Relationship Between Integer Order Systems and Fractional Order Systems and Its Two Applications

Xuefeng Zhang

***Abstract*—Existence of periodic solutions and stability of fractional order dynamic systems are two important and difficult issues in fractional order systems (FOS) field. In this paper, the relationship between integer order systems (IOS) and fractional order systems is discussed. A new proof method based on the above involved relationship for the non existence of periodic solutions of rational fractional order linear time invariant systems is derived. Rational fractional order linear time invariant autonomous system is proved to be equivalent to an integer order linear time invariant non-autonomous system. It is further proved that stability of a fractional order linear time invariant autonomous system is equivalent to the stability of another corresponding integer order linear time invariant autonomous system. The examples and state figures are given to illustrate the effects of conclusion derived.**

***Index Terms*—Existence, equivalence, periodic solutions, rational fractional order systems, stability.**

I. INTRODUCTION

THE concept of fractional differentiation appeared first in a famous correspondence between L' Hopital and Leibniz, in 1695. Fractional calculus has had a 300 years old history, the development of fractional calculus theory is a matter of almost exclusive interest for few mathematicians and theoretical physicists. In recent years, researchers have noticed that the description of some phenomena is more accurate when the fractional derivative is introduced. Many practical control system models can be described by fractional differential equations. It is worth mentioning that many physical phenomena having memory and genetic characteristics can be described by modeling as fractional order systems. Fractional order systems have attracted much attention. In what concerns automatic control, T. T. Hartley and C. F. Lorenzo [1] studied the fractional order algorithms for the control of dynamic systems. Podlubny [2] proposed a generalization of the PID controller, namely the PI$^\lambda$D$^\mu$ controller, involving an integrator of order λ and a differentiator of order μ. L. Yan and Y. Q. Chen [3] propose the definition of Mittag-Leffler stability and introduce the fractional Lyapunov direct method. Fractional comparison principle is introduced and the application of Riemann-Liouville fractional order systems is extended by using Caputo fractional order systems. C.P Li and F.R. Zhang [4] give a survey on the stability of fractional differential equations based on analytical methods.

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Fractional-order differential operators present unique and intriguing peculiarities, not supported by their integer-order counterpart, which raise exciting challenges and opportunities related to the development of control and estimation methodologies involving fractional order dynamics. In recent years, most of papers are devoted to the solvability of the linear fractional equation in terms of a special function and to problems of analyticity in the complex domain. Fractional system and its control has become one of the most popular topics in control theory [5]−[8]. The number of applications where fractional calculus has been used rapidly grows. These mathematical phenomena allow to describe a real object more accurately than the classical integer-order methods [9]−[12]. Paper [10] gives the non existence of periodic solutions in fractional order systems with Mellin transform. But for singular fractional order systems, the Mellin transform method is invalid because of singularity of systems.

In this paper, we will show that rational fractional order linear time invariant autonomous system is equivalent to an integer order linear time invariant non-autonomous system but cannot be equivalent to any integer order linear time invariant autonomous system with any system parameters. The nonexistence of periodic solutions of fractional order dynamic systems are proved by means of contradiction method. Stability of a fractional order linear time invariant autonomous system is equivalent to the stability of another corresponding integer order linear time invariant autonomous system. The examples and state figures are given to illustrate the effects of the conclusions derived. The conclusions provided in the paper can be easily extended to singular fractional order linear time invariant systems.

II. PRELIMINARIES

Let us denote by $\mathbb{Z}^+$ the set of positive integer numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{R}^{n\times n}$ the set of $n\times n$ dimension real numbers. We denote the real part of complex number α by Re(α).

Caputo derivative has been often used in fractional order systems since it has the practical initial states like that of integer order systems.

Definition 1: The Caputo fractional order derivative with order α of function $x(t)$ is defined as

$$ {}_0^C D_t^\alpha x(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} x^{(n)}(\tau) d\tau $$

where $n-1 < \alpha < n \in \mathbb{Z}^+, \Gamma$ is well-known Euler Gamma function.

Definition 2: The Riemann-Liouville derivative of fractional order α of function $x(t)$ is defined as

$$ {}_0^{RL}D_t^{\alpha}x(t) = \frac{1}{\Gamma(n-\alpha)}(\frac{d}{dt})^n \int_0^t (t-\tau)^{n-\alpha-1}x(\tau)d\tau $$

where $n-1<\alpha<n\in\mathbb{Z}^+$.

Definition 3: The Grunwald-Letnikov derivative of fractional order α of function $x(t)$ is defined as

$$ {}_0^{GL}D_t^{\alpha}x(t) = \lim_{h\to 0} h^{-\alpha} \sum_{r=0}^{(t-\alpha)/h} (-1)^r C_\alpha^r x(t-rh) $$

where $n-1<\alpha<n\in\mathbb{Z}^+$.

Definition 4: The Mittag-Leffler function is defined as

$$ E_\alpha(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(k\alpha+1)} $$

where $\mathrm{Re}(\alpha)>0, t\in\mathbb{C}$. The two-parameter Mittag-Leffler function is defined as

$$ E_{\alpha,\beta}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(k\alpha+\beta)} $$

where $\mathrm{Re}(\alpha)>0, \beta, t\in\mathbb{C}$.

Property 1: The Laplace transform of Caputo derivative of function $x(t)$ is

$$ L({}_0^{C}D_t^{\alpha}x(t)) = s^\alpha X(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1}x^k(0), $$

where $X(s)=L[x](s), n-1<\alpha<n\in\mathbb{Z}^+$.

Property 2: If let $\alpha\in(0,\infty)\setminus\mathbb{N}$. Then, we have

$$ {}_0^{RL}D_t^{\alpha}x(t) = {}_0^{GL} D_t^{\alpha}x(t) = {}_0^{C} D_t^{\alpha}x(t) + \sum_{i=0}^{n-1} \frac{x^{(i)}(0)}{\Gamma(i-\alpha+1)}t^{i-\alpha} $$

where $n-1<\alpha<n\in\mathbb{Z}^+$.

Lemma 1: The Laplace transform of $t_+^{\alpha-1}/\Gamma(\alpha)$ is:

$$ L(\frac{t_+^{\alpha-1}}{\Gamma(\alpha)}) = s^{-\alpha} $$

and

$$ t_+^{\alpha-1} = \begin{cases} t^{\alpha-1}, & t>0 \\ 0, & t\le 0. \end{cases} $$

Lemma 2: The Laplace transform of $\frac{e^{-at}}{\sqrt{b-a}}\mathrm{erf}(\sqrt{(b-a)t})$ is:

$$ L(\frac{e^{-at}}{\sqrt{b-a}}\mathrm{erf}(\sqrt{(b-a)t})) = \frac{1}{\sqrt{s+b}(s+a)} $$

where $\mathrm{erf}(t)$ is the error function for each element of t, $\mathrm{erf}(t)=\frac{2}{\sqrt{\pi}}\int_0^t e^{-\tau^2}d\tau$.

Lemma 3: The Laplace transform of $\frac{1}{\sqrt{\pi t}}-\frac{2}{\sqrt{\pi}}\mathrm{daw}(\sqrt{t})$ is:

$$ L(\frac{1}{\sqrt{\pi t}}-\frac{2}{\sqrt{\pi}}\mathrm{daw}(\sqrt{t})) = \frac{\sqrt{s}}{s+1} $$

where $\mathrm{daw}(t)$ is Dawson function for each element of t, $\mathrm{daw}(t)=e^{-t^2}\int_0^t e^{\tau^2}d\tau$.

Lemma 4: The Laplace transform of $A\cos(\omega t)$ is:

$$ L(A\cos(\omega t)) = \frac{As}{s^2+\omega^2}. $$

Lemma 5: The Laplace transform of $t^{\beta-1}E_{\alpha,\beta}(-\omega t^\alpha)$ is:

$$ L(t^{\beta-1}E_{\alpha,\beta}(-\omega t^\alpha)) = \frac{s^{\alpha-\beta}}{s^\alpha+\omega}. $$

Lemma 6: The Laplace transform of n order derivative $f^n(t)$ is:

$$ L(f^{(n)}(t)) = s^nF(s) - \sum_{i=0}^{n-1} s^{n-1-i}f^{(i)}(0). $$

III. Main Results

A. Equivalence Between FOS and IOS

Integer order linear time invariant (LTI) systems have been developed quite maturely. Fractional order LTI system is a subsystem of dynamic control system and is less discussed due to its difficulty. In order to obtain the better control cost index, the control components and devices with fractional order properties are needed to be introduced. Algorithms in measurement technology sometimes process the fractional order characteristics. Some control plants are more difficult to be modeled than integer order systems. By the above reason, fractional order dynamic control systems are essential to be introduced. From Fig. 1, we can see that state figures of $\dot{x}(t)=tx(t)$, and those of $D^\alpha x(t)=x(t), \alpha=0.2, 0.4, \cdots, 1$, are similar to each other, but they are not identically coincided with each other. An obvious question is whether there exists an integer order LTI System (1) equivalent to a fractional order LTI System (2) with any appropriate parameters or be equivalent to a fractional order LTV System (3) with any appropriate parameters or not. It is an important problem for the reason that if the answer is 'yes' the fractional order systems can be regarded as a part of integer order systems and if the answer is 'no' the fractional order systems cannot be ignored so that the research of FOS is magnificently innovative. From the theorems in Section III, it is found that the answer is negative. It can hold only if the state is zero solution. Actually, System (1) is equivalent to System (4) in some cases. In the following subsection, we can have that if $\alpha=1/2$, System (4) reduces to System (5).

$$ D^\alpha x(t) = Ax(t) \tag{1} $$

$$ \dot{x}(t) = A_1x(t) \tag{2} $$

$$ \dot{x}(t) = A_2(t)x(t) \tag{3} $$

$$ \dot{x}(t) = \frac{A}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}x'(\tau)d\tau + \frac{Ax(0)t_+^{\alpha-1}}{\Gamma(\alpha)} \tag{4} $$

$$ \dot{x}(t) = A^2x(t) + \frac{Ax(0)t_+^{-\frac{1}{2}}}{\Gamma(\frac{1}{2})}. \tag{5} $$

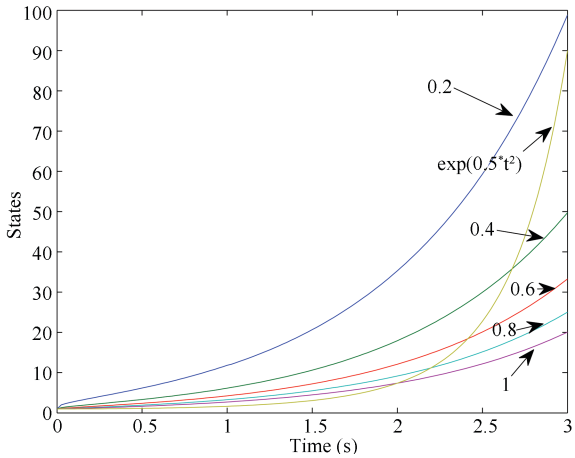

Fig. 1. Plot of states of $\dot{x}(t) = tx(t)$ and $D^\alpha x(t) = x(t)$.

From the discussion on relationship between FOS and IOS in the above subsection, it is easy to introduce its two applications i.e., non existence of periodic solutions for FOS and stability between FOS and IOS.

B. Non Existence of Periodic Solutions for FOS

Theorem 1: While $\alpha = 1/2$, System (1) is equivalent to (5).

Proof: Using Laplace transform for System (1), taking into account the Caputos definition for the fractional-order derivatives in (2), and applying Property 1 in the case that $0 < \alpha < 1$, it yields that

$$s^\alpha X(s) - s^{\alpha-1}x(0) = AX(s).$$

Pre- and post-multiplying above equation by $s^{1-\alpha}$, it follows that

$$\begin{aligned} sX(s) - x(0) =& As^{1-\alpha}X(s) \\ =& A(s^{1-\alpha}X(s) - s^{-\alpha}x(0)) + As^{-\alpha}x(0). \end{aligned}$$

By Lemma 1 and taking inverse Laplace transform in above equation, we have (4).

When $\alpha = 1/2$, we have System (5). If we denote $B = Ax(0), u(t) = t_+^{-\frac{1}{2}}/\Gamma(\frac{1}{2})$, then (5) changes as (6)

$$\dot{x}(t) = A^2x(t) + Bu(t). \tag{6}$$

When $\alpha = p/q, p, q \in \mathbb{Z}^+$, System (1) can be proved to be equivalent to (7).

$$\begin{aligned} x^{(p)}(t) =& A^q x(t) + \sum_{i=1}^{q-1} A^{q-i} \frac{t_+^{-\frac{ip}{q}}}{\Gamma(1-\frac{ip}{q})} x(0) \\ &+ \sum_{i=1}^{p-1} \delta^{(p-1-i)}(t)x^{(i)}(0) \end{aligned} \tag{7}$$

where δ is the unit pulse function.

Theorem 2: While $\alpha = p/q$, System (1) is equivalent to (7).

Proof: Using Laplace transform in (1), taking into account the Caputos definition for the fractional-order derivatives in (2), and applying Property 1 in the case that $0 < \alpha < 1$, we have that

$$s^\alpha X(s) - s^{\alpha-1}x(0) = AX(s)$$

i.e.,

$$s^{\frac{p}{q}}X(s) - s^{p/q-1}x(0) = AX(s).$$

Pre- and post-multiplying above equation by $s^{p/q}$, it follows that

$$\begin{aligned} & s^{\frac{2p}{q}}X(s) - s^{\frac{2p}{q}-1}x(0) \\ &= As^{\frac{p}{q}}X(s) = A(s^{\frac{p}{q}}X(s) - s^{\frac{p}{q}-1}x(0)) + As^{\frac{p}{q}-1}x(0) \\ &= A^2X(s) + As^{\frac{p}{q}-1}x(0). \end{aligned}$$

Keeping on pre- and post-multiplying above equation by $s^{\frac{p}{q}}$ till q times, it follows that

$$s^pX(s) - s^{p-1}x(0) = A^qX(s) + \sum_{i=1}^{q-1} A^{q-i}s^{\frac{ip}{q}-1}x(0)$$

i.e.,

$$\begin{aligned} & s^pX(s) - \textstyle\sum_{i=0}^{p-1} s^{p-1-i}x^{(i)}(0) \\ &= A^qX(s) + \textstyle\sum_{i=1}^{q-1} A^{q-i}s^{\frac{ip}{q}-1}x(0) - \sum_{i=1}^{p-1} s^{p-1-i}x^{(i)}(0). \end{aligned}$$

By Lemma 1 and Property 1 and taking inverse Laplace transform in above equation, we have (7).

Theorem 3: Linear time invariant fractional system (1) with order $0 < \alpha < 1, \alpha = p/q, p, q \in \mathbb{Z}^+$ has no periodic solution.

Proof: By contradiction, suppose linear time invariant fractional system (1) has a periodic solution $x(t)$. For $T-$periodic function $x(t+T) = x(t)$, from

$$\frac{d}{dt}x(t+T) = \frac{d}{d(t+T)}x(t+T)\frac{d}{dt}(t+T) = x'(t+T)$$

it is easy to see that $x^{(k)}(t+T) = x^{(k)}(t)$. From Theorem 2, we know that (1) is equivalent to (7). If we denote

$$\begin{aligned} & f(t) = x^{(p)}(t) - A^qx(t) \\ & g(t) = \textstyle\sum_{i=1}^{q-1} A^{q-i}\frac{t_+^{-\frac{i}{q}}}{\Gamma(1-\frac{i}{q})}x(0) + \sum_{i=1}^{p-1} \delta^{(p-1-i)}x^{(i)}(0) \end{aligned}$$

then $f(t) = g(t)$. However, $f(t)$ is periodic function but $g(t)$ is a non-periodic function. So, there does not exist any periodic solution for System (1). ■

Remark 1: From Theorem 2, we know that there does not exist any integer order LTI System (2) be equivalent to a fractional order LTI System (7) with any appropriate parameters. It means the properties of fractional order LTI systems may be different from those of integer order LTI systems. It can attract researchers to explore the distinct properties of fractional order LTI systems.

Remark 2: Theorem 3 gives a concise and effective proof that there does not exist periodic solutions for fractional order LTI System (7).

Remark 3: With the equivalence between the integer order LTI System (7) and the fractional order LTI System (1), we succeed in finding a new research approach of discussing the difficult fractional order LTI System (1). However, relative to System (1), it is easy and there exists extensive results to discuss the integer order LTI System (7). For example, we can further discuss the stability and robust stability of fractional order LTI System (1) in the future.

Remark 4: From Theorem 3, we can see that only if α is an integer, it follows $g(t) = 0$. This means only if α is an integer, it is possible for System (1) to satisfy periodic solutions.

C. Stabilities Between FOS and IOS

Lemma 7: [12] System (1) is asymptotically stable if and only if there exist two matrices $X, Y \in \mathbb{R}^{n\times n}$, such that

$$\begin{bmatrix} X & Y \\ -Y & X \end{bmatrix} > 0$$
$$aAX + bAY + aXA^T - bYA^T < 0$$

where $a = \sin(\alpha\pi/2), b = \cos(\alpha\pi/2)$.

Lemma 8: [12] System (1) is asymptotically stable if and only if there exist two matrices $X, Y \in \mathbb{R}^{n\times n}$, such that

$$\begin{bmatrix} X & Y \\ -Y & X \end{bmatrix} > 0, \quad \begin{bmatrix} \Pi_1 & \Pi_2 \\ -\Pi_2 & \Pi_1 \end{bmatrix} < 0$$

where

$$\Pi_1 = aAX + bAY + aXA^T - bYA^T$$
$$\Pi_2 = aAY - bAX + bXA^T + aYA^T$$

and a, b are the same as those in Lemma 7.

Lemma 9: [12] A complex matrix $X \in \mathbb{C}^{n\times n}$ satisfies $X < 0$ if and only if

$$\begin{bmatrix} \mathrm{Re}(X) & \mathrm{Im}(Y) \\ -\mathrm{Im}(Y) & \mathrm{Re}(X) \end{bmatrix} < 0. \tag{8}$$

Consider the following specific complex integer order linear time invariant system

$$\dot{x}(t) = (a + jb)A^T x(t) \tag{9}$$

where system matrix $A \in \mathbb{R}^{n\times n}$, $\jmath$ is the imaginary unit.

Using Lyapunov theory of integer order systems and Lemmas 7 and 8, it is easy to obtain the following equivalence stability criterion.

Theorem 4: Fractional order system (1) is asymptotically stable if and only if integer order system (9) is asymptotically stable.

Proof For the specific complex integer order LTI system (9), we choose the quadratic Lyapunov candidate function as

$$V(x(t)) = x^T(t)(X + jY)x(t)$$

where $X+jY > 0$. Then, differentiating $V(x(t))$ with respect to time t along to the solution of (9), we obtain

$$\begin{aligned}\dot{V} &= x^T(t)[(a - jb)A(X + jY) + (X + jY)(a + jb)A^T]x(t) \\ &= x^T(t)(\Pi_1 + j\Pi_2)x(t) < 0.\end{aligned}$$

Using Lyapunov theory of complex integer order systems and considering (8) in Lemma 9, this completes the proof.

IV. Numerical Examples

Example 1: Consider integer order System (2) with parameters as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

from Fig. 2, we can see that the solutions are periodic. But if we consider System (1) with the same above parameters and $\alpha = 1/2$, then by Laplace transform for System (1) we have

$$\begin{aligned} s^{\frac{1}{2}}X_1(s) - X_2(s) &= s^{-\frac{1}{2}} \\ s^{\frac{1}{2}}X_2(s) + X_1(s) &= s^{-\frac{1}{2}}. \end{aligned}$$

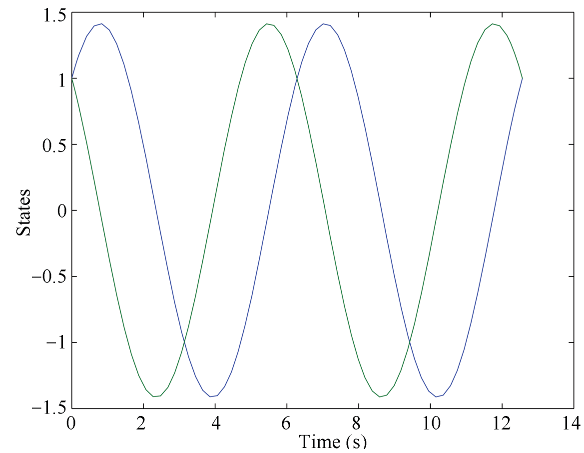

Fig. 2. State curves of IOS in Example 1.

It is easy to obtain the solutions of the above equations as follows:

$$X_1(s) = \frac{s^{-\frac{1}{2}+1}}{s+1}, \quad X_2(s) = \frac{s^{\frac{1}{2}}+1}{s+1} - s^{\frac{1}{2}}.$$

Consider Lemma 1 and 3, and take the inverse Laplace transform for $X_1(s)$ and $X_2(s)$, it follows that:

$$\begin{aligned} x_1(t) &= e^{-t} + \tfrac{2}{\sqrt{\pi}}\mathrm{daw}(\sqrt{t}) \\ x_2(t) &= e^{-t} + \tfrac{1}{\sqrt{\pi t}} - \tfrac{2}{\sqrt{\pi}}\mathrm{daw}(\sqrt{t}) - \tfrac{t^{-\frac{1}{2}}}{\Gamma(\frac{1}{2})}. \end{aligned}$$

With Lemma 5, it also follows that:

$$x_1(t) = e^{-t} + t^{\frac{1}{2}}E_{1,\frac{3}{2}}(-t)$$

$$x_2(t) = e^{-t} - t^{\frac{1}{2}}E_{1,\frac{1}{2}}(-t) - \frac{t^{-\frac{1}{2}}}{\Gamma(\frac{1}{2})}.$$

It is easy to see from Fig. 3 that the state curves of fractional order System (1) with parameter $\alpha = 1/2$ do not possess periodic dynamic orbits.

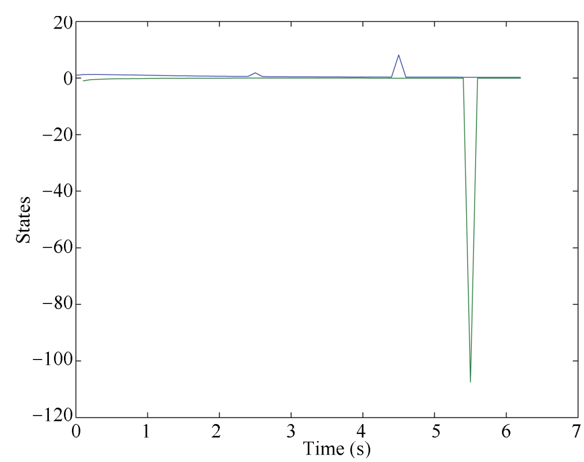

Fig. 3. State curves of FOS in Example 1.

Example 2: By Theorem 2, for $\alpha = 1/3$, we have that System (1) is equivalent to

$$\dot{x}(t) = x(t) + \frac{t_+^{-\frac{1}{3}}}{\Gamma(1-\frac{1}{3})}x(0) + \frac{t_+^{-\frac{2}{3}}}{\Gamma(1-\frac{2}{3})}x(0).$$

From Fig. 4, we can see the state curves of fractional order System (1) with parameter $\alpha = 1/3$ are completely identical to the corresponding state curves of integer order System (7).

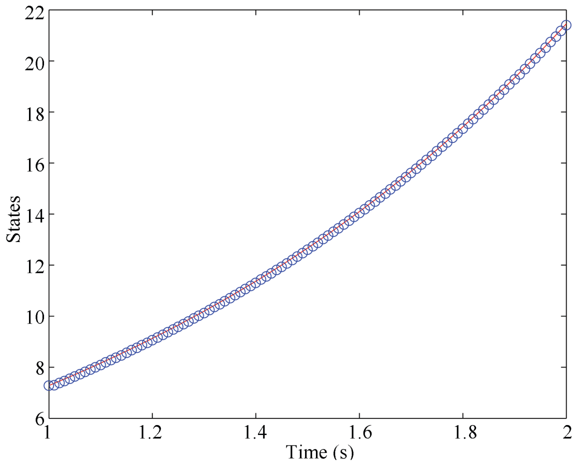

Fig. 4. State curves of FOS in Example 2.

V. CONCLUSIONS

Many systems exhibit the fractional phenomena, such as motions in complex media or environments, random walk of bacteria in fractal substance, etc. These models can be obtained by solving modified fractional order systems. In this paper, we discuss the relationship between rational fractional order systems and integer order systems and conclude that the two kind of systems cannot be substituted for each other. The criteria of nonexistence of periodic solution of fractional order systems are addressed. The proof approach is based on the properties of Laplace transform of fractional order systems. Stability of a fractional order linear time invariant autonomous system is equivalent to the stability of another corresponding integer order linear time invariant autonomous system. Some numerical examples are given to verify the feasibility of results presented. The methods provided in the paper can be extended to singular fractional order linear time invariant systems in the future.

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