An Exploration on Adaptive Iterative Learning Control for a Class of Commensurate High-order Uncertain Nonlinear Fractional Order Systems

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***Abstract*—This paper explores the adaptive iterative learning control method in the control of fraction order systems for the first time. An adaptive iterative learning control (AILC) scheme is presented for a class of commensurate high-order uncertain nonlinear fractional order systems in the presence of disturbance. To facilitate the controller design, a sliding mode surface of tracking errors is designed by using sufficient conditions of linear fraction order. To relax the assumption of the identical initial condition in iterative learning control (ILC), a new boundary layer function is proposed by employing Mittag-Leffler function. The uncertainty in the system is compensated for by utilizing radial basis function neural network. Fractional order differential type updating laws and difference type learning law are designed to estimate unknown constant parameters and time-varying parameter, respecitvely. The hyperbolic tangent function and a convergent series sequence are used to design robust control term for neural network approximation error and bounded disturbance, simultaneously guaranteeing the learning convergence along iteration. The system output is proved to converge to a small neighborhood of the desired trajectory by constructing Lyapnov-like composite energy function (CEF) containing new integral type Lyapunov function, while keeping all the closed-loop signals bounded. Finally, a simulation example is presented to verify the effectiveness of the proposed approach.**

***Index Terms*—Adaptive iterative learning control, fractional order nonlinear systems, Mittag-Leffler function, boundary layer function, composite energy function, fractional order differential learning law .**

I. Introduction

PAST decades have witnessed tremendous research efforts aiming at the development of systematic design methods for the iterative learning control (ILC) of nonlinear systems performing control task over a finite interval repeatedly. ILC has been proven to be the most suitable and effective control scheme for such repeatable control tasks owing to its capacity of achieving perfect tracking by learning along iteration. Generally, according to the stability analysis tool, ILC can be categorized into two classes: traditional ILC [1]−[5] and adaptive ILC (AILC) [6]−[16]. The basic idea of traditional ILC is to use information of the previous execution to design the control signal for current operation by a learning mechanism, which allows to achieve improving performance from iteration to iteration. Furthermore, the stability conclusion of traditional ILC is usually obtained by using contraction mapping theorem and fixed point theorem, which enables traditional ILC to deal with nonlinear plants without needing any information of the system. Traditional ILC has been developed greatly in theory and application because of its simplicity and availability. However, the main drawback of traditional ILC lies in the requirement of the global Lipschitz continuous condition, which restricts its application to certain nonlinear systems. Besides, the usage of contraction mapping theorem rather than Lyapunov method as the key tool of stability analysis in traditional ILC makes it difficult to relax the global Lipschitz condition to local Lipschitz or even non-Lipschitz condition and cooperate with the mainstream methods of nonlinear control theory, such as adaptive control and neural control. To overcome the constraints of traditional ILC, some researchers tried to introduce the idea of adaptive control into ILC and proposed adaptive iterative learning control (AILC) [6], [7]. AILC takes advantage of both adaptive control and ILC, which successfully overcomes the restriction of global Lipschitz condition, thus it enables us to use fuzzy logic systems or neural networks as approximators to deal with nonlinear uncertainties. In general, the control parameters of AILC methods are tuned along the iteration axis, and the so-called composite energy function (CEF) [8] is usually constructed to analyze the stability and convergence property of the closed-loop systems. The past decade has witnessed great progress in AILC of uncertain nonlinear systems [9]−[16].

Fractional calculus is a promising topic for more than 300 years. But the researches are mainly in the field of mathematical sciences [17], [18]. Until recent decade, the applications of fractional calculus develop rapidly [19], [20]. Fractional order systems allow us to describe and model a real object more accurately than the classical integer order dynamical systems. Among the investigations of fractional order systems in the past decades, control design for some fractional order systems has been a hot topic. Many different control methods have been proposed for various kinds of fractional systems [20]−[28]. Especially, the research on control and synchronization control design for fractional order chaotic systems is very active [29]−[39].

Comparing with such a large number of results, the papers

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on the ILC control of fractional order systems are relatively less. Only a few works are reported in the filed of ILC [40]-[53]. Moreover, all these literatures are from the viewpoint of traditional ILC and the stability conclusions are obtained by using contraction mapping theorem method. Therefore, as results of integer-order systems, global Lipschitz condition is required for traditional ILC schemes. As for AILC problem of fractional order systems, to the best of our knowledge, there are no results having been reported.

In this paper, we present an AILC scheme for a class of nonlinear fractional order system with both parametric and nonparametric uncertainties in the presence of disturbance. As far as we know, up till now no works have been presented for such a problem. In the proposed AILC scheme, adaptive iterative learning controller with fractional order differential type and difference type learning laws are presented and the CEF containing new integral type Lyapunov function is constructed to analyze the stability and convergence property. The main contributions of the proposed AILC scheme and stability analysis are highlighted as follows: 1) To the best of our knowledge, it is the first time, in the literature, that AILC problem of fractional order system is investigated. 2) A sliding mode surface of tracking errors is designed by using the sufficient condition for linear fractional order systems. 3) A new boundary layer function using Mittag-Leffler function is designed to deal with the initial condition problem in the ILC design of fractional order system. 4) Fractional order differential type learning laws with alignment method for unknown constant parameters is used in the AILC method for the first time and integral type Lyapunov function is used to analyze the convergence of estimation errors. 5) The hyperbolic tangent function is used to design robust control term for neural network approximation error and bounded disturbance, and a convergent series is introduced to guarantee the learning convergence along iteration index.

The rest of this paper is organized as follow. The problem formulation and preliminaries are given in Section II. The AILC design with parameter updating laws is developed in section III. In Section IV, the CEF-based stability analysis is presented. A simulation example is presented to verify the validity of the proposed scheme in Section V, followed by conclusion in Section VI.

II. Problem Formulation and Preliminaries

A. Preliminaries

In this subsection, some basic definitions and useful lemmas are given.

Definition 1 [18]: Fractional calculus is a generalization of integration and differentiation to noninteger-order fundamental operator ${}_aD_t^{\alpha}$, where a and t are the bounds of the operation and $\alpha \in \mathbf{R}$. The continuous integro-differential operator is defined as

$$ {}_aD_t^{\alpha} = \begin{cases} \frac{d^{\alpha}}{\mathrm{d}t^{\alpha}}, & \alpha > 0, \\ 1, & \alpha = 0, \\ \int_a^t (\mathrm{d}\sigma)^{\alpha}, & \alpha < 0. \end{cases} \tag{1} $$

Definition 2 [17]: The most important function used in fractional calculus-Euler's gamma function is defined as

$$ \Gamma(\alpha) = \int_0^{\infty} \mathrm{e}^{-\sigma} \sigma^{\alpha-1} \mathrm{d}\sigma. \tag{2} $$

Definition 3 [17]: Another important function in the fractional calculus named Mittag-Leffler type with two parameters is defined as

$$ E_{\alpha,\ \beta}(z) = \sum_{j=0}^{\infty} \frac{z^j}{\Gamma(\alpha j + \beta)}, \alpha > 0, \beta > 0. \tag{3} $$

Especially, when $\beta = 1$, we obtain the Mittag-Leffler function with one parameter

$$ E_{\alpha,\ 1}(z) = \sum_{j=0}^{\infty} \frac{z^j}{\Gamma(\alpha j + 1)} \triangleq E_{\alpha}(z). \tag{4} $$

For integer values of α, (4) reduces to the well-known Cauchy repeated integration formula.

The three most frequently used definitions for the general fractional differintegral are: The Grünwald-Letnikov (GL) definition, the Riemann-Liouville (RL) and Caputo definitions.

Definition 4 [17]: The Grünwald-Letnikov derivative definition of order α is described as

$$ {}_aD_t^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\infty} (-1)^j \left(\begin{array}{c} \alpha \\ j \end{array} \right) f(t - jh) \tag{5} $$

with

$$ \left(\begin{array}{c} \alpha \\ j \end{array} \right) = \frac{\alpha!}{j!(\alpha - j)!} = \frac{\Gamma(\alpha+1)}{\Gamma(j-1)\Gamma(\alpha - j + 1)}. \tag{6} $$

Definition 5 [54]: The Riemann-Liouville fractional integral of order α of function $f(t)$ at a time instant $t \geq 0$ is defined as:

$$ {}_aI_t^{\alpha} f(t) = {}_aD_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t f(\sigma)(t-\sigma)^{\alpha-1} \mathrm{d}\sigma. \tag{7} $$

From (7) we can write formula for the Riemann-Liouville definition of fractional derivative of order α in the following form

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_a^t \frac{f(\sigma)}{(t-\sigma)^{\alpha-n+1}} \mathrm{d}\sigma, \tag{8} $$

for $n-1 < \alpha < n$.

Definition 6 [17]: The Caputo fractional integral of order α of function $f(t)$ at time $t \geq 0$ is defined as

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\sigma)}{(t-\sigma)^{\alpha-n+1}} \mathrm{d}\sigma, \tag{9} $$

for $n-1 < \alpha < n$.

Remark 1: Actually, the above three definitions are equivalent under some conditions. We will use the Caputo definition in this paper. In the rest of this paper, the notation $D^{\alpha}(\cdot)$ indicates the Caputo derivative of order α with $a = 0$, i.e., $D^{\alpha}(\cdot) \triangleq {}_0D_t^{\alpha}(\cdot)$.

Lemma 1 [55], [56]: Consider the following fractional order autonomous system

$$ D^{\alpha} x(t) = Ax(t), x(0) = x_0, \tag{10} $$

where $0 < \alpha < 1$, $x \in \mathbf{R}^n$ and $A \in \mathbf{R}^{n\times n}$. This system is asymptotically stable if and only if $|\arg(\lambda(A))| > \alpha\frac{\pi}{2}$. In this situation, the components of the state vector decay toward zero like $t^{-\alpha}$.

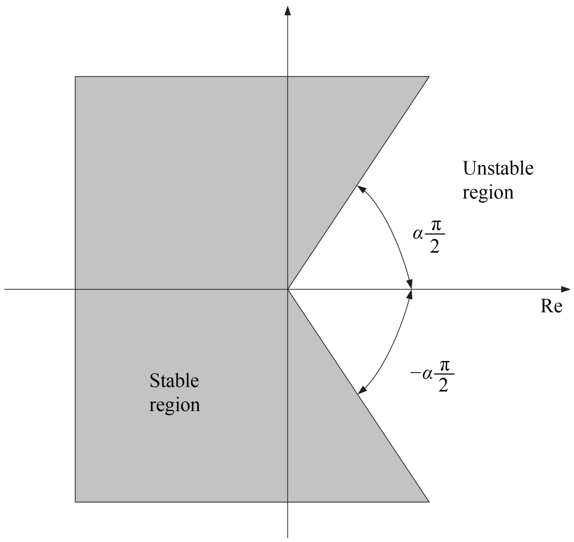

Fig. 1. Stability domain for fractional order linear systems with $0 < \alpha < 1$.

Lemma 1 [57]: The fractional system $D^\alpha y(t) = u(t)$, $0 < \alpha < 1$, is equivalent to the following continuous frequency distributed model

$$\begin{cases} \frac{\partial z(\omega,t)}{\partial t} = -\omega z(\omega,t) + u(t), \\ y(t) = \int_0^\infty \mu(\omega) z(\omega,t) \mathrm{d}\omega, \end{cases} \tag{11}$$

with weighting function $\mu(\omega) = \frac{\sin(\alpha\pi)}{\pi\omega^\alpha}$, $z(\omega,t) \in \mathbf{R}$.

B. Problem Formulation

In this paper, we consider a class of commensurate high-order uncertain nonlinear systems in the presence of disturbance which runs on a finite interval $[0,T]$ repeatedly given by

$$\begin{cases} D^\alpha x_{i,k}(t) = x_{i+1,k}(t), i = 1,\cdots,n-1, \\ D^\alpha x_{n,k}(t) = f(x_k(t)) + \theta(t)\xi(x_k(t)) + u_k(t) + d(t), \\ y_k(t) = x_{1,k}(t), \end{cases} \tag{12}$$

where $t \in [0,T]$ is the time, $k \in \mathbf{N}$ denotes the times of iteration; $x_{i,k}(t) \in \mathbf{R}$ $i = 1,\cdots,n$ and $y_k(t)$ are the pseudo state and output variables, respectively; $x_k(t) = \left[x_{1,k}(t), x_{2,k}(t), \cdots, x_{n,k}(t)\right]^{\mathrm{T}} \in \mathbf{R}^n$ is the pseudo state vector; $f(\cdot)$ is an unknown smooth function. $d(t)$ is unknown bounded external disturbance. $u_k(t) \in \mathbf{R}$ is the control input. The control objective of this paper is to design the adaptive iterative learning controller to steer the output $y_k(t)$ to follow the desired reference signal $r(t)$.

Define $r_1(t) = r(t)$ and $r_{i+1}(t) = D^\alpha r_i(t)$, $i = 1,2,\cdots,n-1$. Then we can write the desired reference vector as $x_d(t) = [r_1(t), r_2(t), \cdots, r_n(t)]^{\mathrm{T}}$. Define the tracking errors as $e_{i,k}(t) = x_{i,k}(t) - r_i(t)$, $i = 1,2,\cdots,n$. Then the tracking error vector can be given by $e_k(t) = x_k(t) - x_d(t) = \left[e_{1,k}(t), e_{2,k}(t), \cdots, e_{n,k}(t)\right]^{\mathrm{T}}$. In the rest of this paper, the denotation t will be omitted when no confusion arises.

Choose the sliding surface as $e_{s,k} = \left[\Lambda^{\mathrm{T}}\ 1\right] e_k$, where $\Lambda = [\lambda_1, \lambda_2, \cdots, \lambda_{n-1}]^{\mathrm{T}}$ and $\lambda_1, \cdots, \lambda_{n-1}$ are chosen suitably such that the eigenvalues of the matrix B satisfy condition of Lemma 1, where the matrix B is given by

$$B = \begin{bmatrix} 0 & & \\ \vdots & I_{n-2} & \\ 0 & & \\ -\lambda_1 & \cdots & -\lambda_{n-1} \end{bmatrix}, \tag{13}$$

with I_{n-2} as unit matrix of $n-2$ dimensions. Then keeping the system's errors on this surface leads to the asymptotic stability of error systems and therefore output tracking of the desired reference signal.

To facilitate control system design, the following reasonable assumptions are made.

Assumption 1: The unknown external disturbance is bounded.

Assumption 2: The desired state trajectory $x_d(t)$ is continuous, bounded and available.

Assumption 3: The initial state errors $e_{i,k}(0)$ at each iteration are not necessarily zero, small and fixed, but assumed to be bounded.

C. RBF Neural Networks

In control engineering, two types of artificial neural networks are usually used to approximate unknown smooth functions, which specifically are linearly parameterized neural networks (LPNNs) and multilayer neural networks (MNNs). As a kind of LPNNs, the radial basis function (RBF) neural network (NN) [58] is usually used as a tool to model unknown nonlinear functions owing to its nice approximation capabilities. The RBF NN can be seen as a two-layer network in which hidden layer performs a fixed nonlinear transformation with no adjustable parameters, i.e., the input space is mapped into a new space. The output layer then combines the outputs in the latter space linearly. Generally, the RBF NN approximates the continuous function $Q(Z): \mathbf{R}^q \to \mathbf{R}$ as follows

$$Q_{nn}(Z) = W^{\mathrm{T}}\phi(Z),$$

where $Z \in \Omega_Z \subset \mathbf{R}^q$ is the input vector, $W = [w_1, w_2, \cdots, w_l]^{\mathrm{T}} \in R^l$ is the weight vector, the NN node number $l > 1$; and $\phi(Z) = [\varphi_1(Z), \cdots, \varphi_l(Z)]^{\mathrm{T}}$, with $\varphi_i(Z)$ as the commonly used Gaussian functions, i.e., $\varphi_i(Z) = e^{-(Z-\mu_i)^{\mathrm{T}}(Z-\mu_i)/\sigma_i^2}$, $i = 1,\cdots,l$, where $\mu_i = [\mu_{i1}, \mu_{i2}, \cdots, \mu_{iq}]$ is the center of the receptive field and σ_i is the width of the Gaussian function. It has been proven that if l is chosen sufficiently large, $W^{\mathrm{T}}\phi(Z)$ can approximate any continuous function, $Q(Z)$, to any desired accuracy over a compact set $\Omega_Z \subset \mathbf{R}^q$ in the form of $Q(Z) = W^{*\mathrm{T}}\phi(Z) + \varepsilon(Z)$, $\forall Z \in \Omega_Z \subset \mathbf{R}^q$ where W^* is the ideal constant weight vector, and $\varepsilon(Z)$ is the approximation error which is bounded over the compact set, i.e., $|\varepsilon(Z)| \le \varepsilon^*$, $\forall Z \in \Omega_Z$, where $\varepsilon^* > 0$ is an unknown constant. The ideal weight vector W^* is an artificial quantity required for analytical purposes. W^* is defined as the value of W that minimizes $|\varepsilon(Z)|$ for all $Z \in \Omega_Z \subset \mathbf{R}^q$, i.e., $W^* := \arg\min_{W\in\mathbf{R}^l}\left\{\sup_{Z\in\Omega_Z}\left|h(Z) - W^{\mathrm{T}}\phi(Z)\right|\right\}$.

III. AILC Scheme Design

According to the systems dynamic equation (10) and definition of tracking errors, we can have the dynamics of tracking errors

$$\begin{cases} D^{\alpha}e_{i,k}(t) = & e_{i+1,k}(t), i=1,\cdots,n-1, \\ D^{\alpha}e_{n,k}(t) = & f(x_k(t))+\theta(t)\xi(x_k(t))+u_k(t) \\ & +d(t)-D^{\alpha}r_n. \end{cases} \tag{14}$$

By taking the derivative of order α of sliding surface, one has

$$\begin{aligned} D^{\alpha}e_{s,k} &= D^{\alpha}e_{n,k}+D^{\alpha}\left(\sum_{i=1}^{n-1}\lambda_i e_{i,k}\right) \\ &= D^{\alpha}e_{n,k}+\sum_{i=1}^{n-1}\lambda_i D^{\alpha}e_{i,k} \\ &= f(x_k(t))+\theta(t)\xi(x_k(t))+u_k(t)+d(t) \\ &\quad -D^{\alpha}r_n+\sum_{i=1}^{n-1}\lambda_i e_{i+1,k}. \end{aligned} \tag{15}$$

According to Assumption 3, there exist known constants ε_i, such that, $\left|e_{i,k}(0)\right|\le\varepsilon_i$, $i=1,2,\cdots,n$ for any $k\in\mathbf{N}$. In order to overcome the uncertainty from initial resetting errors, we define a novel boundary layer function by employing Mittag-Leffler function

$$\eta(t)=\varepsilon E_{\alpha}(-Kt), K>0, \tag{16}$$

where $\varepsilon=\left[\Lambda^{\mathrm{T}}\ 1\right][\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_n]^{\mathrm{T}}$.

Remark 2: As the boundary layer function[13−15] in integer order case, $\eta(t)$ has good property of decreasing along time axis with initial condition $\eta(0)=\varepsilon$. Moreover, it is clear that $D^{\alpha}\eta(t)=\varepsilon D^{\alpha}E_{\alpha}(-Kt)=-K\varepsilon E_{\alpha}(-Kt)=-K\eta(t)$.

Then we can define an auxiliary error signal as

$$s_k(t)=e_{s,k}(t)-\eta(t)\mathrm{sat}\left(\frac{e_{s,k}(t)}{\eta(t)}\right), \tag{17}$$

where $\mathrm{sat}(\cdot)$ is the saturation function which is defined as

$$\mathrm{sat}(\cdot)=\mathrm{sgn}(\cdot)\cdot\min\{|\cdot|,1\}, \tag{18}$$

with $\mathrm{sgn}(\cdot)=\begin{cases} 1, \text{if } \cdot>0 \\ 0, \text{if } \cdot=0 \\ -1, \text{if } \cdot<0 \end{cases}$ as the sign function.

Subsequently, it can be easily obtained that

$$\begin{aligned} |e_{sk}(0)| &= \left|\lambda_1 e_{1,k}(0)+\lambda_2 e_{2,k}(0)+\cdots+e_{n,k}(0)\right| \\ &\le \lambda_1\left|e_{1,k}(0)\right|+\lambda_2\left|e_{2,k}(0)\right|+\cdots+\left|e_{n,k}(0)\right| \\ &\le \lambda_1\varepsilon_1+\lambda_2\varepsilon_2+\cdots+\varepsilon_n=\eta(0), \end{aligned} \tag{19}$$

which implies that $s_k(0)=e_{sk}(0)-\eta(0)\frac{e_{sk}(0)}{\eta(0)}=0$ is satisfied for all $k\in\mathbf{N}$. Moreover, there exists the fact that

$$\begin{aligned} s_k(t)\,\mathrm{sat}\left(\frac{e_{sk}(t)}{\eta(t)}\right) &= \begin{cases} 0, & \text{if } \left|\frac{e_{sk}(t)}{\eta(t)}\right|\le 1 \\ s_k(t)\,\mathrm{sgn}(e_{sk}(t)), & \text{if } \left|\frac{e_{sk}(t)}{\eta(t)}\right|>1 \end{cases} \\ &= s_k(t)\,\mathrm{sgn}(s_k(t))=|s_k(t)|. \end{aligned} \tag{20}$$

To overcome the design difficulty from uncertainty $f(x_k(t))$, we employ radial basis function neural network to approximate $f(x_k(t))$ in the form of

$$f(x_k(t))=W^{*\mathrm{T}}\phi(x_k)+\varepsilon(x_k). \tag{21}$$

From Lemma 2, we can obtain the equivalent continuous frequency distributed model of dynamical system of s_k

$$\begin{cases} \frac{\partial z_k(\omega,t)}{\partial t}=-\omega z_k(\omega,t)+D^{\alpha}s_k, \\ s_k(t)=\int_0^{\infty}\mu(\omega)z_k(\omega,t)\mathrm{d}\omega, \end{cases} \tag{22}$$

with weighting function $\mu(\omega)=\frac{\sin(\alpha\pi)}{\pi\omega^{\alpha}}$, $z_k(\omega,t)\in\mathbf{R}$ is the true error variable.

Define a smooth scalar positive function as

$$V_{s,k}(t)=\frac{1}{2}\int_o^{\infty}\mu(\omega)z_k^2(\omega,t)\mathrm{d}\omega. \tag{23}$$

The time derivative of $V_{s,k}(t)$ can be expressed as

$$\begin{aligned} \dot{V}_{s,k}(t) &= \int_0^{\infty}\mu(\omega)z_k(\omega,t)\frac{\partial z_k(\omega,t)}{\partial t}\mathrm{d}\omega \\ &= \int_0^{\infty}\mu(\omega)z_k(\omega,t)(-\omega z_k(\omega,t)+D^{\alpha}s_k)\mathrm{d}\omega \\ &= -\int_0^{\infty}\mu(\omega)\omega z_k^2(\omega,t)\mathrm{d}\omega+s_kD^{\alpha}s_k \\ &= \begin{cases} -\int_0^{\infty}\mu(\omega)\omega z_k^2(\omega,t)\mathrm{d}\omega \\ +s_k\left(D^{\alpha}e_{s,k}-D^{\alpha}\eta(t)\,\mathrm{sgn}(s_k)\right), \left|e_{s,k}\right|>\eta(t) \\ -\int_0^{\infty}\mu(\omega)\omega z_k^2(\omega,t)\mathrm{d}\omega, \left|e_{s,k}\right|\le\eta(t) \end{cases} \\ &= -\int_0^{\infty}\mu(\omega)\omega z_k^2(\omega,t)\mathrm{d}\omega+s_k\left(D^{\alpha}e_{s,k}-D^{\alpha}\eta(t)\,\mathrm{sgn}(s_k)\right) \\ &= -\int_0^{\infty}\mu(\omega)\omega z_k^2(\omega,t)\mathrm{d}\omega+s_k\big(W^{*\mathrm{T}}\phi(x_k)+\varepsilon(x_k) \\ &\quad +\theta(t)\xi(x_k)+u_k+d(t)-D^{\alpha}r_n+\sum_{i=1}^{n-1}\lambda_i e_{i+1,k} \\ &\quad +K\eta(t)\,\mathrm{sgn}(s_k)\big) \\ &= -\int_0^{\infty}\mu(\omega)\omega z_k^2(\omega,t)\mathrm{d}\omega+s_k\big(W^{*\mathrm{T}}\phi(x_k)+\varepsilon(x_k) \\ &\quad +\theta(t)\xi(x_k)+u_k+d(t)-D^{\alpha}r_n+\sum_{i=1}^{n-1}\lambda_i e_{i+1,k} \\ &\quad -Ke_{s,k}+Ke_{s,k}+K\eta(t)\,\mathrm{sgn}(s_k)\big) \\ &= -\int_0^{\infty}\mu(\omega)\omega z_k^2(\omega,t)\mathrm{d}\omega+s_k\Big(W^{*\mathrm{T}}\phi(x_k)+\theta(t)\xi(x_k) \\ &\quad +u_k+\bar{d}(t)-D^{\alpha}r_n+\sum_{i=1}^{n-1}\lambda_i e_{i+1,k}+Ke_{s,k}\Big)-Ks_k^2, \end{aligned} \tag{24}$$

where $\bar{d}(t)=d(t)+\varepsilon(x_k)$ and using the equality

$$\begin{aligned} & s_k(t)(-Ke_{sk}(t)+K\eta(t)\,\mathrm{sgn}(s_k(t))) \\ &= s_k(t)\left(-Ks_k(t)-K\eta(t)\,\mathrm{sat}\left(\frac{e_{sk}(t)}{\eta(t)}\right)\right. \\ &\quad \left.+K\eta(t)\,\mathrm{sgn}(s_k(t))\right) \\ &= -Ks_k^2(t)-K\eta(t)|s_k(t)|+K\eta(t)|s_k(t)| \\ &= -Ks_k^2(t). \end{aligned} \tag{25}$$

Obviously, $\bar{d}(t)$ is bounded, i.e., there exists an unknown positive constant ρ such that $\left|\bar{d}(t)\right|\le\rho$.

Then we can determine the control law as

$$\begin{aligned}u_k(t)=&D^{\alpha}r_n-\sum_{i=1}^{n-1}\lambda_i e_{i+1,k}-Ke_{s,k}-\hat{W}_k^{\mathrm{T}}\phi(x_k)\\&-\hat{\theta}_k(t)\xi(x_k)-\hat{\rho}_k\tanh\left(\frac{s_k\hat{\rho}_k}{\Delta_k}\right),\end{aligned}\tag{26}$$

where $\hat{W}_k$, $\hat{\theta}_k(t)$ and $\hat{\rho}_k$ are the estimates of W^*, $\theta(t)$ and ρ, respectively, Δ_k is a convergent series sequence which is defined as $\Delta_k=\frac{q}{k^m}$, l and q are constant design parameters and $q(\in\mathbf{R})>0$, $m(\in\mathbf{Z}_+)\geq 2$. For preceding analysis, we need the following lemmas.

Lemma 3 [59]: For any $\Delta_k>0$ and $x\in\mathbf{R}$, the inequality $|x|-x\tanh(x/\Delta_k)\leq\gamma\Delta_k$ is established, where γ is a positive constant and $\gamma=\mathrm{e}^{-(\gamma+1)}$ or $\gamma=0.2785$.

Lemma 4 [60]: $\lim_{k\to\infty}\sum_{j=1}^{k}\Delta_j\leq 2q$.

The adaptive learning laws for unknown parameters are designed as

$$\begin{cases}D^{\alpha}\hat{W}_k(t)=\Gamma_W s_k(t)\phi(x_k),\\ \hat{W}_k(0)=\hat{W}_{k-1}(T),\hat{W}_1(0)=0,\end{cases}\tag{27}$$

$$\begin{cases}\hat{\theta}_k(t)=\hat{\theta}_{k-1}(t)+q_{\theta}s_k(t)\xi(x_k),\\ \hat{\theta}_0(t)=0,t\in[0,T],\end{cases}\tag{28}$$

$$\begin{cases}D^{\alpha}\hat{\rho}_k(t)=q_{\rho}|s_k(t)|,\\ \hat{\rho}_k(0)=\hat{\rho}_{k-1}(T),\hat{\rho}_1(0)=0,\end{cases}\tag{29}$$

where $\Gamma_W\in\mathbf{R}^{l\times l}$ is a positive square matrix and $q_{\theta},q_{\rho}>0$ are design parameters. In the following parts, we define the estimation error of $\Theta(t)$ as $\tilde{\Theta}_k(t)=\hat{\Theta}_k(t)-\Theta(t)$ where $\Theta(t)$ denotes W^*, $\theta(t)$ and ρ.

Substituting the controller (26) back into (24) yields

$$\begin{aligned}\dot{V}_{s,k}(t)\leq&-\int_0^{\infty}\mu(\omega)\omega z_k^2(\omega,t)\mathrm{d}\omega-s_k\tilde{W}_k^{\mathrm{T}}\phi(x_k)\\&-s_k\tilde{\theta}_k(t)\xi(x_k)+|s_k|\rho-|s_k|\hat{\rho}_k\\&+|s_k|\hat{\rho}_k-s_k\hat{\rho}_k\tanh\left(\frac{s_k\hat{\rho}_k}{\Delta_k}\right)-Ks_k^2\\\leq&-\int_0^{\infty}\mu(\omega)\omega z_k^2(\omega,t)\mathrm{d}\omega-s_k\tilde{W}_k^{\mathrm{T}}\phi(x_k)\\&-s_k\tilde{\theta}_k(t)\xi(x_k)-|s_k|\tilde{\rho}_k+\gamma\Delta_k-Ks_k^2\\\leq&-s_k\tilde{W}_k^{\mathrm{T}}\phi(x_k)-s_k\tilde{\theta}_k(t)\xi(x_k)\\&-|s_k|\tilde{\rho}_k+\gamma\Delta_k-Ks_k^2.\end{aligned}\tag{30}$$

From adaptive updating laws (27) and (29) it follows

$$D^{\alpha}\tilde{W}_k=D^{\alpha}\hat{W}_k-D^{\alpha}W^*=D^{\alpha}\hat{W}_k,\tag{31}$$

$$D^{\alpha}\tilde{\rho}_k=D^{\alpha}\hat{\rho}_k-D^{\alpha}\rho=D^{\alpha}\hat{\rho}_k.\tag{32}$$

According to Lemma 2, we can obtain the distributed frequency model of (31) and (32) as follows

$$\begin{cases}\frac{\partial z_{W,k}(\omega,t)}{\partial t}=-\omega z_{W,k}(\omega,t)+\Gamma_W s_k(t)\phi(x_k),\\ \tilde{W}_k(t)=\int_0^{\infty}\mu(\omega)z_{W,k}(\omega,t)\mathrm{d}\omega,\end{cases}\tag{33}$$

$$\begin{cases}\frac{\partial z_{\rho,k}(\omega,t)}{\partial t}=-\omega z_{\rho,k}(\omega,t)+q_{\rho}|s_k(t)|,\\ \tilde{\rho}_k(t)=\int_0^{\infty}\mu(\omega)z_{\rho,k}(\omega,t)\mathrm{d}\omega,\end{cases}\tag{34}$$

where $z_{W,k}(\omega,t)\in\mathbf{R}^l$ and $z_{\rho,k}(\omega,t)\in\mathbf{R}$ are the true estimation error variables.

Define a positive scalar positive function of parameter estimation errors as

$$\begin{aligned}V_{p,k}(t)=&\frac{1}{2}\int_o^{\infty}\mu(\omega)z_{W,k}^{\mathrm{T}}(\omega,t)\Gamma_W^{-1}z_{W,k}(\omega,t)\mathrm{d}\omega\\&+\frac{1}{2q_{\rho}}\int_o^{\infty}\mu(\omega)z_{\rho,k}^2(\omega,t)\mathrm{d}\omega.\end{aligned}\tag{35}$$

Taking the time derivative of $V_{p,k}(t)$ results in

$$\begin{aligned}\dot{V}_{p,k}(t)=&\int_o^{\infty}\mu(\omega)z_{W,k}^{\mathrm{T}}(\omega,t)\Gamma_W^{-1}\frac{\partial z_{W,k}(\omega,t)}{\partial t}\mathrm{d}\omega\\&+\frac{1}{q_{\rho}}\int_o^{\infty}\mu(\omega)z_{\rho,k}(\omega,t)\frac{\partial z_{\rho,k}(\omega,t)}{\partial t}\mathrm{d}\omega\\=&-\int_o^{\infty}\mu(\omega)\omega z_{W,k}^{\mathrm{T}}(\omega,t)\Gamma_W^{-1}z_{W,k}(\omega,t)\mathrm{d}\omega\\&+\Gamma_W^{-1}D^{\alpha}\tilde{W}_k^{\mathrm{T}}\int_o^{\infty}\mu(\omega)z_{W,k}(\omega,t)\mathrm{d}\omega\\&-\frac{1}{q_{\rho}}\int_o^{\infty}\mu(\omega)\omega z_{\rho,k}^2(\omega,t)\mathrm{d}\omega\\&+\frac{1}{q_{\rho}}D^{\alpha}\tilde{\rho}_k\int_o^{\infty}\mu(\omega)z_{\rho,k}(\omega,t)\mathrm{d}\omega\\\leq&s_k\tilde{W}_k^{\mathrm{T}}\phi(x_k)+|s_k|\tilde{\rho}_k.\end{aligned}\tag{36}$$

Define a Lyapunov candidate as $V_k(t)=V_{s,k}(t)+V_{p,k}(t)$. Hence, we can obtain the derivative of $V_k(t)$ with respect to time by combining (30) and (36)

$$\dot{V}_k\leq-Ks_k^2-s_k\tilde{\theta}_k(t)\xi(x_k)+\gamma\Delta_k.\tag{37}$$

IV. Analysis of Stability and Convergence

In this section, we will prove that the controller can guarantee the stability of the closed-loop system and the convergence of tracking errors.

The stability of the proposed AILC scheme is summarized as follows.

Theorem 1: Considering the fractional order system (12), and designing adaptive iterative learning controller (26) and with parameter adaptive learning algorithms (27)−(29), the following properties can be guaranteed: 1) all the signals of the closed-loop system are bounded; 2) the pseudo error variable $s_k(t)$ converges to zero as $k\to\infty$, i.e., $\lim_{k\to\infty}\int_0^T(s_k(\sigma))^2\mathrm{d}\sigma\leq 0$.

Proof: Define the Lyapunov-like CEF as

$$E_k(t)=V_k(t)+\frac{1}{2q_{\theta}}\int_0^t\tilde{\theta}_k^2(\sigma)\mathrm{d}\sigma.\tag{38}$$

The proof includes four parts.

1) Difference of $E_k(t)$

Compute the difference of $E_k(t)$, which is

$$\begin{aligned}\Delta E_k(t)=&E_k(t)-E_{k-1}(t)\\=&V_k(t)-V_{k-1}(t)\\&+\frac{1}{2q_{\theta}}\int_0^t\left[\tilde{\theta}_k^2(\sigma)-\tilde{\theta}_{k-1}^2(\sigma)\right]\mathrm{d}\sigma.\end{aligned}\tag{39}$$

Considering (37), one has

$$V_k(t) \le V_k(0) + \int_0^t \left(-Ks_k^2 - s_k\tilde{\theta}_k(t)\xi(x_k) + \gamma\Delta_k\right)\mathrm{d}\sigma$$
$$= V_{p,k}(0) - K\int_0^t s_k^2 \mathrm{d}\sigma - \int_0^t s_k\tilde{\theta}_k(\sigma)\xi(x_k)\mathrm{d}\sigma + \gamma\Delta_k t. \quad (40)$$

Utilizing the algebraic relation $(\boldsymbol{a}-\boldsymbol{b})^{\mathrm{T}}(\boldsymbol{a}-\boldsymbol{b}) - (\boldsymbol{a}-\boldsymbol{c})^{\mathrm{T}}(\boldsymbol{a}-\boldsymbol{c}) = (\boldsymbol{c}-\boldsymbol{b})^{\mathrm{T}}[2(\boldsymbol{a}-\boldsymbol{b})+(\boldsymbol{b}-\boldsymbol{c})]$ and taking the adaptive learning laws (28) into consideration, we have

$$\frac{1}{2q_\theta}\int_0^t \left[\tilde{\theta}_k^2(\sigma) - \tilde{\theta}_{k-1}^2(\sigma)\right]\mathrm{d}\sigma$$
$$= \int_0^t s_k\tilde{\theta}_k(\sigma)\xi(x_k)\mathrm{d}\sigma - \frac{q_\theta}{2}\int_0^t s_k^2(\sigma)\xi^2(x_k)\mathrm{d}\sigma. \quad (41)$$

Substituting (40) and (41) back into (39), it follows that

$$\Delta E_k(t) \le V_{p,k}(0) - V_{k-1}(t) - K\int_0^t s_k^2 \mathrm{d}\sigma + \gamma\Delta_k t. \quad (42)$$

Let $t=T$ in (42). From the adaptive parameter updating laws we know $V_{p,k}(0) = V_{p,k-1}(T)$. Therefore, it follows from (42) that

$$\Delta E_k(T) \le V_{p,k}(0) - V_{p,k-1}(T) - V_{s,k-1}(T)$$
$$- K\int_0^t s_k^2 \mathrm{d}\sigma + \gamma\Delta_k T$$
$$\le -K\int_0^t s_k^2 \mathrm{d}\sigma + \gamma\Delta_k T. \quad (43)$$

2) The finiteness of $E_k(T)$

According to (38), we know

$$E_1(t) = V_1(t) + \frac{1}{2q_\theta}\int_0^t \tilde{\theta}_1^2(\sigma)\mathrm{d}\sigma. \quad (44)$$

Recalling adaptive updating law (28), we can have $\hat{\theta}_1(t) = q_\theta s_1(t)\xi(x_1)$, which leads to time derivative of $E_1(t)$ as follows

$$\dot{E}_1(t) = \dot{V}_1(t) + \frac{1}{2q_\theta}\tilde{\theta}_1^2(t)$$
$$\le -Ks_1^2 - s_1\tilde{\theta}_1(t)\xi(x_1) + \gamma\Delta_1$$
$$+ \frac{1}{2q_\theta}\left(\tilde{\theta}_1^2(t) - 2\tilde{\theta}_1(t)\hat{\theta}_1(t)\right) + \frac{1}{q_\theta}\tilde{\theta}_1(t)\hat{\theta}_1(t)$$
$$= -Ks_1^2 - s_1\tilde{\theta}_1(t)\xi(x_1) + \gamma\Delta_1 + s_1\tilde{\theta}_1(t)\xi(x_1)$$
$$+ \frac{1}{2q_\theta}\left[\left(\hat{\theta}_1(t) - \theta(t)\right)^2 - 2\left(\hat{\theta}_1(t) - \theta(t)\right)\hat{\theta}_1(t)\right]$$
$$\le -Ks_1^2 + \gamma\Delta_1 + \frac{1}{2q_\theta}\theta^2(t). \quad (45)$$

Denote $c = \max_{t\in[0,T]}\left\{\theta^2(t)/(2q_\theta)\right\}$. Integrating the above inequality over $[0,t]$ yields

$$E_1(t) - E_1(0) \le -K\int_0^t s_1^2(\sigma)\mathrm{d}\sigma + t\cdot c + \theta\Delta_1 t. \quad (46)$$

According to the adaptive updating laws it is clear that $E_1(0) = V_{p,1}(0)$, which is determined by W^* and ρ. Thus the boundedness of $E_1(t)$ can be ensured since

$$E_1(t) \le -K\int_0^t s_1^2(\sigma)\mathrm{d}\sigma + t\cdot c + \theta\Delta_1 t + V_{p,1}(0), t\in[0,T]. \quad (47)$$

Letting $t=T$ in (47), we can obtain the boundedness of $E_1(T)$

$$E_1(T) \le -K\int_0^T s_1^2(\sigma)\mathrm{d}\sigma + T(c+\gamma\Delta_1) + V_{p,1}(0)$$
$$< \infty. \quad (48)$$

Applying (43) repeatedly, we may have

$$E_k(T) = E_1(T) + \sum_{j=2}^k \Delta E_j(T)$$
$$\le -K\sum_{j=1}^k\int_0^T s_j^2(\sigma)\mathrm{d}\sigma + T\cdot c_{\max} + \gamma T\sum_{j=1}^k \Delta_k + V_{p,1}(0)$$
$$\le T\cdot c_{\max} + \gamma T\sum_{j=1}^k \Delta_k + V_{p,1}(0). \quad (49)$$

Recalling Lemma 4 we have $\gamma T\sum_{j=1}^k \Delta_k \le \lim_{k\to\infty}\gamma T\sum_{j=1}^k \Delta_k \le 2\gamma T q$, which further implies the boundedness of $E_k(T)$.

3) The finiteness of $E_k(t)$

Next we will prove the boundedness of $E_k(t)$ by induction. The boundedness of $E_k(T)$ is guaranteed for all iterations. Consequently, $\forall k\in\mathbf{N}$, there exists a constant M_1 satisfying $\int_0^{\mathrm{T}}\tilde{\theta}_k^2(\sigma)\mathrm{d}\sigma \le M_1$, thus it follows

$$E_k(t) = V_k(t) + \int_0^t \tilde{\theta}_k^2(\sigma)\mathrm{d}\sigma$$
$$\le V_k(t) + \int_0^T \tilde{\theta}_k^2(\sigma)\mathrm{d}\sigma$$
$$\le V_k(t) + M_1. \quad (50)$$

On the other hand, from (42), we obtain

$$\Delta E_{k+1}(t) \le V_{p,k+1}(0) - V_k(t) - K\int_0^t s_{k+1}^2 d\sigma + \gamma\Delta_{k+1}t. \quad (51)$$

Adding (51) to (50) leads to

$$E_{k+1}(t) = E_k(t) + \Delta E_{k+1}(t)$$
$$\le V_k(t) + M_1 + V_{p,k+1}(0) - V_k(t)$$
$$- K\int_0^t s_{k+1}^2\mathrm{d}\sigma + \gamma\Delta_{k+1}t$$
$$\le M_1 + V_{p,k}(T) + \gamma\Delta_{k+1}t. \quad (52)$$

As we have proven that $E_1(t)$ is bounded, therefore $E_k(t)$ is finite by induction. In the sequel, we can obtain the boundedness of $\hat{W}_k(t)$, $\hat{\theta}_k(t)$ and $\hat{\rho}_k$.

4) Learning convergence property

Rewrite inequality (49) as

$$\sum_{j=1}^k\int_0^T s_j^2(\sigma)\mathrm{d}\sigma$$
$$\le \frac{\left(T\cdot c_{\max} + \gamma T\sum\limits_{j=1}^k \Delta_k + V_{p,1}(0) - E_k(T)\right)}{K}. \quad (53)$$

Taking the limitation of (53), it follows that

$$\begin{aligned}&\lim_{k\to\infty}\sum_{j=1}^{k}\int_{0}^{T}s_j^2(\sigma)\mathrm{d}\sigma\\&\leq\lim_{k\to\infty}\left(T\cdot c_{\max}+\gamma T\sum_{j=1}^{k}\Delta_k+V_{p,1}(0)-E_k(T)\right)/K\\&\leq T\cdot c_{\max}+2\gamma qT+V_{p,1}(0).\end{aligned}\tag{54}$$

According to the convergence theorem of the sum of series, $\lim_{k\to\infty}\int_0^T s_k^2(\sigma)\mathrm{d}\sigma=0$. Since x_d is bounded, the boundedness of x_k is established. Based on the above reasoning, we can arrive at that $u_k(t)$ is bounded. □

V. Simulation Study

In this section, a simulation study is presented to verify the effectiveness of the AILC scheme. Consider the following second-order nonlinear fractional order system:

$$\begin{cases}D^{\alpha}x_{1,k}(t)=x_{2,k}(t),\\D^{\alpha}x_{2,k}(t)=f(x_k)+\theta(t)\xi(x_k)+u_k(t)+d(t),\\y_k(t)=x_{1,k}(t),\end{cases}$$

where $\alpha=0.9$, $f(x_k)=-x_{1,k}x_{2,k}\sin\left(x_{1,k}x_{2,k}\right)$, $\theta(t)=1+0.5\sin t$, $\xi(x_k)=\sin\left(x_{1,k}\right)\cos\left(x_{2,k}\right)$, $d(t)=0.1*rand*\sin(t)$ with *rand* presenting a Gaussian white noise. The desired reference trajectory is given by $r(t)=\sin t$. The design parameters are chosen as $\varepsilon_1=\varepsilon_2=1$, $\lambda=2$, $K=6$, $\Gamma_W=\mathrm{diag}\{0.6\}$, $q_\theta=2$, $q_\rho=0.8$, $\varepsilon=\lambda\varepsilon_1+\varepsilon_2=3$. It is clear that $|\lambda|>\frac{\alpha}{2}\pi$. Additionally, the boundary layer function is specified by $\eta(t)=3E_{0.9}(-Kt)$, a graphic representation of $\eta(t)$ is shown in Fig. 2.

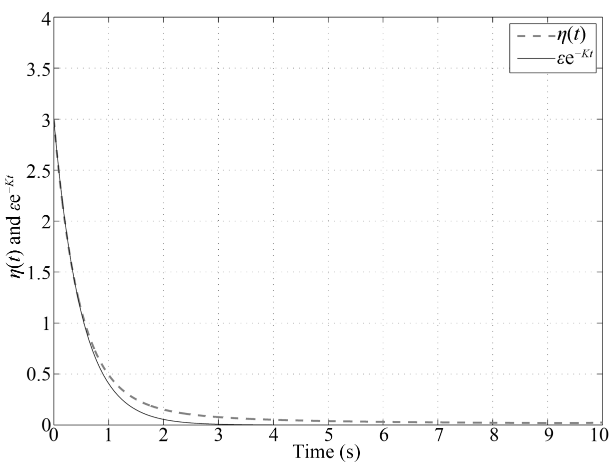

Fig. 2. Mittag-Leffler type boundary layer function $\eta(t)$.

The parameters for neural network are chosen as $l=30$, $\mu_j=\frac{1}{l}(2j-l)[2,3]$, $\sigma_j=2$, $j=1,2,\cdots,l$. The initial condition $x_{1,k}(0)$ and $x_{2,k}(0)$ are randomly taken as $r(0)+0.5(1-2rand)$ and $r_1(0)+0.5(1-2rand)$, respectively. For ease of programming, we use the Grünwald-Letnikov definition in the simulation. The system runs on $[0,2\pi]$ repeatedly. Parts of the simulation results are shown in Fig. 3 ∼ Fig. 7.

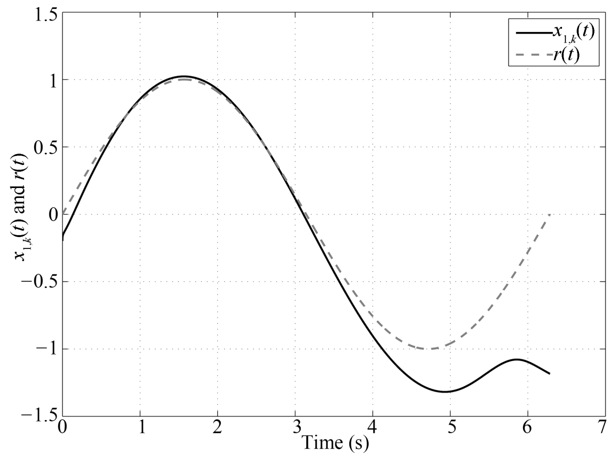

Fig. 3. System output $y_k(t)$ on $r(t)$ $(k=1)$.

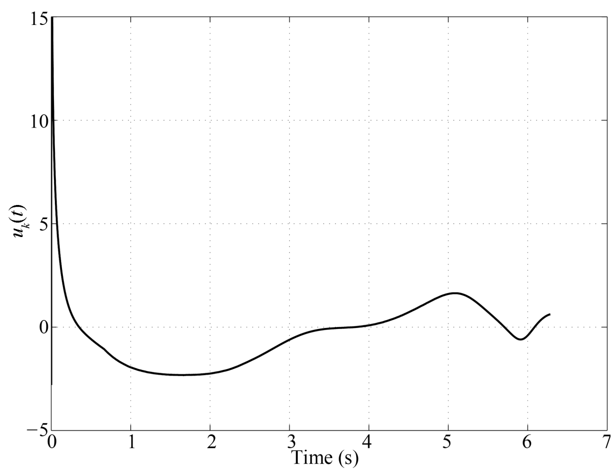

Fig. 4. Control input $(k=1)$.

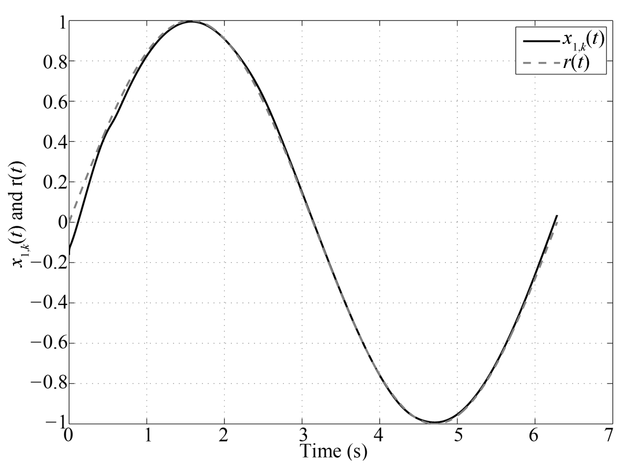

Fig. 5. System output $y_k(t)$ on $r(t)$ $(k=30)$.

Figs. 3 ∼ 4 and Figs. 5 ∼ 6 show the output tracking trajectory and control input of the 1st and the 30th iteration. Obviously, the signals are bounded and the tracking performance of 1st iteration is much worse than that of 30th iteration. Fig. 7 gives the convergence of $\int_0^{\mathrm{T}} s_k^2(t)\,\mathrm{d}t$ along the iteration axis, which indicates that the proposed AILC scheme achieves perfect tracking by learning.

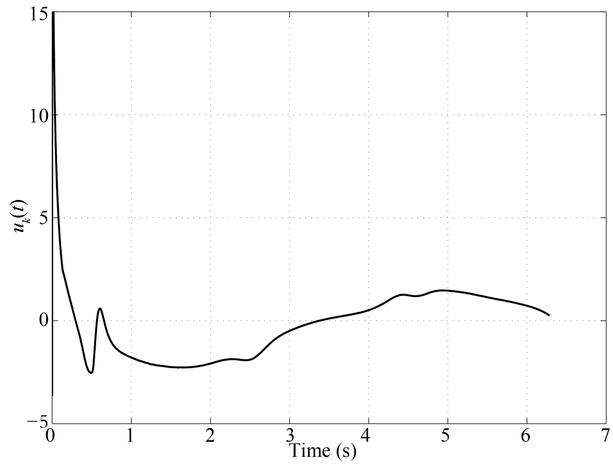

Fig. 6. Control input ($k = 30$).

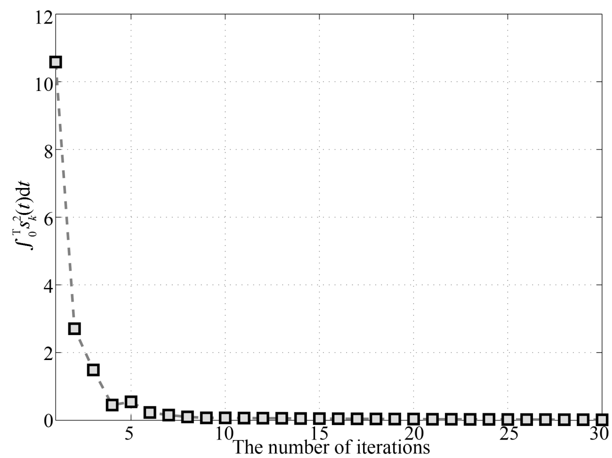

Fig. 7. $\int_0^T s_k^2(t)\,\mathrm{d}t$ versus the number of iterations.

VI. Conclusions

In this paper, an adaptive iterative learning control scheme has been presented for a class of nonlinear fractional order systems in the presence of disturbance. A new boundary layer function by introducing Mittag-Leffler function is designed to deal with the initial condition problem of ILC. RBF NN is utilized to approximate the system uncertainty while fractional order differential type updating laws are designed to estimate ideal neural weight and the upper bound of neural approximation error and disturbance. The hyperbolic tangent function with a convergent series sequence is employed to form the robust control term. Theoretical analysis by constructing Lyapunov-like CEF has been presented to show the boundedness of all signals and convergence along iteration of tracking error. Simulation results have been provided to show the validity the proposed scheme. This is the first time consideration of the AILC problem of fractional order system. Compared with traditional ILC of fractional systems, our AILC scheme relaxes the global Lipschitz condition and a new framework of stability analysis by using Lyapunov-like CEF is presented. Although we only consider the class of fraction as (12), the idea of the proposed AILC method can be applied to more kinds of fractional order systems and provide a reference for AILC design of fractional order systems.

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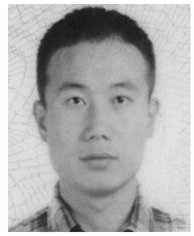

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