortega, R. (1991). Passivity properties for stabilization of cascaded nonlinear systems. *Automatica*, *27*(2), 423–424.
Podlubny, I. (1999). *Fractional differential equations*. San Diego: Academic Press.

Rakhshan, M., Gupta, V., & Goodwine, B. (2017). *On passivity of fractional order systems*. Retrieved from arXiv:1706.07551 [math.DS].

Sabatier, J., Farges, C., Merveillaut, M., & Feneteau, L. (2012). On observability and pseudo state estimation of fractional order systems. *European Journal of Control*, *18*(3), 260–271.

Sara, D., & Reza, M. H. (2013). Passivity-based fractional-order integral sliding-mode control design for uncertain fractional-order nonlinear systems. *Mechatronics*, *23*(7), 880–887.

Sun, K., Mou, S., Qiu, J., Wang, T., & Gao, H. (2018). Adaptive fuzzy control for non-triangular structural stochastic switched nonlinear systems with full state constraints. *IEEE Transactions on Fuzzy Systems*. doi:10.1109/TFUZZ.2018.2883374

Surendra, K., & Nagarajan, S. (2013). Controllability of fractional order system with nonlinear term having integral contractor. *Fractional Calculus and Applied Analysis*, *16*(4), 791–801.

Tepljakov, A. (2017). *Fractional-order modeling and control of dynamic systems*. Berlin: Springer.

Wag, M., Qiu, J., Chadli, M., & Wang, M. (2016). A switched system approach to exponential stabilization of sampled-data t–s fuzzy systems with packet dropouts. *IEEE Transactions on Cybernetics*, *46*(12), 3145–3156.

Wang, M., Qiu, J., & Feng, G (2018). Finite frequency filtering design for uncertain discrete-time systems using past output measurements. *IEEE Transactions on Circuits and Systems I: Regular Papers*.

Wang, J., & Zhou, Y. (2011). A class of fractional evolution equations and optimal controls. *Nonlinear Analysis-Real World Applications*, *12*(1), 262–272.

West, B. (2016). *Fractional calculus view of complexity: Tomorrows science*. America: CRC Press.

Xia, M., Antsaklis, P., Gupta, V., & Zhu, F. (2017). Passivity and dissipativity analysis of a system and its approximation. *IEEE Transactions on Automatic Control*, *62*(2), 620–635.

Xie, L. (1996). Output feedback h_∞ control of systems with parameter uncertainty. *International Journal Control*, *63*(4), 741–750.

Xue, D. (2017). *Fractional-order control systems: Fundamentals and numerical implementations*. Berlin: De Gruyter.

Zeng, J., Zhang, Z., & Qiao, W. (2014). An interconnection and damping assignment passivity-based controller for a DC–DC boost converter with a constant power load. *IEEE Transactions on Industry Applications*, *50*(4), 2314–2322.