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MESA LAB

Fractional Order Thinking and an Overview of Fractional Order Mechanics

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2020 The 8th Int. Conf. on Control, Mechatronics and Automation

Moscow, Russia. November 6-8, 2020

17:40-18:20 11/7/2020 Saturday (GMT+3)

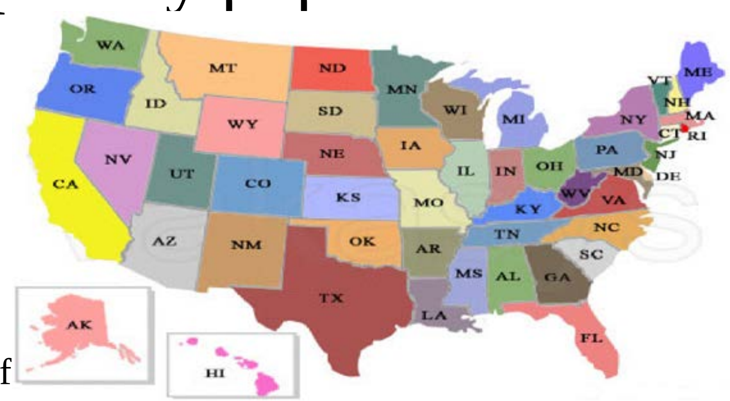
Thank you

- **ICCMA2020 organizers!**
  - **In particular Lily L. Chen and Kate Wong**

- **You all, for coming!**

University of California, Merced

- The Research University of the Central Valley
- Centrally Located
  - Sacramento – 2 hrs
  - San Fran. – 2 hrs
  - Yosemite – 1.5 hrs
  - LA – 4 hrs
- Surrounded by farmlands and sparsely populated areas



University of California is the world #1 public university system

- UCLA professor **Andrea Ghez**, Reinhard Genzel, a professor emeritus of physics at the University of California, Berkeley, and director of the Max Planck Institute for Extraterrestrial Physics in Garching, Germany. Ghez is only the fourth woman to win a Nobel physics prize.
- University of California, Berkeley, biochemist **Jennifer Doudna** won the 2020 Nobel Prize in chemistry
- So far, 67 Nobel Prize winners in UC

- UC Merced (est. 2005)
  - No. 97 among all universities (US News & World Report)
  - No. 40 among public universities (US News & World Report)
  - No. 3 in U.S. among universities younger than 50 years (Times Higher Education Young University Rankings 2020)

Outline

- What and Why Fractional Order Thinking
- Fractional Order Modeling and Controls
- Introduction to Fractional Order Mechanics
- Take Home Messages

Information Item

- Clara M. Ionescu, Riccardo Caponetto, YangQuan Chen. Special Issue "*Fractional Order Modelling and Control in Mechatronic Applications*". **Mechatronics**, Volume 23, Issue 7, October 2013, Editorial. Pages 739-740. https://www.sciencedirect.com/journal/mechatronics/vol/23/issue/7

What is "Fractional Calculus"?

- **Calculus**: integration and differentiation.
- **"Fractional Calculus":** integration and differentiation of non-integer orders.
  - Orders can be real numbers (and even complex numbers!)
  - Orders are not constrained to be "integers" or even "fractionals"

Interpolation of operations

$$f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \frac{d^3 f}{dt^3}, \quad \ldots$$

$$f, \quad \int f(t)dt, \quad \int dt \int f(t)dt, \quad \int dt \int dt \int f(t)dt, \quad \ldots$$

$$\ldots, \quad \frac{d^{-2} f}{dt^{-2}}, \quad \frac{d^{-1} f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \ldots$$

- **How this is possible?**
- **Why should I care?**
- **Any (good) consequences (to me)?**

… from integer to non-integer …

-2 -1 0 1 2

$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n}$$

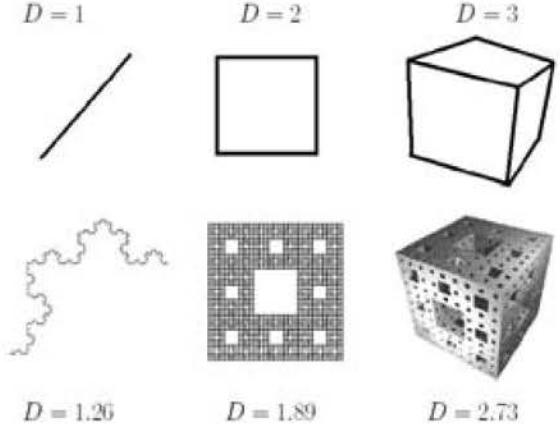
$$x^n = e^{n \ln x}$$

$$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n,$$

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \qquad x > 0,$$

$$\Gamma(n+1) = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n = n!$$

… from integer to non-integer …



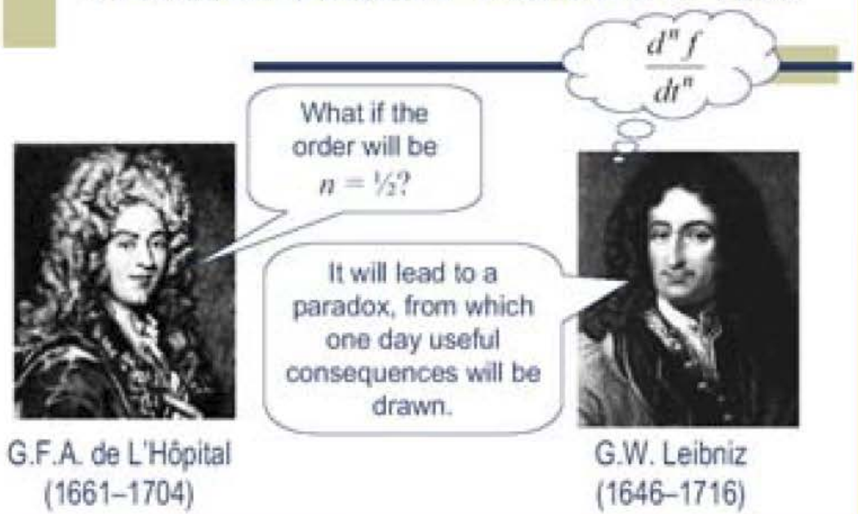
Interpolation of operations

$$f, \ \frac{df}{dt}, \ \frac{d^2 f}{dt^2}, \ \frac{d^3 f}{dt^3}, \ \ldots$$

$$f, \ \int f(t)dt, \ \int dt \int f(t)dt, \ \int dt \int dt \int f(t)dt, \ \ldots$$

$$\ldots, \ \frac{d^{-2} f}{dt^{-2}}, \ \frac{d^{-1} f}{dt^{-1}}, \ f, \ \frac{df}{dt}, \ \frac{d^2 f}{dt^2}, \ \ldots$$

Fractional Calculus was born in 1695



The beginning of a new stage

static models	dynamical models	fractional order modeling
geometry, algebra	differential and integral calculus	***Do better than*** fractional calculus ***the best doable before!***

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Integer-Order Calculus

Fractional-Order Calculus

Fractional Order Mechanics!

Hooke's law: $F = kx$

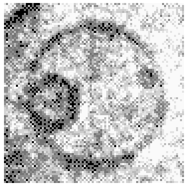
Newton's fluid: $F = kx'$

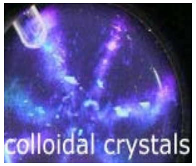
Newton's 2nd law: $F = kx''$

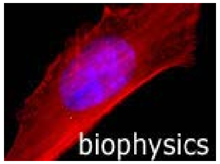
$\rightarrow F(t) = k x^{(\alpha)}(t)$

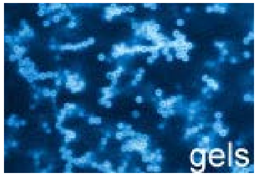
Going in-between: interpolation of operators:

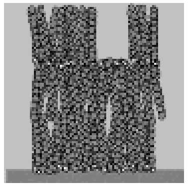
$$\ldots, \frac{d^{-2}f}{dt^{-2}}, \frac{d^{-1}f}{dt^{-1}}, f, \frac{df}{dt}, \frac{d^2 f}{dt^2}, \ldots$$

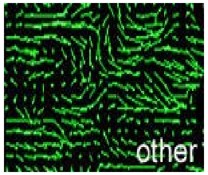


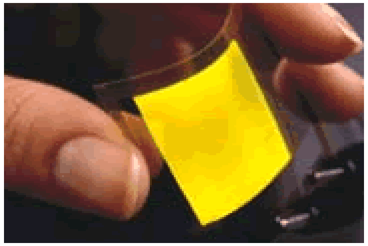


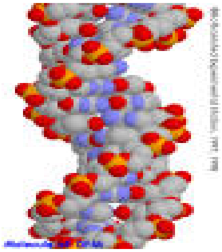


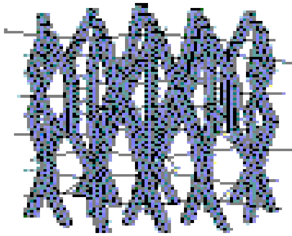


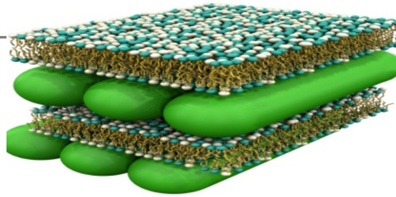






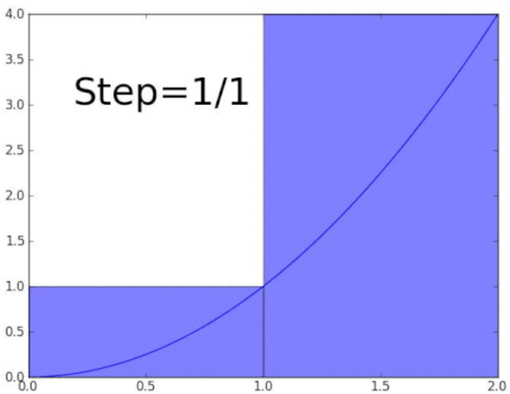


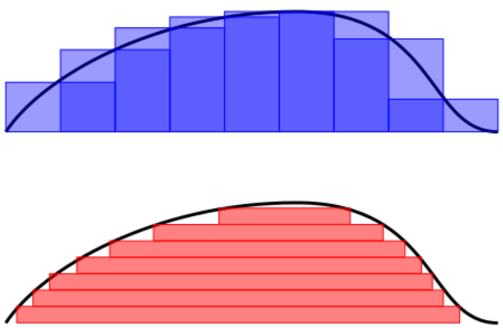




Soft matters, **complex** fluids

Interpretation of Fractional Integral





Riemann integral   Lebesgue integral

Riemann–Stieltjes integral,

Lebesgue–Stieltjes integral,

Itô integral and Stratonovich integral,

rough path integral,

line/surface/contour ...

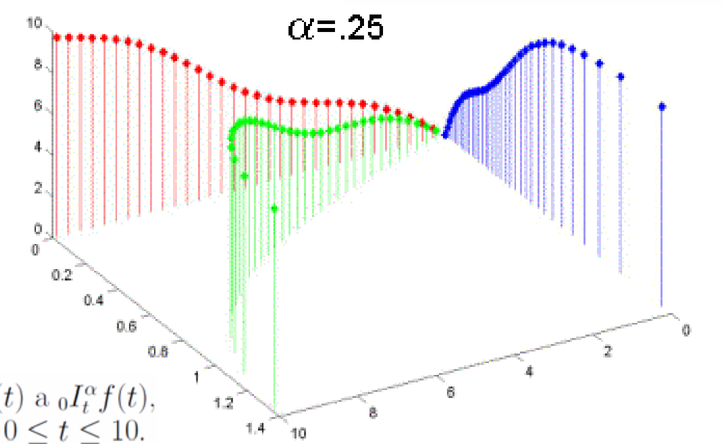
$$I^{\alpha} f(t) = \left(\frac{1}{t^{1-\alpha}}\right) * f(t) / \Gamma(\alpha)$$

$$ {}_0I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t f(\tau)(t-\tau)^{\alpha-1}\, d\tau, \quad t \geq 0,$$

$$ {}_0I_t^{\alpha} f(t) = \int_0^t f(\tau)\, dg_t(\tau),$$

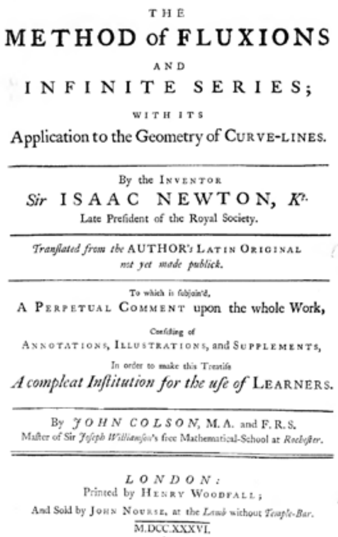
$$g_t(\tau) = \frac{1}{\Gamma(\alpha+1)} \{t^{\alpha} - (t-\tau)^{\alpha}\}.$$

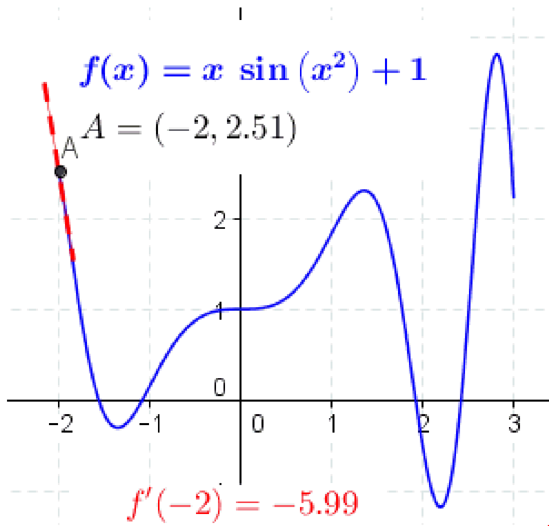
$$g_{t_1}(\tau_1) = g_{kt}(k\tau) = k^{\alpha} g_t(\tau)$$



"Live fence" and its shadows: ${}_0I_t^1 f(t)$ a ${}_0I_t^{\alpha} f(t)$, for $\alpha = 0.75$, $f(t) = t + 0.5\sin(t)$, $0 \leq t \leq 10$.

Interpretation of Fractional Derivative


THE
METHOD of FLUXIONS
AND
INFINITE SERIES;
WITH ITS
Application to the Geometry of CURVE-LINES.
By the INVENTOR
Sir ISAAC NEWTON, Kt.
Late President of the Royal Society.
Translated from the AUTHOR's LATIN ORIGINAL
not yet made publick.
To which is subjoin'd,
A PERPETUAL COMMENT upon the whole Work,
Consisting of
ANNOTATIONS, ILLUSTRATIONS, and SUPPLEMENTS,
In order to make this Treatise
A compleat Institution for the use of LEARNERS.
By JOHN COLSON, M.A. and F.R.S.
Master of Sir Joseph Williamson's free Mathematical-School at Rochester.
LONDON:
Printed by HENRY WOODFALL;
And Sold by JOHN NOURSE, at the Lamb without Temple-Bar.
M.DCC.XXXVI.



https://en.wikipedia.org/wiki/Fluxion

https://en.wikipedia.org/wiki/Celerity

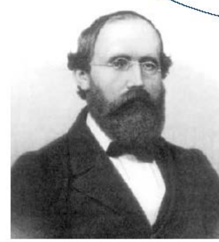
Newton introduced the concept of how to quantify "change" in 1665

(integer-order) derivative
is a very local/localized concept

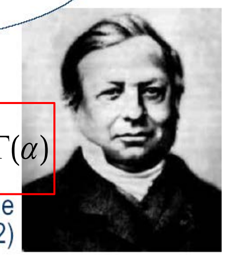
Riemann–Liouville definition

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)\,d\tau}{(t-\tau)^{\alpha-n+1}} $$

$$ (n-1 \le \alpha < n) $$



$$ I^{\alpha} f(t) = \left(\frac{1}{t^{1-\alpha}}\right) * f(t) / \Gamma(\alpha) $$



G.F.B. Riemann (1826–1866) J. Liouville (1809–1882)

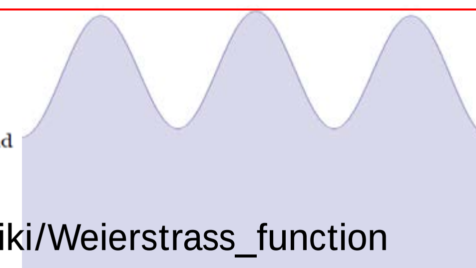
$$ D^{\alpha} f(t) = \frac{d}{dt}[I^{1-\alpha} f(t)] = \frac{d}{dt}\left[\left(\frac{1}{t^{\alpha}}\right) * f(t)\right] / \Gamma(1-\alpha) $$

$$ {}^{C}D^{\alpha} f(t) = \left[\left(\frac{1}{t^{\alpha}}\right) * \left(\frac{d}{dt} f(t)\right)\right] / \Gamma(1-\alpha) $$: Caputo

$$ f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x), $$

where $0 < a < 1$, b is a positive odd integer, and

$$ ab > 1 + \frac{3}{2}\pi. $$



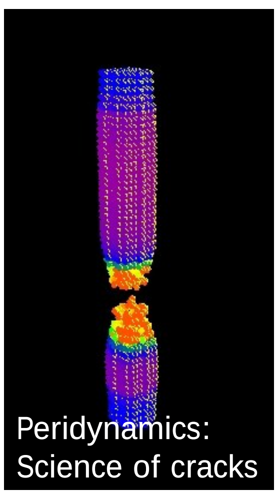
https://en.wikipedia.org/wiki/Weierstrass_function

(fractional-order) derivative
is a very nonlocal(ized) concept

Non-local operators

Qiang Du
Columbia University
New York, New York

Nonlocal Modeling, Analysis, and Computation



SIAM. SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS PHILADELPHIA

A recap. An important message from the above discussion is that one may see nonlocal models as more general models that serve as *bridges* connecting local continuum, nonlocal discrete, and fractional differential equations. That is, for the special examples considered here, the nonlocal operator

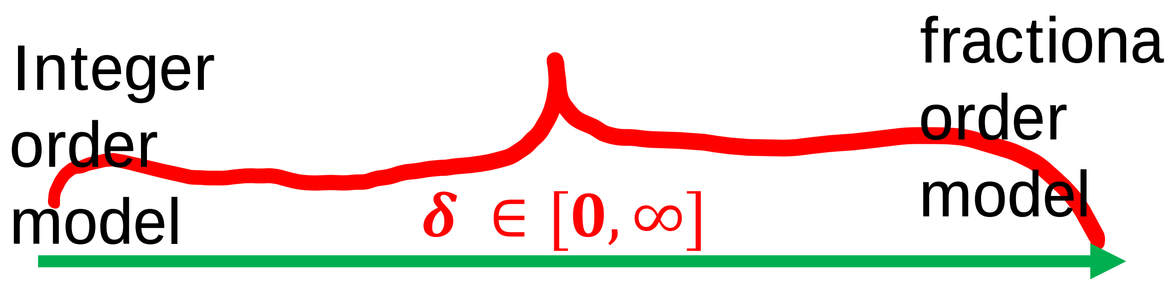
$$-\mathcal{L}_\delta u(x) = -\int_{-\delta}^{\delta} \frac{u(x+s) - 2u(x) + u(x-s)}{s^2} \underline{\omega}_\delta(s)\, ds = f(x)$$

recovers respectively the following special cases:

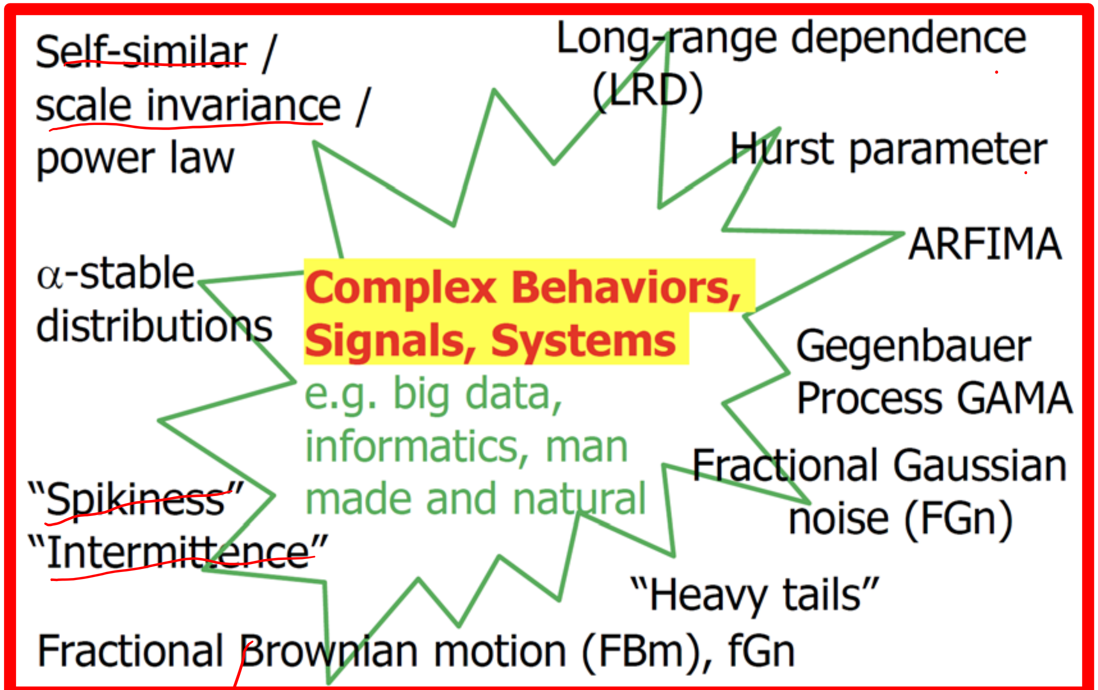
$$\underline{\omega}_\delta(s) = \underline{\omega}_0(s) = \text{Dirac delta at } s = 0 \quad \Rightarrow \quad \mathcal{L}_0 = \frac{d^2}{dx^2};$$

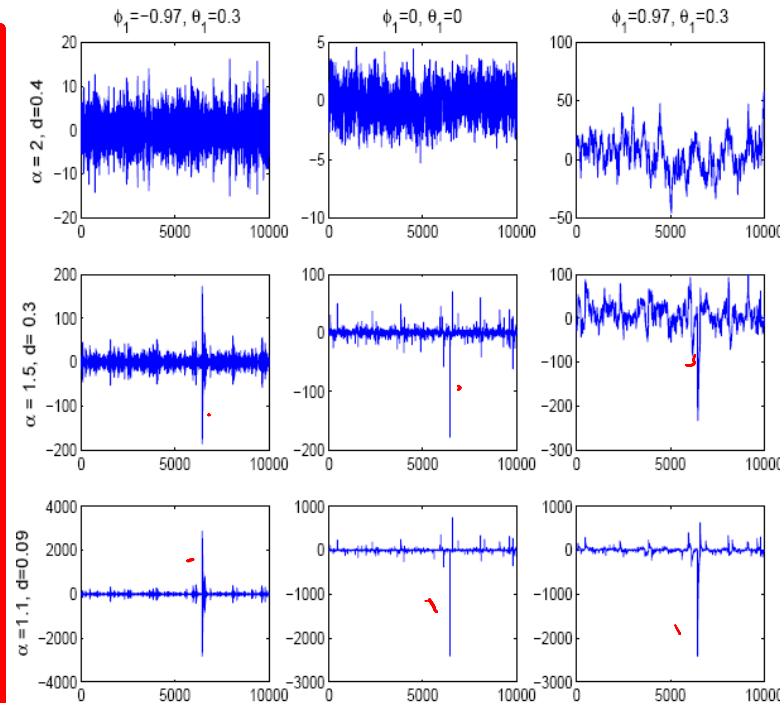
$$\underline{\omega}_\delta(s) = \underline{\omega}_h(s) = \text{Dirac delta at } s = h \quad \Rightarrow \quad \mathcal{L}_h = D_h^2;$$

$$\underline{\omega}_\delta(s) = \underline{\omega}_\infty(s) = \frac{c_{1,\alpha}}{s^{1-2\alpha}}, \ \delta \to \infty \quad \Rightarrow \quad \mathcal{L}_\alpha = -\left(-\frac{d^2}{dx^2}\right)^\alpha.$$

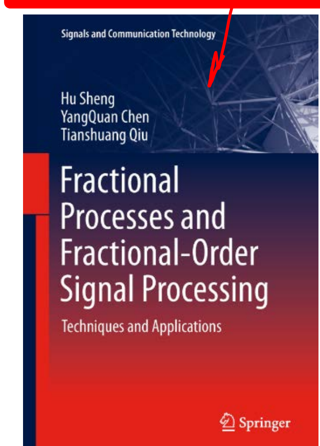


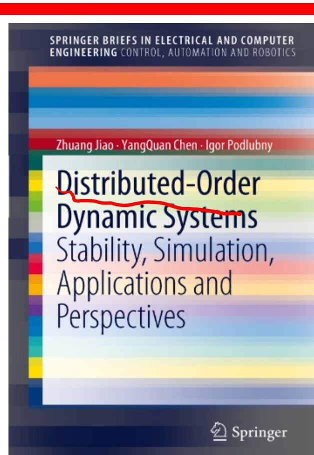
Fractional Order Stochasticity

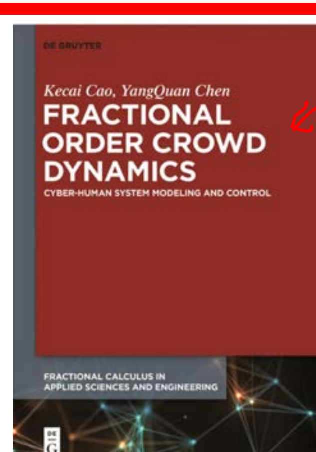




FARIMA(1, d, 1)







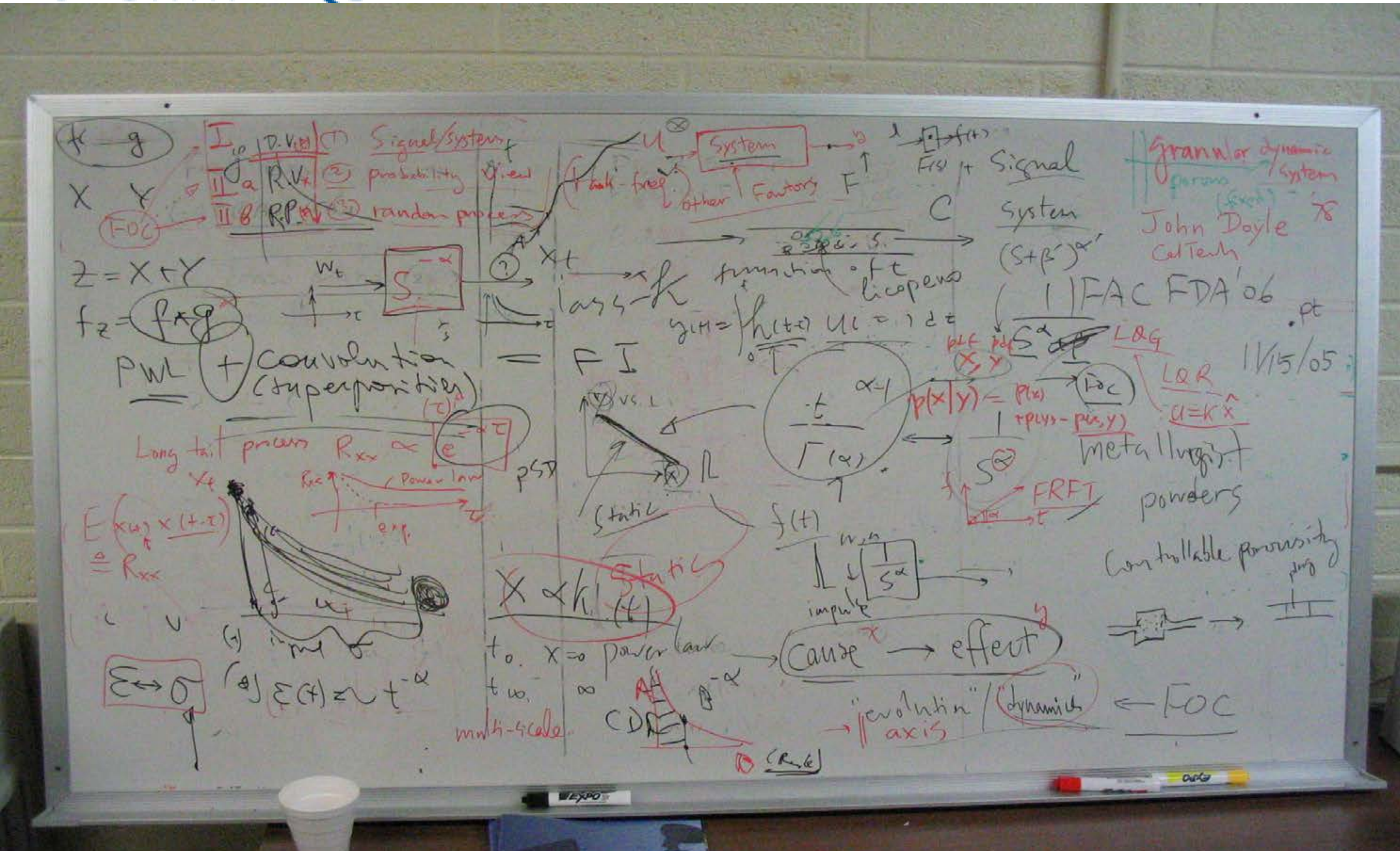
[7] S. Stoev, and M.S. Taqqu, "Simulation Methods for Linear Fractional Stable Motion and FARIMA Using the Fast Fourier Transform". *Fractals*, 2004.

"Fractional Order Thinking" or, "In Between Thinking"

- For example
  - Between integers there are non-integers;
  - Between logic 0 and logic 1, there is the "**fuzzy logic**";
  - Between integer order splines, there are "**fractional order splines**"
  - Between integer high order moments, there are **noninteger order moments (e.g. FLOS)**
  - Between "integer dimensions", there are **fractal dimensions**
  - Fractional Fourier transform (FrFT) – in-between time-n-freq.
  - Non-Integer order calculus (fractional order calculus – abuse of terminology.) (FOC)

Rule of thumb for "Fractional Order Thinking"

- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- *1/f* ^{a} noise
- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …



A snap shot of discussion board of Igor Podlubny and YangQuan Chen in Sept. 2005

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Optimal stochasticity entails fractional calculus that enlightens big data and machine learning

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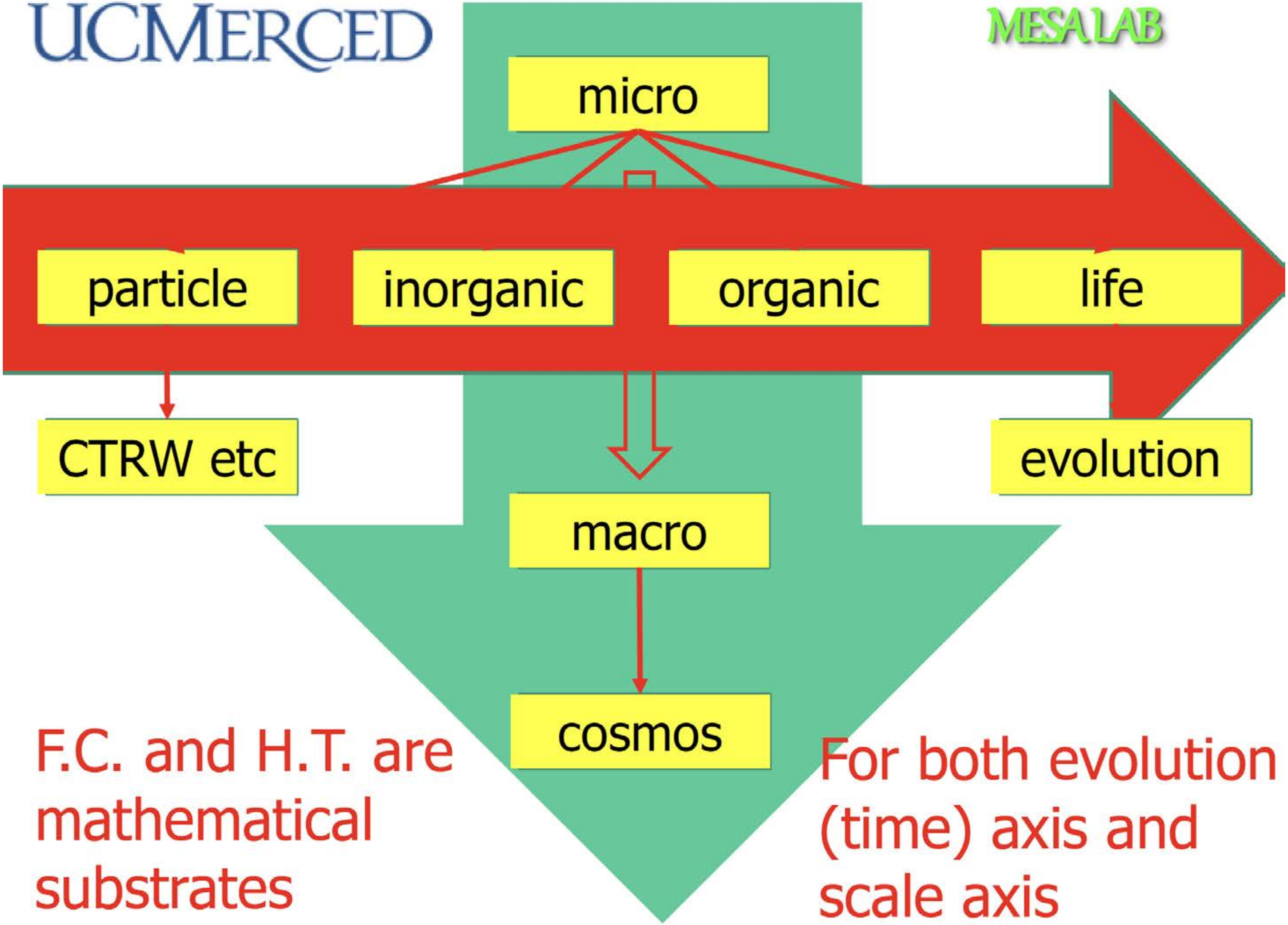
April 2, 2019. 8:10 -9:00 pm

IWDNS-2019 Xi'an, China



F.C. and H.T. are mathematical substrates

For both evolution (time) axis and scale axis

Fractional Order Stochasticity

Self-similar / scale invariance / power law

Long-range dependence (LRD)

Hurst parameter

ARFIMA

α-stable distributions

Complex Behaviors, Signals, Systems

e.g. big data, informatics, man made and natural

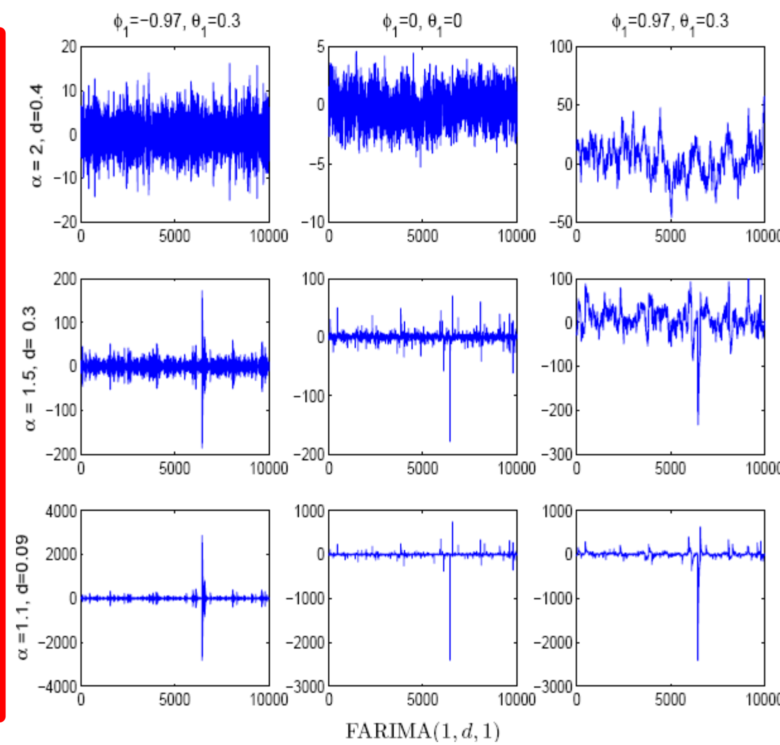
Gegenbauer Process GAMA

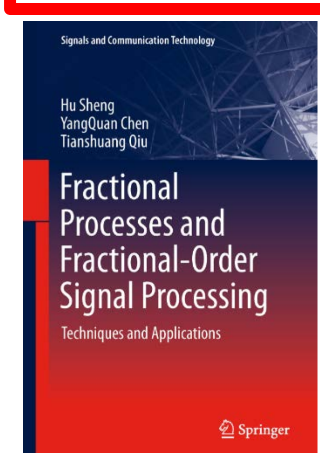
Fractional Gaussian noise (FGn)

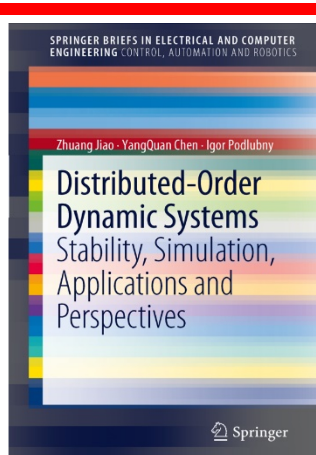
"Spikiness" "Intermittence"

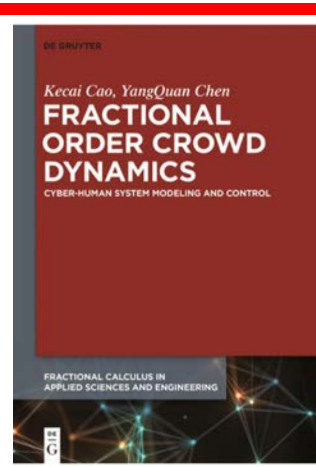
"Heavy tails"

Fractional Brownian motion (FBm), fGn









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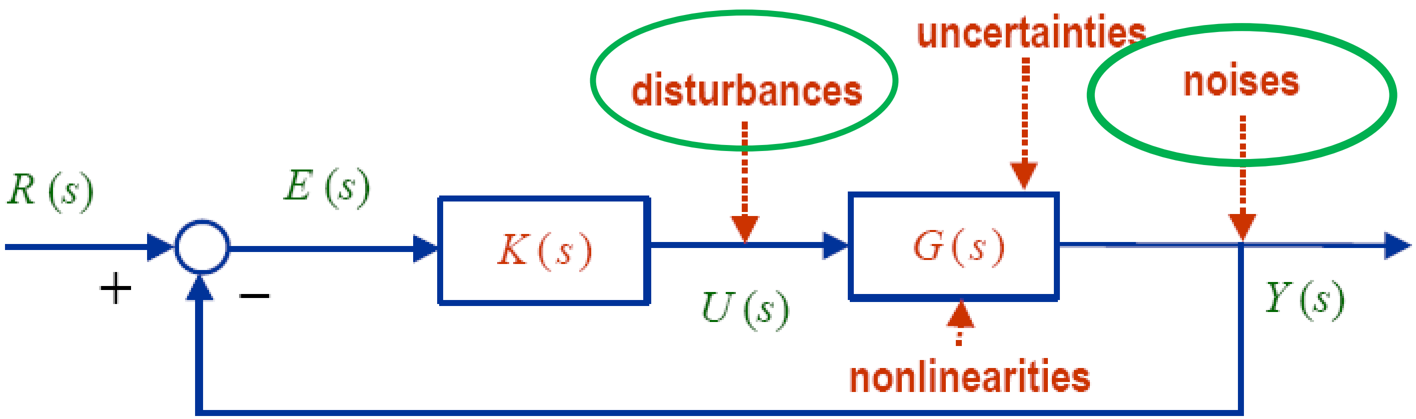
Fractional Order Systems and Controls

- IO Controller + IO Plant
- FO Controller + IO Plant
- FO Controller + FO Plant
- IO Controller + FO Plant

4 + 4 + ...

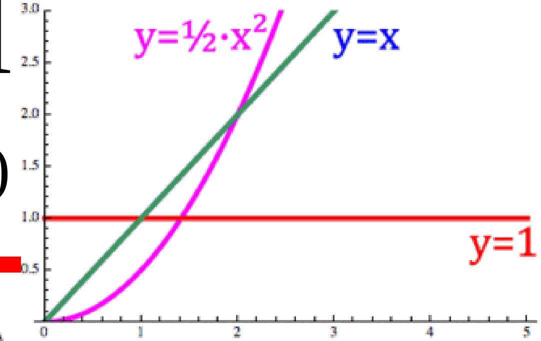


D. Xue and **Y. Chen***, "**A Comparative Introduction of Four Fractional Order Controllers**". Proc. of The 4th IEEE World Congress on Intelligent Control and Automation (WCICA02), June 10-14, 2002, Shanghai, China. pp. 3228-3235.

Want to be "More Optimal"?
(Better than the best?)
Go "fractional"!

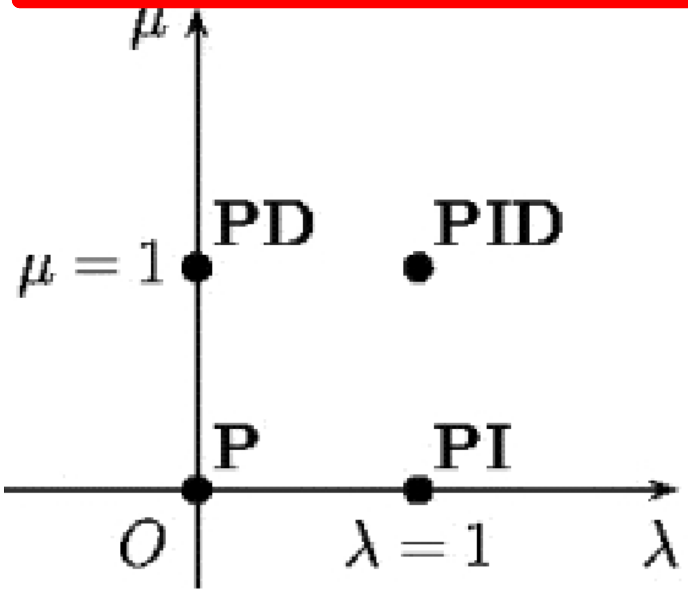
$$\dot{x}(t) \Rightarrow x^{(\alpha)}(t)$$

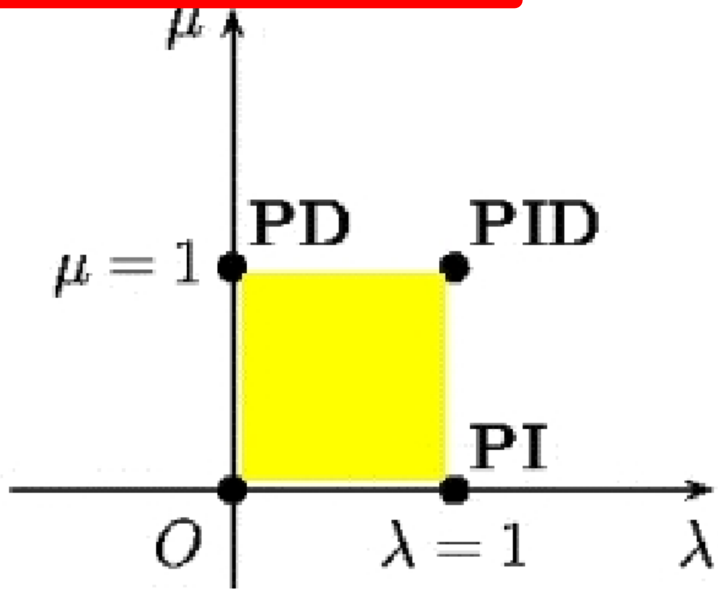
Fractional order PID control



- **90% are PI/PID type in industry** (**Ubiquitous**)
- **80% are for temperature control**

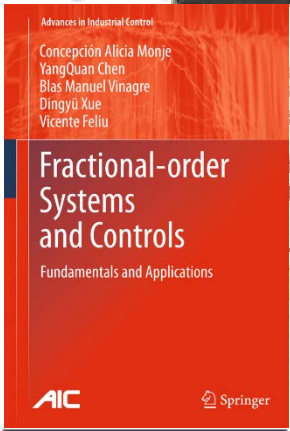
$$u(t) = K_p\left(e(t) + T_i \mathrm{D}_t^{-\lambda} e(t) + \frac{1}{T_d}\mathrm{D}_t^{\mu} e(t)\right). \qquad (\mathrm{D}_t^{(*)} \equiv {}_0\mathrm{D}_t^{(*)}).$$

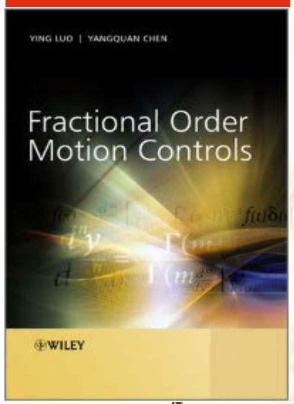


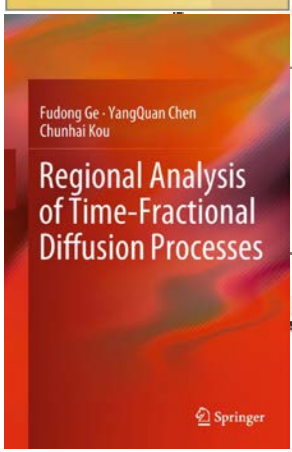


Igor Podlubny. "Fractional-order systems and $PI^{\lambda}D^{\mu}$-controllers". IEEE Trans. Automatic Control,44(1): 208–214, 1999.

*YangQuan Chen, Dingyu Xue, and Huifang Dou. "**Fractional Calculus and Biomimetic Control**". IEEE Int. Conf. on Robotics and Biomimetics (RoBio04), August 22-25, 2004, Shenyang, China.*







- Fractional Order System – official keyword of IFAC
- pid12.ing.unibs.it/

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Fractional order PID control: better than the best issue and what's next

YangQuan Chen, Ph.D., Director,

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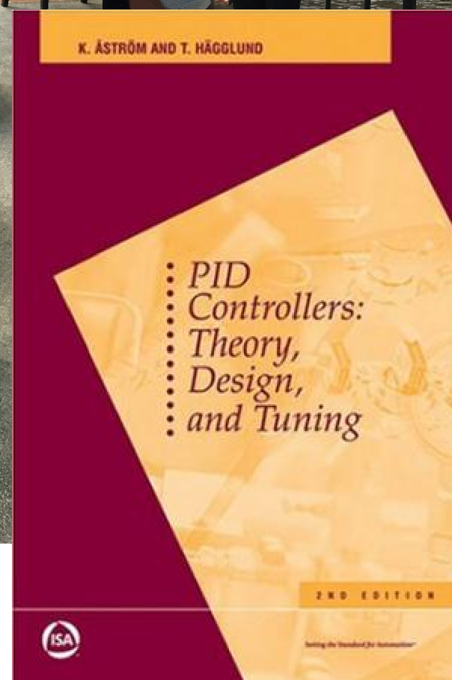
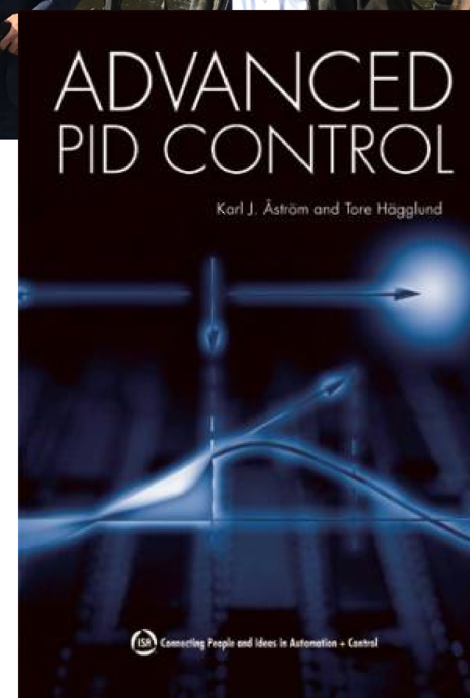
May 11, 2018. 11-12am, The Global

IFAC PID2018, Ghent, Belgium

- Slides and video: http://www.pid18.ugent.be/Quest.html


K. ÅSTRÖM AND T. HÄGGLUND
PID Controllers: Theory, Design, and Tuning
2ND EDITION
ISA


ADVANCED PID CONTROL
Karl J. Åström and Tore Hägglund
ISA Connecting People and Ideas in Automation + Control

8 hours recording PID @ 2020 IFAC World Congress Preconference Tutorial Workshop (10 speakers)
https://youtu.be/pgeOEq51RRk

All slides are online too
https://www.ifac2020.org/program/workshops/advanced-topics-in-pid-control-system-design-automatic-tuning-and-applications/

What left to be done for PID control research? 2020 IFAC World Congress full day PID pre-conference workshop and beyond

PID控制研究还有什么好做的？从2020 IFAC世界大会全天PID会前研讨会谈起

YangQuan Chen, Ph.D., Director,

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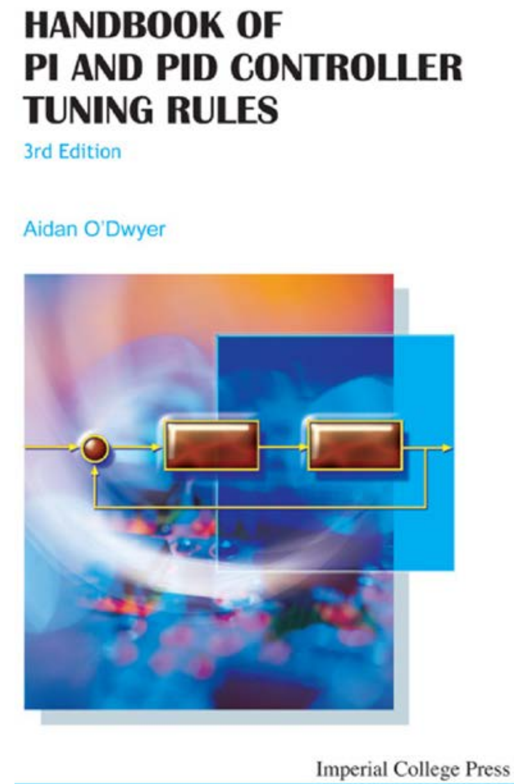
Web Seminar

What's next?

- **Fractional Order PID tuning,**
- **Smarter PID (digital twin, edge computing, embedded AI etc.)**

Can PID still be PHD topics?

- Yes. **Starting from this slide!**



- 3rd ed, 1935-2008, 600+ pages, 2009.
- 33 pages of references

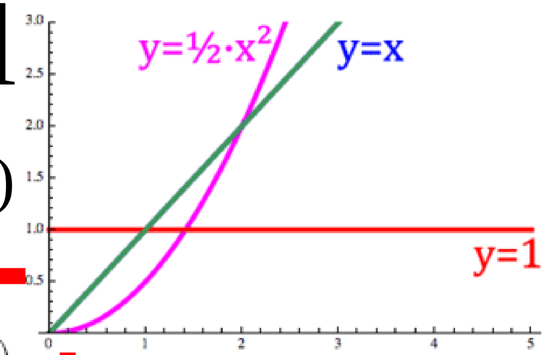
[1] IFAC PID 2018 Conference Plenary talk: "Fractional order PID control: better than the best issue and what's next" https://youtu.be/B3BurjUYPOA

[2] AA Dastjerdi, BM Vinagre, YQ Chen, SH HosseinNia. "Linear fractional order controllers; A survey in the frequency domain". **Annual Reviews in Control**, https://doi.org/10.1016/j.arcontrol.2019.03.8(49), 51-70, 2019

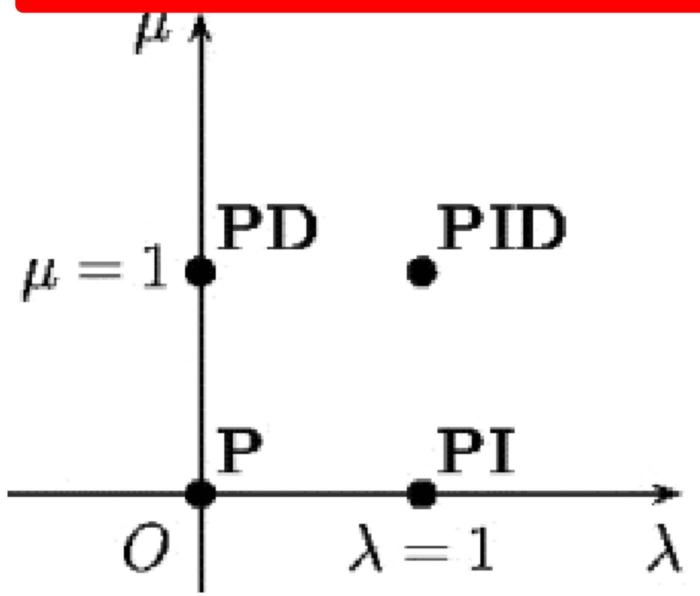
[3] Shah, P., and Agashe, S. (2016). Review of fractional PID controller. **Mechatronics**, volume (38), 29-41.

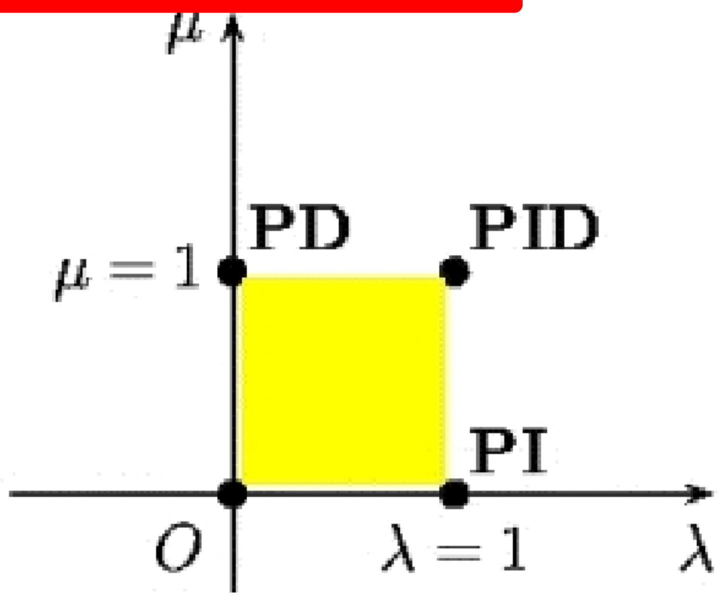
Fractional order PID control

- **90% are PI/PID type in industry** (**Ubiquitous**)
- **80% are for temperature control**



$$u(t) = K_p(e(t) + T_i \mathrm{D}_t^{-\lambda} e(t) + \frac{1}{T_d} \mathrm{D}_t^{\mu} e(t)). \qquad (\mathrm{D}_t^{(*)} \equiv_0 \mathrm{D}_t^{(*)}).$$





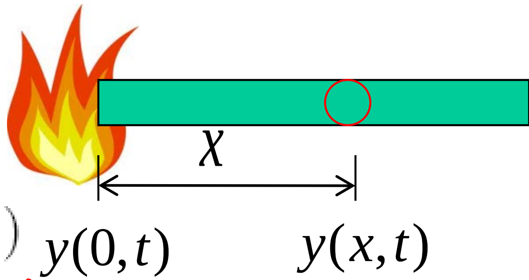
Igor Podlubny. "Fractional-order systems and $PI^{l}D^{\mu}$-controllers". IEEE Trans. Automatic Control,44(1): 208–214, 1999.

*YangQuan Chen, Dingyu Xue, and Huifang Dou. "**Fractional Calculus and Biomimetic Control**". IEEE Int. Conf. on Robotics and Biomimetics (RoBio04), August 22-25, 2004, Shenyang, China.*

Modeling: heat transfer

$$\frac{\partial^2 y(x,t)}{\partial x^2} = k^2 \frac{\partial y(x,t)}{\partial t},$$

$$(t > 0, \quad 0 < x < \infty)$$



Boundary condition (BC):

$$y(0,t) = m(t)$$

$$y(x,0) = 0$$

Initial condition (IC)

$$\left| \lim_{x \to \infty} y(x,t) \right| < \infty$$

Physical limit

Transfer function:

$$\frac{\mathrm{d}^2 Y(x,s)}{\mathrm{d}x^2} = k^2 s Y(x,s)$$

$$Q(0,s) = M(s)$$

$$\left| \lim_{x \to \infty} Y(x,s) \right| < \infty$$

$$Y(x,s) = A(s)e^{-kx\sqrt{s}} + B(s)e^{kx\sqrt{s}}$$

$$\begin{aligned} A(s) &= Y(0,s) = M(s) \\ B(s) &= 0 \end{aligned}$$

$$Y(x,s) = M(s)e^{-kx\sqrt{s}}$$

$$G(s) = \frac{Y(x,s)}{M(s)} = e^{-kx\sqrt{s}}$$

think about transfer function $e^{-\sqrt{s}}$!

Irrational Transfer Function.

Taylor series expansion: polynomial of half order integrators $s^{0.5}$!!

Ideal physical plant model:

$$G_p(s) = e^{-\sqrt{s}}$$

First Order Plus Time Delay (FOPTD) Model:

$$G_{IO}(s) = \frac{K_1}{T_1 s + 1} e^{-L_1 s}$$

Time Delay with Single Fractional Pole Model:

$$G_{FO}(s) = \frac{K_2}{T_2 s^{0.5} + 1} e^{-L_2 s}$$

All models are wrong but some are useful.
George E. P. Box

All models are wrong but some are dangerous ...
Leonard A. Smith

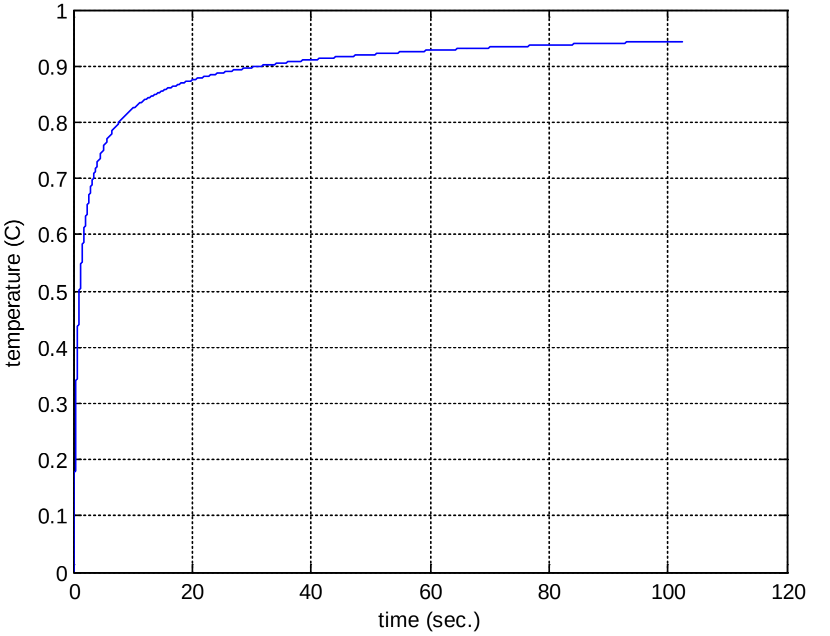
Step response of the "Ideal Plant"

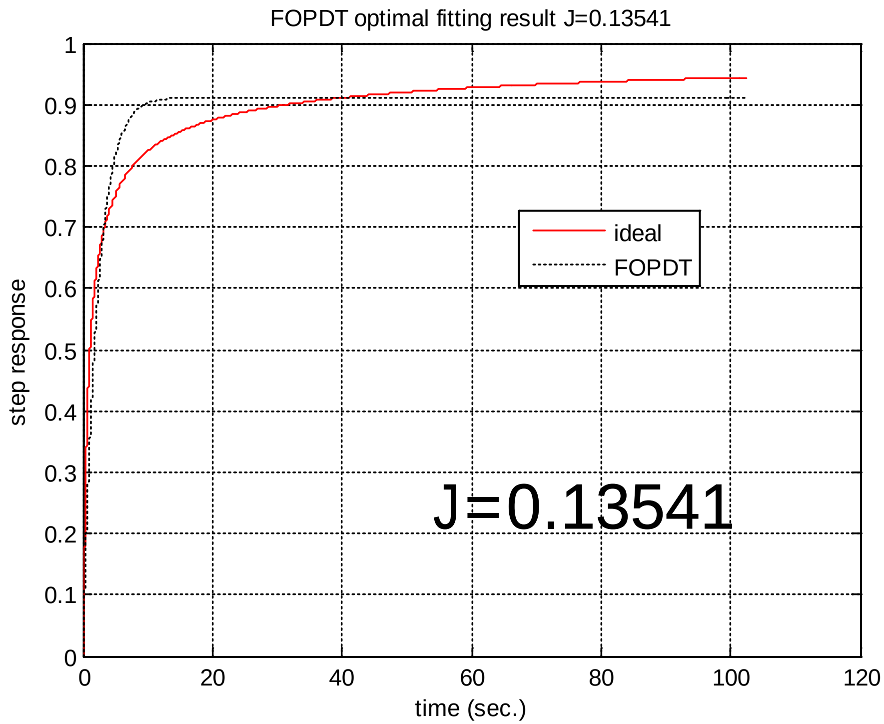
$$y(0,t) = m(t) = 1u(t), M(s) = \frac{1}{s}$$

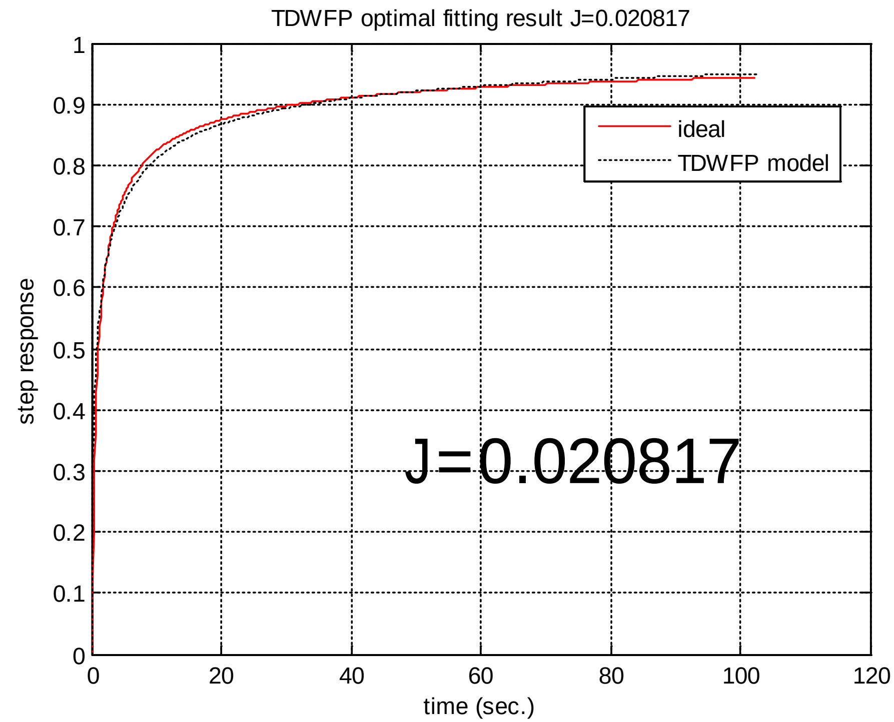
$$Y(x,s)\big|_{x=1} = G(x,s)\big|_{x=1} M(s) = G_p(s) M(s) = \frac{1}{s} e^{-\sqrt{s}}$$

So, "Reaction-Curve" or Step response of the "Ideal Plant"

$$y(t) = L^{-1}[\frac{1}{s} e^{-\sqrt{s}}]$$







K1	T1	L1
0.9120	2.2393	0

K2	T2	L2
1.0197	1.2312	0.0001

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Benefits of FOM

- Captures (more) physics $G_p(s) = e^{-\sqrt{s}} \rightarrow G_{FO}(s) = \frac{K_2}{T_2 s^{0.5} + 1} e^{-L_2 s}$
- Reaction curve fitting: **Better than the best** FOPDT model $G_{IO}(s) = \frac{K_1}{T_1 s + 1} e^{-L_1 s}$
- Could be a nice starting point for better controller design?
  - Reminder: Among all control tasks, 80% of them are for temperature controls that calls for $\sqrt{s}$
  - **Lots of process control papers may be re-written.**

Double check the "Reaction Curve" by $G_{FO}(s) = \frac{K}{T s^{\alpha} + 1} e^{-Ls}$

https://www.mathworks.com/matlabcentral/fileexchange/69790-fractional-order-alpha-scanning-code

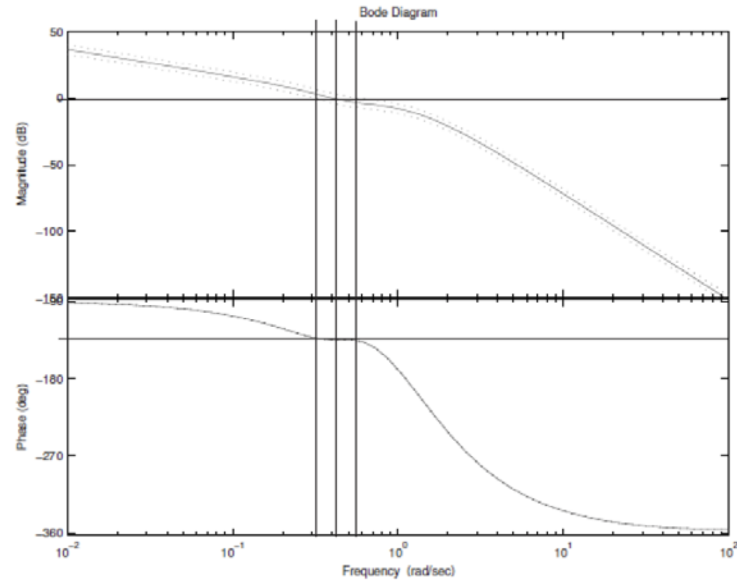
The Idea of "More Flat Phase" Design

Based on the discussions above, a robust fractional order controller should consider the following specifications :

① **A specified gain crossover frequency, ω_{gc};**

② **A specified phase margin, ϕ_m;**

③ **A flat phase constraint, $\frac{d\varphi}{d\omega} = 0$.**



(a) Basic idea: a flat phase curve at gain crossover frequency

$$K_p + \frac{K_i}{s} + K_d s$$

IO-PID

$$C(s) = K_p + \frac{K_i}{s^r}$$

FO-PI

However, when the parameter number of a robust fractional order PID controller is larger, how can we design a robust FOPID systematically and theoretically?

① A specified gain crossover frequency, ω_{gc};

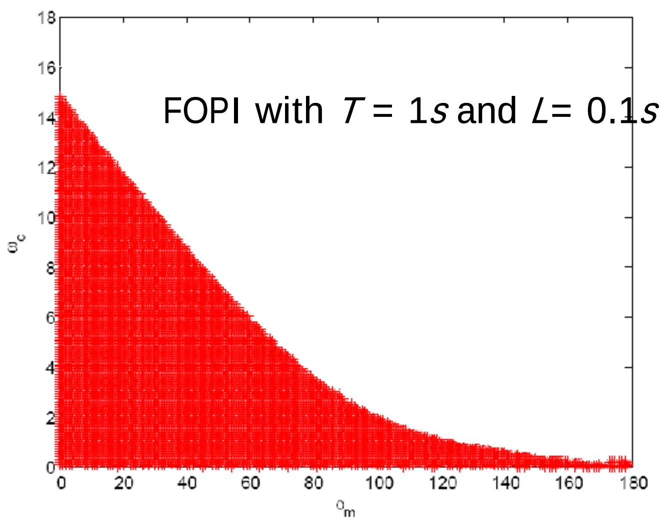
② A specified phase margin, ϕ_m;

③ **More flat phase constraints,** $\begin{cases} \frac{d\varphi}{d\omega} = 0 \\ \cdots \\ \frac{d^{(n)}\varphi}{d\omega^{(n)}} = 0 \end{cases}$, n is the integer order > 1.

$$C(s) = \left(K_p + \frac{K_i}{s} + K_d s\right)^r$$

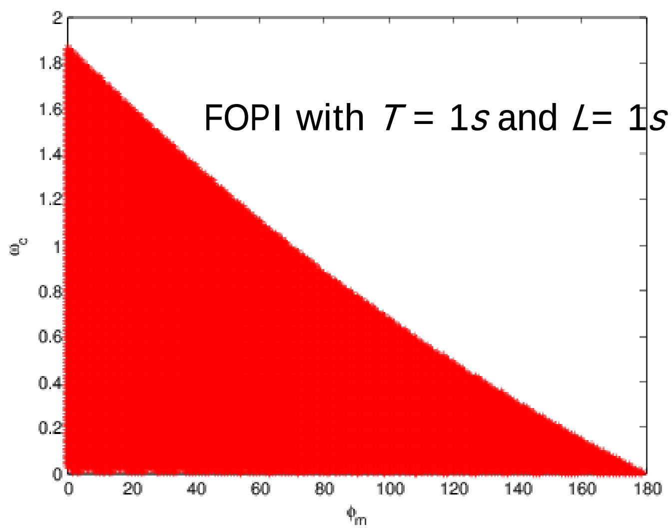
FO-[PID]

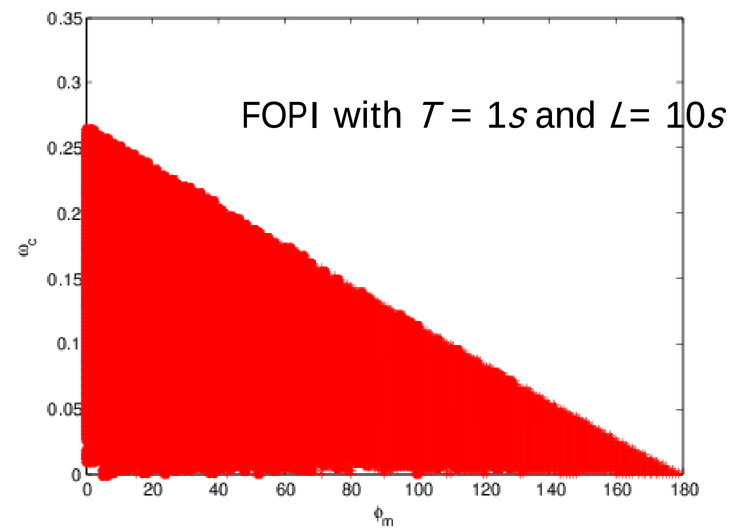
Example: Achievable region for FO-PI $C(s) = K_p + \frac{K_i}{s^r}$



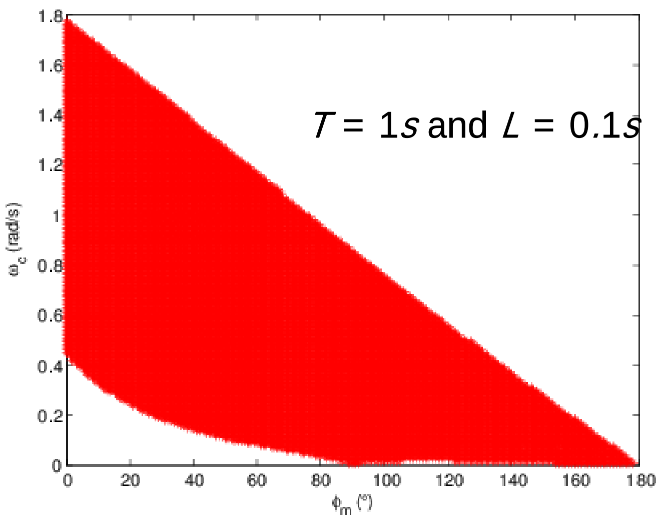
*Ying Luo and YangQuan Chen, "Stabilizing and Robust FOPI Controller Synthesis for First Order Plus Time Delay Systems." **Automatica**. Vol. 48, no. 9, pages: 2159–2167. Published Sept. 2012.*
http://dx.doi.org/doi:10.1016/j.automatica.2012.05.072

$$G(s) = \frac{1}{s+1} e^{-Ls}$$



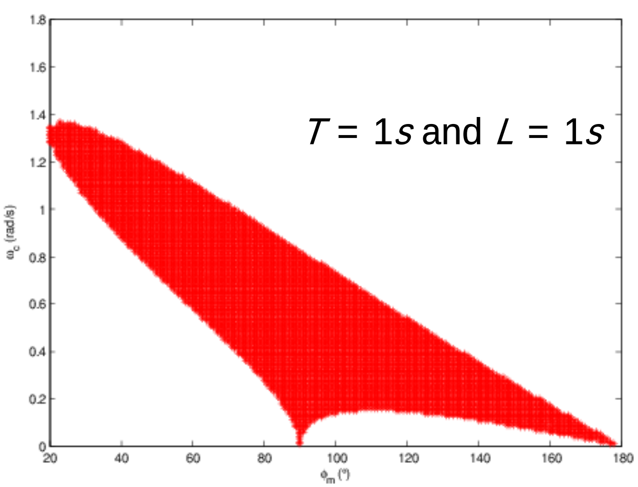


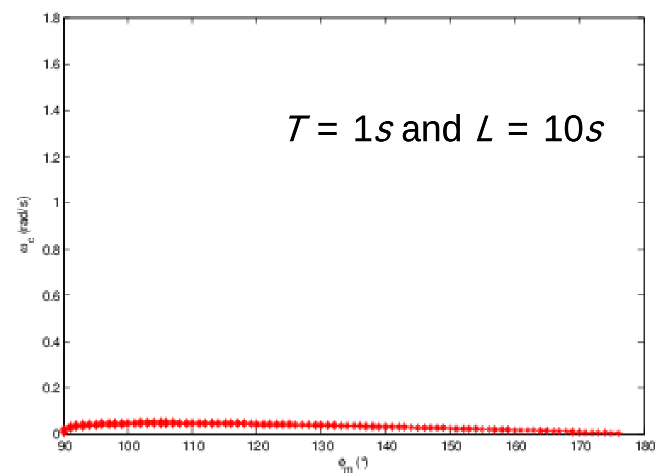
Achievable region for IO-PID $C(s) = K_p + \frac{K_i}{s} + K_d s$



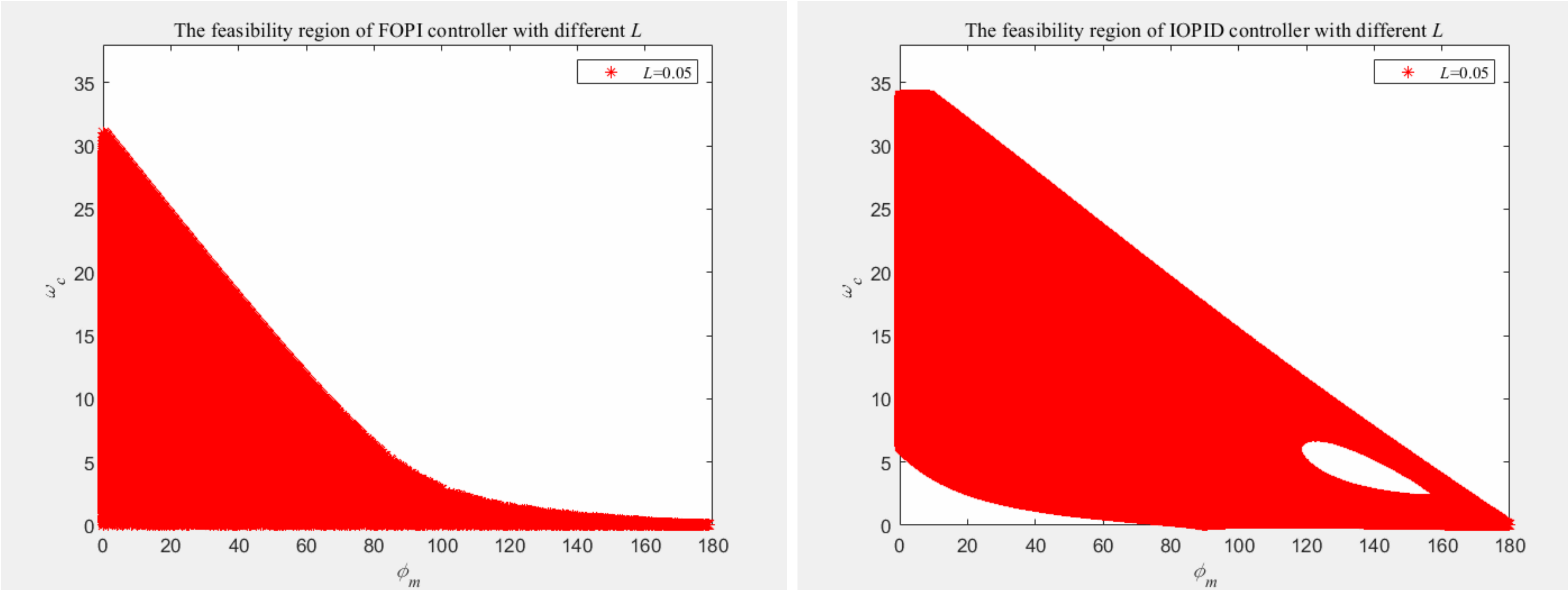
*Ying Luo and YangQuan Chen, "Stabilizing and Robust FOPI Controller Synthesis for First Order Plus Time Delay Systems." **Automatica**. Vol. 48, no. 9, pages: 2159–2167. Published Sept. 2012.*
http://dx.doi.org/doi:10.1016/j.automatica.2012.05.072

$$G(s) = \frac{1}{s+1} e^{-Ls}$$





The Animation of Feasible Regions of FOPI and IO-PID



(a) The FOPI with different L.

(b) The IOPID with different L.

Fig. 15. The feasibility regions of ω_c and ϕ_m for FOPI and IOPID design with $T \in [0.05,10]$.

$$G(s) = \frac{1}{s+1} \mathrm{e}^{-Ls}$$

$$\frac{1}{Ts+1} e^{-(L/T)Ts} = \frac{1}{s'+1} e^{-L's'}$$

In summary:

- When $L/(L+T) \to 1$, FOPI is more needed than IOPID.
- When $L/(L+T) \to 0$, FOPI is an extended option of IOPID.

Atherton, D.P. (2007). Feedback. *IEEE Control Systems*, 27(4), 17–18. `http://ieeexplore.ieee.org/document/4272322/`.

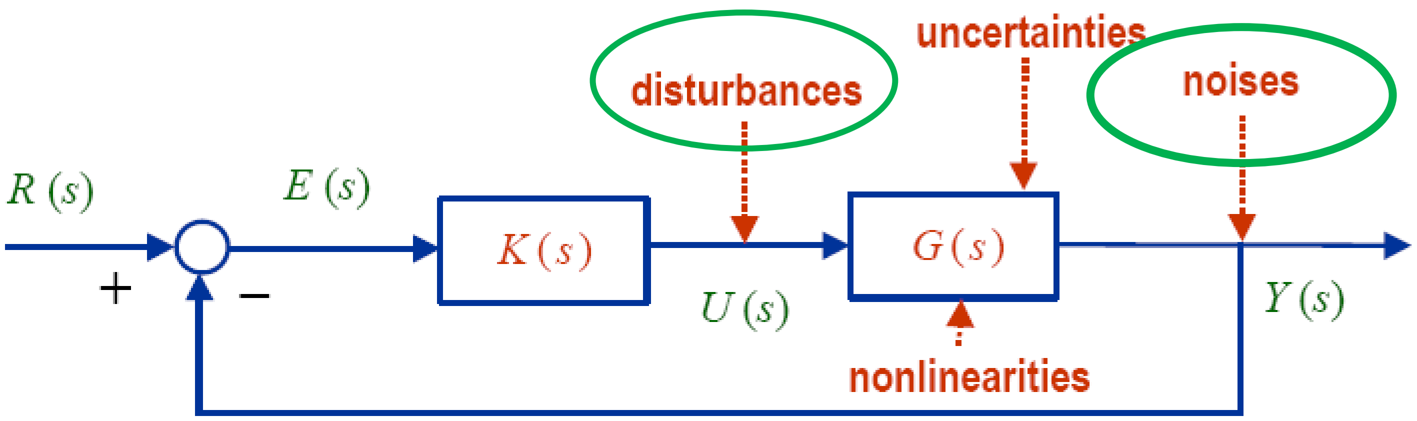
$$\mathcal{L}[x(at)] = \frac{1}{|a|} X\left(\frac{s}{a}\right)$$

Credit: Animation made by Dr. Zhenlong Wu

Fractional Order Systems and Controls

- IO Controller + IO Plant
- FO Controller + IO Plant
- FO Controller + FO Plant
- IO Controller + FO Plant

4 + 4 + ...



D. Xue and **Y. Chen***, "**A Comparative Introduction of Four Fractional Order Controllers**". Proc. of The 4th IEEE World Congress on Intelligent Control and Automation (WCICA02), June 10-14, 2002, Shanghai, China. pp. 3228-3235.

Outline

- What and Why Fractional Order Thinking
- Fractional Order Modeling and Controls
- Introduction to Fractional Order Mechanics
- Take Home Messages

ME280 "Fractional Order Mechanics" Course description

- This course prepares students with fractional calculus (differentiation or integration of non-integer order) and fractional dynamic modeling of complex mechanical systems such as porous medias, particulate systems, soft matters etc. that have inherent nature of memory, heredity, or long-range dependence (LRD), or long range interactions at or across various scales.

Fractional Order Mechanics: WHY?

- Softmatter / hardmatter
- Softbody / Rigidbody
- Lumped / distributed
- Granular, particulate, porous, disordered … materials
- …

Soft matter?

- Soft matters, also known as ***complex fluids***, behave unlike ideal solids and fluids.
- ***Mesoscopic*** macromolecule rather than microscopic elementary particles play a more important role.

Typical soft matters

- Granular materials
- Colloids, liquid crystals, emulsions, foams,
- Polymers, textiles, rubber, glass
- Rock layers, sediments, oil, soil, DNA
- Multiphase fluids
- Biopolymers and biological materials

highly deformable, porous, thermal fluctuations play major role, highly unstable

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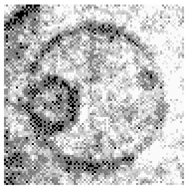
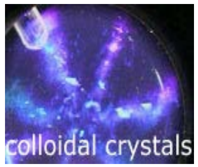
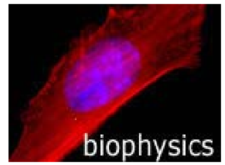
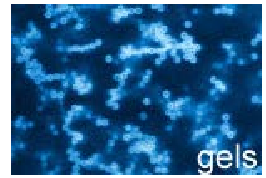
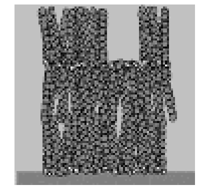
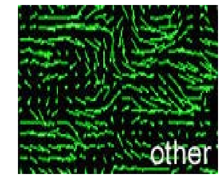


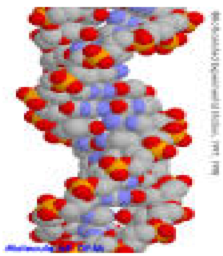


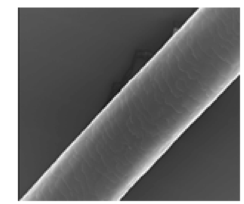


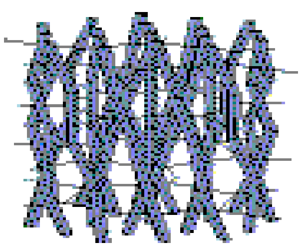


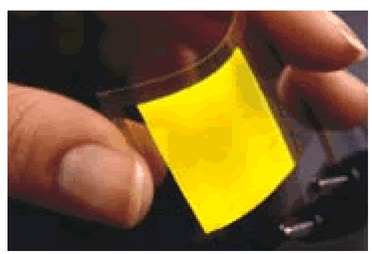


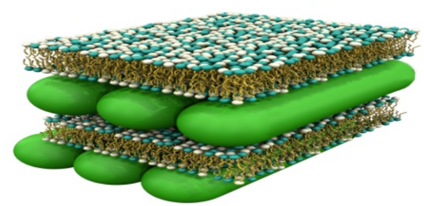


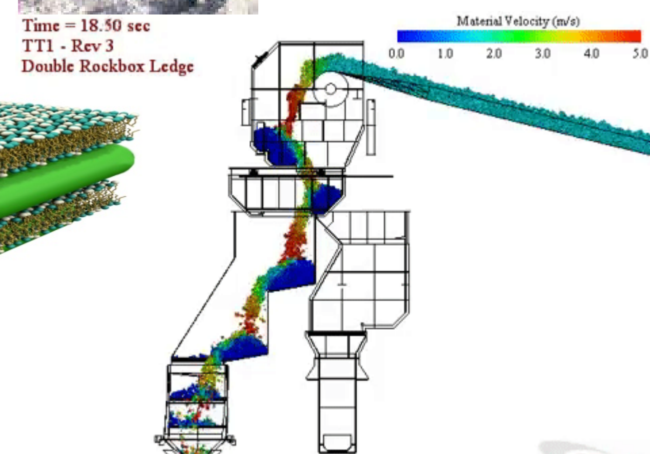












AC-Tek

Slide credit: Wen Chen of HHU

Constitutive relationships

- **Hookian law in ideal solids:** $F = kx$
- **Ideal Newtonian fluids:** $F = \upsilon \frac{\partial u}{\partial y}$
- **Newtonian 2nd law for rigid solids:** $F = m \frac{d^2 x}{dt^2}$
- **One model of soft matter:** $F = \rho \frac{\partial^\alpha x}{\partial t^\alpha}$ $\quad 0 \le \alpha \le 2$

Slide credit: Wen Chen of HHU

Fractional Order Mechanics!

Hooke's law: $F = kx$

Newton's fluid: $F = kx'$

Newton's 2nd law: $F = kx''$

$$F(t) = k x^{(\alpha)}(t)$$

Going in-between: interpolation of operators:

$$\ldots, \quad \frac{d^{-2}f}{dt^{-2}}, \quad \frac{d^{-1}f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2f}{dt^2}, \quad \ldots$$

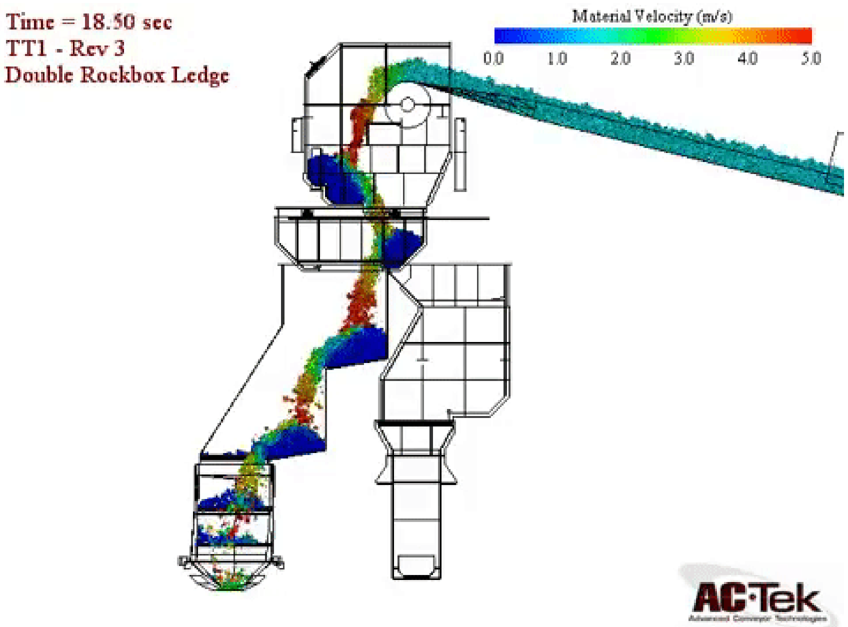
Conclusion of Talk

Integer-Order Calculus

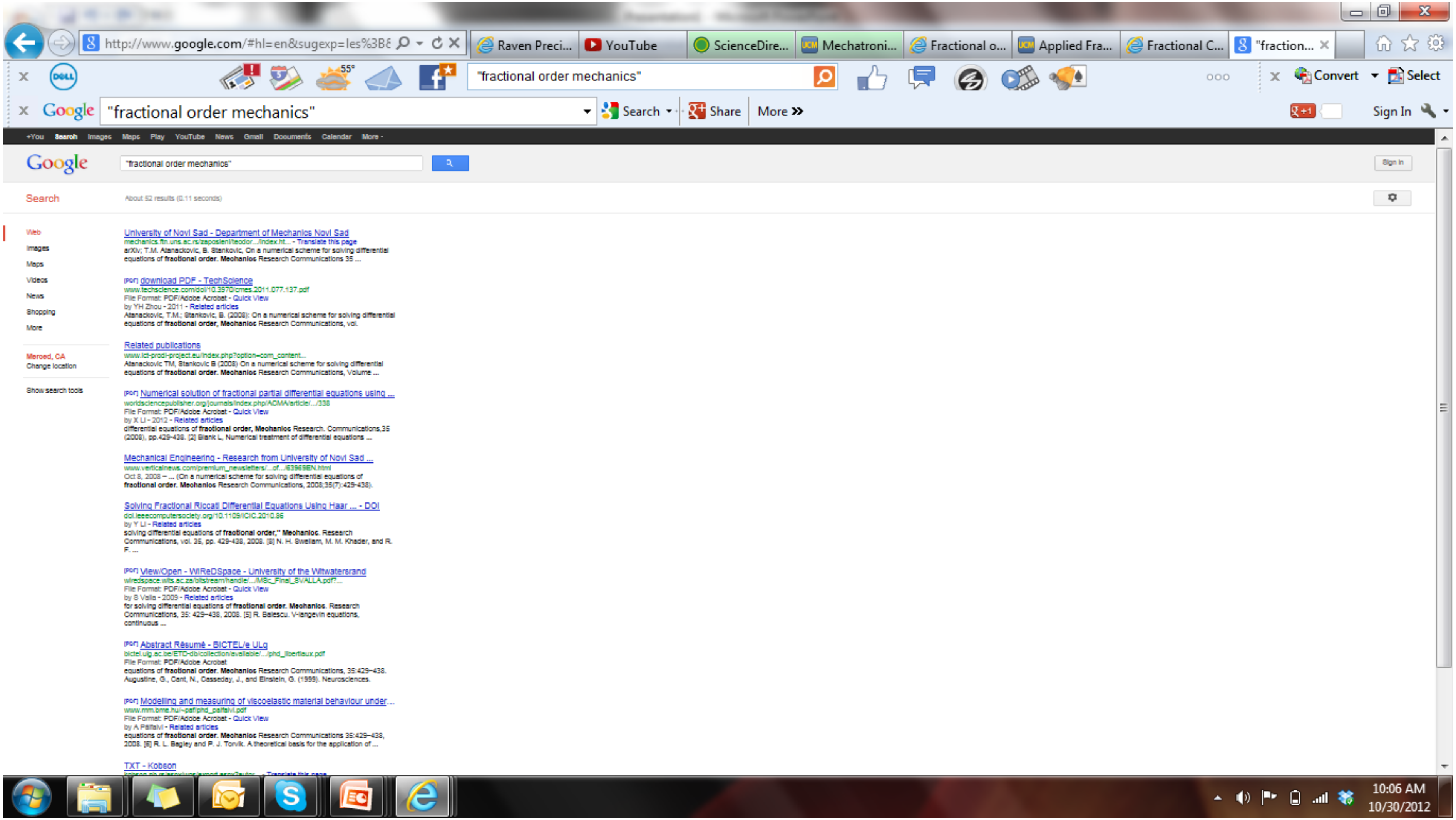
Fractional-Order Calculus

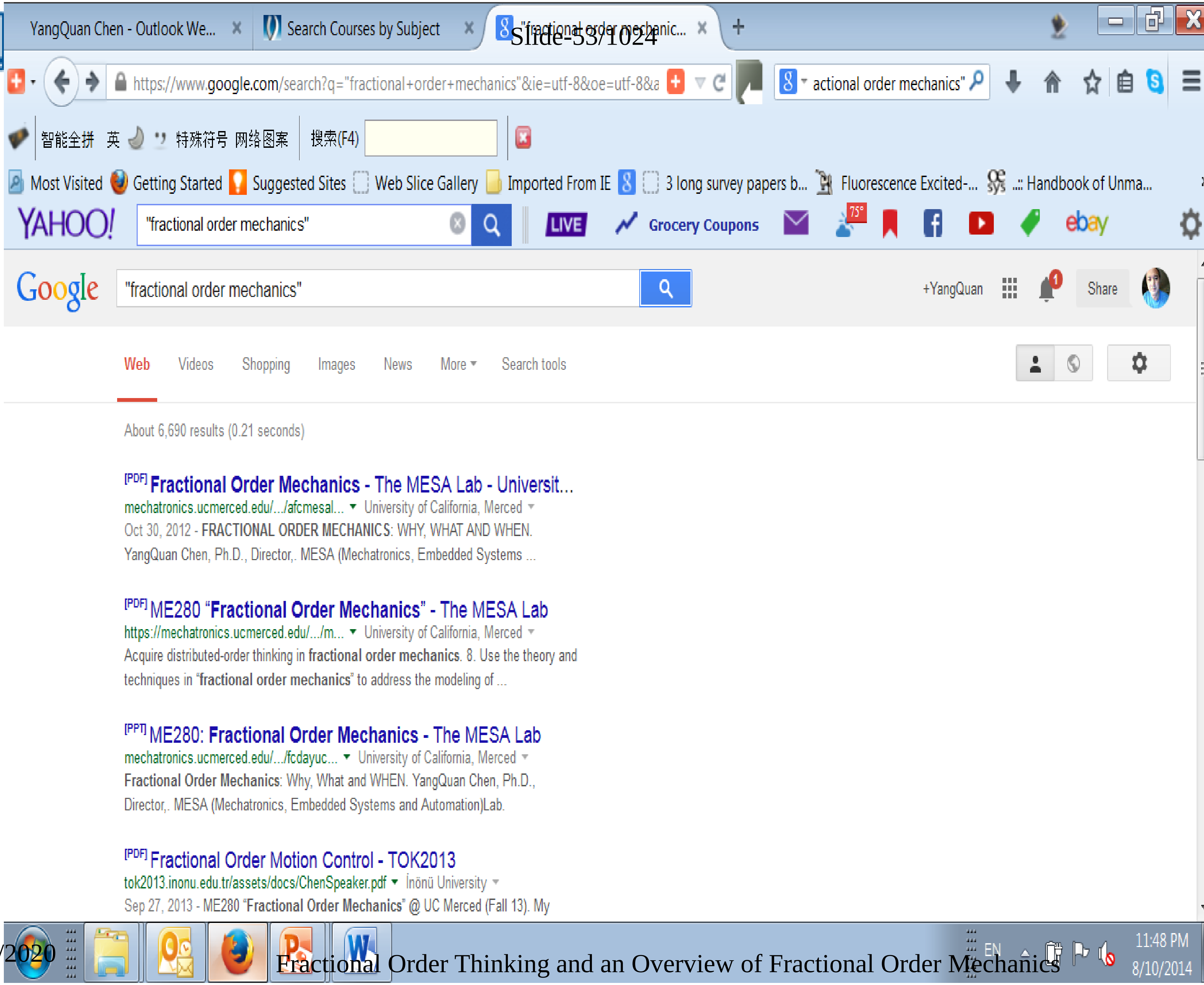
Slide credit: Richard L. Magin, ICCC12

- Kurt Lewin: “There is nothing so practical as good theory" (p. 169).
  - Lewin, K. (1951). Field theory in social science. New York: Harper & Row.



Google says“Fractional Order Mechanics”

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Oct 30, 2012 - FRACTIONAL ORDER MECHANICS: WHY, WHAT AND WHEN.
YangQuan Chen, Ph.D., Director,. MESA (Mechatronics, Embedded Systems ...
[PDF] ME280 "Fractional Order Mechanics" - The MESA Lab
https://mechatronics.ucmerced.edu/.../m... University of California, Merced
Acquire distributed-order thinking in fractional order mechanics. 8. Use the theory and
techniques in "fractional order mechanics" to address the modeling of ...
[PPT] ME280: Fractional Order Mechanics - The MESA Lab
mechatronics.ucmerced.edu/.../fcdayuc... University of California, Merced
Fractional Order Mechanics: Why, What and WHEN. YangQuan Chen, Ph.D.,
Director,. MESA (Mechatronics, Embedded Systems and Automation)Lab.
[PDF] Fractional Order Motion Control - TOK2013
tok2013.inonu.edu.tr/assets/docs/ChenSpeaker.pdf İnönü University
Sep 27, 2013 - ME280 "Fractional Order Mechanics" @ UC Merced (Fall 13). My
EN
11:48 PM
8/10/2014

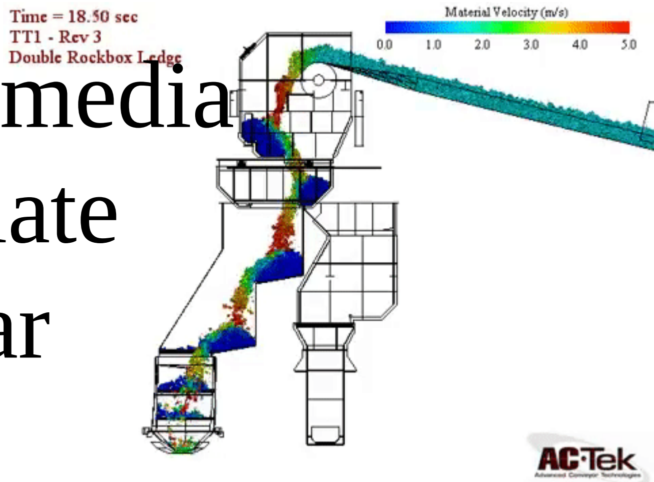
G.W. Scott Blair (1950)

- “We may express our concepts in Newtonian terms if we find this convenient but, if we do so, we must realize that we have made a translation into a language which is foreign to the system which we are studying.”

TOC of ME280 FOMech

- FC basics; Interpretations; FC signals and systems
- *Bagley-Torvik* mechanics
- CTRW – Continuous Time Random Walk
- FC modeling (ML fitting) of complex relaxation processes
- Fractional Euler-Lagrange Equation
- Advanced topics (application oriented) (FISP – focused independent study and presentation)
  - Battery system models, biological signal processing
  - Fractional Order ESC, nanomaterial modeling
  - Salinity dynamics, complexity quantification
  - Hysteresis modeling and compensation
  - FO stochastic mechanics for evolving complex networks etc.

FOMech: WHEN?



- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- *1/f* ^{a} noise

- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …

- **TOPICS (2013, 2014, 2017, 2020):**
  - Course admin, motivations and real world needs; (2 weeks) 1 2
  - FOMech Motivations: FOT and fractional stochasticity; (2 weeks) 3 4
  - Fractional mechanics in classical sense (Bagley-Torvik) (1 week) 5
  - Fundamentals of FC and Geometrical/Physical Interpretations (1 week) 6
  - CTRW and Anomalous Diffusion (2 weeks) 7 8
  - Fractional order system modeling; (1 weeks) 9
  - Fractional order damping (1 week) 10
  - Variable-Order and Distributed Order Mechanics (1 week) 11
  - Fractional-Order Analytical Mechanics (2 weeks) 12 13
  - Integer-Order Analytical Mechanics and A Dark Cloud
  - Integer-Order Optimal Control, Integer Order Calculus of Variation and RIOTS_95
  - Fractional Order Analytical Mechanics (FO Euler-Lagrange mechanics and fractional variational principle)
  - Semester Summary and Looking Into The Future (1 week Thanksgiving week) 14
  - FISP – Weeks 15, 16 and 17 (final exam week) plus guest lectures

School of Engineering, University of California, Merced
ME-280 Fractional Order Mechanics (FOMech) (32641) Fall 2013 Weekly Schedule
Lectures: 9:00-10:15am KOLLIG 217. LAB (32834) Weds, 9:00-11:50am, SCIENG 172

Week#	Date	Lecture Topic(s)	Homework/Lab assignments	Remarks/readings (M:Magin; P:Podlubny)
01				
01	08/29	Course admin, SLOs, labs. Introducing class members. Why FOMech? Motivations. MITOCW "Signals and Systems"	HW01: Web search and analysis and essay writing Quiz#01	www Preface (M) Chap 01. (M)
02	09/03	Why FOMech? Motivations Grading policy, General Motivations on Fractional Calculus and Fractional Order Thinking (FOT) –Part-1	Quiz#02 Lab#01	MITOCW "Signals and Systems"
02	09/05	General Motivations on Fractional Calculus and Fractional Order Thinking (FOT) –Part-2	Quiz#03 HW02:	MITOCW "Signals and Systems"
03	09/10	General Motivations on Fractional Calculus and Fractional Order Thinking (FOT) –Part-3	Quiz#04	MITOCW "Signals and Systems"
03	09/12	Fractional Order Mechanics: Motivations	Quiz#05 HW03:	MITOCW "Signals and Systems"

[1] http://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/lecture-notes/ (Please go through LEC# 1-7, 20, 21, 25). Go through the corresponding problem sets and solutions as well http://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/assignments/

04	09/17	No lecture. Replaced by Bruce J. West's Monday 9/16 MTS Seminar 3-4:30PM. **Optional lecture:** (Integer-Order) Signals and Systems (with introduction to stochastic processes)	Quiz#06 Lab#02	9/16 Monday @ KL232 3PM. Chancellor's conference room
04	09/19	Fractional signals and systems http://www.sciencedirect.com/science/article/pii/S0165168410003233 Fractional Order Mechanics: Motivations Final Front: **Fractional Order Stochasticity - Power Law, Scale-Free, Heavy-Tailedness, Long Range Dependence, Long Memory, and Complexity due to Fractional Dynamics** (two papers to read)	Quiz#07 HW04; Lab#03	Read this review paper by Magin, Ortigueira, Podlubny and Trujillo
05	09/24	Bagley-Torvik FOMech (part-1) (guest lectured by Dr. JG Liu)	Quiz#08 HW05	Dr. Chen TOK13 trip
05	09/26	Bagley-Torvik FOMech (part-2) (guest lectured by Dr. JG Liu)	Quiz#09 Lab#04	Dr. Chen TOK13 trip
06	10/01	FC fundamentals (Gorenflo's post-FDA10 lecture)	Quiz#10 HW06	
06	10/03	FC fundamentals (Podlubny's Geometrical and physical interpretations of FC) http://people.tuke.sk/igor.podlubny/ravello2012/recorded/lecture1/ravello-lecture-1-recorded-web.mov	Quiz#11	Lab01 due 10/31 Lab02 due

07	10/08	CTRW fundamentals and its link to FC (Gorenflo's post-FDA10 lecture)	Quiz#12 HW07	
07	10/10	Integer-Order Diffusion and CTRW (Derivation in one dimension-heat-equation) (Bruce Henry)	Quiz#13	
08	10/15	CTRW and Anomalous Diffusion (Bruce Henry)	Quiz#14 HW08	
08	10/17	CTRW and Anomalous Diffusion (Bruce Henry)	Quiz#15	Lab03 due
09	10/22	Fractional order modeling (Integer-Order System ID)	Quiz#16 HW09 (forward modeling)	Take-home Mid-term Exam
09	10/24	Fractional order modeling (Fractional-Order Model Fitting)	Quiz#17	FISP Proposal Due Extended to 10/29/2013

10	10/29	Fractional Order Damping	Quiz#18 HW10 (reverse modeling) Lab#05	
10	10/31	Fractional Order Damping	Quiz#19	Lab04 due.
11	11/05	Variable-Order and Distributed Order Mechanics	Quiz#20 HW11	
11	11/07	Variable-Order and Distributed Order Mechanics	Quiz#21 Lab#06	
12	11/12	Integer-Order Analytical Mechanics and A Dark Cloud	Quiz#22 HW12	
12	11/14	Integer-Order Optimal Control, Integer Order Calculus of Variation and RIOTS_95	Quiz#23	
13	11/19	Fractional Order Analytical Mechanics (FO Euler-Lagrange mechanics and fractional variational principle)	Quiz#24 HW13	Guest Lectured by Prof. Zhanbing Bai (TBD)
13	11/21	Fractional Order Analytical Mechanics (FO Euler-Lagrange mechanics and fractional variational principle)	Quiz#25	Guest Lectured by Prof. Zhanbing Bai (TBD)
14	11/26	Semester Summary and Outlook of FOMech	Quiz#26 HW14 Lab#07	
14	11/28	Thanksgiving		

15	12/03	No class.		Replaced by Friday ME/EECS Seminar by Richard Magin
15	12/05	No class. FISP preparation Attending both. Richard L. Magin guest lectures on Friday b12/06/2013 - EECS 12:00-1:20PM ME 1:30PM – 2:30PM		Replaced by Friday ME/EECS Seminar by Richard Magin
16	12/10	**FISP** Presentations (recorded). (75 min)		3 students x25min (75 min)
16	12/12	**FISP** Presentations (recorded). (75 min)		3 students x25min (75 min)
Final	12/17	**FISP** Presentations (recorded). 3:00-6:00pm, KOLLIG 217 @ 17-DEC-2013 Tue NO FINAL. (180 min)		4 students x25min + 4 observers x20 min (180 min)

Labs (ME280 is lab intensive)

- Week-02 ME280 Lab01 **Numerical Studies of Mittag-Leffler Function (MLF)**
- Week-04 ME280 Lab02 **Numerical Inverse Laplace Transform (NILT) and Fractional Calculus**
- Week-04 ME280 Lab03 **On Fractional Processes and Hurst Parameter Estimation**
- Week-05 ME280 Lab04 **Numerical Inverse Laplace Transform (NILT) to Visualize Fractional Order Vibration Equations**
- Week-08 ME280 Lab05 **Numerical Study of CTRW**
- Week-11 ME280 Lab06 **Numerical Studies of Variable Order Differentiation - t(t) as a benchmark example**
- Week-13 ME280 Lab07 (last lab!) **Fractional Order Calculus of Variations: A Numerical Solution Example**

Homeworks and Quizzes

- Every week 2 to 5 problems
- Every lecture, end of lecture quiz

New ideas

- Optimal Foraging
- Human operator modeling
- Spatial variability analysis (DRONEMATH)
- PSO using Levy strategy and ML distribution
- Big data optimal random searching/matching?
- Crowd science and fractional calculus
- Opinion formation
- Attention quantification
- Solar power industry and irradiance variability modeling …

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Outline

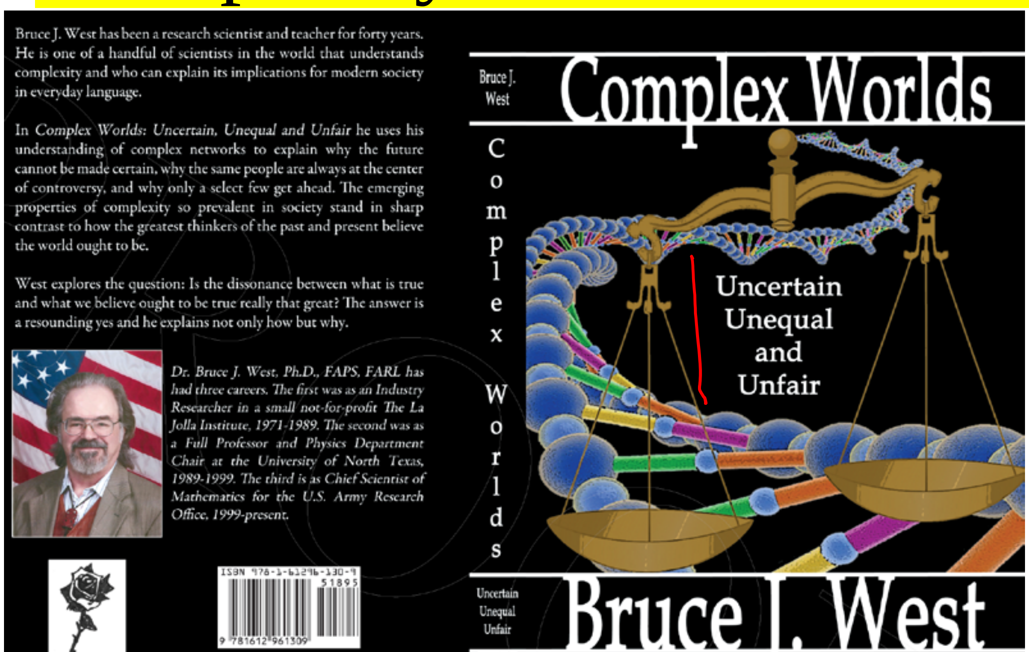
- What and Why Fractional Order Thinking
- Fractional Order Modeling and Controls
- Introduction to Fractional Order Mechanics
- Take Home Messages

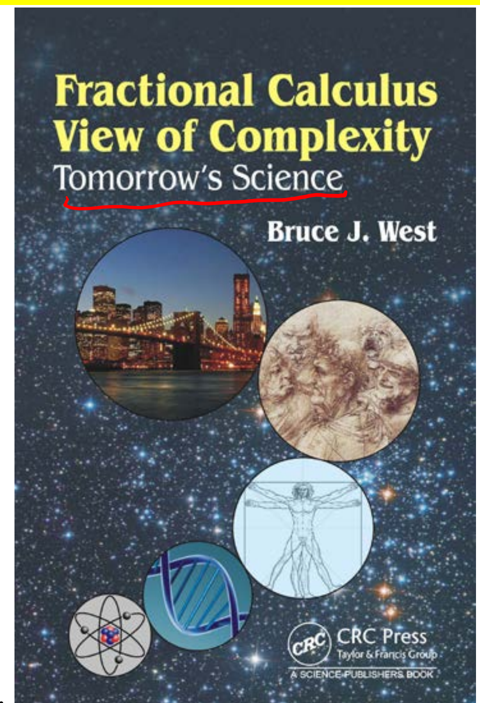
Take home message - 1

- “Integer-order” is a special case of “Fractional Order” – it’s possible to do better than the best in both modeling and control.

Take home message - 2

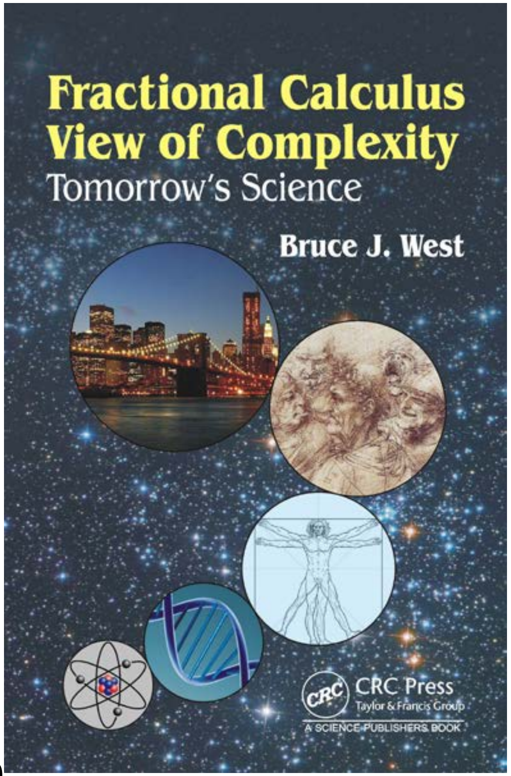
- “Fractional Order Thinking” can lead us to a better life by better understanding and embracing “complexity” 积极入世的人生态度如王阳明

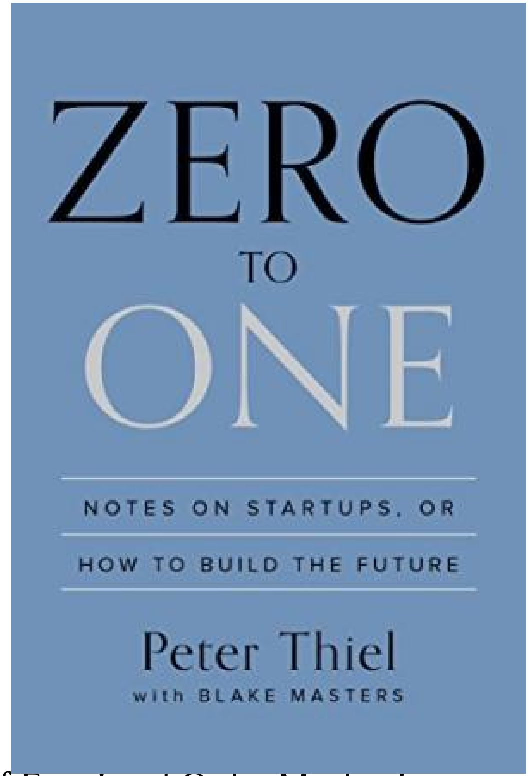




Take home message - 3

- Think from “0” to “1” not from “1” to “100”
- Think in-between: go fractional, think non-local!





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Why big data and machine learning must meet fractional calculus?

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July 6th, 2020. Monday 2-3PM

Seminar over internet hosted by Beijing Jiaotong University

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Fractional Calculus and Its Connection to Creativity

分数阶微积分和创造性之间的联系

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Dec. 28, 2019

The First FOSCC, Jinan, China

- http://169.236.9.29/ME280-Fall2013-release/
- 2013 all recorded Youtube playlist https://www.youtube.com/playlist?list=PL-euleXgwWUN_r5hersTDQn18MvPK2nUT

Thank you for attending my talk!

For more information, check

http://mechatronics.ucmerced.edu/