EMPIRICAL OBSERVABILITY-BASED SOURCE TERM ESTIMATION USING MOBILE SENSOR TRAJECTORIES

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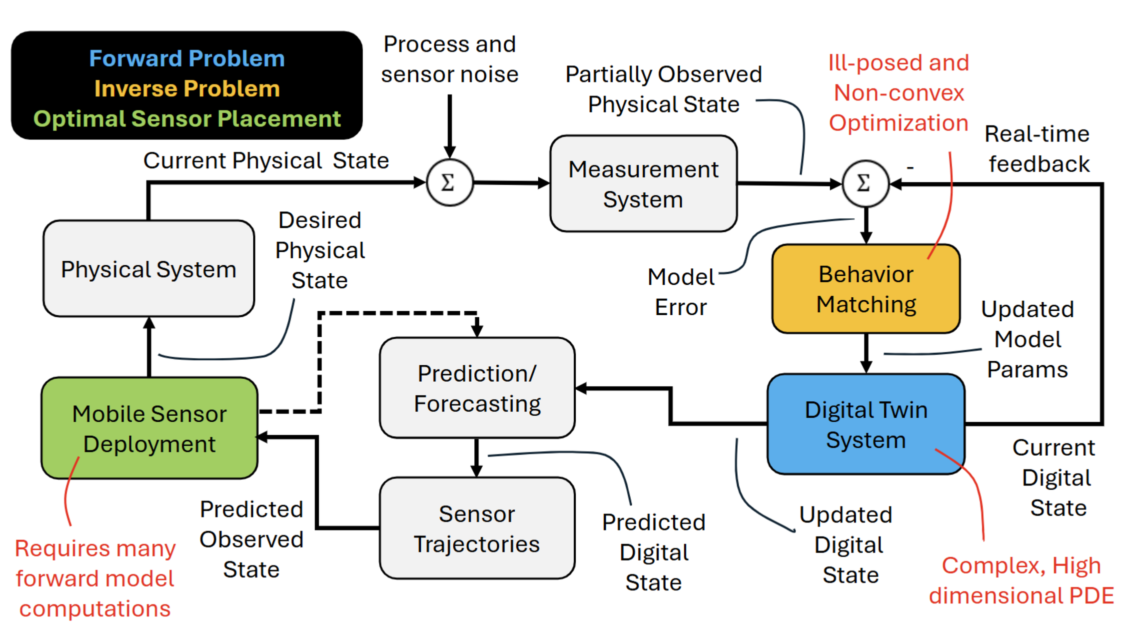
ABSTRACT

Real time parameter estimation relies on a fast running models of the system and robust optimization schemes. The source term estimation problem of a trace atmospheric gas in complex terrains, requires many PDE forward solves – rendering it very difficult to solve in a real-time. Utilizing the empirical Gramian, a fast running low fidelity surrogate model, and a high fidelity digital twin of the emission source, the optimal trajectory can be explored, in the observability sense, to estimate the parameters of the system.

PROBLEM AND APPROACH

Source term estimation is the process of estimating emission source rate, location, and other dispersion parameters from a set of observations based on mobile trajectories. Digital twins can be used to behavior match observation data to provide a forecast dispersion patterns for optimal sensor placement [1]. Estimating parameters from observation data is often ill-posed and computationally expensive.



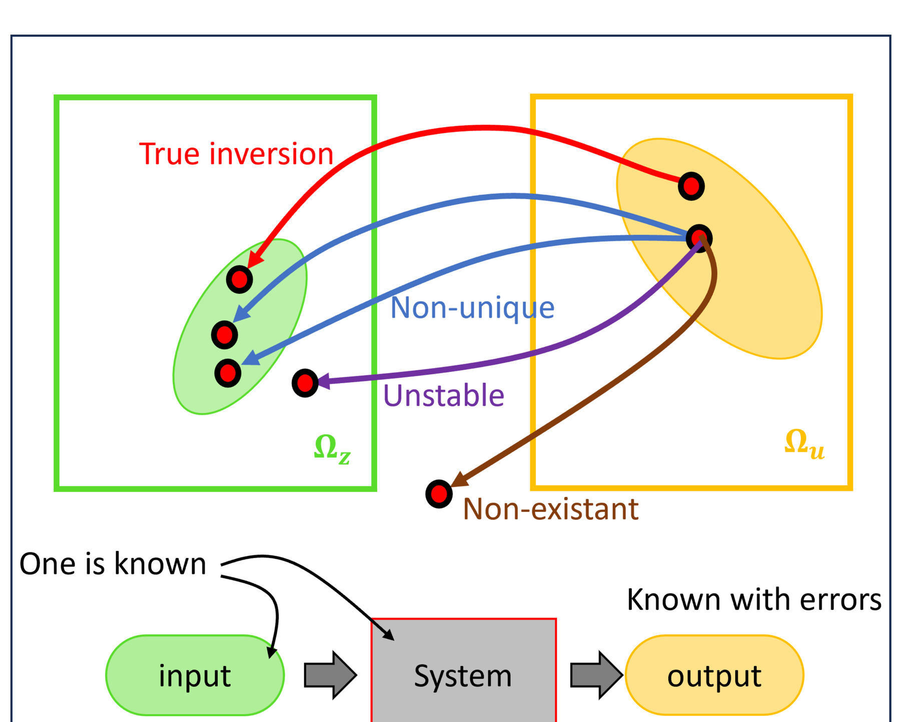


Figure 1: (Top) Diagram depicting the Digital Twin life cycle. (Bottom) Diagram depicting the ill-posedness of inverse problems.

HIGH AND LOW FIDELITY DIGITAL TWIN DISPERSION MODELS

To represent the dispersion of a trace atmospheric gas, we utilize both a high and low fidelity model. The low fidelity model is represented by the Gaussian plume model (GPM),

$$y_L(\mathbf{x}_m, t) = \mathcal{M}_L(\mathbf{x}_m, t, \theta_L) = \frac{Q}{\bar{u}} D_2 D_3, \quad (1)$$

where D_2 and D_3 are the lateral and vertical dispersion functions with respect to the mean wind direction, and $\theta_L = [Q, \mathbf{x}_s, \tau]$ included the source rate, location, and scale factors.
The MOABS/DT platform [1] is computed by implicitly solving the wind field $\mathbf{u}$, and stochastically solving the advection diffusion equation for y,

$$\partial_t \mathbf{u} = -\mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{K} \cdot \nabla^2 \mathbf{u}, \quad (2)$$

$$\partial_t y = -\mathbf{u} \cdot \nabla y + \nabla \cdot (D \nabla y) + Q\delta(\mathbf{x} - \mathbf{x}_s), \quad (3)$$

such that, the model can be expressed as, $y_L(\mathbf{x}_m, t) = \mathcal{M}_L(\mathbf{x}_m, t, \theta_H)$, where the parameters include the number of filament particles, source rate, location, etc.

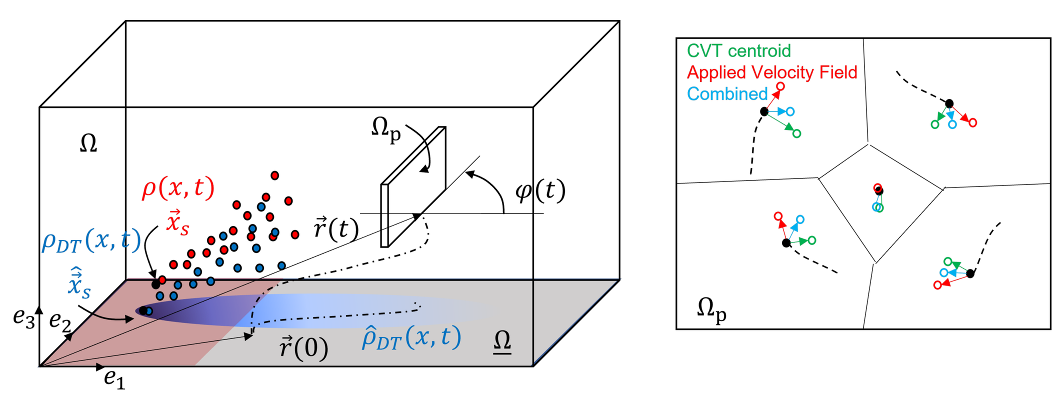


Figure 2: Diagram depicting the MOABS/DT model and projection for EMGR-based sensor placement.

SOURCE TERM ESTIMATION AND EMPIRICAL GRAMIANS

Source term estimation – To estimate the parameters of the plume, the modified near-field Gaussian plume inversion method (mod-NGI) is applied [2]. This method uses the GPM with a continuous centroidal Voronoi tessellations (CVT) coverage control approach, to extract the source rate, location within the plane, and the scale factors.

$$\mathcal{H}(\mathbf{z}, t) = \sum_{i=1}^{N} \int_{\mathcal{V}_i} \mathcal{M}(q, t) |\mathbf{z}_i - q|^2 dq, \quad \text{for } q \in \Omega_p, \quad (4)$$

where $\mathcal{M}(\cdot)$ is a concentration map defined by the mod-NGI algorithm in the plane, Ω_p. The partial derivative can be used with the mass, m_i, and center of mass, c_i, to identify the critical point, $\frac{\partial \mathcal{H}}{\partial \mathbf{z}_i} = 2m_i(\mathbf{z}_i - c_{m_i})^T$, of which, $c_i = \mathbf{z}_i$ for all $i = 1, 2, ..N$ is a minimizer of the CVT and thus Llyod's algorithm can be used to update their positions.
Empirical Observability Gramian – can be defined as [3, 4],

$$\hat{W}_O = \frac{1}{|S_x|} \sum_{l=1}^{|S_x|} \frac{1}{d_l^2} \int_0^\infty \Psi^l(t) dt, \quad (5)$$

where $\Psi^l = (y^{li}(t) - \bar{y}^{li})^T (y^{li}(t) - \bar{y}^{li})$. The initial state configuration is given as, $x_0^{li} = d_l \epsilon^i + \bar{x}$, $u(t) = \bar{u}$, and $\bar{y} = 1/T \int_0^T y(t)dt$. The term $S_x = \{d_l \in \mathbb{R} : l = 1...L, d_l \neq 0\}$.
The Gramian only requires the system's simulation ability for computation, without needing a closed-form analytical model. Consequently, it has become a significant tool for assessing the observability of nonlinear systems, especially in terms of acquiring quantitative metrics, exploring effective regions of observability [4], or source seeking [5].

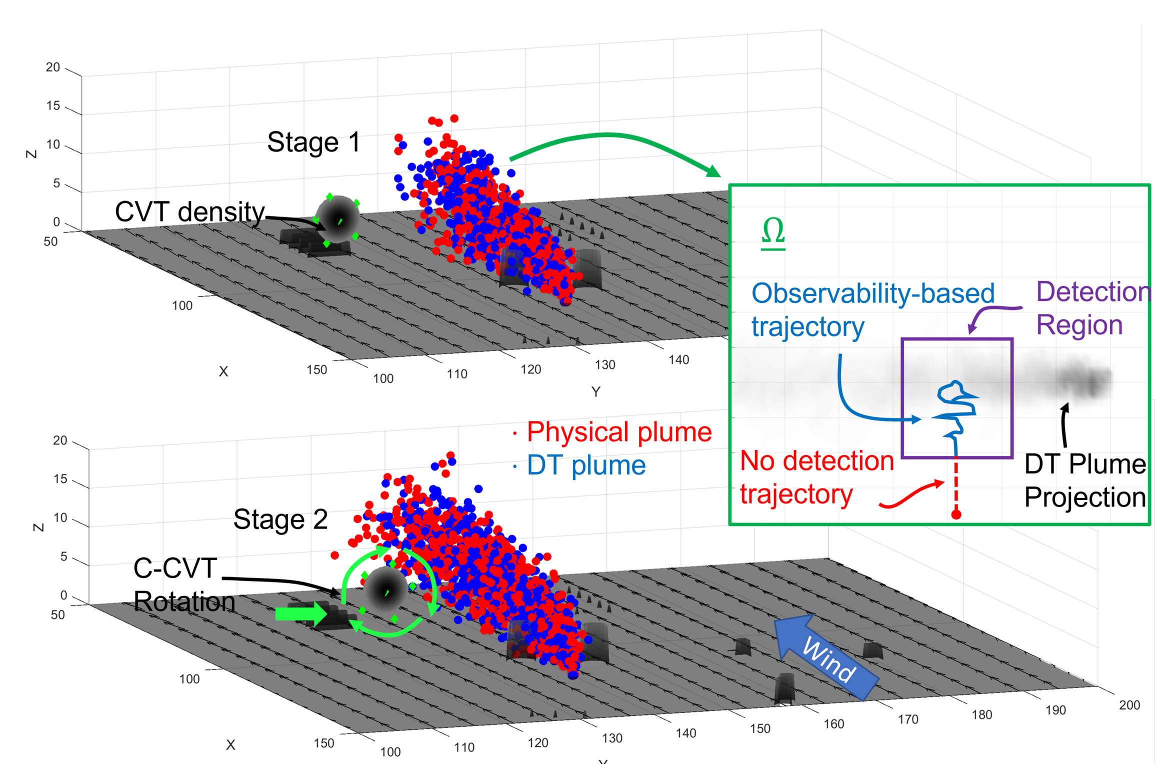


Figure 3: The MOABS/DT simulation is initialized by letting the physical and DT plumes develop past the measurement point, then initializes the continuous CVT and commands the Ω_p plane to cross the plume. Once a detection is made the sensor placement and steering is activated and begins using the DT projection onto Ω for the empirical observability Gramian computation.

SIMULATION RESULTS

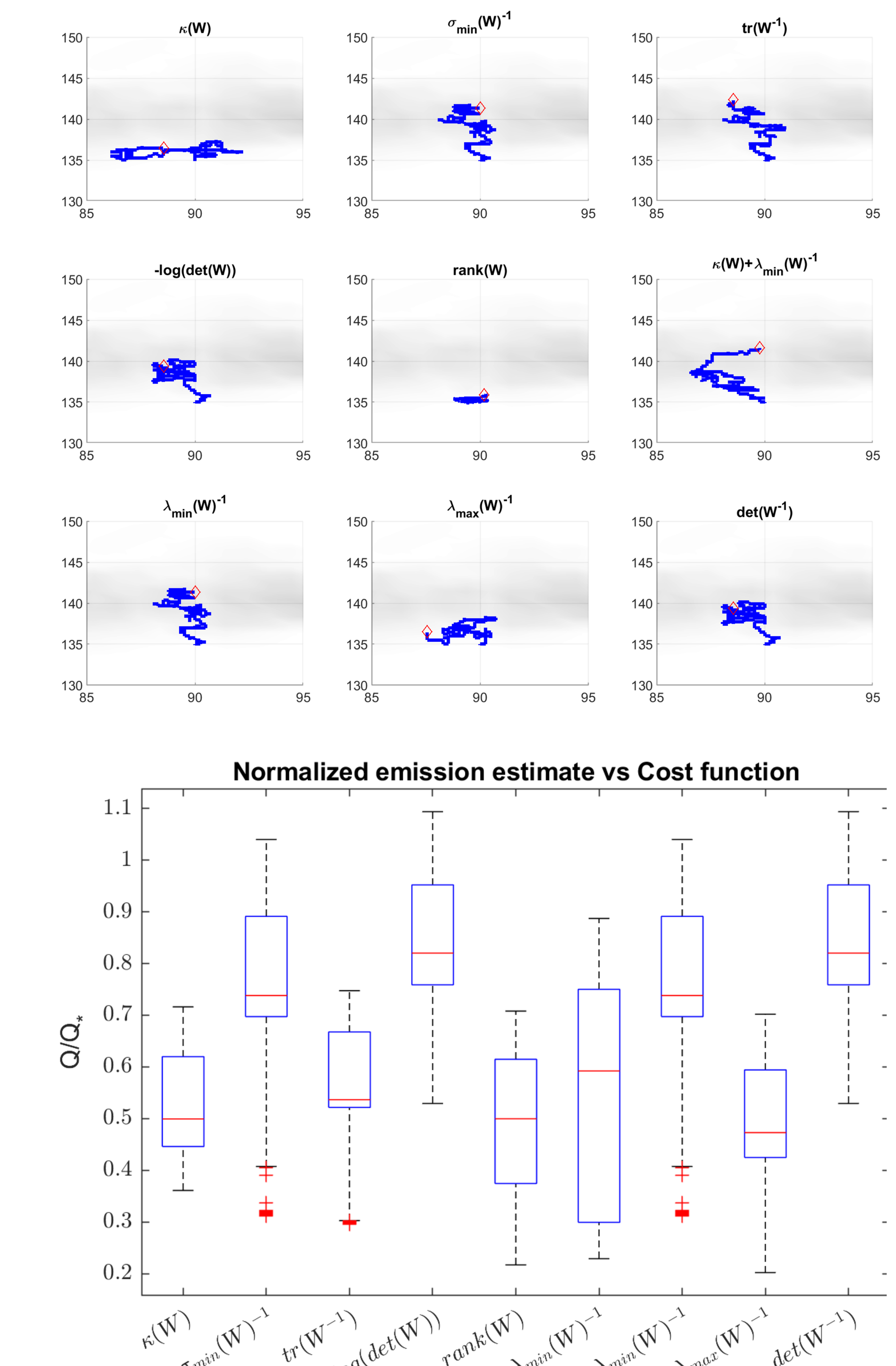


Figure 4: (Top) The trajectories of the plane control algorithm based on EMGR. (Bottom) The normalized emission rate quantification based on EMGR observability metric.

ON-GOING DEVELOPMENTS

1) Investigate optimal downwind distance for measurements.
2) Integrate method with sUAS hardware-in-the-loop
3) Develop real-time sensor placement optimization capabilities.

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