Using Fractional Order Method to Generalize Strengthening Buffer Operator and Weakening Buffer Operator

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***Abstract*—To reveal the relationship between the weakening buffer operator and strengthening buffer operator, the traditional integer order buffer operator is extended to fractional order one. Fractional order buffer operator not only can generalize the weakening buffer operator and the strengthening buffer operator, but also realize tiny adjustment of buffer effect. The effectiveness of grey model (GM(1,1)) with the fractional order buffer operator is validated by six cases.**

***Index Terms*—Fractional order, grey system theory, strengthening buffer operator, weakening buffer operator.**

I. INTRODUCTION

DUE to the growing demand for reliable small sample statistics, small sample prediction is of great importance topic. Over the years, many scholars have carried out vigorous programs[1−4]. Among these programs, it is reported that the forecasting performance of grey model is better than many conventional methods with incomplete or insufficient data[4−6]. Grey system theory is developed by Deng[7]. As the primary forecasting method of grey system theory, GM(1,1) has been applied in many fields[4−7]. However, GM(1,1) is suitable for the stable time series, how to predict the non-stationary series is a difficult problem which deserves to be researched.

For non-stationary time series prediction problem, the theory on how to select model would lose its validity. That is not the problem of selecting better model; instead, when a system is severely affected by shock, the available data of the past cannot truthfully reflect the law of the system. Under the circumstances, buffer operator of grey system theory[7] has been successfully used in many fields to overcome the above difficulties[8−13], it combines quantitative and judgmental forecast (qualitative analysis). Many kinds of buffer operators have been proposed simultaneously[14−18], how to choose a suitable kind of buffer operator is very important in practice. In this paper, many kinds of buffer operators are unified and generalized based on fractional order method.

The rest of this paper is organized as follows. Section II is a compendium of grey buffer operator. In Section III, the inherent relationship between weakening buffer operator and strengthening buffer operator based on fractional order method is revealed. In Section IV, real examples for fractional order buffer operator are discussed. Some conclusions of this study are provided in the final section.

II. WEAKENING BUFFER OPERATOR AND STRENGTHENING BUFFER OPERATOR

Assume that $X=\{x(1),x(2),\dots,x(n)\}$ is the true behavior sequence of a system, the observed behavior sequence of the system is $Y=\{x(1)+\epsilon_1,x(2)+\epsilon_2,\dots,x(n)+\epsilon_n\}$, where $(\epsilon_1,\epsilon_2,\dots,\epsilon_n)$ is a term for the shocking disturbance. To correctly discover and recognize the true behavior sequence X of the system from the shock-disturbed sequence Y, one first has to go over the hurdle $(\epsilon_1,\epsilon_2,\dots,\epsilon_n)$ (That is to say that cleaning up the disturbance). If we directly use the severely impacted data Y to construct model and to make predictions, then our prediction is likely to fail, because what the model described was not the true situation X of the underlying system.

The wide existence of severely shocked systems often causes quantitative predictions disagree with the outcomes of intuitive qualitative analysis. Hence, seeking an equilibrium between qualitative analysis and quantitative predictions by eliminating these disturbances is an important task in order to discover the true situation of the system. Grey buffer operator proposed by Liu can address the problem, its definition is as follows.

Definition 1[7]**.** Assume that raw data sequence is $X=\{x(1),x(2),\dots,x(n)\}$. If $\forall k=2,3,\dots,n$, $x(k)-x(k-1)>0$, then X is called as a monotonic increasing sequence. If $\forall k=2,3,\dots,n$, $x(k)-x(k-1)<0$, then X is called as a monotonic decreasing sequence. If there are $k,k'\in\{k=2,3,\dots,n\}$ such that $x(k)-x(k-1)>0$, $x(k')-x(k'-1)<0$, then X is defined as a random vibrating or fluctuating sequence. If $M=\max\{x(k)|k=1,2,\dots,n\}$ and $m=\min\{x(k)|k=1,2,\dots,n\}$, then $M-m$ is called as the amplitude of the sequence X.

Lemma 1[7]**.** $X=\{x(1),\,x(2),\,\dots,\,x(n)\}$ is a monotonic increasing sequence. Then, $XD=\{x(1)d,\,x(2)d,\,\dots,\,x(n)d\}$

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is a weakening buffer operator(WBO), iff $x(k)d \geq x(k)$, $k = 1,2,\ldots,n$; $XD = \{x(1)d,\ x(2)d,\ \ldots,\ x(n)d\}$ is a strengthening buffer operator(SBO), iff $x(k)d \leq x(k)$, $k = 1,2,\ldots,n$.

Lemma 2[7]**.** Assume that $X = \{x(1),\ x(2),\ \ldots,\ x(n)\}$ is a monotonic decreasing sequence. Then, $XD = \{x(1)d,\ x(2)d,\ \ldots,\ x(n)d\}$ is a WBO, iff $x(k)d \leq x(k)$, $k = 1,2,\ldots,n$; $XD = \{x(1)d, x(2)d, \ldots, x(n)d\}$ is a SBO, iff $x(k)d \geq x(k)$, $k = 1,2,\ldots,n$.

Lemma 3[7]**.** Assume that $X = \{x(1), x(2), \ldots, x(n)\}$ is a fluctuating sequence, $XD = \{x(1)d, x(2)d, \ldots, x(n)d\}$ is a WBO, iff $\max\{x(k)|k = 1,2,\ldots,n\} \geq \max\{x(k)d|k = 1,2,\ldots,n\}$ and $\min\{x(k)|k = 1,2,\ldots,n\} \leq \min\{x(k)d|k = 1,2,\ldots,n\}$; $XD = \{x(1)d,\ x(2)d,\ \ldots,\ x(n)d\}$ is a SBO, iff $\max\{x(k)|k = 1,2,\ldots,n\} \leq \max\{x(k)d|k = 1,2,\ldots,n\}$ and $\min\{x(k)|k = 1,2,\ldots,n\} \geq \min\{x(k)d|k = 1,2,\ldots,n\}$.

Definition 2[7]**.** Assume that raw data sequence is $X = \{x(1), x(2), \ldots, x(n)\}$, $XD = \{x(1)d, x(2)d, \ldots, x(n)d\}$, where

$$x(k)d = \frac{x(k) + x(k+1) + \ldots + x(n)}{n-k+1}, \qquad (1)$$

D is a first order WBO no matter whether X is monotonic decreasing, increasing, or vibrating. If $XD^2 = XDD = \{x(1)dd, x(2)dd, \ldots, x(n)dd\}$, D^2 is a second order WBO. Similarity, D^3 is a third order WBO.

If

$$x(k)d = \frac{x(1) + x(2) + \ldots + x(k-1) + kx(k)}{2k-1}, \qquad (2)$$

then D is a first order SBO when sequence X is either monotonic decreasing or increasing. If $XD^2 = XDD = \{x(1)dd, x(2)dd, \ldots, x(n)dd\}$, D^2 is a second order SBO. Similarity, D^3 is a third order SBO.

$x^{(0)}(k)d = x^{(0)}(k)$ of WBO is consistent with the results of above studies, that is they all suggested that more emphasis should be placed on the most recent and most relevant information.

III. The Relationship Between WBO and SBO

Due to traditional weakening buffer operators cannot tune the effect intensity to a small extent, which leads to problems that the buffer effect may be too strong or too weak. Considering this situation, and like the fractional-order systems[19−21], fractional weakening buffer operator is constructed. Then (1) can be expressed by

$$\begin{aligned} XD =& \{x(1)d, x(2)d, \ldots, x(n)d\} \\ =& [x(1), x(2), \ldots, x(n)] \begin{bmatrix} \frac{1}{n} & 0 & \ldots & 0 \\ \frac{1}{n} & \frac{1}{n-1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n} & \frac{1}{n-1} & \ldots & 1 \end{bmatrix} \end{aligned}$$

then second order WBO can be expressed by

$$XD^2 = [x(1), x(2), \ldots, x(n)] \begin{bmatrix} \frac{1}{n} & 0 & \ldots & 0 \\ \frac{1}{n} & \frac{1}{n-1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n} & \frac{1}{n-1} & \ldots & 1 \end{bmatrix}^2$$

Similarly, $\frac{p}{q}(\frac{p}{q} \in R^+)$ order WBO is

$$XD^{\frac{p}{q}} = [x(1), x(2), \ldots, x(n)] \begin{bmatrix} \frac{1}{n} & 0 & \ldots & 0 \\ \frac{1}{n} & \frac{1}{n-1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n} & \frac{1}{n-1} & \ldots & 1 \end{bmatrix}^{\frac{p}{q}}$$

Theorem 1. For original data $X = [x(1), x(2), \ldots, x(n)]$, $-\frac{p}{q}(\frac{p}{q} \in R^+)$ order WBO from (1) is the $\frac{p}{q}$ order SBO.

Proof. Set

$$\begin{bmatrix} \frac{1}{n} & 0 & \ldots & 0 \\ \frac{1}{n} & \frac{1}{n-1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n} & \frac{1}{n-1} & \ldots & 1 \end{bmatrix} = A,$$

since $-\frac{p}{q}(\frac{p}{q} \in R^+)$ order WBO is

$$\begin{aligned} XD^{-\frac{p}{q}} =& X \begin{bmatrix} \frac{1}{n} & 0 & \ldots & 0 \\ \frac{1}{n} & \frac{1}{n-1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n} & \frac{1}{n-1} & \ldots & 1 \end{bmatrix}^{-\frac{p}{q}} \\ =& XA^{-\frac{p}{q}} \\ =& X \begin{bmatrix} n & 0 & 0 & \ldots & 0 \\ -(n-1) & n-1 & 0 & \ldots & 0 \\ 0 & -(n-2) & n-2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix}^{\frac{p}{q}} \end{aligned}$$

The result of $XA^{\frac{p}{q}}$ is a vector. When each component of $XA^{\frac{p}{q}}$ is not less than the corresponding component of X, we can write as $XA^{\frac{p}{q}} \geq X$. If sequence X is either monotonically decreasing or increasing, because $XA^{\frac{p}{q}} \geq X$ and A is an invertible matrix, we have $XA^{\frac{p}{q}}A^{-\frac{p}{q}} \geq XA^{-\frac{p}{q}}$, that is $X \geq XA^{-\frac{p}{q}}$. So $-\frac{p}{q}$ order WBO is the $\frac{p}{q}$ order SBO when sequence X is either monotonically decreasing or increasing.

If sequence $X = [x(1), x(2), \ldots, x(n)]$ is a fluctuating sequence, $x(l) = \max\{x(k)|k = 1,2,\ldots,n\}$, $x(h) = \min\{x(k)|k = 1,2,\ldots,n\}$, because $[x(l), x(l), \ldots, x(l)]A^{\frac{p}{q}} \geq [x(l), x(l), \ldots, x(l)]$ and A is an invertible matrix, we have $[x(l), x(l), \ldots, x(l)]A^{\frac{p}{q}}A^{-\frac{p}{q}} \geq [x(l), x(l), \ldots, x(l)]A^{-\frac{p}{q}}$, that is $[x(l), x(l), \ldots, x(l)] \geq [x(l), x(l), \ldots, x(l)]A^{-\frac{p}{q}}$; Similarly, we have $[x(h), x(h), \ldots, x(h)] \leq [x(h), x(h), \ldots, x(h)]A^{-\frac{p}{q}}$. So $-\frac{p}{q}$ order WBO is the $\frac{p}{q}$ order SBO when sequence X is a fluctuating sequence.

So $-\frac{p}{q}$ $(\frac{p}{q} \in R^+)$ order WBO from (1) is the $\frac{p}{q}$ order SBO. ■

Corollary 1. For original data $X = [x(1), x(2), \ldots, x(n)]$, $-\frac{p}{q}(\frac{p}{q} \in R^+)$ order SBO from (2) is the $\frac{p}{q}$ order WBO.

Corollary 2. For original data $X = [x(1), x(2), \ldots, x(n)]$, if nonnegative matrix B satisfies $XB^{-\frac{p}{q}}$ $(\frac{p}{q} \in R^+) > 0$ and $XD^{-\frac{p}{q}} = XB^{-\frac{p}{q}}$ is SBO (WBO), then $XD^{\frac{p}{q}} = XB^{\frac{p}{q}}$ is WBO (SBO).

The procedures of GM(1,1) model with $\frac{p}{q}$ order WBO ($\frac{p}{q}$WGM(1,1)) are more complex than the traditional GM(1,1), because more work must be done before forecasting. The procedures can be summarized as follows:

Step 1: Given a raw data sequence $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$, $\frac{p}{q}$ order WBO sequence is $X^{(0)}D^{\frac{p}{q}} = \{x^{(0)}(1)d^{\frac{p}{q}}, x^{(0)}(2)d^{\frac{p}{q}}, \ldots, x^{(0)}(n)d^{\frac{p}{q}}\}$.

Step 2: Sequence $\{x^{(0)}(1)d^{\frac{p}{q}}, x^{(0)}(2)d^{\frac{p}{q}}, \ldots, x^{(0)}(n)d^{\frac{p}{q}}\}$ is used to establish GM(1,1), accumulated generating operator $x^{(1)}(k)d^{\frac{p}{q}} = \sum_{i=1}^{k} x^{(0)}(i)d^{\frac{p}{q}}$, $k = 1, 2, \ldots, n$.

Step 3: The parameter a and b can be obtained by

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (A^T A)^{-1} A^T Y$$

where

$$Y = \begin{bmatrix} x^{(0)}(2)d^{\frac{p}{q}} \\ x^{(0)}(3)d^{\frac{p}{q}} \\ \vdots \\ x^{(0)}(n)d^{\frac{p}{q}} \end{bmatrix}, \; A = \begin{bmatrix} -\frac{x^{(1)}(1)d^{\frac{p}{q}} + x^{(1)}(2)d^{\frac{p}{q}}}{2} & 1 \\ -\frac{x^{(1)}(2)d^{\frac{p}{q}} + x^{(1)}(3)d^{\frac{p}{q}}}{2} & 1 \\ \vdots & \vdots \\ -\frac{x^{(1)}(n-1)d^{\frac{p}{q}} + x^{(1)}(n)d^{\frac{p}{q}}}{2} & 1 \end{bmatrix}$$

Step 4: After substituting $\hat{a}$ and $\hat{b}$ into $\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = [x^{(0)}(1) - \frac{\hat{b}}{\hat{a}}](1 - e^{\hat{a}})e^{-\hat{a}k}$ $(k = 1, 2, \ldots, n-1)$, we can make prediction $x^{(0)}(n+1), x^{(0)}(n+2), \ldots$.

Step 5: If the predicted value $x^{(0)}(n+1), x^{(0)}(n+2), \ldots$ is not consistent with the result of qualitative analysis, then change the order number $\frac{p}{q}$. (If we want to pay more attention to the recent data, the order number $\frac{p}{q}$ must be the larger one. If we want to pay more attention to the old data, the order number $\frac{p}{q}$ must be the smaller one. Because the strengthening buffer operator reflects the priority of old data[22]).

Step 6: Repeat Step 1-5 until the predicted values $x^{(0)}(n+1), x^{(0)}(n+2), \ldots$ are consistent with the result of qualitative analysis.

IV. Experimentation results

To test the proposed model, mean absolute percentage error (MAPE $= 100\% \times \frac{1}{n}\sum_{k=1}^{n}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|$) is used to evaluate the precision.

Case 1. Energy consumption forecasting in China[23]

The data from 1998 to 2005 ($X^{(0)} = \{$13.22, 13.38, 13.86, 14.32, 15.18, 17.50, 20.32, 22.47$\}$) are used to establish different GM(1,1) models with different WBO, and the data from 2006 to 2007 are used to determine the optimal order of WBO. The results are shown in Table I.

As can be seen from Table I, 0.1WGM(1,1) is the best model among the above models in out-of sample data. So 0.1WGM(1,1) is used to predict the data from 2008 to 2009. The results are listed in Table II. As can be seen from Table II, 0.1WGM(1,1) yielded the lowest MAPE in out-of-sample data. This implies that 0.1WGM(1,1) can improve the prediction precision.

TABLE II
THE RESULTS OF TWO GREY MODELS

Year	Actual value	0.1WGM(1,1)	The result of Reference[23]
2008	29.10	28.86	28.59
2009	31.00	31.41	31.23
MAPE		0.98	1.26

Case 2. Electricity consumption per capita forecasting in China[24]

The data from 2000 to 2005 ($X^{(0)} = \{$132.4, 144.6, 156.3, 173.7, 190.2, 216.7$\}$) are used to obtain different GM(1,1) models with different WBO, and the data of 2006 is predicted by these models. The results are shown in Table III.

As can be seen from Table III, both WGM(1,1) models are better than the best result of Reference[23], as a conclusion, fractional order WBO has a perfect forecasting capability.

Case 3. The qualified discharge rate of industrial wastewater forecasting in Jiangxi in China[17]

The data from 2000 to 2005 ($X^{(0)} = \{$68.63, 75.9, 77.59, 83.06, 88.66, 92.13$\}$) are used to construct two GM(1,1) models with WBO, and the data from 2006 to 2007 are predicted by these models. The results are shown in Table IV.

As can be seen from Table IV, the WGM(1,1) model is better than the best result of Reference[17], so fractional order WBO can improve the prediction accuracy of conventional GM(1,1) model.

TABLE IV
THE FITTED VALUES AND MAPE OF TWO GREY MODELS

Year	Actual value	GM(1,1)	WGM(1,1)
2006	93.23	93.4	93.95
2007	93.89	94.5	95.77
MAPE		0.42	1.75

Case 4. The electricity consumption forecasting in Vietnam[25]

The data from 2000 to 2003 ($X^{(0)} = \{$1927, 2214, 2586, 2996$\}$, unit: KTOE) are used to construct four models with WBO, and the data from 2004 to 2007 are predicted by these models. The results are shown in Table V.

As can be seen from Table V, the WGM(1,1) model is better than the best result of Reference[17], so fractional order WBO can improve the prediction accuracy of conventional GM(1,1) model.

Case 5. The logistics demand forecasting in Jiangsu[26]

The data from 2005 to 2008 are used to construct three grey models with WBO, and the data from 2009 are predicted by these models. The results are shown in Table VI.

As can be seen from Table VI, the WGM(1,1) model is better than the traditional grey model, so fractional order WBO can improve the prediction accuracy of conventional GM(1,1).

TABLE VI
THE FITTED VALUES AND MAPE OF THREE GREY MODELS

Year	Actual value	GM(1,1)[26]	1WGM(1,1)	0.5WGM(1,1)
2009	5154.46	5330	5008	5138
MAPE		3.41	2.84	0.32

Case 6: The energy production forecasting in China[27]

The 1985-1989 data are used for model building, while the 1990-1995 data are used as an ex-post testing data set. The

TABLE I
THE RESULTS OF DIFFERENT GREY MODELS

Year	Actual value	0.3WGM(1,1)	0.1WGM(1,1)	The best result of Reference[23]
2006	24.63	23.95	24.05	27.95
2007	26.56	25.97	26.34	26.16
MAPE		2.43	1.55	2.12

TABLE III
THE FITTED VALUES AND MAPE OF DIFFERENT GREY MODELS

Year	Actual value	-0.6WGM(1,1)	-0.7WGM(1,1)	The best result of Reference[24]
2006	249.4	248.3	250.8	241.21
MAPE		0.44	0.56	3.28

TABLE V
THE FITTED VALUES AND MAPE OF FOUR GREY MODELS

Year	Actual value	GM(1,1)	AGM(1,1)[25]	1WGM(1,1)	0.1WGM(1,1)
2004	3437	3477	3334	3215	3439
2005	3967	4042	3807	3452	3953
2006	4630	4699	4347	3706	4544
2007	5256	5462	4963	3979	5224
MAPE		2.12	4.68	15.92	0.72

results given by the GM(1,1) model and 1.5WGM(1,1) as well as the observed values are shown in Table VII.

TABLE VII
THE FITTED VALUES AND MAPE OF TWO GREY MODELS

Year	Actual value	GM(1,1)[27]	1.5WGM(1,1)
1990	103922	106069	103407
1991	104844	111296	105320
1992	107265	116781	107270
1993	111059	122536	109255
1994	118729	128574	111277
1995	129034	134910	113337
MAPE		6.71	3.50

Table VII shows that the 1.5WGM(1,1) model is better for forecasting the energy production in China. The forecasted values are more precise than the GM(1,1) model, for data sequence with large random fluctuation.

V. CONCLUSION

Let us now return to the name of the fractional calculus. The fractional calculus is a name for the theory of integrals and derivatives of arbitrary order. which unifies and generalizes the notions of integer-order differential and integral. Similarly, fractional order WBO unifies and generalizes the notions of WBO and SBO. As can be seen from Table II-VII, GM(1,1) with the fractional order buffer operator can predict the development trend of the system accurately.

Six real cases were seen to obtain good results, however, the order $\frac{p}{q}$ may be not optimal. In this paper, the order $\frac{p}{q}$ is chosen from more computational experiments. In future studies, it is suggested that the particle swarm algorithm should be used to determine the optimal order.

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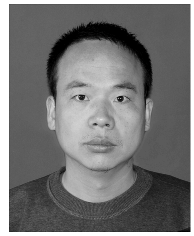

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