Parameter Estimation and Topology Identification of Uncertain General Fractional-order Complex Dynamical Networks with Time Delay

Xiaojuan Chen, Jun Zhang, and Tiedong Ma

***Abstract*—Complex networks have attracted much attention from various fields of sciences and engineering in recent years. However, many complex networks have various uncertain information, such as unknown or uncertain system parameters and topological structure, which greatly affects the system dynamics. Thus, the parameter estimation and structure identification problem has theoretical and practical importance for uncertain complex dynamical networks. This paper investigates identification of unknown system parameters and network topologies in uncertain fractional-order complex network with time delays (including coupling delay and node delay). Based on the stability theorem of fractional-order differential system and the adaptive control technique, a novel and general method is proposed to address this challenge. Finally two representative examples are given to verify the effectiveness of the proposed approach.**

***Index Terms*—Complex networks, fractional-order, parameter estimation, structure identification, time delay.**

I. INTRODUCTION

COMPLEX networks widely exist in the world, from Internet to World Wide Web, from communication networks to social network, etc.. All the above networks can be represented in terms of nodes and edges, where edges indicate connections between nodes. Due to the tremendous potentials in real applications, the research of complex networks has become a hot topic in modern scientific research[1−4]. In recent years, synchronization in complex network, as collective behavior, has received increasing attention and been extensively investigated due to its potential applications in many fields, including secure communization, image processing, neural networks, information science, etc.[5−9]. However, there exists much uncertain information in real-world complex networks[10−11], such as the unknown or uncertain topological structure and node dynamics, as it is often difficult to exactly know all system parameters beforehand in many practical applications. Moreover, the uncertainty would greatly affect the modeling, understanding and controlling of the complex networks. Therefore, the issue of network structure and parameter identification is of theoretical and practical importance for uncertain complex dynamical networks. However, due to the nonlinear, complex, and high dimensional nature of the practical complex networks, it is very difficult to exactly identify its topological structure and system parameters by using the traditional approaches. Recently, some researchers have made great effort to address this problem and some valuable results have been obtained[12−14]. Wu[12] proposed an adaptive feedback control method to identify the exact topology of weighted general complex dynamical networks with time delay. Zhou et al.[13] investigated the topology identification of weighted complex dynamical networks. Liu et al.[14] proposed a novel adaptive feedback control approach to simultaneously identify the unknown or uncertain network topological structure and system parameters of uncertain delayed general complex dynamical networks. It is noted that the mentioned references [12−14] mainly contribute to the control or identification of networks with nodes of conventional integer-order dynamics.

On the other hand, the study of complex network with fractional-order dynamic nodes also begins to attract increasing interest among the researchers. It is well known that the fractional calculus is a classical mathematical notion, and is a generalization of ordinary differentiation and integration to arbitrary order[15]. However, the fractional calculus did not attract much attention for a long time due to lack of application background. Nowadays, many known systems can be described by fractional-order systems, such as viscoelastic system, dielectric polarization, electromagnetic waves[16−18]. Compared with the classical integer-order models, fractional-order derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. Therefore, it may be more accurate to model by fractional-order derivatives than integer-order ones. It is demonstrated that many fractional-order differential systems behave chaotically or hyperchaotically, such as the fractional-order Chua circuit[19], the fractional-order Lorenz system[20], the fractional-order chaotic and hyperchaotic Rössler system[21], etc.. Following these findings, synchronization of chaotic fractional-order differential systems becomes a challenging and interesting problem due to the potential applications in secure communication and control processing.

Not surprisingly, a complex network with nodes modeled by fractional-order differential systems has currently been one of the most promising research topics. However, due to the limited theories for the coupled fractional-order dynamical

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systems, the synchronization between fractional-order complex networks is still a challenging research topic. Compared with integer-order complex networks, the fractional-order complex networks related studies are still few[22−29]. For example, Chai et al.[22] investigated synchronization of general fractional-order complex dynamical networks by adaptive pinning method. In [23−25], the authors discussed the cluster synchronization in fractional-order complex networks. Wu and Lu[26] investigated outer synchronization between two different fractional-order general complex networks. The above mentioned literatures concentrated on the research of fractional-order network with known system parameters and network structures. So far, there are very few studies on the parameter estimation and topology identification of uncertain fractional-order complex networks.

Time delay is ubiquitous in many physical systems due to the finite switching speed of amplifiers, the finite signal propagation time in biological networks, traffic congestions and so forth. Time delay in the interaction may influence the dynamical behavior of the system. Si et al.[27] has investigated the identification of fractional-order complex network with unknown system parameters and network topologies. Yang and Jiang[28] has discussed the drive-response fractional-order complex dynamical network with uncertainty. Unfortunately, time delay is ignored. Although Ma et al.[29] discussed parameter identification and synchronization problem of fractional-order neural networks with time delays, but only the case of state variables $x \in \mathbf{R}$ is discussed, and the case for state vector $x \in \mathbf{R}^n$ has not been investigated.

Motivated by the above discussion, in this paper, we will study the identification of unknown system parameters and network topologies in uncertain fractional-order complex network with coupling delay and node delay. The paper is organized as follows. In Section II, some fractional-order definitions and lemmas are given. Sections III and VI study the parameter estimation and topology identification method for delayed fractional-order complex networks with different nodes. In Section V, two representative examples are given to demonstrate the effectiveness of the proposed method. Finally, some concluding remarks are given in Section VI.

Throughout this paper, the following notations are used. $\|\cdot\|$ is the Euclidean norm of a vector. $A^{\rm T}$ means the transpose of the matrix A. I_n denotes the identity matrix with dimension n. $\otimes$ represents the Kronecker product of two matrices.

II. Preliminaries and Notations

A. The Definition of Fractional Calculus

The fractional-order integer-differential operator is the generalized concept of an integer-order integer-differential operator, which is denoted by a fundamental operator as follows:

$$ {}_aD_t^q = \begin{cases} \dfrac{\mathrm{d}^q}{\mathrm{d}t^q}, & R(q) > 0, \\ 1, & R(q) = 0, \\ \int_a^t (\mathrm{d}\tau)^{-q}, & R(q) < 0, \end{cases} \tag{1}$$

where q is the fractional-order calculus operator which can be a complex number, a and t are the limits of the operation. The commonly used definitions are Grunwald-Letnikov (GL), Riemann-Liouville (RL), and Caputo (C) definitions. In the rest of this paper, the notation $\dfrac{\mathrm{d}^q}{\mathrm{d}t^q}$ is chosen as the Caputo fractional derivation operator.

Definition 1. The Caputo fractional derivative is defined as follows

$$ D^q x(t) \doteq {}_a^cD_t^q x(t) = \begin{cases} \frac{1}{\Gamma(n-q)} \int_a^t (t-\tau)^{n-q-1} x^{(n)}(\tau)\mathrm{d}\tau, \\ \qquad\qquad\qquad\qquad n-1 < q < n, \\ \dfrac{\mathrm{d}^n}{\mathrm{d}t^n} x(t), \qquad\qquad q = n, \end{cases} \tag{2}$$

where $\Gamma(\cdot)$ is the Gamma function which is defined by $\Gamma(z) = \int_0^\infty \mathrm{e}^{-z} t^{z-1}\mathrm{d}t$.

It should be noted that the advantage of the Caputo approach is that the initial conditions for fractional differential equations with Caputo derivatives take on the same form as those for integer-order ones, which have well understood physical meaning. Therefore, in the rest of this paper, the notation $\dfrac{\mathrm{d}^q}{\mathrm{d}t^q}$ is chosen as the Caputo fractional derivation operator.

B. Mathematical Preliminaries

Consider uncertain dynamical systems

$$ D^q x_i(t) = \bar{f}_i(t, x_i(t), \alpha_i), \tag{3}$$

or rewrite systems (3) in the following form:

$$ D^q x_i(t) = f_i(t, x_i(t)) + F_i(t, x_i(t))\alpha_i, \tag{4}$$

where $x_i(t) \in \mathbf{R}^n$ are state vectors, $\alpha_i \in \mathbf{R}^{m_i}$ are unknown system parameter vectors for $i = 1, 2, \ldots, N$, in which m_i are positive integers. $f_i(t, x_i(t)) \in \mathbf{R}^n$ is a continuous vector function and $F_i(t, x_i(t)) \in \mathbf{R}^{n\times m_i}$ is a continuous matrix function.

Assumption 1 (A1). Suppose that there exist positive constants L_i such that

$$ \left\|\bar{f}_i(t, x(t), \alpha_i) - \bar{f}_i(t, y(t), \alpha_i)\right\| \le L_i \left\|x(t) - y(t)\right\|, \tag{5}$$

where $x(t), y(t) \in \mathbf{R}^n$ are time-varying vectors, and α_i is the parameter vector of function $\bar{f}_i(\cdot)$.

Assumption 2 (A2). Denote $F_i(t, x_i(t)) = (F_i^{(1)}(t, x_i(t)), F_i^{(2)}(t, x_i(t)), \ldots, F_i^{(m_i)}(t, x_i(t)))$. Suppose that $F_i^{(j)}(t, x_i(t)) \in \mathbf{R}^n$ for $j = 1, 2, \ldots, m_i$, and $\{\{F_i^{(j)}(t, x_i(t))\}_{j=1}^{m_i}, \{Ax_j(t-\tau)\}_{j=1}^N\}$ are linearly independent on the orbit $\{x_i(t), x_i(t-\tau)\}_{i=1}^N$ of synchronization manifold.

If time delay τ is considered, similar to (3) and (4), we can get the following delayed uncertain dynamical systems:

$$ D^q x_i(t) = \bar{g}_i(t, x_i(t), x_i(t-\tau), \beta_i), \quad i = 1, 2, \ldots, N, \tag{6}$$

or

$$ \begin{aligned} D^q x_i(t) &= \bar{g}_i(t, x_i(t), x_i(t-\tau), \beta_i) \\ &= g_i(t, x_i(t), x_i(t-\tau)) \\ &\quad + G_i(t, x_i(t), x_i(t-\tau))\beta_i, \end{aligned} \tag{7}$$

where $x_i(t)$, $x_i(t-\tau) \in \mathbf{R}^n$ are the state vectors, $\beta_i \in \mathbf{R}^{q_i}$ are the unknown parameter vector. $g_i(t, x_i(t), x_i(t-\tau)) \in \mathbf{R}^n$ is a continuous vector function and $G_i(t, x_i(t), x_i(t-\tau)) \in \mathbf{R}^{n\times q_i}$ is a continuous matrix function.

Assumption 3 (A3). Assume that there exists a nonnegative constant M satisfying

$$\begin{aligned}&\|\bar{g}_i(t,x(t),x(t-\tau),\beta_i)-\bar{g}_i(t,y(t),y(t-\tau),\beta_i)\|\\&\quad\le\sqrt{M}\left(\|x(t)-y(t)\|^2+\|x(t-\tau)-y(t-\tau)\|^2\right)^{\frac{1}{2}}.\end{aligned}\tag{8}$$

Assumption 4 (A4). Denote $G_i(t,x_i(t),x_i(t-\tau)) = (G_i^{(1)}(t,x_i(t),x_i(t-\tau)), G_i^{(2)}(t,x_i(t),x_i(t-\tau)),\cdots,G_i^{(q_i)}(t, x_i(t),x_i(t-\tau)))$. Assume that $G_i^{(j)}(t,x_i(t),x_i(t-\tau))\in\mathbf{R}^n$ for $j=1,2,\cdots,q_i$, and $\{\{G_i^{(j)}(t,x_i(t),x_i(t-\tau))\}_{j=1}^{q_i}, \{Ax_j(t)\}_{j=1}^{N}\}$ are linearly independent on the orbit $\{x_i(t), x_i(t-\tau)\}_{j=1}^{N}$ of synchronization manifold.

Lemma 1[30]**.** Consider a delayed fractional order system:

$$D^q x(t)=f(x(t),x(t-\tau)),\tag{9}$$

where fractional order $0<q\le 1$, $x(t)=(x_1,x_2,\dots,x_n)^{\mathrm T}\in\mathbf{R}^n$ is the state vector. $f(x(t),x(t-\tau))=(f_1(x(t),x(t-\tau_1)),f_2(x(t),x(t-\tau_2)),\dots,f_n(x(t),x(t-\tau_n)))^{\mathrm T}$ is nonlinear vector function satisfying Lipschitz condition and the delay time $\tau=(\tau_1,\tau_2,\dots,\tau_n)^{\mathrm T}\in\mathbf{R}^n$. If there exist a positive definite matrix P and a semi positive definite matrix Q such that

$$x^{\mathrm T}(t)PD^q x(t)+x^{\mathrm T}(t)Qx(t)-x^{\mathrm T}(t-\tau)Qx(t-\tau)\le 0,\tag{10}$$

then the delayed fractional system (9) is Lyapunov stable.

Lemma 2[26]**.** For any vector $x,\ y\in\mathbf{R}^n$, the inequality $2x^{\mathrm T}y\le x^{\mathrm T}x+y^{\mathrm T}y$ holds.

III. Structure Identification of Uncertain General Fractional-Order Complex Dynamical Networks with Coupling Delay

Consider a complex dynamical network with time-varying coupling delay and N different nodes, which is described by

$$D^q x_i(t)=\bar{f}_i(t,x_i(t),\alpha_i)+\sum_{j=1}^{N}c_{ij}Ax_j(t-\tau),\tag{11}$$

or it can be rewritten in the following form:

$$\begin{aligned}&D^q x_i(t)=\\&\quad f_i(t,x_i(t))+F_i(t,x_i(t))\alpha_i+\sum_{j=1}^{N}c_{ij}Ax_j(t-\tau),\end{aligned}\tag{12}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\dots,x_{in}(t))^{\mathrm T}\in\mathbf{R}^n$ is the state vector of the ith node, $i=1,\dots,N$, τ is the constant time delay. $C=(c_{ij})_{N\times N}\in\mathbf{R}^{N\times N}$ is an unknown or uncertain coupling configuration matrix, and c_{ij} is the weight or coupling strength. If there exists a link from nodes i to j ($j\ne i$), then $c_{ij}\ne 0$, otherwise, $c_{ij}=0$. $A\in\mathbf{R}^{n\times n}$ is an inner-coupling matrix which determines the interaction variables.

Hereafter, the coupling configuration matrix C need not be symmetric, irreducible, or diffusive. Of course, it is necessary to ensure the boundedness of complex dynamical networks in this paper. The main goal is to identify these unknown or uncertain coupling strengths, namely the network topological structure, and all unknown system parameter vectors α_i of the complex dynamical networks.

Consider another complex dynamical network which will be referred to as the response network with coupling delay as follows:

$$\begin{aligned}D^q\hat{x}_i(t)=&\,f_i(t,\hat{x}_i(t))+F_i(t,\hat{x}_i(t))\hat{\alpha}_i\\&+\sum_{j=1}^{N}\hat{c}_{ij}A\hat{x}_j(t-\tau)+u_i,\end{aligned}\tag{13}$$

where $\hat{x}_i(t)=(\hat{x}_{i1}(t),\hat{x}_{i2}(t),\dots,\hat{x}_{in}(t))^{\mathrm T}\in\mathbf{R}^n$ is the response state vector of the i-th node, $u_i\in\mathbf{R}^n$ is its controller, $\hat{c}_{ij}$ is the estimated value of weight c_{ij}, and vector $\hat{\alpha}_i$ is the estimated value of the unknown parameter vector α_i.

Denote $\tilde{x}_i=\hat{x}_i-x_i$, $\tilde{c}_{ij}=\hat{c}_{ij}-c_{ij}$, $\tilde{\alpha}_i=\hat{\alpha}_i-\alpha_i$. The systems (12) and (13) achieve synchronization if $\tilde{x}_i\to 0$ as $t\to\infty$. Then the error system is given by

$$\begin{aligned}\tilde{x}_i(t)=&\,f_i(t,\hat{x}_i(t))+F_i(t,\hat{x}_i(t))\hat{\alpha}_i-f_i(t,x_i(t))\\&-F_i(t,x_i(t))\alpha_i+\sum_{j=1}^{N}\hat{c}_{ij}A\hat{x}_j(t-\tau)\\&-\sum_{j=1}^{N}c_{ij}Ax_j(t-\tau)+u_i.\end{aligned}\tag{14}$$

That is,

$$\begin{aligned}D^q\tilde{x}_i(t)=&\,\bar{f}_i(t,\hat{x}_i(t),\alpha_i)-\bar{f}_i(t,x_i(t),\alpha_i)\\&+F_i(t,\hat{x}_i(t))\tilde{\alpha}_i+\sum_{j=1}^{N}\tilde{c}_{ij}A\hat{x}_j(t-\tau)\\&-\sum_{j=1}^{N}c_{ij}A\tilde{x}_j(t-\tau)+u_i.\end{aligned}\tag{15}$$

Theorem 1. Suppose that Assumptions A1 and A2 hold. Then the uncertain coupling configuration matrix C and parameter vectors α_i of uncertain general delayed complex dynamical network (12) can be identified by the estimated values $\hat{C}$ and $\hat{\alpha}_i$ via the response network

$$\begin{cases}D^q\hat{x}_i=f_i(t,\hat{x}_i(t))+F_i(t,\hat{x}_i(t))\hat{\alpha}_i\\\qquad\quad+\sum\limits_{j=1}^{N}\hat{c}_{ij}A\hat{x}_j(t-\tau)+u_i,\\u_i=-k_i\tilde{x}_i(t),\\D^q k_i=d_i\|\tilde{x}_i\|^2,\\D^q\hat{\alpha}_i=-F_i^{\mathrm T}(t,\hat{x}_i(t))\tilde{x}_i(t),\\D^q\hat{c}_{ij}=-\delta_{ij}\tilde{x}_i(t)^{\mathrm T}A\hat{x}_j(t-\tau),\end{cases}\tag{16}$$

where $i,\ j\in\{1,2,\dots,N\}$ and d_i, δ_{ij} are any positive constants.

Proof. Denote $\tilde{k}_i=k_i-k_i^*$, and k_i^* is a positive constant. Further denote $X=(\tilde{X}^{\mathrm T},\tilde{\alpha}^{\mathrm T},\tilde{c}^{\mathrm T},\tilde{k}^{\mathrm T})^{\mathrm T}$, where

$$\begin{cases}\tilde{X}=\left(\tilde{x}_1^{\mathrm T},\tilde{x}_2^{\mathrm T},\dots,\tilde{x}_N^{\mathrm T}\right)^{\mathrm T},\quad\tilde{x}_i=(\tilde{x}_{i1},\tilde{x}_{i2},\dots,\tilde{x}_{in})^{\mathrm T},\\\tilde{\alpha}=\left(\tilde{\alpha}_1^{\mathrm T},\tilde{\alpha}_2^{\mathrm T},\dots,\tilde{\alpha}_N^{\mathrm T}\right)^{\mathrm T},\quad\tilde{\alpha}_i=(\tilde{\alpha}_{i1},\tilde{\alpha}_{i2},\dots,\tilde{\alpha}_{im_i})^{\mathrm T},\\\tilde{c}=\left(\tilde{c}_1^{\mathrm T},\tilde{c}_2^{\mathrm T},\dots,\tilde{c}_N^{\mathrm T}\right)^{\mathrm T},\qquad\tilde{c}_i=(\tilde{c}_{i1},\tilde{c}_{i2},\dots,\tilde{c}_{iN})^{\mathrm T},\\\tilde{k}=\left(\tilde{k}_1,\tilde{k}_2,\dots,\tilde{k}_N\right)^{\mathrm T}.\end{cases}\tag{17}$$

Choose the real symmetric positive definite matrix P as

$$P=\operatorname{diag}\left\{\underbrace{1,\ldots,1}_{nN+\sum_{i=1}^{N}m_i},\frac{1}{\delta_{11}},\ldots,\frac{1}{\delta_{NN}},\frac{1}{d_1},\ldots,\frac{1}{d_N}\right\},\quad(18)$$

$$Q=\operatorname{diag}\left\{\underbrace{1,\ldots,1}_{nN},\ \underbrace{0,\ldots,0}_{\sum_{i=1}^{N}m_i+N^2+N}\right\}.\quad(19)$$

Then we have

$$\begin{aligned}J=&\,X^{\mathrm T}(t)PD^qX(t)+X^{\mathrm T}(t)QX(t)\\&-X^{\mathrm T}(t-\tau)QX(t-\tau)\\=&\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t)D^q\tilde x_i(t)+\sum_{i=1}^{N}\tilde\alpha_i^{\mathrm T}D^q\tilde\alpha_i\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\delta_{ij}}\tilde c_{ij}D^q\tilde c_{ij}+\sum_{i=1}^{N}\frac{1}{d_i}\tilde k_iD^q\tilde k_i\\&+\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t)\tilde x_i(t)-\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t-\tau)\tilde x_i(t-\tau)\\=&\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t)\left\{\bar f_i(t,\hat x_i(t),\alpha_i)-\bar f_i(t,x_i(t),\alpha_i)\right\}\\&+\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t)F_i(t,\hat x_i(t))\tilde\alpha_i\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}\tilde x_i^{\mathrm T}(t)A\tilde x_j(t-\tau)-\sum_{i=1}^{N}k_i\|\tilde x_i(t)\|^2\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde c_{ij}\tilde x_i^{\mathrm T}(t)A\hat x_j(t-\tau)\\&+\sum_{i=1}^{N}\tilde\alpha_i^{\mathrm T}D^q\tilde\alpha_i+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\delta_{ij}}\tilde c_{ij}D^q\tilde c_{ij}\\&+\sum_{i=1}^{N}(k_i-k^*)\|\tilde x_i(t)\|^2\\&+\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t)\tilde x_i(t)-\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t-\tau)\tilde x_i(t-\tau)\\\le&\sum_{i=1}^{N}L_i\tilde x_i^{\mathrm T}(t)\tilde x_i(t)\\&+\sum_{i=1}^{N}\tilde x_i(t)F_i(t,\hat x_i(t))\tilde\alpha_i+\sum_{i=1}^{N}\tilde\alpha_i^{\mathrm T}D^q\tilde\alpha_i\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde c_{ij}\tilde x_i^{\mathrm T}(t)A\hat x_j(t-\tau)\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\delta_{ij}}\tilde c_{ij}D^q\tilde c_{ij}\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}\tilde x_i^{\mathrm T}(t)A\tilde x_j(t-\tau)-\sum_{i=1}^{N}k^*\|\tilde x_i(t)\|^2\\&+\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t)\tilde x_i(t)-\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t-\tau)\tilde x_i(t-\tau)\\\le&\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t)\tilde x_i(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}\tilde x_i(t)A\tilde x_j(t-\tau)\\&-\sum_{i=1}^{N}k^*\|\tilde x_i(t)\|^2+\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t)\tilde x_i(t)\\&-\sum_{i=1}^{N}\tilde x_i^{\mathrm T}(t-\tau)\tilde x_i(t-\tau),\\\le&\,L\tilde X^{\mathrm T}(t)\tilde X(t)+\tilde X^{\mathrm T}(t)(C\otimes A)\tilde X(t-\tau)\\&-k^*\tilde X^{\mathrm T}(t)\tilde X(t)+\tilde X^{\mathrm T}(t)\tilde X(t)\\&-\tilde X^{\mathrm T}(t-\tau)\tilde X(t-\tau)\\\le&\,L\tilde X^{\mathrm T}(t)\tilde X(t)+\frac12\tilde X^{\mathrm T}(t)(CC^{\mathrm T}\otimes AA^{\mathrm T})\tilde X(t)\\&+\frac12\tilde X^{\mathrm T}(t-\tau)\tilde X(t-\tau)-k^*\tilde X^{\mathrm T}(t)\tilde X(t)\\&+\tilde X^{\mathrm T}(t)\tilde X(t)-\tilde X^{\mathrm T}(t-\tau)\tilde X(t-\tau)\\=&\left(L-k^*+1+\frac12\lambda_{\max}(CC^{\mathrm T}\otimes AA^{\mathrm T})\right)\tilde X^{\mathrm T}(t)\tilde X(t)\\&-\frac12\tilde X^{\mathrm T}(t-\tau)\tilde X(t-\tau),\end{aligned}\quad(20)$$

where $L=\max\{L_i|1\le i\le N.\}$. Lemma 2 is used in the last inequality of (20). It is obvious that there exists sufficiently large positive constant k^* such that J is negative definite. Namely, $X^{\mathrm T}(t)PD^qX(t)+X^{\mathrm T}(t)QX(t)-X^{\mathrm T}(t-\tau)QX(t-\tau)\le 0$ holds, which implies the Lyapunov stability of error system (14) or (15) by Lemma 1. □

Remark 1. It should be especially pointed out that the coupling configuration matrix C need not be symmetric, irreducible, even diffusive.

Remark 2. The positive constants δ_{ij}, d_i in the updating laws D^qk_i and $D^q\hat c_{ij}$ can control the convergence speed of the synchronization and identification.

Remark 3. Assumption A2 is a very essential condition for guaranteeing the success of identification. Without this condition, it may cause false identification result. Similarly, Assumption A4 guarantees the identification of the next section.

IV. Structure Identification of an Uncertain General Complex Dynamical Network with Node Delay

Consider an uncertain general complex dynamical network consisting of N different nodes with time delay τ, called the drive network, which is described by

$$D^qx_i(t)=\bar g_i(t,x_i(t),x_i(t-\tau),\beta_i)+\sum_{j=1}^{N}c_{ij}Ax_j(t),\quad(21)$$

where the node dynamics can be rewritten as follows

$$\begin{aligned}&\bar{g}_i\left(t,x_i(t),x_i\left(t-\tau\right),\beta_i\right)\\&\quad=g_i\left(t,x_i\left(t\right),x_i\left(t-\tau\right)\right)+G_i\left(t,x_i\left(t\right),x_i\left(t-\tau\right)\right)\beta_i,\end{aligned}\tag{22}$$

and β_i $(i=1,2,\dots,N)$ are unknown or uncertain system parameter vectors.

Construct another controlled general fractional-order complex network, called response network, which is given by

$$D^q\hat{x}_i=\bar{g}_i\left(t,\hat{x}_i(t),\hat{x}_i\left(t-\tau\right),\hat{\beta}_i\right)+\sum_{j=1}^{N}\hat{c}_{ij}A\hat{x}_j(t)+u_i,\tag{23}$$

where $\hat{x}_i(t)=(\hat{x}_{i1}(t),\hat{x}_{i2}(t),\dots,\hat{x}_{in}(t))^{\rm T}\in\mathbf{R}^n$ is the response state vector of the i-th node, $u_i\in\mathbf{R}^n$ is its control input, $\hat{c}_{ij}$ and $\hat{\beta}_i$ are the estimated values of c_{ij} and β_i, respectively. Denote $\tilde{x}_i=\hat{x}_i-x_i$, $\tilde{c}_{ij}=\hat{c}_{ij}-c_{ij}$, $\tilde{\beta}_i=\hat{\beta}_i-\beta_i$. Thus the error system is described by

$$\begin{aligned}D^q\tilde{x}_i\left(t\right)=&\ \bar{g}_i\left(t,\hat{x}_i\left(t\right),\hat{x}_i\left(t-\tau\right),\beta_i\right)\\&-\bar{g}_i\left(t,x_i\left(t\right),x_i\left(t-\tau\right),\beta_i\right)\\&+G_i\left(t,\hat{x}_i\left(t\right),\hat{x}_i\left(t-\tau\right)\right)\tilde{\beta}_i\\&+\sum_{j=1}^{N}\tilde{c}_{ij}A\hat{x}_j\left(t\right)+\sum_{j=1}^{N}c_{ij}A\tilde{x}_j\left(t\right)+u_i.\end{aligned}\tag{24}$$

Theorem 2. Suppose that Assumptions A3 and A4 hold. Then uncertain coupling configuration matrix C and system parameter vectors β_i can be identified by using the estimated values $\hat{C}$ and $\hat{\beta}_i$ via the response network

$$\begin{cases}D^q\hat{x}_i\left(t\right)=g_i\left(t,\hat{x}_i\left(t\right),\hat{x}_i\left(t-\tau\right)\right)\\\qquad+\ G_i\left(t,\hat{x}_i\left(t\right),\hat{x}_i\left(t-\tau\right)\right)\hat{\beta}_i+\sum\limits_{j=1}^{N}\hat{c}_{ij}A\hat{x}_j\left(t\right)+u_i,\\u_i=-k_i\tilde{x}_i\left(t\right),\\D^qk_i=d_i\left\|\tilde{x}_i\left(t\right)\right\|^2,\\D^q\hat{\beta}_i=-G_i^{\rm T}\left(t,\hat{x}_i\left(t\right),\hat{x}_i\left(t-\tau\right)\right)\tilde{x}_i\left(t\right),\\D^q\hat{c}_{ij}=-\delta_{ij}\tilde{x}_i^{\rm T}\left(t\right)A\hat{x}_j\left(t\right),\end{cases}\tag{25}$$

where i, $j\in\{1,2,\dots,N\}$, d_i, δ_{ij} are any positive constants.

Proof. Denote $\tilde{k}_i=k_i-k_i^*$, k_i^* is a positive constant. Further denote $X=(\tilde{X}^{\rm T},\tilde{\beta}^{\rm T},\tilde{c}^{\rm T},\tilde{k}^{\rm T})^{\rm T}$, where

$$\begin{cases}\tilde{X}=\left(\tilde{x}_1^{\rm T},\tilde{x}_2^{\rm T},\dots,\tilde{x}_N^{\rm T}\right)^{\rm T}, & \tilde{x}_i=\left(\tilde{x}_{i1},\tilde{x}_{i2},\dots,\tilde{x}_{in}\right)^{\rm T},\\\tilde{\beta}=\left(\tilde{\beta}_1^{\rm T},\tilde{\beta}_2^{\rm T},\dots,\tilde{\beta}_N^{\rm T}\right)^{\rm T}, & \tilde{\beta}_i=\left(\tilde{\beta}_{i1},\tilde{\beta}_{i2},\dots,\tilde{\beta}_{im_i}\right)^{\rm T},\\\tilde{c}=\left(\tilde{c}_1^{\rm T},\tilde{c}_2^{\rm T},\dots,\tilde{c}_N^{\rm T}\right)^{\rm T}, & \tilde{c}_i=\left(\tilde{c}_{i1},\tilde{c}_{i2},\dots,\tilde{c}_{iN}\right)^{\rm T},\\\tilde{k}=\left(\tilde{k}_1,\tilde{k}_2,\dots,\tilde{k}_N\right)^{\rm T}.\end{cases}\tag{26}$$

Choose the real symmetric positive definite matrix P as

$$P=\mathrm{diag}\left(\underbrace{1,\dots,1}_{nN+\sum_{i=1}^{N}m_i},\frac{1}{\delta_{11}},\dots,\frac{1}{\delta_{NN}},\frac{1}{d_1},\dots,\frac{1}{d_N}\right),\tag{27}$$

$$Q=\mathrm{diag}\left(\underbrace{\frac{M}{2},\dots,\frac{M}{2}}_{nN},\ \underbrace{0,\dots,0}_{\sum_{i=1}^{N}m_i+N^2+N}\right).\tag{28}$$

Then, we have

$$\begin{aligned}J=&\ X^{\rm T}(t)PD^qX(t)+X^{\rm T}\left(t\right)QX\left(t\right)\\&-X^{\rm T}\left(t-\tau\right)QX\left(t-\tau\right)\\=&\sum_{i=1}^{N}\tilde{x}_i^{\rm T}\left(t\right)D^q\tilde{x}_i\left(t\right)+\sum_{i=1}^{N}\tilde{\beta}_i^{\rm T}D^q\tilde{\beta}_i\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\delta_{ij}}\tilde{c}_{ij}D^q\tilde{c}_{ij}+\sum_{i=1}^{N}\frac{1}{d_i}\tilde{k}_iD^qk_i\\&+\sum_{i=1}^{N}\frac{M}{2}\tilde{x}_i^{\rm T}\left(t\right)\tilde{x}_i\left(t\right)-\sum_{i=1}^{N}\frac{M}{2}\tilde{x}_i^{\rm T}\left(t-\tau\right)\tilde{x}_i\left(t-\tau\right)\\=&\sum_{i=1}^{N}\tilde{x}_i^{\rm T}\left(t\right)\left(\bar{g}_i\left(t,\hat{x}_i\left(t\right),\hat{x}_i\left(t-\tau\right),\beta_i\right)\right.\\&-\bar{g}_i\left(t,x_i\left(t\right),x_i\left(t-\tau\right),\beta_i\right)\\&+G_i\left(t,\hat{x}_i\left(t\right),\hat{x}_i\left(t-\tau\right)\right)\tilde{\beta}_i\\&\left.+\sum_{j=1}^{N}\tilde{c}_{ij}A\hat{x}_j\left(t\right)+\sum_{j=1}^{N}c_{ij}A\tilde{x}_j\left(t\right)-k_i\tilde{x}_i\left(t\right)\right)\\&-\sum_{i=1}^{N}\tilde{\beta}_i^{\rm T}G_i^{\rm T}\left(t,\hat{x}_i\left(t\right),\hat{x}_i\left(t-\tau\right)\right)\tilde{x}_i\left(t\right)\\&-\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}\tilde{x}_i^{\rm T}\left(t\right)A\hat{x}_j\left(t\right)+\sum_{i=1}^{N}\left(k_i-k^*\right)\left\|\tilde{x}_i^{\rm T}\left(t\right)\right\|^2\\&+\sum_{i=1}^{N}\frac{M}{2}\tilde{x}_i^{\rm T}\left(t\right)\tilde{x}_i\left(t\right)-\sum_{i=1}^{N}\frac{M}{2}\tilde{x}_i^{\rm T}\left(t-\tau\right)\tilde{x}_i\left(t-\tau\right)\\=&\sum_{i=1}^{N}\tilde{x}_i^{\rm T}(t)(\bar{g}_i(t,\hat{x}_i(t),\hat{x}_i(t-\tau),\beta_i)\\&-\bar{g}_i(t,x_i(t),x_i(t-\tau),\beta_i))\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{x}_i^{\rm T}\left(t\right)c_{ij}A\tilde{x}_j\left(t\right)-k^*\sum_{j=1}^{N}\left\|\tilde{x}_i\left(t\right)\right\|^2\\&+\sum_{i=1}^{N}\frac{M}{2}\tilde{x}_i^{\rm T}\left(t\right)\tilde{x}_i\left(t\right)-\sum_{i=1}^{N}\frac{M}{2}\tilde{x}_i^{\rm T}\left(t-\tau\right)\tilde{x}_i\left(t-\tau\right)\\\le&\ \frac{1}{2}\sum_{i=1}^{N}\tilde{x}_i^{\rm T}(t)\tilde{x}_i(t)+\frac{1}{2}\|\bar{g}_i(t,\hat{x}_i(t),\hat{x}_i(t-\tau),\beta_i)\\&-\bar{g}_i(t,x_i(t),x_i(t-\tau),\beta_i)\|^2\\&+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{x}_i^{\rm T}\left(t\right)c_{ij}A\tilde{x}_j\left(t\right)-k^*\sum_{j=1}^{N}\left\|\tilde{x}_i\left(t\right)\right\|^2\\&+\sum_{i=1}^{N}\frac{M}{2}\tilde{x}_i^{\rm T}\left(t\right)\tilde{x}_i\left(t\right)-\sum_{i=1}^{N}\frac{M}{2}\tilde{x}_i^{\rm T}\left(t-\tau\right)\tilde{x}_i\left(t-\tau\right)\\\le&\ \frac{1}{2}\sum_{i=1}^{N}\tilde{x}_i^{\rm T}\left(t\right)\tilde{x}_i\left(t\right)+\frac{M}{2}\left(\left\|\tilde{x}_i\left(t\right)\right\|^2+\left\|\tilde{x}_i\left(t-\tau\right)\right\|^2\right)\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{x}_i^{\mathrm{T}}(t)\,c_{ij}A\tilde{x}_j(t)-k^*\sum_{j=1}^{N}\|\tilde{x}_i(t)\|^2\\
&+\sum_{i=1}^{N}\frac{M}{2}\tilde{x}_i^{\mathrm{T}}(t)\,\tilde{x}_i(t)-\sum_{i=1}^{N}\frac{M}{2}\tilde{x}_i^{\mathrm{T}}(t-\tau)\,\tilde{x}_i(t-\tau)\\
=&\left(\frac{1}{2}+M-k^*\right)\tilde{X}^{\mathrm{T}}(t)\,\tilde{X}(t)+\tilde{X}^{\mathrm{T}}(t)\,(C\otimes A)\,\tilde{X}(t)\\
\le&\left(\frac{1}{2}+M-k^*+\lambda_{\max}(C\otimes A)\right)\tilde{X}^{\mathrm{T}}(t)\tilde{X}(t). \qquad (29)
\end{aligned}
$$

It is obvious that there exists sufficiently large positive constant k^* such that J is negative definite. Namely, $X^{\mathrm{T}}(t)P \times D^qX(t)+X^{\mathrm{T}}(t)QX(t)-X^{\mathrm{T}}(t-\tau)QX(t-\tau)\le 0$ holds, which implies the Lyapunov stability of error system (24) by Lemma 1. □

V. Numerical simulations

In this section, two representative examples are given to verify the effectiveness of the proposed parameters estimation and structure identification approaches.

A. Identification with Coupling Time Delay

The well-known Lü system with fractional order derivative is used as the node dynamics in the uncertain network, which is described as

$$D^qx_i(t)=f_i\left(t,x_i\left(t\right)\right)+F_i\left(t,x_i(t)\right)\alpha_i, \qquad (30)$$

where $q=0.9$, $x_i(t)=(x_{i1}(t),x_{i2}(t),x_{i3}(t))^{\mathrm{T}}$ is state vector, $f_i(t,x_i(t)) = (0,-x_{i1}(t)x_{i3}(t),x_{i1}(t)x_{i2}(t))^{\mathrm{T}}$, $F_i(t,x_i(t)) = \mathrm{diag}\{x_{i2}(t)-x_{i1}(t),x_{i2}(t),-x_{i3}(t)\}$, and $\alpha_i=(\alpha_{i1},\alpha_{i2},\alpha_{i3})^{\mathrm{T}}$, $i=1,\ldots,4$. Fig. 1 shows the chaotic attractor of fractional-order Lü system.

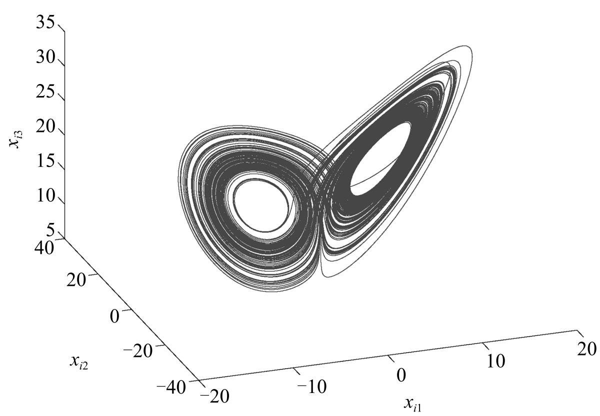


Fig. 1. Chaotic attractor of fractional-order Lü system.

The weight configuration matrix is set as

$$C=\begin{pmatrix}-5 & 1 & 4 & 0\\ 3 & -4 & 1 & 0\\ 0 & 1 & -3 & 2\\ 1 & 3 & 0 & -4\end{pmatrix}. \qquad (31)$$

Let $\alpha_i=(\alpha_{i1},\alpha_{i2},\alpha_{i3})^{\mathrm{T}}=(36,\ 20+i,\ 3)^{\mathrm{T}}$ for $i=1,\ldots,4$. $\tau=0.2$, and networks inner-coupling matrix $A=\mathrm{diag}\{1,\ 1,\ 1\}$.

According to Theorem 1, the coupling configuration matrix C and system parameter vectors α_i of complex networks (12) can be identified by using adaptive control laws (16). Fig. 2 shows the identification of the uncertain system parameters, while Fig. 3 illustrates the identification of the unknown network topology.

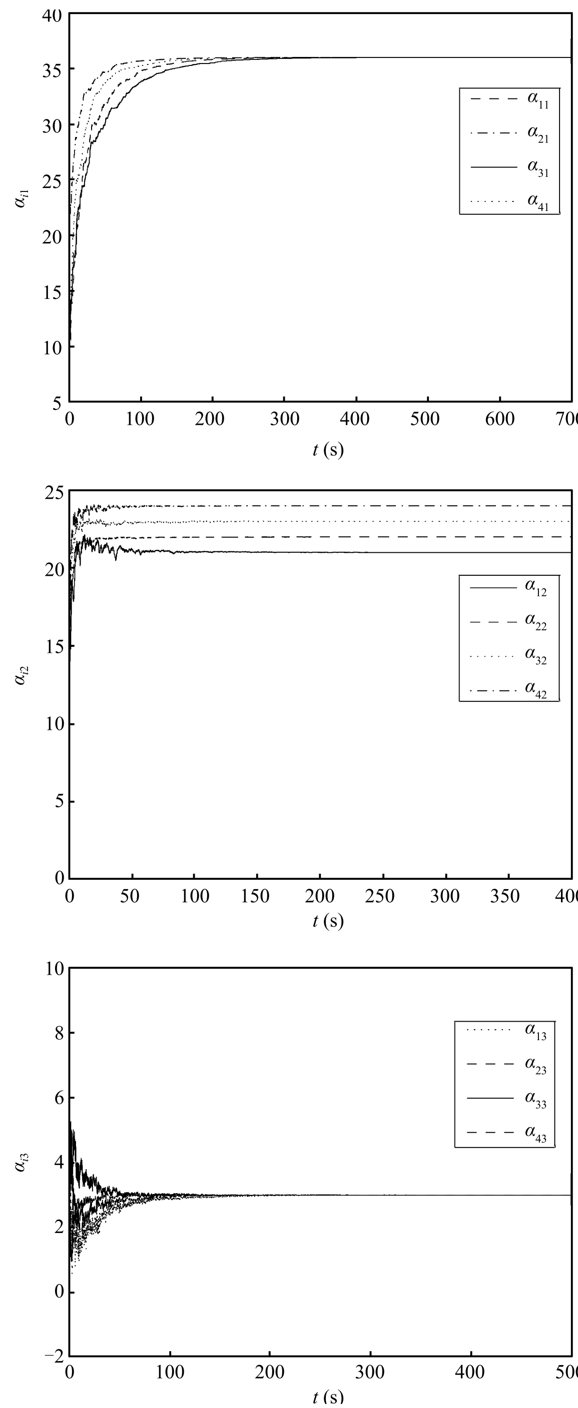


Fig. 2. Identification of uncertain parameters.

B. Identification with Node Time Delay

In this subsection, we consider the uncertain network (21) with four nonidentical delayed Lü systems, and the single delayed Lü system is described as

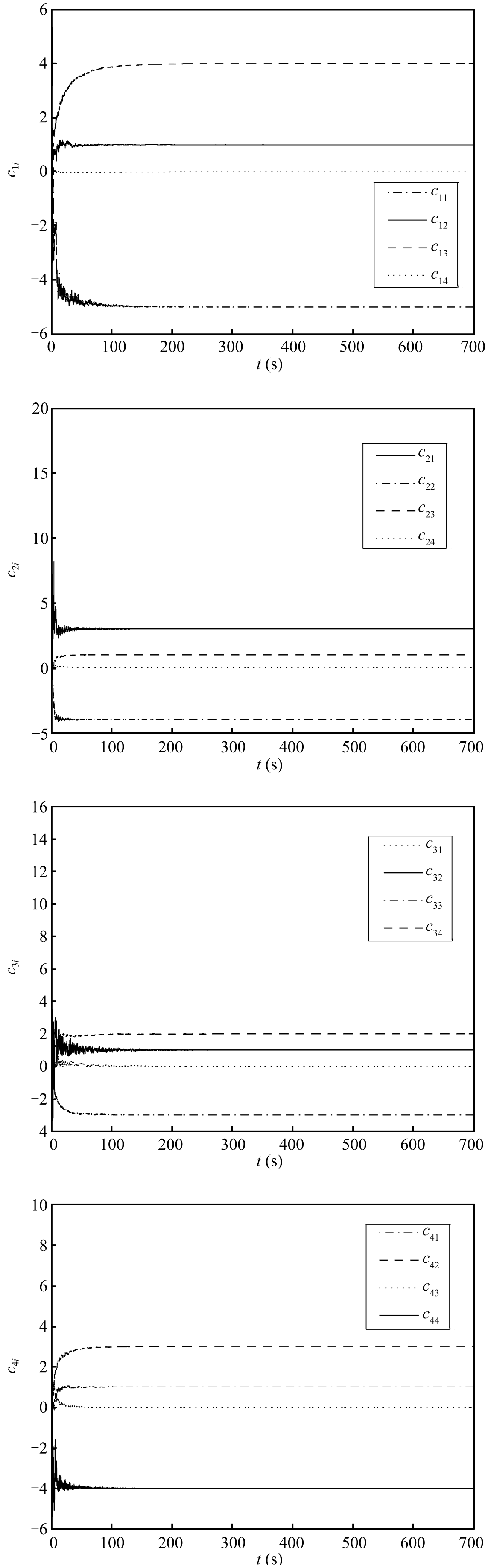


Fig. 3. Identification of network structure.

$$\begin{cases} D^q x_{i1}(t) = \beta_{i1}\left(x_{i2}(t-\tau) - x_{i1}(t-\tau)\right), \\ D^q x_{i2}(t) = -x_{i1}(t-\tau)\, x_{i3}(t-\tau) + \beta_{i2} x_{i2}(t-\tau), \\ D^q x_{i3}(t) = x_{i1}(t-\tau)\, x_{i2}(t-\tau) - \beta_{i3} x_{i3}(t-\tau), \end{cases} \tag{32}$$

where $q = 0.9$, $\beta_i = (\beta_{i1}, \beta_{i2}, \beta_{i3})^{\mathrm{T}} = (36,\ 20+i,\ 3)^{\mathrm{T}}$ for $i = 1, \ldots, 4$. Let $A = \mathrm{diag}\{1, 1, 1\}$ and $\tau = 0.002$. Here, the coupling configuration matrix C is also defined as (31). The chaotic attractor of delayed fractional-order Lü system (32) is shown in Fig. 4. According to Theorem 2, the unknown or uncertain coupling configuration matrix C and system parameter vector β_i can be estimated by using $\hat{C}$ and $\hat{\beta}_i$, respectively. Fig. 5 shows the identification of the uncertain

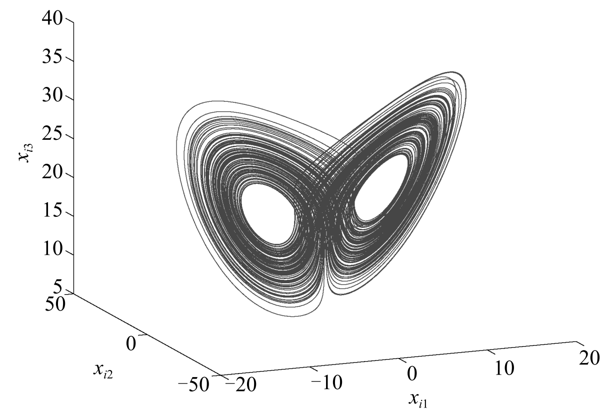
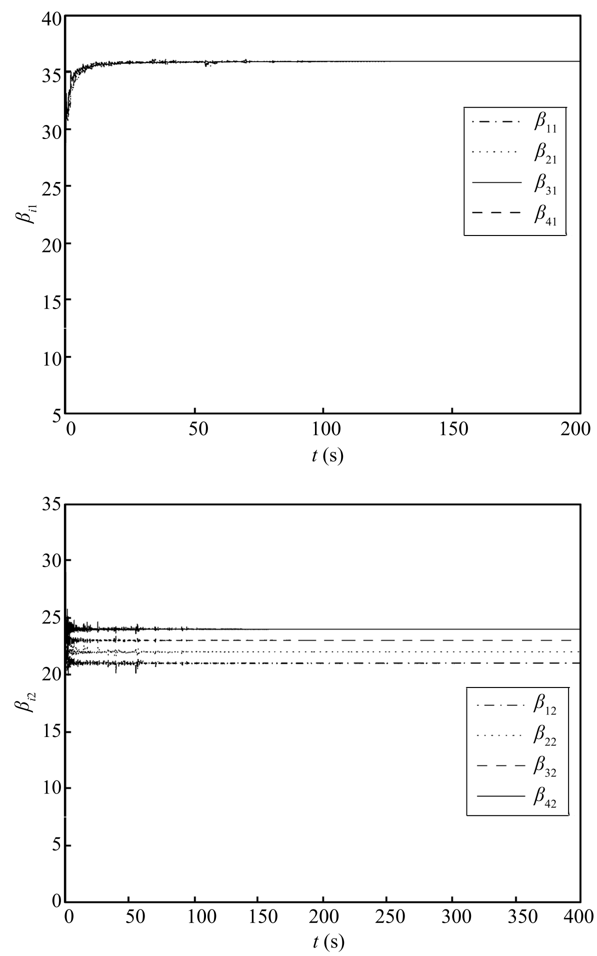


Fig. 4. Chaotic attractor of delayed fractional-order Lü system.



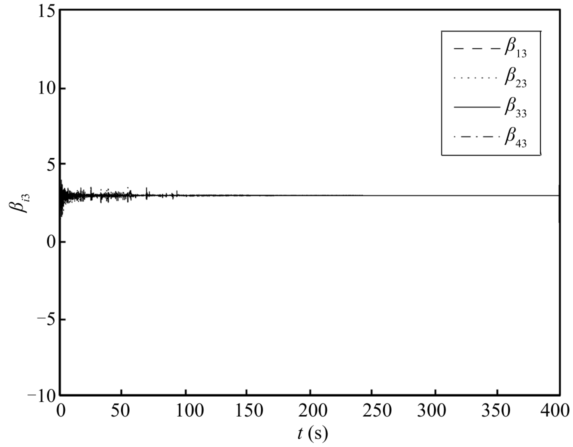
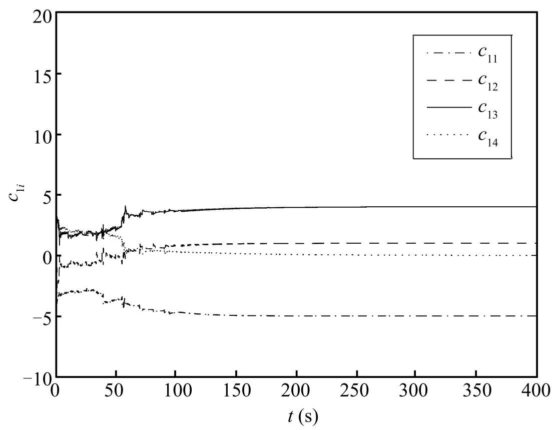
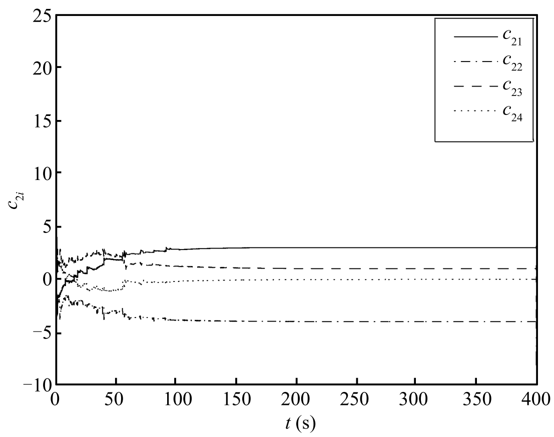
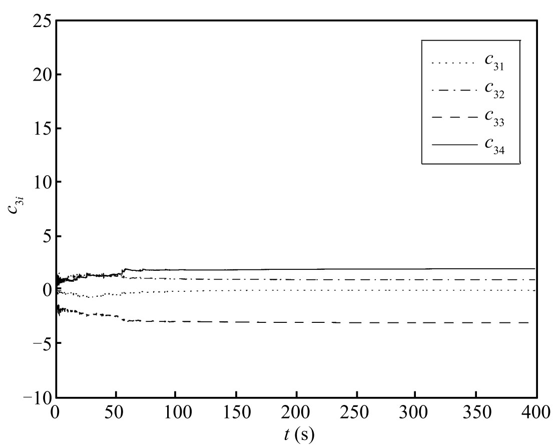


Fig. 5. Identification of uncertain parameters.







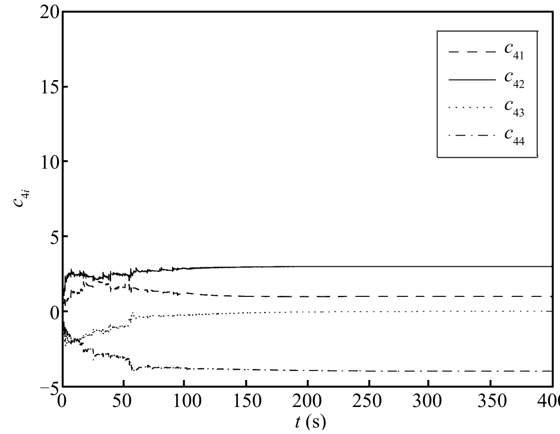


Fig. 6 Identification of network structure.

system parameters, and Fig. 6 illustrates the identification of the unknown network topology.

VI. CONCLUSION

In this paper, a novel and feasible approach to identify the parameters and network topology of fractional-order complex network with time delay is proposed. Based on the stability theorem of fractional-order differential system and the adaptive control technique, two useful identification criteria are derived. Illustrative simulations are provided to verify the correctness and effectiveness of the proposed methods.

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