Dynamics of the Fractional-order Lorenz System Based on Adomian Decomposition Method and Its DSP Implementation

Shaobo He, Kehui Sun, and Huihai Wang

Abstract—**Dynamics and digital circuit implementation of the fractional-order Lorenz system are investigated by employing Adomian decomposition method (ADM). Dynamics of the fractional-order Lorenz system with derivative order and parameter varying is analyzed by means of Lyapunov exponents (LEs), bifurcation diagram, chaos diagram and phase diagram. Results show that the fractional-order Lorenz system has rich dynamical behavior and it is a potential model for application. It is also found that the minimum order is affected by numerical algorithm and time step size. Finally, the fractional-order system is implemented on DSP digital circuit. Phase diagrams generated by the DSP are consistent with that generated by simulation.**

Index Terms—**fractional calculus, Lorenz system, Adomian decomposition method, dynamics, DSP implementation.**

I. INTRODUCTION

IN recent years, the application of fractional calculus to chaotic system has become a hot topic [1], and researchers begin to investigate dynamics and applications of the fractional-order chaotic systems [2,3].

The fractional-order Lorenz system with a new set of parameters is firstly analyzed by Grigorenko I et.al [4], and they reported that the system can generate chaos when the total order is 2.91 by a numerical method they derived. Unfortunately, an error was found in the derived numerical method, thus the result in this paper was not reliable [5]. More recently, Jia H Y et al. [6] analyzed dynamics of this system with order q=0.7, 0.8 and 0.9 and implemented it in analog circuit by employing frequency domain method (FDM) [7]. However, whether this method accurately reflects chaotic characteristics in fractional-order chaotic system was questioned in [8-10]. Another method for solving fractional-order chaotic systems is the Adams-Bashforth-Moulton algorithm (ABM) [11]. It can be used to analyze dynamics with continuous derivative order [12], and some researches of the fractional-order chaos are based on this algorithm [13, 14]. But the calculation speed of this algorithm is very slow, and it consumes too many computer resources [15]. Meanwhile, Adomian decomposition method (ADM)[16] is employed to obtain numerical solution of the fractional-order chaotic system for its high precision and fast speed of convergence [17-19]. For instance, the fractional-order Chen system is investigated by Cafagna D et.al [19] by applying ADM, and the results show that it is a good method for solving the fractional-order chaotic systems. In addition, based on ADM, Lyapunov exponents (LEs) of the fractional-order system are calculated [20]. Furthermore, circuit design is essential for application of fractional-order chaotic systems. Although analog circuit implementation is widely reported by researchers [6], digital circuit realization of the fractional-order chaotic system has better flexibility and repeatability [21]. So, we focus on dynamics of the fractional-order Lorenz system and its DSP implementation by employing ADM in this paper.

The structure of the paper is as follows. In Sec.II, characteristics of the ADM are presented and the numerical solution of fractional-order Lorenz system is obtained. In Sec.III, dynamics of the fractional-order Lorenz system is investigated. In Sec.IV, the fractional-order Lorenz system is realized by employing DSP technology. Finally, we summarize the conclusions.

II. NUMERICAL SOLUTION FOR THE FRACTIONAL-ORDER LORENZ SYSTEM

A. Advantages of Adomian decomposition method

We choose ADM to solve the fractional-order chaotic system since it has some advantages over the following aspects comparing with other standard numerical methods.

i) ADM can get more exact solution of the fractional-order system as it preserves the system nonlinearities. The precision of FDM is within 2dB or 3dB, and a satisfying approximation of the actual system can be obtained within the desired frequency band. But a large error is illustrated at the high and low frequency band [10, 23]. The truncation error of ABM is $O(h^p)$, p=min(2,1+q). It is acceptable, but it is not as effective as ADM [15].

ii) ADM obtains chaos with much lower order. Taking fractional-order Chen system as an example, the minimum order by ADM is 0.24 [19], and this value represents the lowest order reported in literatures. However, the minimum order of this system by ABM is 2.64 [8], and it is difficult for FDM to obtain the minimum order of a fractional-order chaotic system.

iii) ADM provides a potential iterative approach for digital circuit implementation of the fractional-order chaotic system.

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ABM is not suitable for practical application of fractional-order chaotic system since it needs more and more time and memory space for computation as time goes on [15]. FDM is the theoretical basis for the fractional-order chaotic systems implemented in analog circuit.

B. Description of Adomian decomposition method

For a given fractional-order chaotic system with form of $D^q_{t_0}\mathbf{x}(t)$=$f(\mathbf{x}(t))$+$\mathbf{g}(t)$, where $\mathbf{x}(t)$=[$x_1(t)$, $x_2(t)$, ..., $x_n(t)$] is the state variable, $\mathbf{g}(t) = [g_1(t), g_2(t), ..., g_n(t)]$ is the constant in the system, and $D^q_{t_0}$ is the Caputo fractional derivative operator [17]. So it can be divided into three parts as the form

$$D^q_{t_0}\mathbf{x}(t) = L\mathbf{x}(t) + N\mathbf{x}(t) + \mathbf{g}(t), \tag{1}$$

where $m \in N$, $m-1 < q \leq m$. $L\mathbf{x}(t)$ and $N\mathbf{x}(t)$ are the linear and nonlinear terms of the fractional differential equations respectively. Here, let $J^q_{t_0}$ is the inverse operator of $D^q_{t_0}$, thus we have [17].

$$\mathbf{x} = J^q_{t_0}L\mathbf{x} + J^q_{t_0}N\mathbf{x} + J^q_{t_0}\mathbf{g} + \Phi, \tag{2}$$

where $\Phi = \sum_{k=0}^{m-1}\mathbf{b}_k(t-t_0)^k/k!$, $\mathbf{x}^{(k)}(t_0^+) = \mathbf{b}_k, k = 0,\cdots,m-1$, and it involves the initial condition. By applying the recursive relation [17]

$$\left\{\begin{array}{l}\mathbf{x}^0 = J^q_{t_0}\mathbf{g} + \Phi\\ \mathbf{x}^1 = J^q_{t_0}L\mathbf{x}^0 + J^q_{t_0}\mathbf{A}^0(\mathbf{x}^0)\\ \mathbf{x}^2 = J^q_{t_0}L\mathbf{x}^1 + J^q_{t_0}\mathbf{A}^1(\mathbf{x}^0,\mathbf{x}^1)\\ \cdots\\ \mathbf{x}^i = J^q_{t_0}L\mathbf{x}^{i-1} + J^q_{t_0}\mathbf{A}^{i-1}(\mathbf{x}^0,\mathbf{x}^1,\cdots,\mathbf{x}^{i-1})\\ \cdots\end{array}\right., \tag{3}$$

the analytical solution of the fractional-order system is presented as

$$\mathbf{x}(t) = \sum\nolimits_{i=0}^{\infty}\mathbf{x}^i, \tag{4}$$

where $i = 1, 2, ..., \infty$, and the nonlinear terms of the fractional differential equations $N\mathbf{x}(t)$ are evaluated by [22]

$$N\mathbf{x} = \sum_{i=0}^{\infty}\mathbf{A}^i(\mathbf{x}^0,\mathbf{x}^1,\cdots,\mathbf{x}^i), \tag{5}$$

$$\left\{\begin{array}{l}A^i_j = \frac{1}{i!}[\frac{d^i}{d\lambda^i}N(v^i_j(\lambda))]_{\lambda=0}\\ v^i_j(\lambda) = \sum\limits_{k=0}^{i}(\lambda)^k x^k_j\end{array}\right.. \tag{6}$$

Because ADM converges very fast [17-19], we choose $i = 6$ for the approximate solution in this paper. To discretize Eq.(4), a time interval $[t_0, t]$ is divided into subintervals $[t_n, t_{n+1}]$, where $h = t_{n+1} - t_n$. So, the solution of the fractional-order Lorenz system is expressed as

$$\mathbf{x}(t_n) = \sum\nolimits_{i=0}^{6}\mathbf{x}^i(t_{n-1}) = F(\mathbf{x}(t_{n-1})). \tag{7}$$

Then we can obtain the discrete iterative form $\mathbf{x}(t_{n+1}) = F(\mathbf{x}(t_n))$, which is denoted as $\mathbf{x}(n+1) = F(\mathbf{x}(n))$ for general cases.

C. Solution of the fractional-order Lorenz system

The fractional-order chaotic Lorenz system is presented by [4, 6] as

$$\left\{\begin{array}{l}D^q_{t0}x_1 = a(x_2 - x_1)\\ D^q_{t0}x_2 = cx_1 - x_1x_3 + dx_2\\ D^q_{t0}x_3 = x_1x_2 - bx_3\end{array}\right., \tag{8}$$

where a, b, c, and d are system parameters, and q is the derivative order. As the same with [4] and [6], we investigate dynamics and digital circuit realization of this system by fixing $a = 40$, $b = 3$, $c = 10$, and varying d and q. By applying ADM, the numerical solution of the fractional-order Lorenz system is denoted by

$$\left\{\begin{array}{l}x_1(n+1) = \sum_{j=0}^{6}\kappa^j_1 h^{jq}/\Gamma(jq+1)\\ x_2(n+1) = \sum_{j=0}^{6}\kappa^j_2 h^{jq}/\Gamma(jq+1)\\ x_3(n+1) = \sum_{j=0}^{6}\kappa^j_3 h^{jq}/\Gamma(jq+1)\end{array}\right.. \tag{9}$$

where h is the integration step-size, $\Gamma(\cdot)$ is the Gamma function, and $\kappa^j_i(\cdot)$ are defined as

$$\kappa^0_1 = x_1(n), \kappa^0_2 = x_2(n), \kappa^0_3 = x_3(n), \tag{10}$$

$$\left\{\begin{array}{l}\kappa^1_1 = a(\kappa^0_2 - \kappa^0_1)\\ \kappa^1_2 = c\kappa^0_1 + d\kappa^0_2 - \kappa^0_1\kappa^0_3\\ \kappa^1_3 = -b\kappa^0_3 + \kappa^0_1\kappa^0_2\end{array}\right., \tag{11}$$

$$\left\{\begin{array}{l}\kappa^2_1 = a(\kappa^1_2 - \kappa^1_1)\\ \kappa^2_2 = c\kappa^1_1 + d\kappa^1_2 - \kappa^0_1\kappa^1_3 - \kappa^1_1\kappa^0_3\\ \kappa^2_3 = \kappa^1_1\kappa^0_2 + \kappa^0_1\kappa^1_2 - b\kappa^1_3\end{array}\right., \tag{12}$$

$$\left\{\begin{array}{l}\kappa^3_1 = a(\kappa^0_2 - \kappa^0_1)\\ \kappa^3_2 = c\kappa^2_1 + d\kappa^2_2 - \kappa^0_1\kappa^2_3 -\\ \qquad \kappa^1_1\kappa^1_3\frac{\Gamma(2q+1)}{\Gamma^2(q+1)} - \kappa^2_1\kappa^0_3\\ \kappa^3_3 = \kappa^0_1\kappa^2_2 + \kappa^1_1\kappa^1_2\frac{\Gamma(2q+1)}{\Gamma^2(q+1)} +\\ \qquad \kappa^2_1\kappa^0_2 - b\kappa^2_3\end{array}\right., \tag{13}$$

$$\left\{\begin{array}{l}\kappa^4_1 = a(\kappa^3_2 - \kappa^3_1)\\ \kappa^4_2 = c\kappa^3_1 + d\kappa^3_2 - \kappa^0_1\kappa^3_3 - \kappa^3_1\kappa^0_3 -\\ \qquad (\kappa^2_1\kappa^1_3 + \kappa^1_1\kappa^2_3)\frac{\Gamma(3q+1)}{\Gamma(q+1)\Gamma(2q+1)}\\ \kappa^4_3 = \kappa^0_1\kappa^3_2 + \kappa^3_1\kappa^0_2 + b\kappa^3_3 +\\ \qquad (\kappa^2_1\kappa^1_2 + \kappa^1_1\kappa^2_2)\frac{\Gamma(3q+1)}{\Gamma(q+1)\Gamma(2q+1)}\end{array}\right., \tag{14}$$

$$\left\{\begin{array}{l}\kappa^5_1 = a(\kappa^4_2 - \kappa^4_1)\\ \kappa^5_2 = c\kappa^4_1 + d\kappa^4_2 - \kappa^0_1\kappa^4_3 -\\ \qquad (\kappa^3_1\kappa^1_3 + \kappa^1_1\kappa^3_3)\frac{\Gamma(4q+1)}{\Gamma(q+1)\Gamma(3q+1)} -\\ \qquad \kappa^2_1\kappa^2_3\frac{\Gamma(4q+1)}{\Gamma^2(2q+1)} - \kappa^4_1\kappa^0_3\\ \kappa^5_3 = \kappa^0_1\kappa^4_2 + (\kappa^3_1\kappa^1_2 + \kappa^1_1\kappa^3_2)\frac{\Gamma(4q+1)}{\Gamma(q+1)\Gamma(3q+1)}\\ \qquad +\kappa^2_1\kappa^2_2\frac{\Gamma(4q+1)}{\Gamma^2(2q+1)} + \kappa^4_1\kappa^0_2 - b\kappa^4_3\end{array}\right., \tag{15}$$

$$\left\{\begin{array}{l}\kappa^6_1 = a(\kappa^5_2 - \kappa^5_1)\\ \kappa^6_2 = c\kappa^5_1 + d\kappa^5_2 - \kappa^0_1\kappa^5_3 -\\ \qquad (\kappa^1_1\kappa^4_3 + \kappa^4_1\kappa^1_3)\frac{\Gamma(5q+1)}{\Gamma(q+1)\Gamma(4q+1)} -\\ \qquad (\kappa^2_1\kappa^3_3 + \kappa^3_1\kappa^2_3)\frac{\Gamma(5q+1)}{\Gamma(2q+1)\Gamma(3q+1)} - \kappa^5_1\kappa^0_3\\ \kappa^6_3 = \kappa^0_1\kappa^5_2 + \kappa^5_1\kappa^0_2 - b\kappa^5_3 +\\ \qquad (\kappa^1_1\kappa^4_2 + \kappa^4_1\kappa^1_2)\frac{\Gamma(5q+1)}{\Gamma(q+1)\Gamma(4q+1)} +\\ \qquad (\kappa^2_1\kappa^3_2 + \kappa^3_1\kappa^2_2)\frac{\Gamma(5q+1)}{\Gamma(2q+1)\Gamma(3q+1)}\end{array}\right., \tag{16}$$

According to Eq.(9), the chaotic sequences of the fractional-order Lorenz system are obtained with appropriate initial values. Meanwhile, Eq.(9) provides a necessary iterative approach for DSP implementation of the fractional-order Lorenz system.

III. Chaotic dynamics of the fractional-order Lorenz system

A. Design of Lyapunov exponents calculation algorithm

The Lyapunov exponents are calculated based on Jacobian matrix obtained from Eqs. (9)-(16) and QR decomposition method [20]. The QR decomposition method is shown as

$$\begin{aligned} qr(\mathbf{J}_N\mathbf{J}_{N-1}\cdots\mathbf{J}_1) &= qr\left(\mathbf{J}_N\mathbf{J}_{N-1}\cdots\mathbf{J}_2(\mathbf{J}_1\mathbf{Q}_0)\right) \\ &= qr\left(\mathbf{J}_N\mathbf{J}_{N-1}\cdots\mathbf{J}_3(\mathbf{J}_2\mathbf{Q}_1)\right)\mathbf{R}_1 \\ &= \mathbf{Q}_N\mathbf{R}_N\cdots\mathbf{R}_2\mathbf{R}_1 \end{aligned}, \quad (17)$$

where $qr(\cdot)$ is the QR decomposition function, $\mathbf{J}$ is the Jacobian matrix of Eq.(9). The Lyapunov exponents are obtained as

$$\lambda_k = \frac{1}{Nh}\sum_{i=1}^{N}\ln|R_i(k,k)|, \quad (18)$$

where k=1, 2, 3, and N is the iteration times. The flow chart for LEs calculation is shown in Fig.1. Before calculating LEs, parameters, time step size h and number of iterations N should be confirmed. The Jacobian matrix is obtained according to Eq.(9) by applying mathematical software Matlab. LEs are calculated based on the QR decomposition method as illustrated in Eqs. (17) and (18).

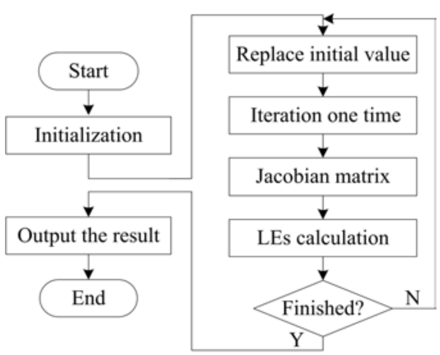
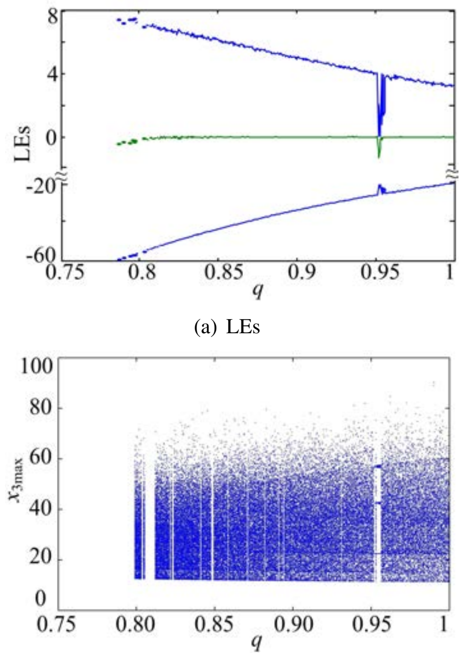


Fig. 1. Flow diagram for LEs calculation algorithm.

B. Dynamics with varying parameters

In this section, dynamics in the fractional-order Lorenz system with varying system parameter d and derivative order q are investigated. Parameter fixed dynamical analysis method and chaos diagram are used. Here, we set N=20000 and h=0.01. Three cases are investigated.

i) Fix $d = 25$, and vary derivative order q from 0.75 to 1 with step size of 0.0005. The bifurcation diagram and LEs are shown in Fig.2. It shows that the system generates chaos for $0.813 \leq q < 1$ except some periodic windows. Thus the minimum total order for fractional-order Lorenz system to generate chaos is 2.439 and the corresponding phase diagram is shown in Fig.3. In addition, the maximum Lyapunov exponent illustrates a decreasing trend as order q increasing.



(a) LEs

(b) Bifurcation diagram

Fig. 2. Dynamics of the fractional-order Lorenz system with $d = 25$ and q varying

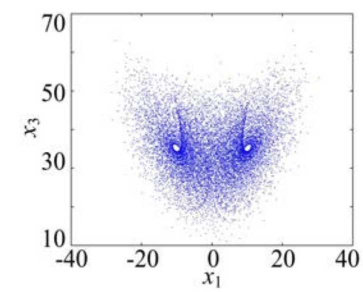
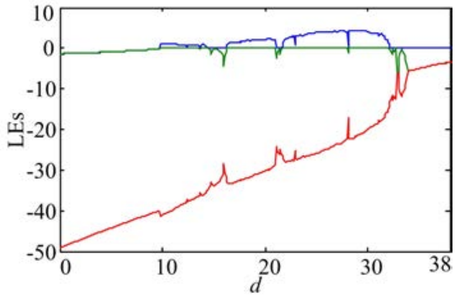


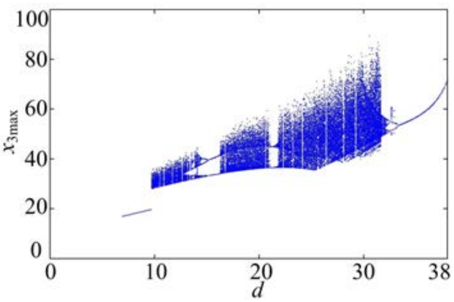
Fig. 3. Phase diagram of the fractional-order Lorenz system with $d = 25$ and $q = 0.813$.

ii) Fix $q = 0.96$ and vary d from 0 to 38 with step size of 0.1. When d decreases from 38, the system presents periodical states until it enters into chaos at $d = 32.1$ by the period-doubling bifurcation as shown in Fig. 4(a). Chaos covers most of the range $d \in [9.8, 32.1]$ with several small periodic widows, such as $d \in [14.5, 16.3] \cup [21.1, 21.5]$. Finally, the system becomes convergent at $d = 9.8$ by a tangent bifurcation. To observe dynamics better, phase diagrams are presented in Fig.5. When d=15, 21.5 and 37, the system is periodic, and the system is chaotic when $d = 20$. It shows that the system presents different states with different values of parameter d.

iii) Vary q from 0.75 to 1 with step size of 0.0025 and vary d from 0 to 38 with step size of 0.38 simultaneously. The maximum Lyapunov exponent based chaos diagram in

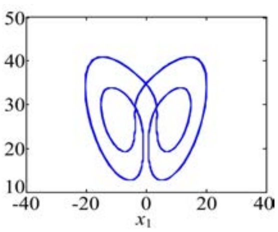


(a) LEs

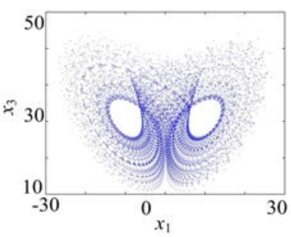


(b) Bifurcation diagram

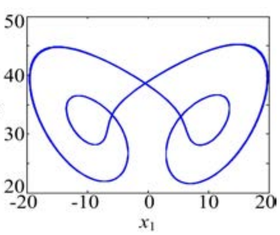
Fig. 4. Dynamics of the fractional-order Lorenz system with $q = 0.96$ and d varying



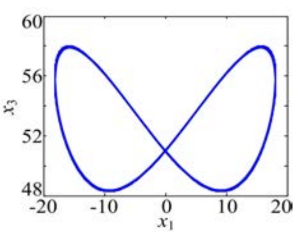
(a) d=15



(b) d=20



(c) d=21.5



(d) d=37

Fig. 5. Phase diagrams of the fractional-order Lorenz system with $q = 0.96$ and d varying

this $q-d$ parameter plane is illustrated in Fig.6. In this figure, we only plot the case when the maximum Lyapunov exponent is larger than zero. According to Fig.6, chaos exists in the range of $d \in [10, 32]$. A high complexity region is observed within $d \in [25, 30]$ and $q \in [0.8, 0.97]$, which is favorable for practical application. So, the fractional-order Lorenz system is a good model for real application. It shows that the chaos diagram provides a parameter selection basis for fractional-order chaotic Lorenz system in practical application.

Compared with bifurcation analysis results based on FDM

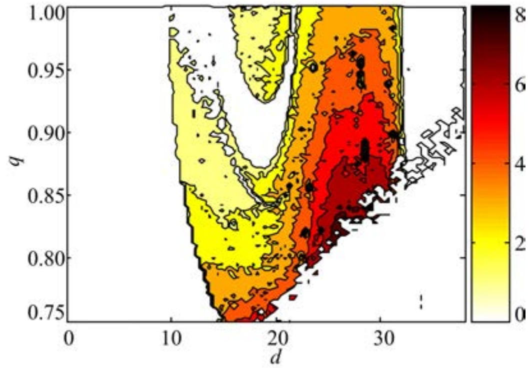


Fig. 6. Maximum Lyapunov exponent based chaos diagram.

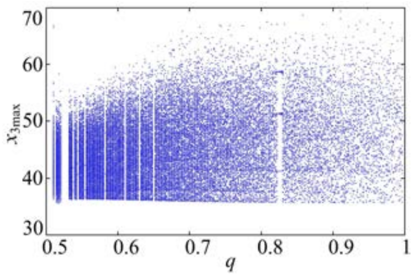
as shown in [6], results based on ADM are more detailed and accurate. It also shows that we can analyze dynamics of the system with q varying continuously, but it is difficult for FDM to do so.

C. Discussion about the minimum order

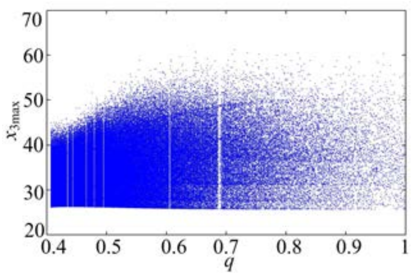
Obviously, the minimum order for chaos is different for different system parameter. But it is also different when the numerical solution algorithm or time step size h is different. Thus these two aspects are discussed as follows.

i) Compared with other approaches, chaotic system has a much lower order if it is solved by ADM algorithm. The equilibrium point of this system is (0, 0, 0) and ($\pm\sqrt{b(c+d)}$, $\pm\sqrt{b(c+d)}$, $c + d$). When $d = 25$, the eigenvalues at (0, 0, 0) are λ_1=-45.6608, λ_2=30.6608, λ_3=-3.0000, and the eigenvalues at $(\pm\sqrt{105}, \pm\sqrt{105}, 35)$ are λ_1=-25.2415, λ_2=3.6207+17.8795i and λ_3=3.6207-17.8795i. According to the stability theory as proposed in Refs. [8] and [9], the lowest order q to generate chaos is $q = 0.8726$. It is not difficult to find out that ABM satisfies this result. However, FDM and ADM do not. According to [6], when $q = 0.7$, the system has rich dynamics and chaos still exists by applying FDM. According to Fig.2, the minimum order of the system is $q = 0.813$ by applying ADM. Actually, the stability theory from [25] is proposed to analyze fractional-order linear systems. For fractional-order nonlinear systems, Li L X et al. [26] proved that the stability theory does not always work when the specified matrix $J(X)$ is time-varying. We believe that it is more complex to analyze stability of fractional-order nonlinear system. Besides, although FDM and ADM do not satisfy the stability theory as presented in [8] and [9], they are widely used and accepted by researchers [7, 15-21]. In addition, it shows in [27] that different results of a fractional-order system may be achieved when simulations are performed based on different numerical methods. Since FDM and ADM can obtain chaos at a much lower order, they extend the parameter space of fractional-order chaotic systems.

ii) The effect of time step h should be further investigated. As for ADM, when h=0.01, the lowest order to generate chaos is q=0.813. We also find that the lowest order decreases with the decrease of the time step size h. As shown in Fig.7, when



(a) h=0.001



(b) $h = 0.0001$

Fig. 7. Bifurcation diagrams of the fractional-order Lorenz system under different h

h=0.001, the lowest order is q=0.505, and the lowest order is q=0.402 for h=0.0001. The system generates chaos with lower order when time step h is smaller, but more memory and computing resources are needed. It is not good for real application of the system. We think $h = 0.01$ is a suitable choice for general cases. However, the reason why the lowest order decreases with the decrease of time h needs further study.

According to the discussion above, when a minimum order for chaos generation of a fractional-order chaotic system is presented, the certain set of parameters, numerical algorithm and time step size should also be specified.

IV. Digital circuit implementation

In this section, the digital circuit of the fractional-order Lorenz system is designed, and the numerical solution applied in DSP board is presented as Eqs.(9)-(16). Hardware block diagram of the digital circuit is shown in Fig.8. The floating-point DSP TMS320F28335 produced by TI is chosen. A 16-bit dual-channel D/A converter DAC8552 is used to convert time series generated by DSP. An oscilloscope is used to capture figures randomly. The flow diagram is presented in Fig.9. Firstly, the DSP is initialized, then the initial values, including h, q, $\mathbf{x}_0$, parameters and iteration number are confirmed. In this step, all $\Gamma(\cdot)$ and h^{nq} are computed and saved before iterative computation to improve the iteration speed. When the data is popped out, the data should be processed before D/A conversion. There are two steps in data processing. At first, a big enough data is added to make sure the data is larger than zero. Then, the data is rescaled and truncated to adapt data width of the DAC8552. It should be pointed out that the iterative computation is not affected by data processing with pushing and popping operation. If the iteration is not finished, the initial value should be replaced before the next iteration.

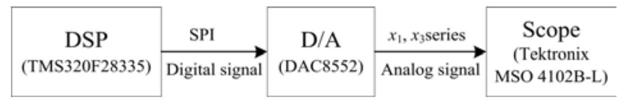


Fig. 8. Hardware block diagram of DSP implementation

Here, the initial value is $\mathbf{x}_0 = [1, 2, 3]$. Setting $q = 0.8130$, $d = 25$, the phase diagram is shown in Fig. 10(a). The corresponding Matlab simulation result is illustrated in Fig.3. Setting $q = 0.96$, $d = 15$, the phase diagram is shown in Fig. 10(b), and its corresponding Matlab simulation result is presented in Fig.5(a). Setting $q = 0.96$ and varying d (d=20 and d=37), the phase diagrams are shown in Figs. 10(c) and (d). It can be seen that they consist of phase diagrams as shown in Fig.5(b) and Fig.5(d). It shows that the fractional-order Lorenz system is implemented in the DSP successfully. It lays a hardware foundation for the applications of the fractional-order Lorenz chaotic system.

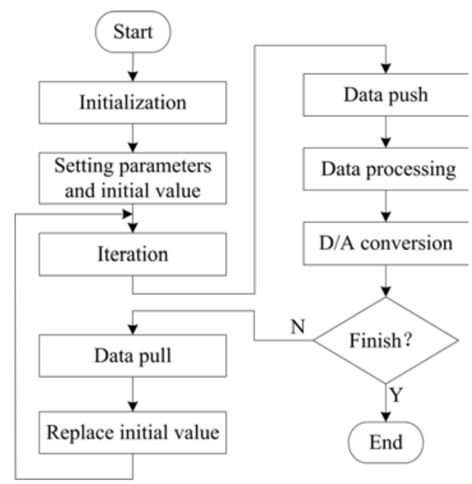
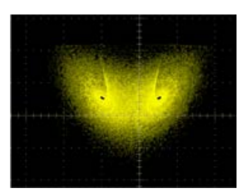
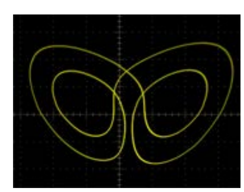


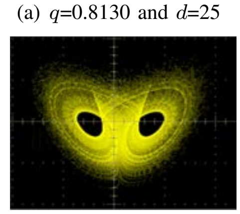
Fig. 9. Flow diagram for DSP implementation of the fractional-order Lorenz system.



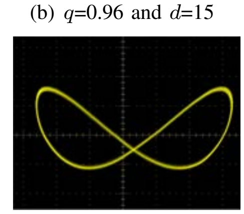
(a) q=0.8130 and d=25



(b) q=0.96 and d=15



(c) q=0.96 and d=20



(d) q=0.96 and d=37

Fig. 10. Phase diagrams of the fractional-order Lorenz system recorded by the oscilloscope

V. CONCLUSIONS

In this paper, based on ADM algorithm, we investigated the dynamics of fractional-order Lorenz system again. It shows that the fractional-order Lorenz system has rich dynamical characteristics. The system is more complex for smaller derivative order q, and the maximum Lyapunov exponent decreases with the increase of q. The lowest order for chaos generation is different according to different numerical algorithms. The fractional-order Lorenz system has a much lower order for chaos if it is solved by ADM algorithm. Meanwhile, the lowest order for chaos is smaller when the time step size h is smaller. Finally, the system is implemented in the digital circuit by employing DSP technology, and phase diagrams generated by the DSP device are consistent with the simulation results. Our further work will focus on real applications of the fractional-order Lorenz system.

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