UCMERCED MESA LAB

Fractional order PID control: better than the best issue and what's next

YangQuan Chen, Ph.D., Director,
MESA(**Mechatronics, Embedded Systems and Automation**)LAB
ME/EECS/SNRI/HSRI/CITRIS, School of Engineering,
University of California, Merced
E: yqchen@ieee.org; *or*, yangquan.chen@ucmerced.edu
T: (209)228-4672; **O**: SE2-273; **Lab**: Castle #22 (**T**: 228-4398)

May 11, 2018. 11-12am, The Global
IFAC PID2018, Ghent, Belgium

Acknowledgements

- **Karl Astrom – model reference in control works**
- **IFAC PID2018 Organizing Committee – Well done!**
  - **Clara, Antonio, Robin …**
- **Other Invited Inspiring Speakers**
- **You all, for coming !**

Skip Ad in a *fractional* hour

University of California, Merced

- The Research University of the Central Valley
- Centrally Located
  - Sacramento – 2 hrs
  - San Fran. – 2 hrs
  - Yosemite – 1.5 hrs
  - LA – 4 hrs
- Surrounded by farmlands and sparsely populated areas

UC Merced

- Established 2005
- 1st research university in 21st century in USA.
- 7,967 Undergraduates
- 592 Grads (mostly Ph.D)
- 233 faculty, 159 lecturers

- Strong Undergraduate Research Presence (HSI, MSI)

2017 U.S. News and World Report Rankings

Campus	Public	National
UC Berkeley	1	20
UCLA	2	24
UC Santa Barbara	8	37
UC Irvine	9	39
UC Davis	10	44
UC San Diego	10	44
UC Santa Cruz	30	79
UC Riverside	56	118
UC Merced	**78**	**152**

https://www.universityofcalifornia.edu/news/6-uc-campuses-named-among-nation-s-top-10-public-universities

UCMERCED w: mechatronics.ucmerced.edu MESA LAB

Dr. YangQuan Chen, yqchen@ieee.org

Mechatronics, Embedded Systems and Automation Lab

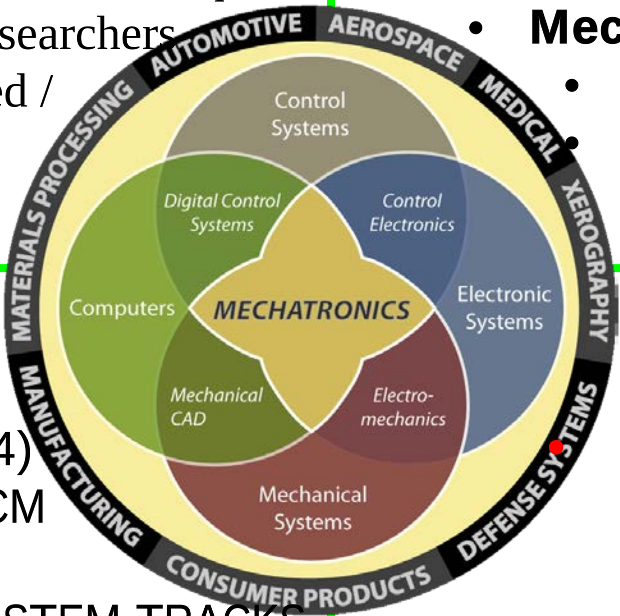
Real solutions for sustainability!

Established Aug. 2012 @ Castle, 4,500+ sq ft
6+2 Ph.D/10+ undergraduate researchers
10+ visiting scholars || sponsored / mentored many capstone teams

Research Areas of Excellence:

(ISI H-index=45, Google H-index=70; i10-index=367)

- Unmanned Aerial Systems & UAV-based Personal Remote Sensing (PRS)
- **Cyber-Physical Systems (CPS)**
  - **Mechatronics**
    - **Applied Fractional Calculus**
    - **Modeling and Control of Renewable Energy Systems**



Education and Outreach Activities:

- Eng Service Learning(Sp14)
- AIAA Student Branch @UCM
- Preview Days, Bobcat Day
- Robots-n-Ribs| MESABox! STEM-TRACKS TEAM-E
- UAS4STEM. USDA/NIFA HSI: 2016-2020
- **ME142 Mechatronics (**take-home labs**)**
- **ME280 Fractional Order Mechanics**
- **ME211 Nonlinear Control**
- **ME143 Unmanned Aerial Systems**

Projects Related to San Joaquin Valley:

- **Energy** [Solar/wind energy, Building efficiency (HVAC lighting), smart grids integration, NG pipelines]
- **Water** (Water/soil salinity management, water sampling UAVs)
- **Precision Ag / Environment** (Crop dynamics, optimal harvesting, pest, methane sniffing/mapping, DH ...)

Outline

- **Fractional calculus: What, Why and When**
- **Better than the best: Example 1 - Modeling**
- **Better than the best: Example 2 – Control**
- **Sample future chances:**
  - **Networked control systems**
  - **Nonlinearities with memory**
  - **Human-in-the-loop model**
  - **Cyber Physical Human Systems**

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What is "Fractional Calculus"?

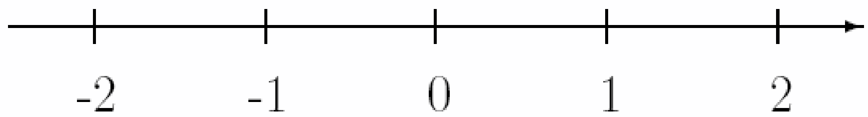
- **Calculus**: integration and differentiation.
- **"Fractional Calculus":** integration and differentiation of non-integer orders.
  - Orders can be real numbers (and even complex numbers!)
  - Orders are not constrained to be "integers" or even "fractionals"

How this is possible?

Why should I care?

Any (good) consequences (to me)?

... from integer to non-integer ...



$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n}$$

$$x^n = e^{n \ln x}$$

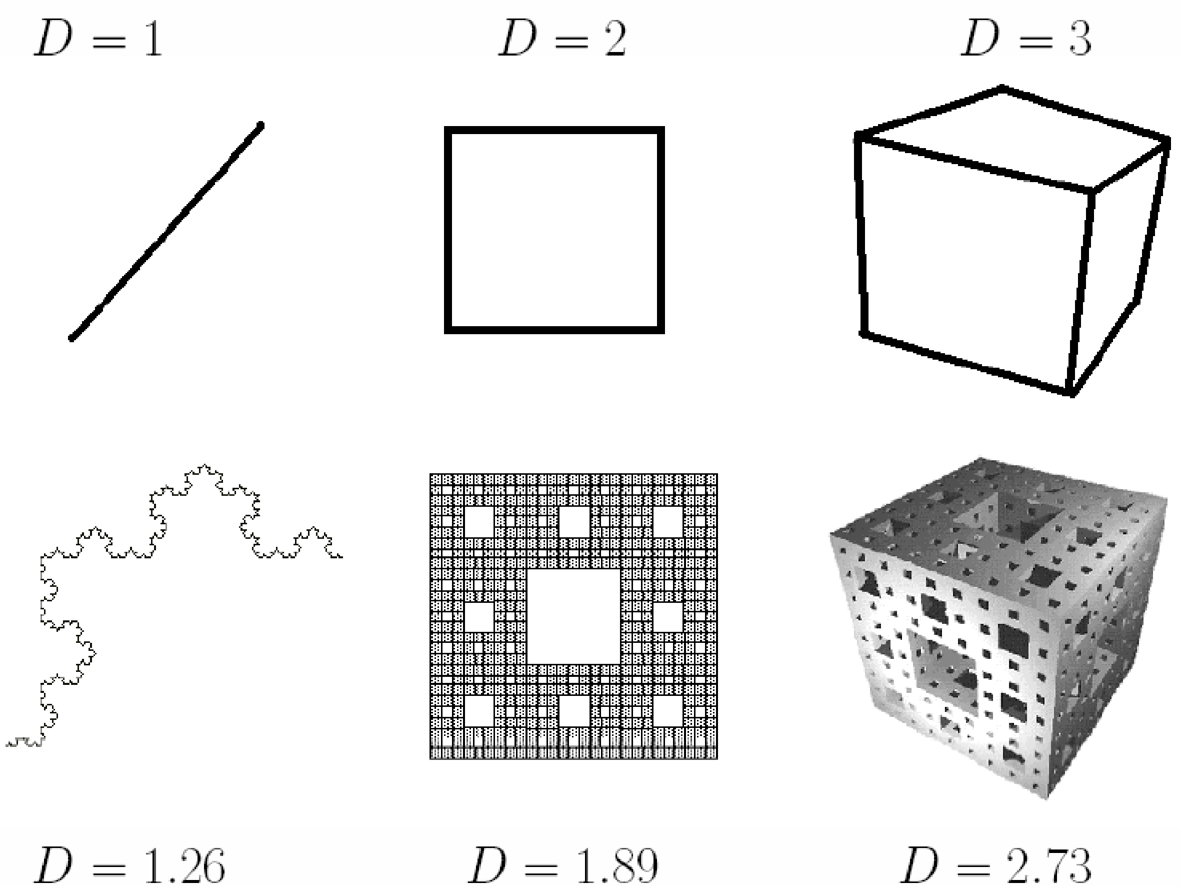
$$n! = 1 \cdot 2 \cdot 3 \cdot \ \ldots \cdot (n-1) \cdot n,$$

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \qquad x > 0,$$

$$\Gamma(n+1) = 1 \cdot 2 \cdot 3 \cdot \ \ldots \cdot n = n!$$

Slide credit: Igor Podlubny

… from integer to non-integer …



Slide credit: Igor Podlubny

Interpolation of operations

$$f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \frac{d^3 f}{dt^3}, \quad \ldots$$

$$f, \quad \int f(t)dt, \quad \int dt \int f(t)dt, \quad \int dt \int dt \int f(t)dt, \quad \ldots$$

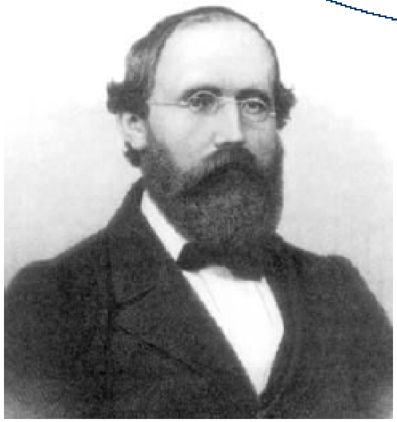
$$\ldots, \quad \frac{d^{-2} f}{dt^{-2}}, \quad \frac{d^{-1} f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \ldots$$

Slide credit: Igor Podlubny

Riemann–Liouville definition

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)\,d\tau}{(t-\tau)^{\alpha-n+1}} $$

$$ (n-1 \le \alpha < n) $$



G.F.B. Riemann
(1826–1866)

J. Liouville
(1809–1882)

$$ I^{\alpha} f(t) = \left(\frac{1}{t^{1-\alpha}}\right) * f(t) / \Gamma(\alpha) $$

$$ D^{\alpha} f(t) = \frac{d}{dt}[I^{1-\alpha} f(t)] = \frac{d}{dt}[\left(\frac{1}{t^{\alpha}}\right) * f(t)] / \Gamma(1-\alpha) $$

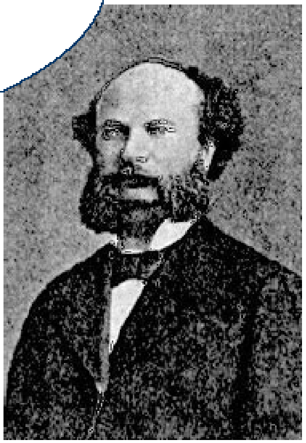
Grünwald–Letnikov definition

$$ {}_aD_t^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t - jh) $$

$[x]$ – integer part of x

A.K. Grünwald

A.V. Letnikov



First Derivative:

$$ f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}. $$

Slide credit: Igor Podlubny

Fractional Calculus was born in 1695

$$\frac{d^n f}{dt^n}$$

What if the order will be $n = ½$?

It will lead to a paradox, from which one day useful consequences will be drawn.

G.F.A. de L'Hôpital
(1661–1704)

G.W. Leibniz
(1646–1716)

Slide credit: Igor Podlubny


Program Includes:
Regular and Interactive Sessions
Panel Debates
Benchmark System
• 163 Papers
• 36 countries
• 127 papers at PID12
TOPICS DISTRIBUTION
Applications
Fractional 28
Mechatronics 14
Benchmark 14
Design 12
Education 12
Optimization 10
Control Structures
Energy
Load Frequency Control
Nonlinear
Tuning

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Good Consequences


ELSEVIER

IFAC

Control Engineering Practice

Best Paper Prize

Awarded jointly by Elsevier Ltd and the International Federation of Automatic Control (IFAC)

for the best paper published in the period 2008-2011

Awarded to

Y. Chen

for the paper

Tuning and auto-tuning of fractional order controllers for industry applications

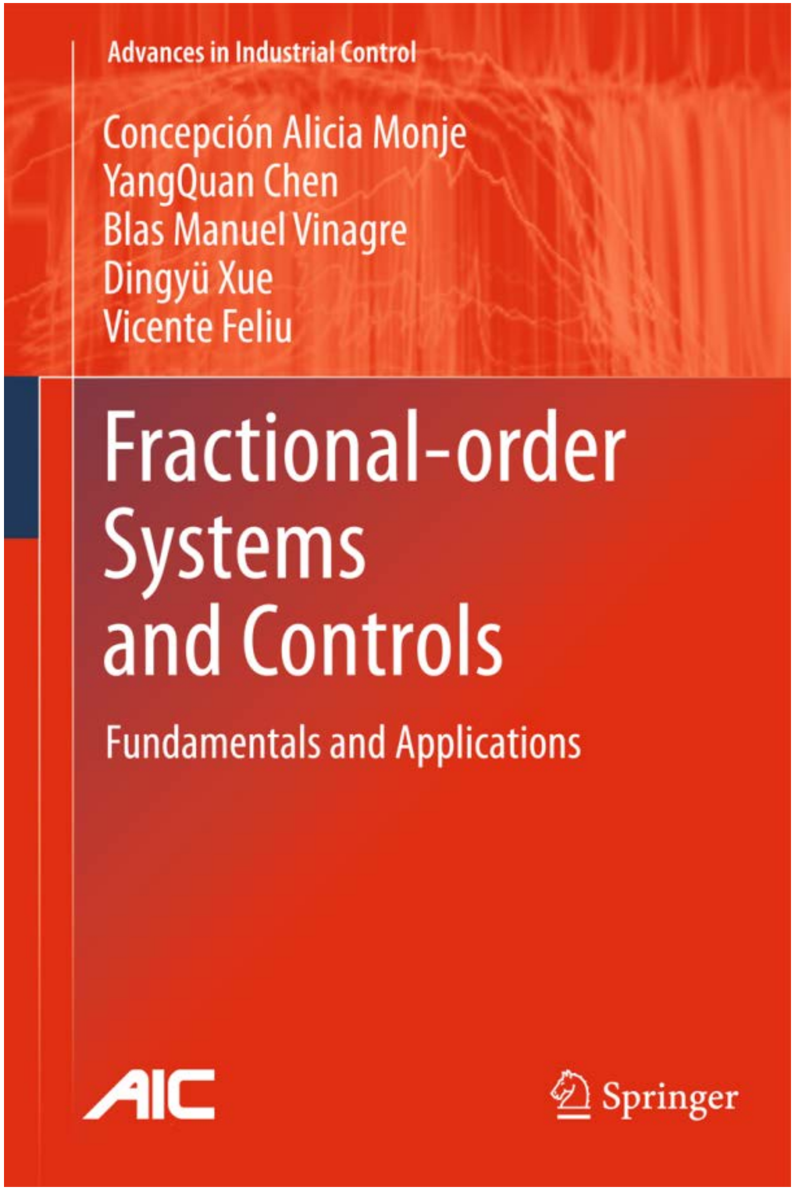
(Vol. 16, No. 7, pp. 798-812)

Christopher Greenwell, Publisher
Elsevier Ltd.

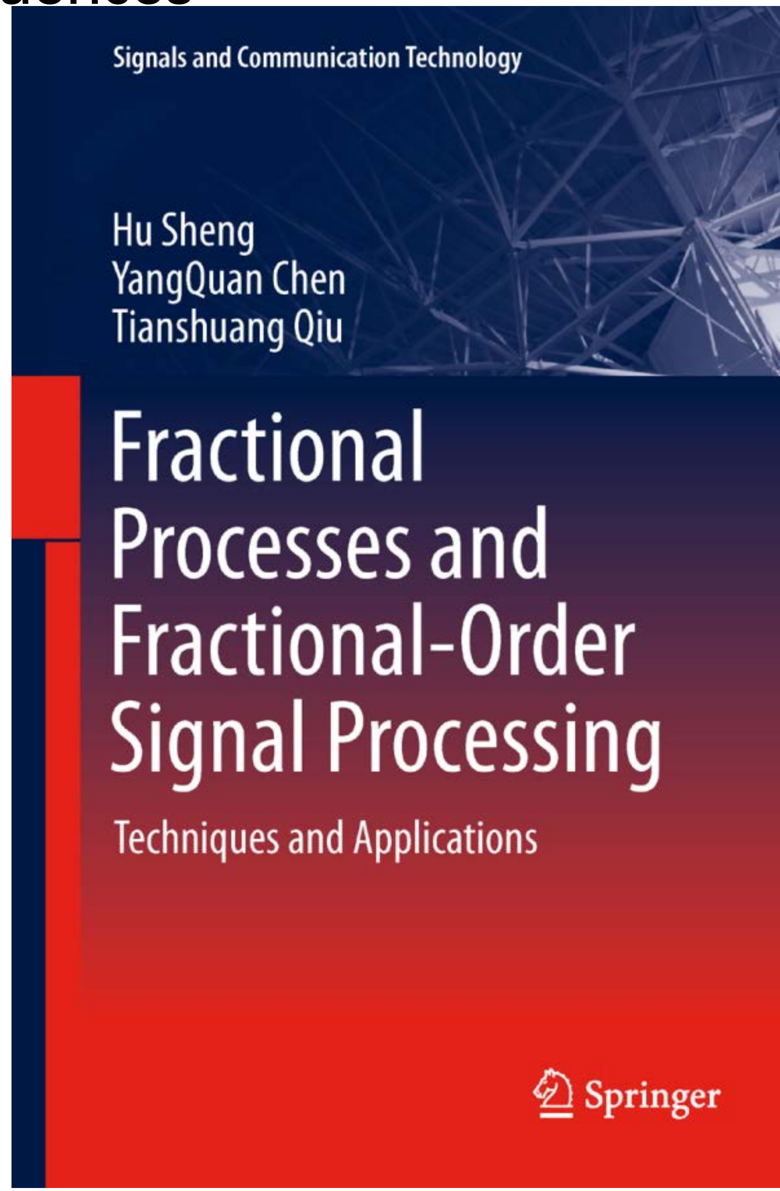
Prof. Andreas Kugi, Editor-in-Chief
Control Engineering Practice

- Fractional Order System – official keyword of IFAC
- pid12.ing.unibs.it/

Good Consequences

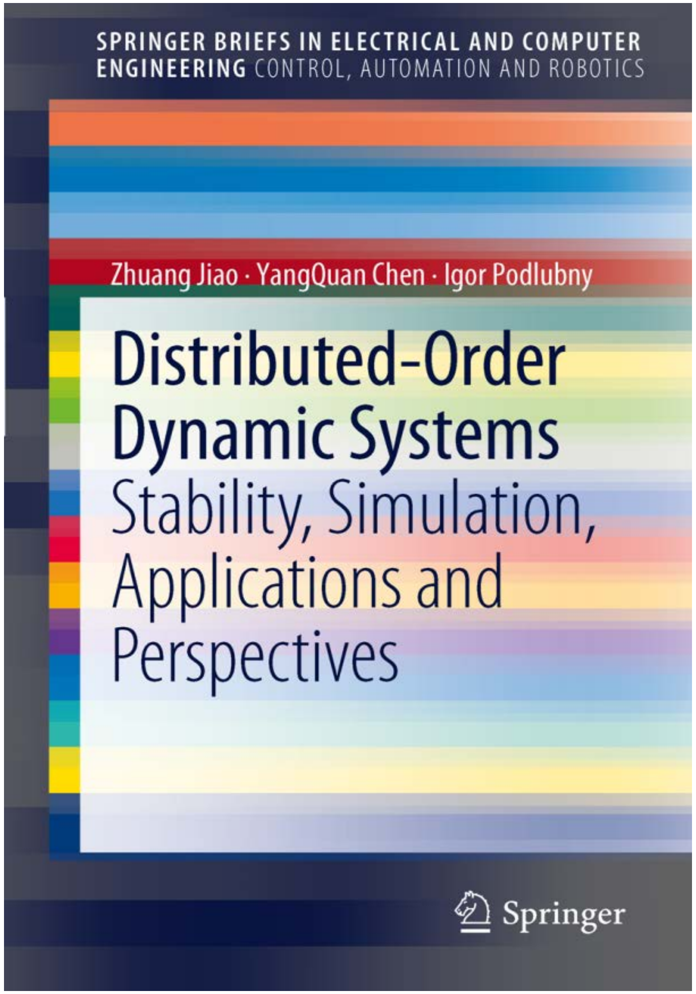


2001-2010

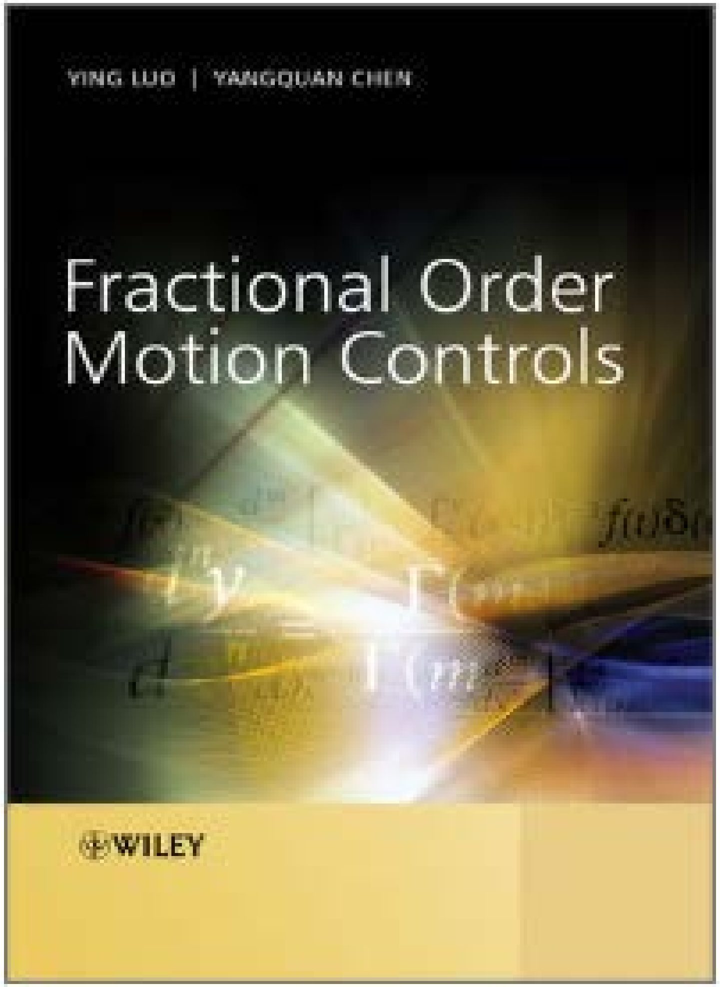


2005-2011

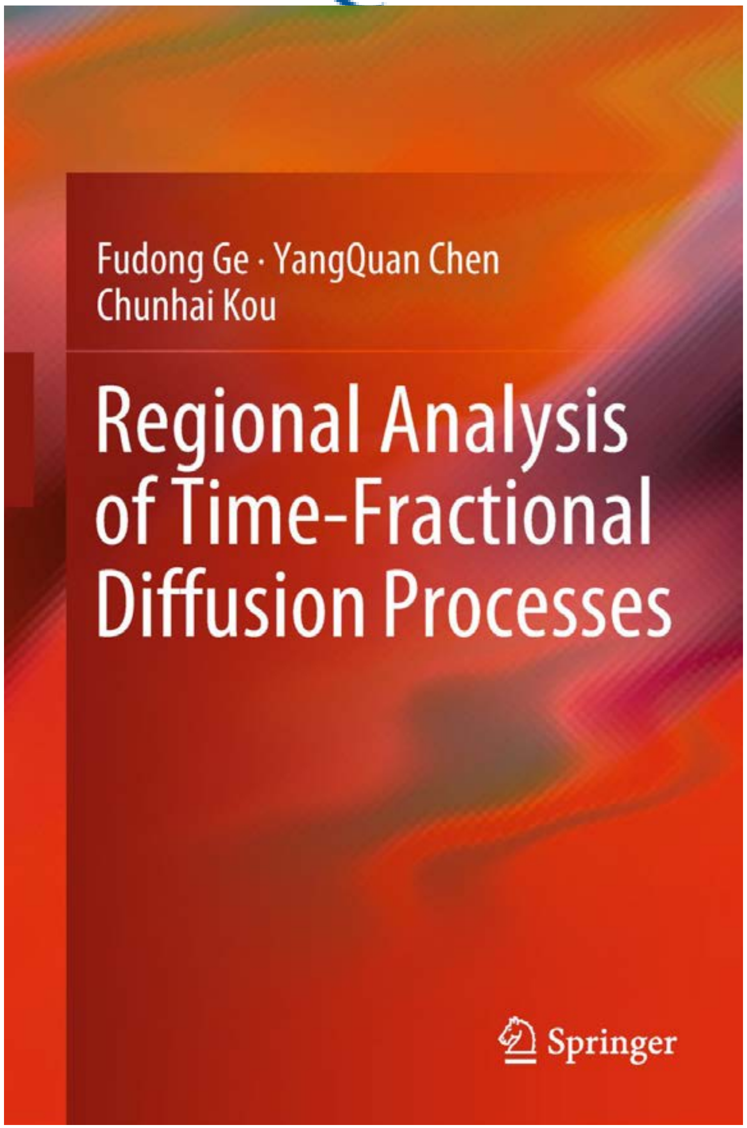
Good Consequences

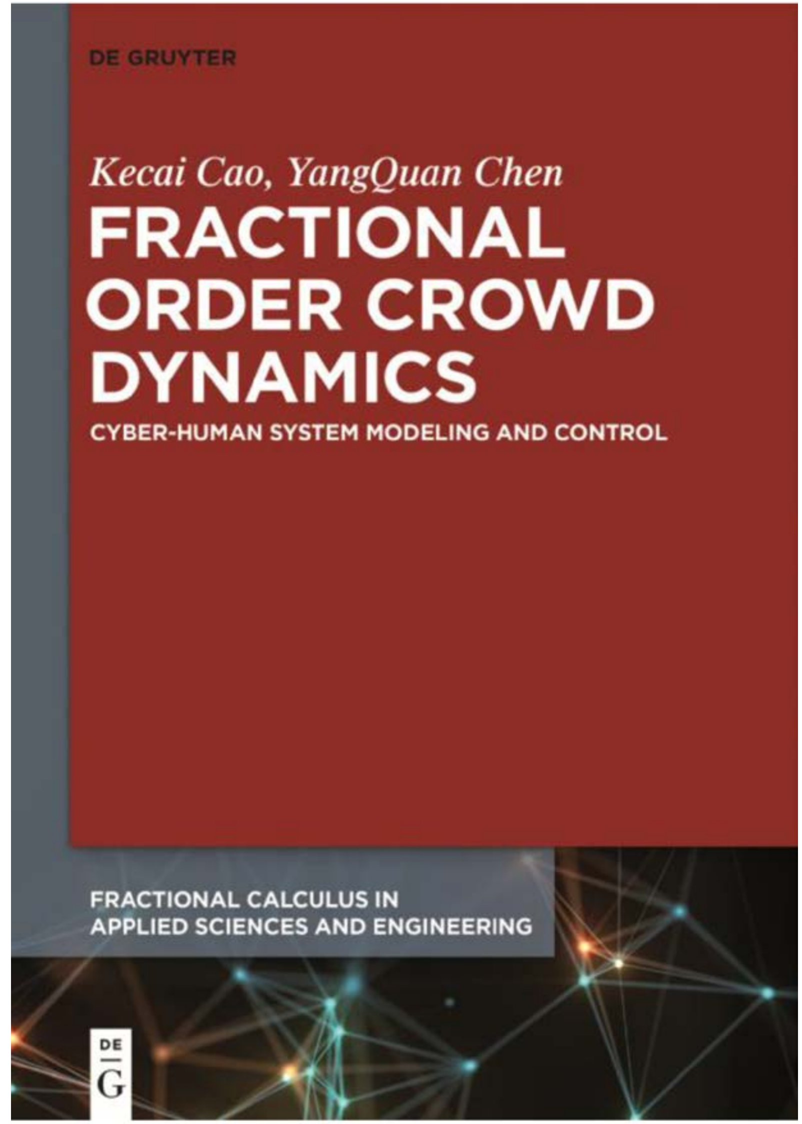


2007-2012



2007-2012

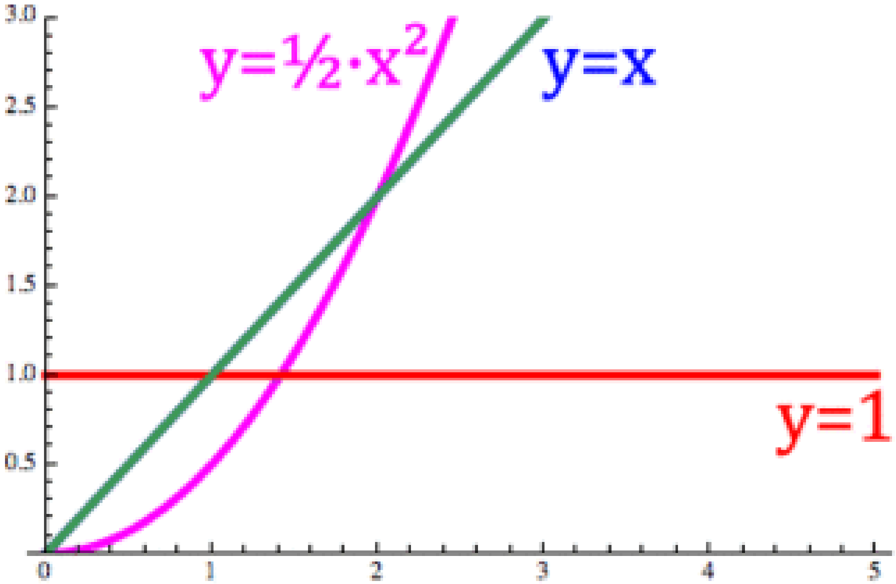


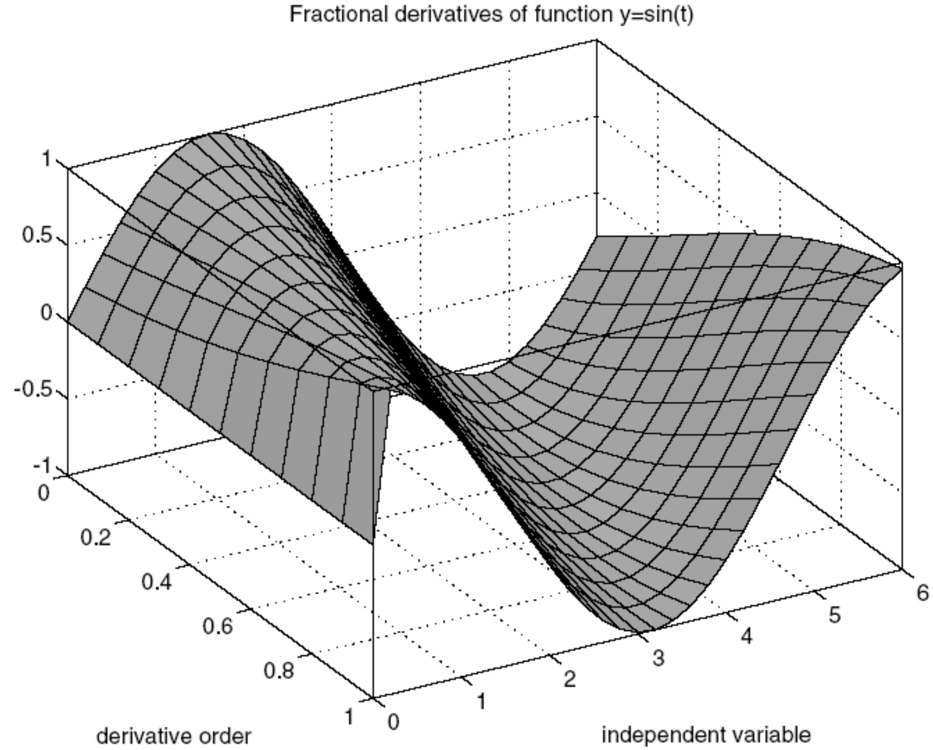


2015-2018

2010-2019

Example: $\sin(t)$





The animation shows the derivative operator oscillating between the antiderivative (α=-1) and the derivative (α=1) of the simple power function y=x continuously.

http://en.wikipedia.org/wiki/Fractional_calculus

Slide credit: Igor Podlubny

"Fractional Order Thinking" or, "In Between Thinking"

- For example
  - Between integers there are non-integers;
  - Between logic 0 and logic 1, there is the "**fuzzy logic**";
  - Between integer order splines, there are "**fractional order splines**"
  - Between integer high order moments, there are **noninteger order moments (e.g. FLOS)**
  - Between "integer dimensions", there are **fractal dimensions**
  - Fractional Fourier transform (FrFT) – in-between time-n-freq.
  - Non-Integer order calculus (fractional order calculus – abuse of terminology.) (FOC)


NEURO FUZZY
Made in Japan
KEEP WARM
EXTENDED KEEP WARM
NS-ZCC10
WHITE RICE
REG/SUSHI
SOFTER
HARDER
QUICK COOKING
18:34
MIXED
PORRIDGE
SWEET
SEMIBROWN
BROWN
COOKING REHEAT
RESET
MENU
TIMER
TIME SETTING
PRE-WASHED
ZOJIRUSHI
0
1

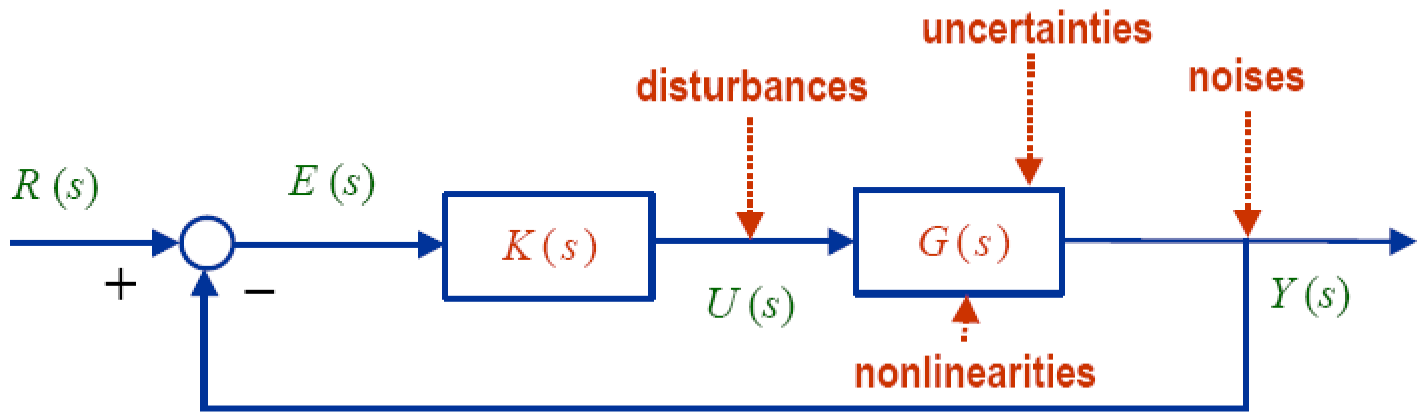
Integer-Order Calculus

Fractional-Order Calculus

Slide credit: Richard L. Magin, ICCC12

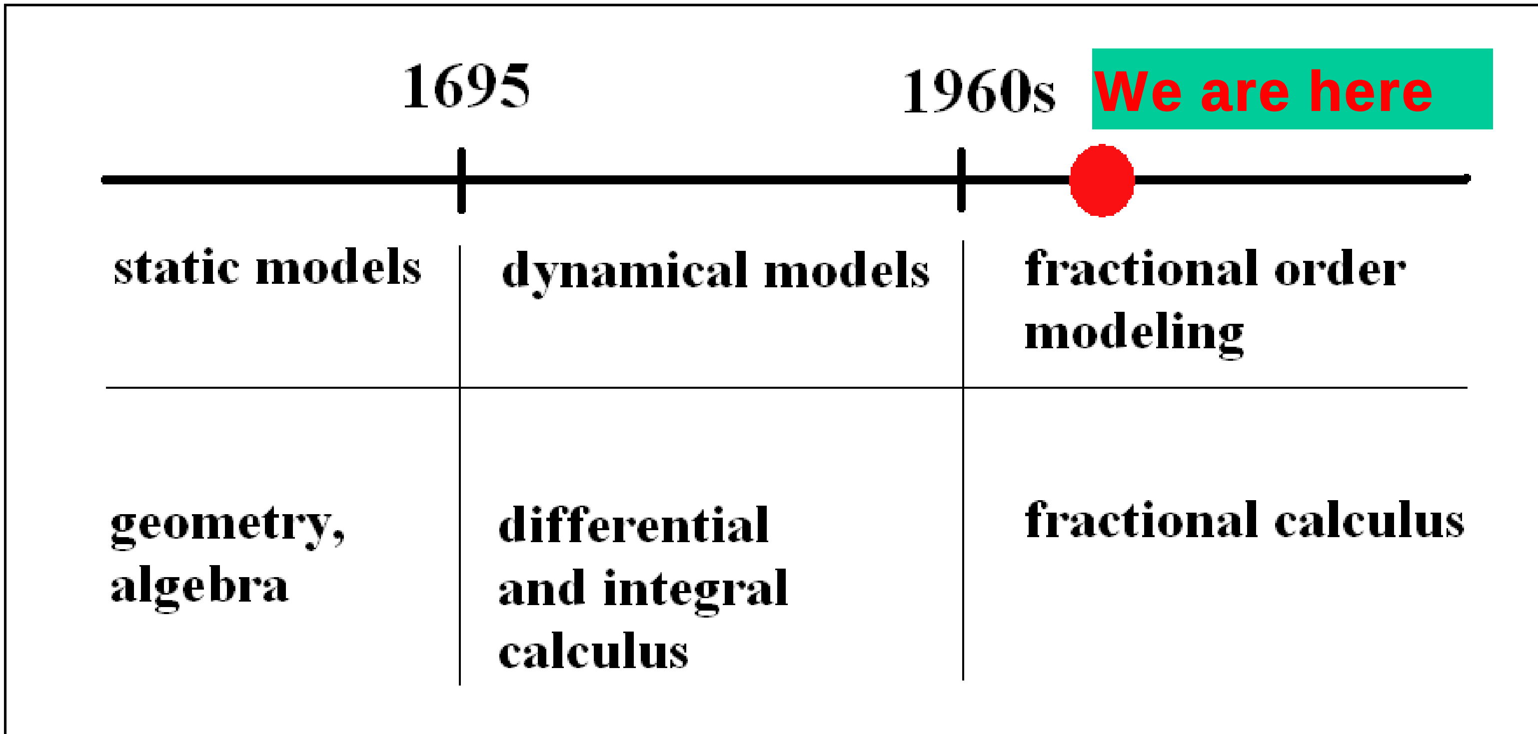
Fractional Order Controls

- IO Controller + IO Plant
- FO Controller + IO Plant
- FO Controller + FO Plant
- IO Controller + FO Plant



D. Xue and **Y. Chen***, "A Comparative Introduction of Four Fractional Order Controllers". Proc. of The 4th IEEE World Congress on Intelligent Control and Automation (WCICA02), June 10-14, 2002, Shanghai, China. pp. 3228-3235.

Fractional Calculus: a response to more advanced characterization of our more complex world at smaller scale



Slide credit: Igor Podlubny

Rule of thumb for "Fractional Order Thinking"

- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- $1/f^{a}$ noise

- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …

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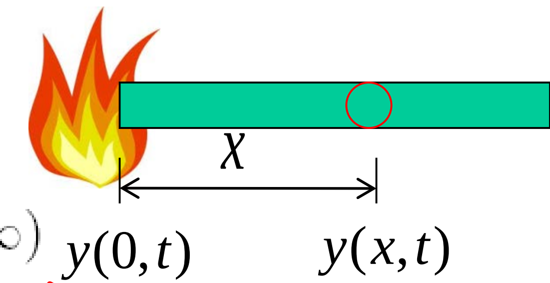
Modeling: heat transfer



$$\frac{\partial^2 y(x,t)}{\partial x^2} = k^2 \frac{\partial y(x,t)}{\partial t},$$

$$(t > 0, \quad 0 < x < \infty)$$

Boundary condition (BC):

$$y(0,t) = m(t)$$

$$y(x,0) = 0$$ Initial condition (IC)

$$\left|\lim_{x\to\infty} y(x,t)\right| < \infty$$ Physical limit

Transfer function:

$$\frac{\mathrm{d}^2 Y(x,s)}{\mathrm{d}x^2} = k^2 s Y(x,s)$$

$$Q(0,s) = M(s)$$

$$\left|\lim_{x\to\infty} Y(x,s)\right| < \infty$$

$$Y(x,s) = A(s)e^{-kx\sqrt{s}} + B(s)e^{kx\sqrt{s}}$$

$$\begin{aligned} A(s) &= Y(0,s) = M(s) \\ B(s) &= 0 \end{aligned}$$

$$Y(x,s) = M(s)e^{-kx\sqrt{s}}$$

$$G(s) = \frac{Y(x,s)}{M(s)} = e^{-kx\sqrt{s}}$$

think about transfer function $e^{-\sqrt{s}}$!

Irrational Transfer Function.

Taylor series expansion: polynomial of half order integrators $s^{0.5}$!!

Ideal physical plant model:

$$G_p(s) = e^{-\sqrt{s}}$$

First Order Plus Time Delay (FOPTD) Model:

$$G_{IO}(s) = \frac{K_1}{T_1 s + 1} e^{-L_1 s}$$

Time Delay with Single Fractional Pole Model:

$$G_{FO}(s) = \frac{K_2}{T_2 s^{0.5} + 1} e^{-L_2 s}$$

All models are wrong but some are useful.

George E. P. Box

All models are wrong but some are dangerous ...

Leonard A. Smith

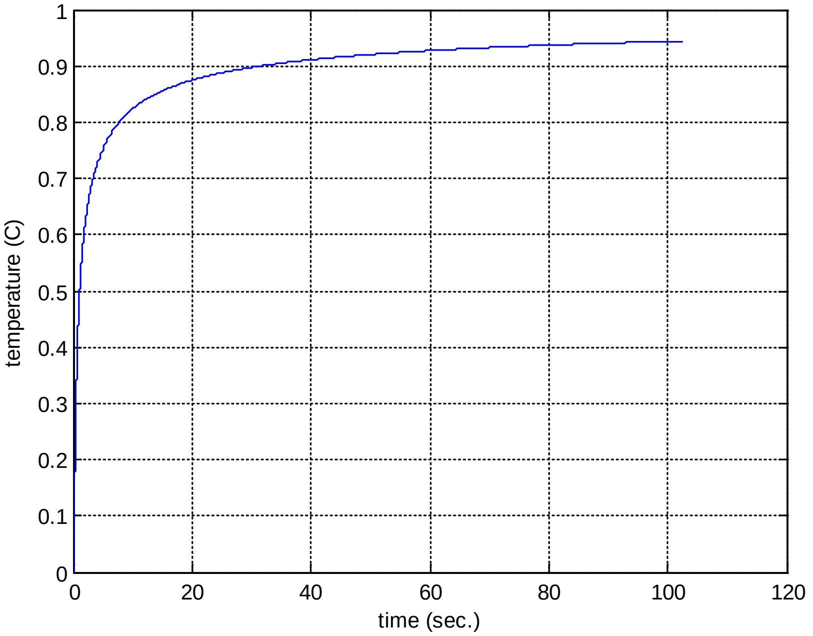
Step response of the "Ideal Plant"

$$y(0,t) = m(t) = 1u(t), M(s) = \frac{1}{s}$$

$$Y(x,s)\Big|_{x=1} = G(x,s)\Big|_{x=1} M(s) = G_p(s) M(s) = \frac{1}{s} e^{-\sqrt{s}}$$

So, "Reaction-Curve" or Step response of the "Ideal Plant"

$$y(t) = L^{-1}[\frac{1}{s} e^{-\sqrt{s}}]$$



Magic code **NILT** to do $y(t) = L^{-1}[\frac{1}{s} e^{-\sqrt{s}}]$

```
% step response of normalized 1D heat equation when x=1
clear all;close all; alpha=.5; Ts=0.1;
F= @(s)  exp(-s.^alpha)./s;
%---------------------------------------------------------------
alfa=0; M=1024; P=20; Er=1e-10; tm=M*Ts; wmax0=2*pi/Ts/2; L = M;
Taxis=[0:L-1]*Ts;  n=1:L-1; n=n*Ts ;
N=2*M; qd=2*P+1; t=linspace(0,tm,M); NT=2*tm*N/(N-2); omega=2*pi/NT;
c=alfa-log(Er)/NT; s=c-i*omega*(0:N+qd-1);
Fsc=feval(F,s); ft=fft(Fsc(1:N)); ft=ft(1:M);
q=Fsc(N+2:N+qd)./Fsc(N+1:N+qd-1); d=zeros(1,qd); e=d;
   d(1)=Fsc(N+1); d(2)=-q(1); z=exp(-i*omega*t);
   for r=2:2:qd-1; w=qd-r; e(1:w)=q(2:w+1)-q(1:w)+e(2:w+1); d(r+1)=e(1);
      if r>2; q(1:w-1)=q(2:w).*e(2:w)./e(1:w-1); d(r)=-q(1);
      end
   end
   A2=zeros(1,M); B2=ones(1,M); A1=d(1)*B2; B1=B2;
   for n=2:qd
   A=A1+d(n)*z.*A2; B=B1+d(n)*z.*B2;A2=A1; B2=B1; A1=A; B1=B;
   end
ht=exp(c*t)/NT.*(2*real(ft+A./B)-Fsc(1));
%---------------------------------------------------------------
figure;tt=0:(length(ht)-1);tt=tt*Ts;plot(tt,ht);
xlabel('time (sec.)');ylabel('temperature (C)');grid on
```

Application of numerical inverse Laplace transform algorithms in fractional calculus
Journal of the Franklin Institute, *Volume 348, Issue 2*, *March 2011*, *Pages 315-330*
Hu Sheng, Yan Li, YangQuan Chen http://dx.doi.org/10.1016/j.jfranklin.2010.11.009 (Check ref [8])

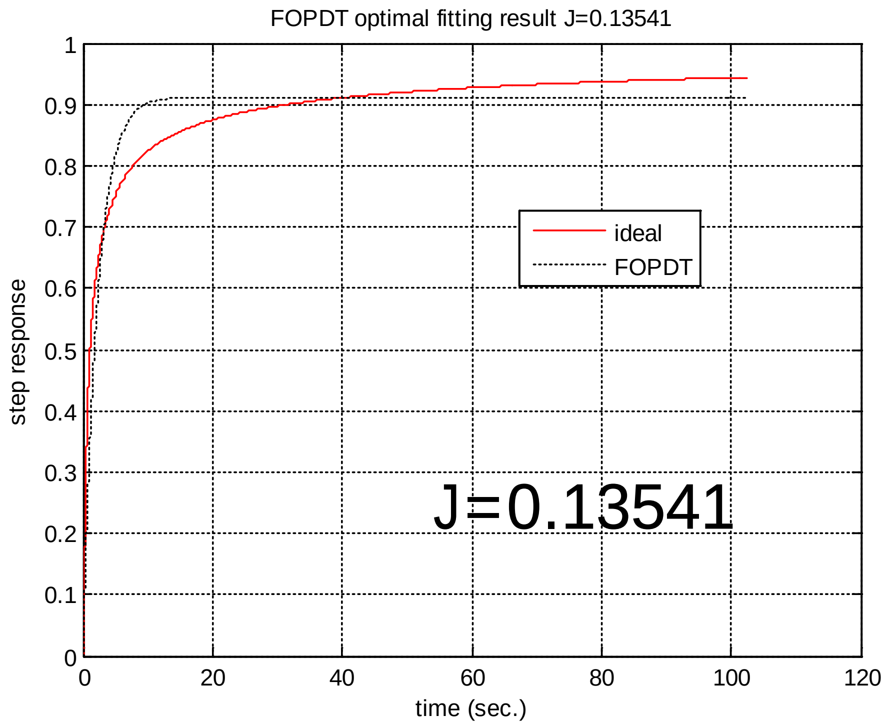
So, let us do fitting!

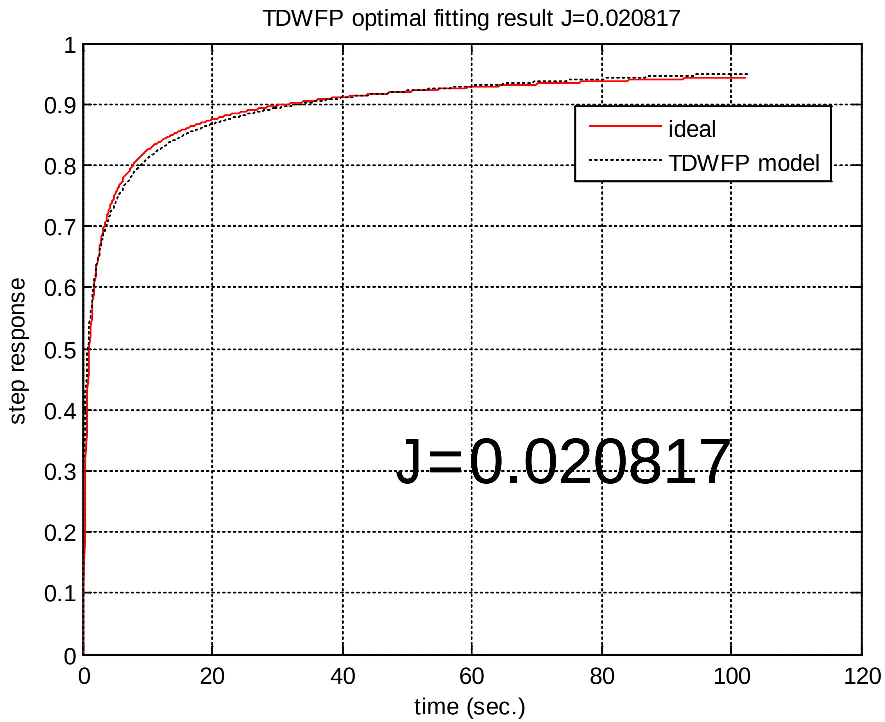
Ideal physical plant model: $G_p(s) = e^{-\sqrt{s}}$

First Order Plus Time Delay (FOPTD) Model: $G_{IO}(s) = \dfrac{K_1}{T_1 s + 1} e^{-L_1 s}$

Time Delay with Single Fractional Pole Model: $G_{FO}(s) = \dfrac{K_2}{T_2 s^{0.5} + 1} e^{-L_2 s}$

All models are wrong but some are useful. **George E. P. Box**





K1	T1	L1
0.9120	2.2393	0

K2	T2	L2
1.0197	1.2312	0.0001

Fitting code for $G_{IO}(s)=\dfrac{K_1}{T_1 s+1}e^{-L_1 s}$

```
% Ts: sampling period; ht: step response (from NILT numerical inverse
% Laplace transform)
% previously we got Ts and ht array (reaction curve)
options=optimset('TolX',1e-10,'TolFun',1e-10);
Tic;[x,FVAL,EXITFLAG] =fminsearch(@(x) fopdtfit(x,ht,Ts),[1,1,0],options);toc
% May need to wait half minute
K1=x(1);T1=x(2);L1=x(3);T=(0:length(ht)-1)*Ts;if L1<0; L1=0; end
sysfoptd=tf([K1],[T1,1],'iodelay',L1);
y=step(sysfoptd,T);plot(T,ht,'r',t,y,'k:');grid on;
title(['FOPDT optimal fitting result J=',num2str(FVAL)]);
xlabel('time (sec.)');ylabel('step response'); legend('ideal', 'FOPDT')
```

```
% fitting using FOPTD model - integral of error square (ISE)
function [J]=foptdfit(x,y0,Ts);
K1=x(1);T1=x(2);L1=x(3);T=(0:length(y0)-1)*Ts;if L1<0; L1=0; end
sysfoptd=tf([K1],[T1,1],'iodelay',L1);
y=step(sysfoptd,T);
J=(y'-y0)*(y-y0')*Ts;
```

Fitting code for $G_{FO}(s) = \frac{K_2}{T_2 s^{0.5} + 1} e^{-L_2 s}$

```
options=optimset('TolX',1e-10,'TolFun',1e-10);
Tic;[x,FVAL,EXITFLAG] =fminsearch(@(x) tdwfpfit(x,ht,Ts),[1,2,0],options);toc
% May need to wait 1000+ seconds!
K1=x(1);T1=x(2);L1=x(3);Np=length(ht);T=(0:Np-1)*Ts;if L1<0; L1=0; end
y=mlf(0.5,1.5,-T.^0.5/T1);y=(K1/T1)*(T.^0.5) .* y;
Nstep=floor(L1/Ts);
y1=zeros(size(y));y1(Nstep+1:Np)=y(1:Np-Nstep);
y=y1;plot(T,ht,'r',t,y,'k:');grid on;
title(['TDWFP optimal fitting result J=',num2str(FVAL)]);
xlabel('time (sec.)');ylabel('step response'); legend('ideal', 'TDWFP model')
```

```
% fitting using TDWFP model - integral of error square (ISE)
function [J]=tdwfpfit(x,y0,Ts);
K1=x(1);T1=x(2);L1=x(3);Np=length(y0);T=(0:Np-1)*Ts;if L1<0; L1=0; end
y=mlf(0.5,1.5,-T.^0.5/T1);y=(K1/T1)*(T.^0.5) .* y;
Nstep=floor(L1/Ts);y1=zeros(size(y));y1(Nstep+1:Np)=y(1:Np-Nstep);
J=(y1-y0)*(y1-y0)'*Ts;
% get Igor Poblubny's MLF.m from
% www.mathworks.com/matlabcentral/fileexchange/8738-mittag-leffler-function
```

Benefits of FOM

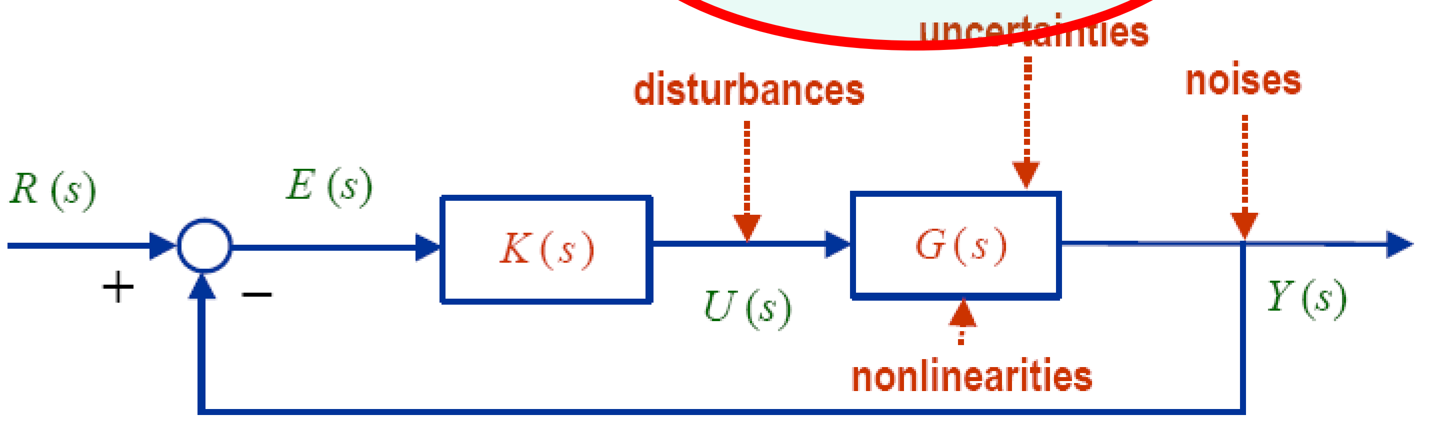
- Captures (more) physics $G_p(s) = e^{-\sqrt{s}} \rightarrow G_{FO}(s) = \frac{K_2}{T_2 s^{0.5} + 1} e^{-L_2 s}$
- Reaction curve fitting: **Better than the best** FOPDT model $G_{IO}(s) = \frac{K_1}{T_1 s + 1} e^{-L_1 s}$
- Could be a nice starting point for better controller design?
  - Reminder: Among all control tasks, 80% of them are for temperature controls that calls for $\sqrt{s}$
  - **Lots of process control papers may be re-written.**

Double check the "Reaction Curve" by $G_{FO}(s) = \frac{K}{T s^{\alpha} + 1} e^{-Ls}$

https://www.mathworks.com/matlabcentral/fileexchange/52061-fractional-order-scanning

FOMs and Fractional Order Controls

- **IO Controller + IO Plant**
- **FO Controller + IO Plant**
- **FO Controller + FO Plant**
- **IO Controller + FO Plant**



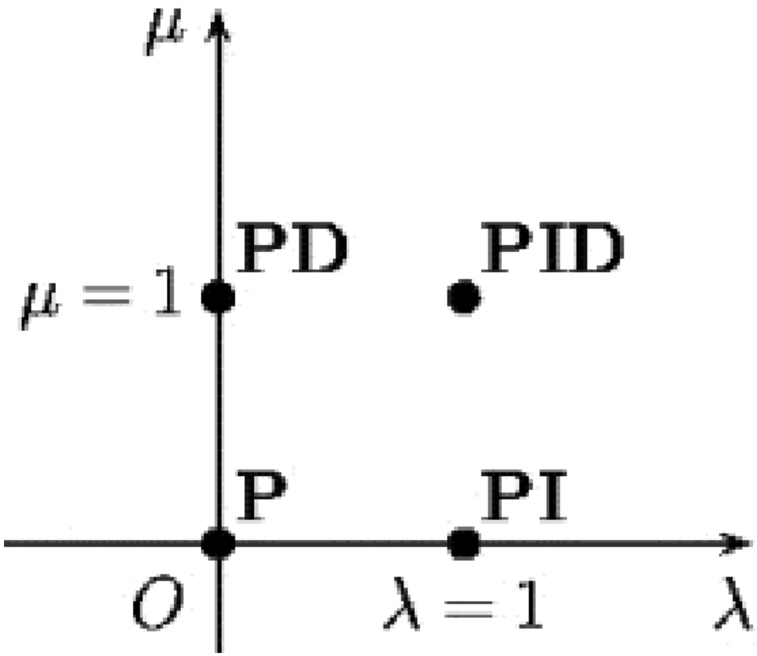
Concepcin A. Monje, YangQuan Chen, Blas Vinagre, Dingyu Xue and Vicente Feliu (2010). "Fractional Order Systems and Controls - Fundamentals and Applications." Advanced Industrial Control Series, Springer-Verlag, www.springer.com/engineering/book/978-1-84996-334-3 (2010), 415 p. 223 ill.19 in color.

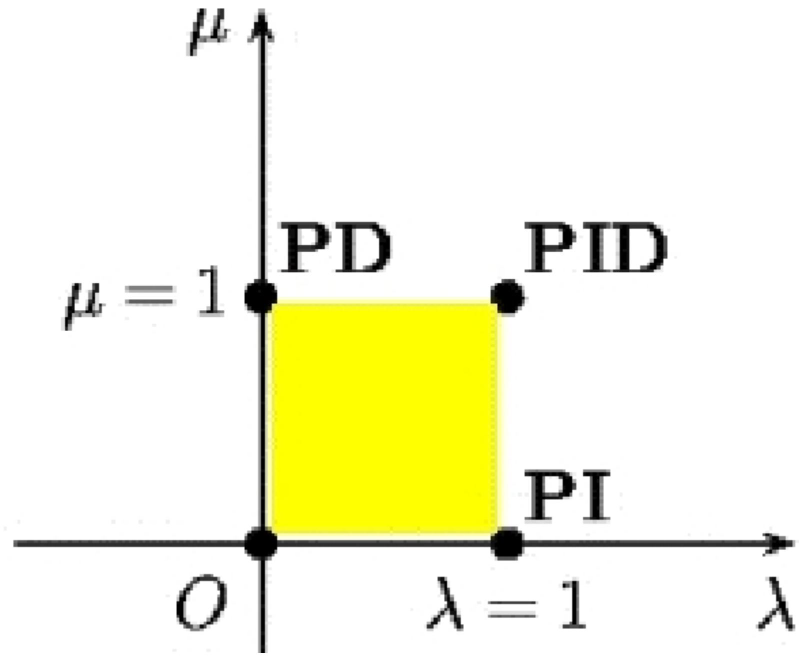
https://www.mathworks.com/matlabcentral/fileexchange/60874-fotf-toolbox

Fractional order PID control

- 90% are PI/PID type in industry. (**Ubiquitous**)

$$u(t) = K_p\left(e(t) + T_i \mathrm{D}_t^{-\lambda} e(t) + \frac{1}{T_d}\mathrm{D}_t^{\mu} e(t)\right). \qquad (\mathrm{D}_t^{(*)} \equiv {}_0\mathrm{D}_t^{(*)}).$$





Igor Podlubny. "Fractional-order systems and $PI^{\lambda}D^{\mu}$-controllers". IEEE Trans. Automatic Control,44(1): 208–214, 1999.

*YangQuan Chen, Dingyu Xue, and Huifang Dou. "**Fractional Calculus and Biomimetic Control**". IEEE Int. Conf. on Robotics and Biomimetics (RoBio04), August 22-25, 2004, Shengyang, China.*

Outline

- **Fractional calculus: What, Why and When**
- **Better than the best: Example 1 - Modeling**
- **Better than the best: Example 2 – Control**
- **Sample future chances:**
  - **Networked control systems**
  - **Nonlinearities with memory**
  - **Human-in-the-loop model**
  - **Cyber Physical Human Systems**

IOPID and FO-PI for FOPDT plants

Plant

$$P(s) = \frac{K}{Ts+1}e^{-Ls}$$

IOPID Controller

$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

FOPI Controller

$$C(s) = K_p + \frac{K_i}{s^r}$$

The gain-phase margin tester

$$M_T(A,\phi) = Ae^{-j\phi}$$

A is the boundary of gain margin, ϕ is the boundary of phase margin

Normalization of FOPDT

$$G(s) = \frac{1}{s+1}\mathrm{e}^{-Ls}$$

$$\frac{1}{Ts+1}e^{-(L/T)Ts} = \frac{1}{s'+1}e^{-L's'}$$

Atherton, D.P. (2007). Feedback. *IEEE Control Systems*, 27(4), 17–18. http://ieeexplore.ieee.org/document/4272322/.

$$\mathcal{L}[x(at)] = \frac{1}{|a|}X(\frac{s}{a})$$

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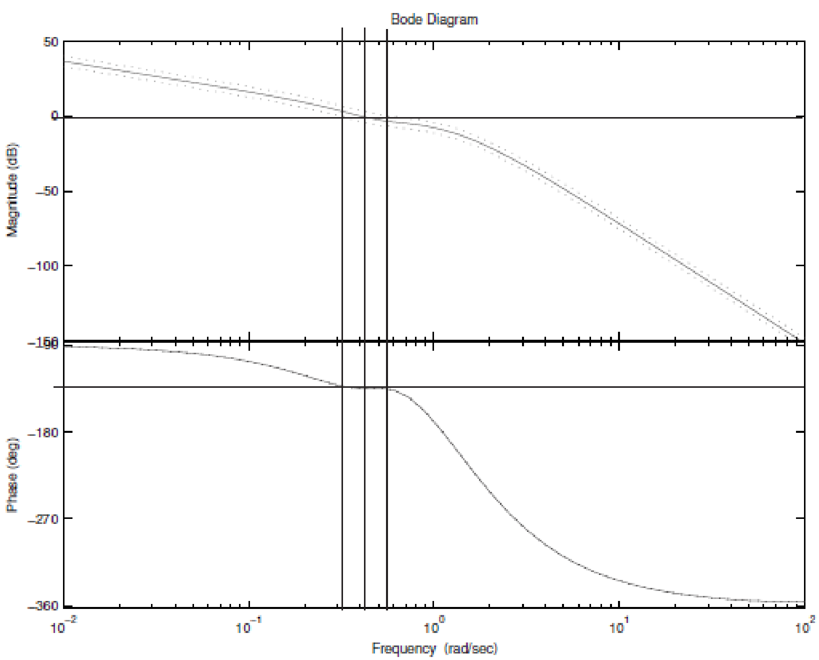
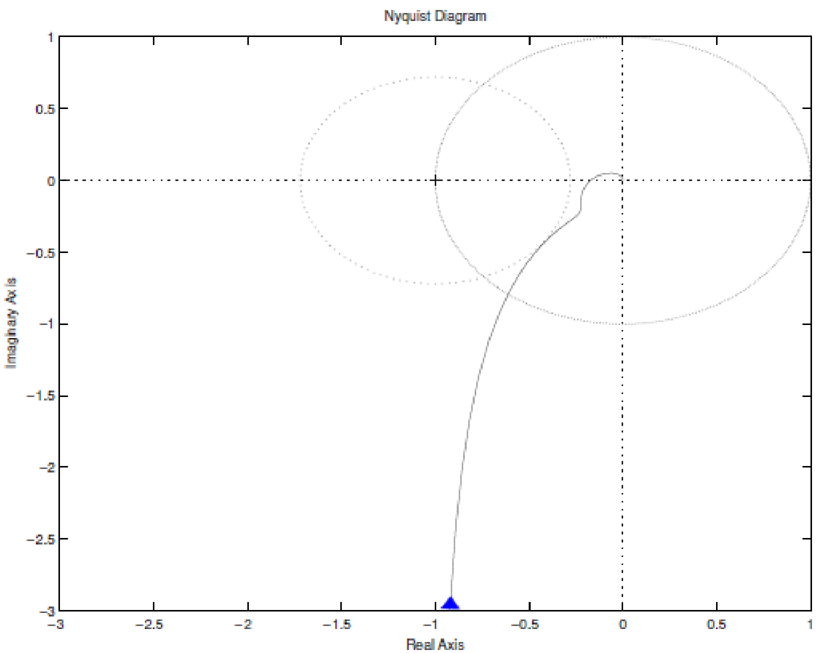
The "flat phase" concept

$$\frac{d\angle G(s)}{ds}\Big|_{s=j\omega_c} = 0$$

Y. Q. Chen and K. L. Moore, "*Relay Feedback Tuning of Robust PID Controllers With Iso-Damping Property*", IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, , Vol. 35. Issue: 1. 2005.



(a) Basic idea: a flat phase curve at gain crossover frequency



(b) Sensitivity circle tangentially touches Nyquist curve at the flat phase

The “flat phase” design concept

The “flat phase” tuning rules

- Gain crossover frequency

$$|G(j\omega_c)| = |C(j\omega_c)P(j\omega_c)| = 0$$

- Phase margin

$$\angle[G(j\omega_c)] = Arg[C(j\omega_c)P(j\omega_c)] = -\pi + \phi_m$$

- Robustness: “flat phase”

$$\left.\frac{d(\angle G(j\omega_c))}{d\omega}\right|_{\omega=\omega_c} = 0$$

The gain-phase margin tester

Definition: Gain-phase margin tester

M_T in figure below is a gain-phase master which provides information for plotting the boundaries of constant gain margin and phase margin in the parameter plane.

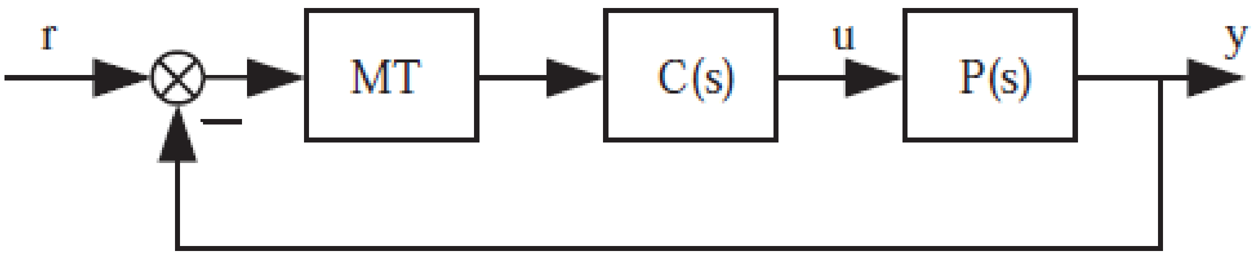


Figure: The feedback control system with the gain-phase margin tester

Open loop

$$G(s) = M_T(A,\phi)C(s)P(s)$$

Closed loop

$$\Phi(s) = \frac{M_T(A,\phi)C(s)P(s)}{1 + M_T(A,\phi)C(s)P(s)}$$
$$= \frac{Ae^{-j\phi}Ke^{-Ls}(K_p s + K_i + K_d s\ )}{s(Ts+1) + Ae^{-j\phi}Ke^{-Ls}(K_p s + K_i + K_d s\ )}$$

Property

Assuming $\phi = 0$ for M_T, the controller parameters can be obtained satisfying a given gain margin A. Vice versa, assuming $A = 1$ for M_T, one can obtain the controller parameters for a given phase margin ϕ.

Stability region of PID for FOPTD

Definition: IRB (Infinity root boundary):

$$D\left(K_p, K_i, K_d, A, \phi; s = \infty\right) = 0$$

$$\Rightarrow K_d = \pm \frac{T}{AK}$$

Definition: RRB (Real root boundary):

$$D\left(K_p, K_i, K_d, A, \phi; s = 0\right) = 0$$

$$\Rightarrow K_i = 0$$

Stability region of PID for FOPTD

Definition: CRB (Complex root boundary):

$$D\left(K_p, K_i, K_d, A, \phi; s = j\omega\right) = 0$$

$$\Rightarrow \begin{cases} K_d = \dfrac{KAsin(\phi + \omega L)K_i - KA\omega K_p \cos(\phi + \omega L) - \omega}{KA\omega^2 \sin(\phi + \omega L)} \\ K_p = \dfrac{T\omega \sin(\phi + \omega L) - \cos(\phi + \omega L)}{KA} \\ K_i = \dfrac{\omega \sin(\phi + \omega L) - T\omega^2 \cos(\phi + \omega L)}{KA} + \omega^2 K_d \end{cases}$$

Stability region of PID for FOPTD

Example

$K_d = 0.5$

Complete stability region

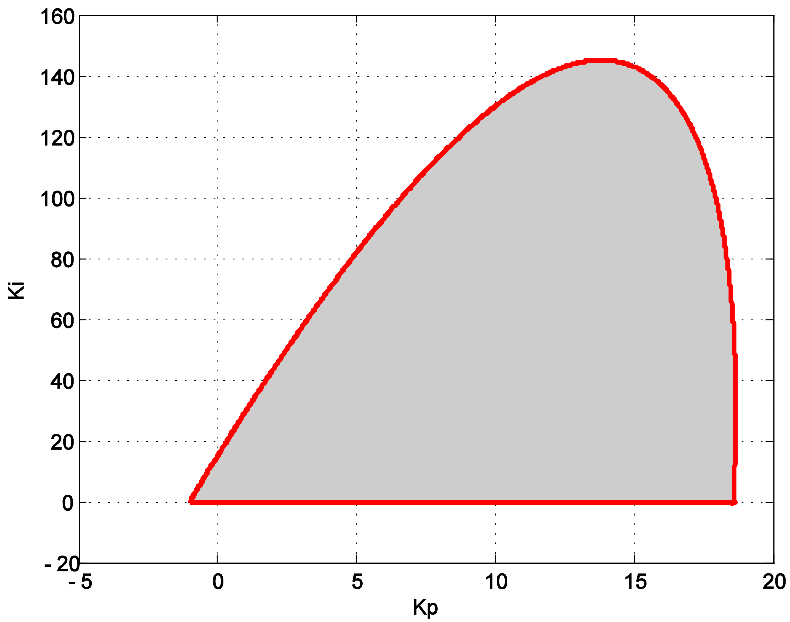
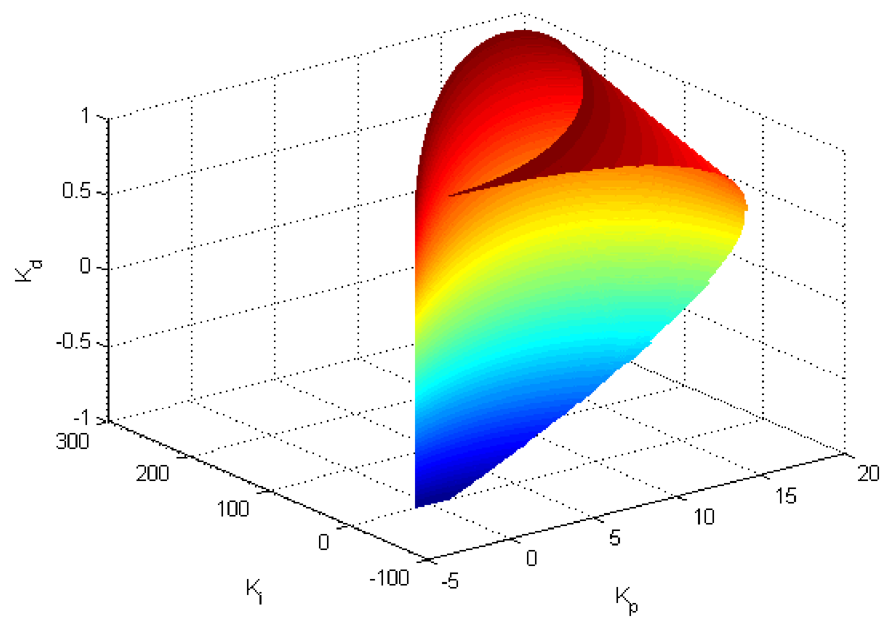


Figure: Stability region of *Ki* with respect to *Kp* with *Kd* = 0.5



Complete stability region of *Ki*, *Kp* and *Kd*

Stability region of PID for FOPTD

Example

$K_d = 0.5$

$\varphi_m = 50°$

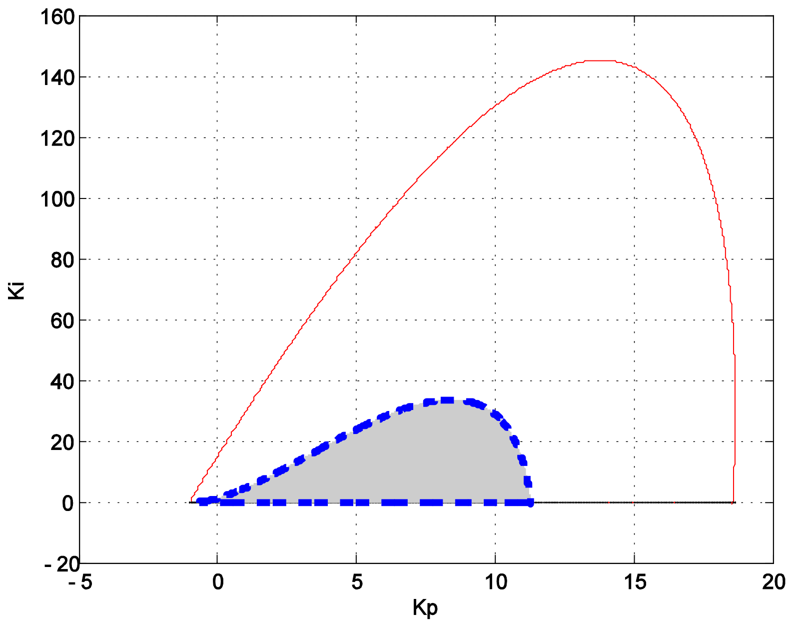


Figure: Stability region of *Ki* with respect to *Kp* with *Kd* = 0.5 and $\varphi_m = 50°$

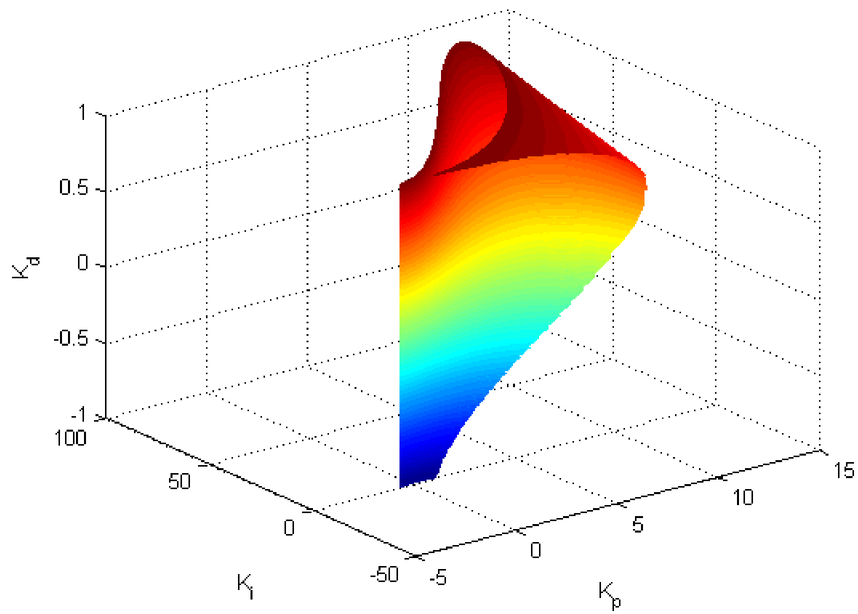
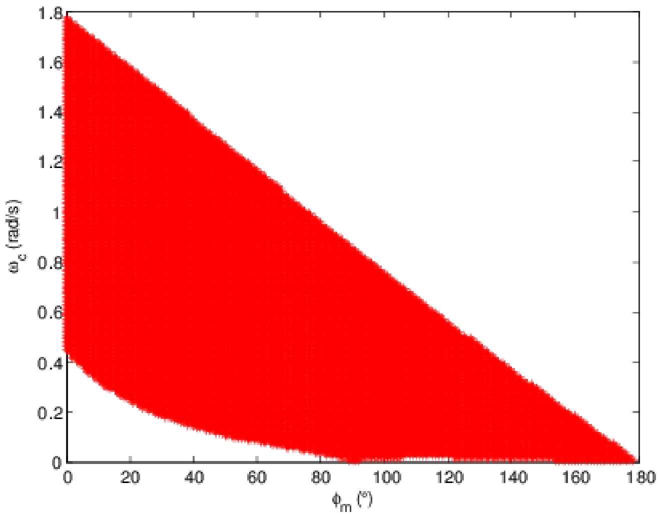
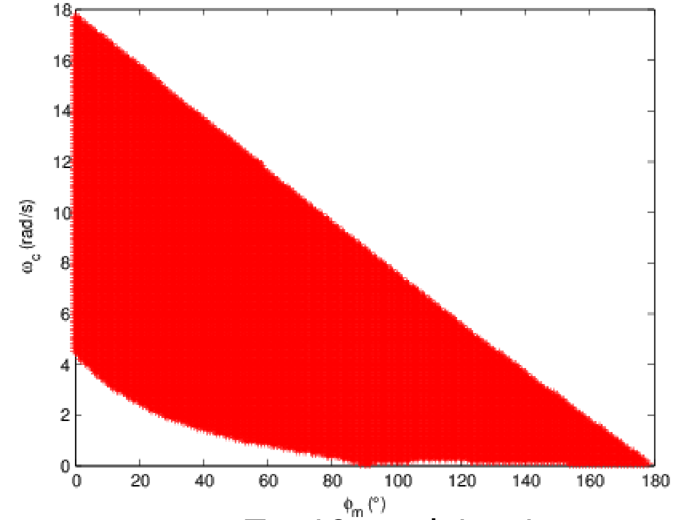


Figure: Complete stability region of *Ki*, *Kp* and *Kd* and $\varphi_m = 50°$

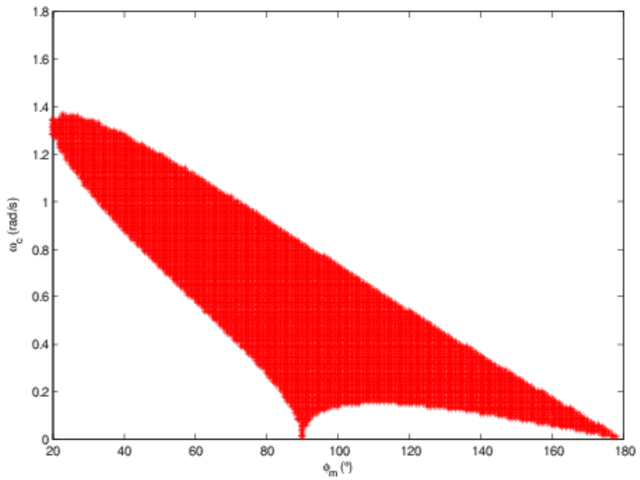
Example: Achievable region for PID



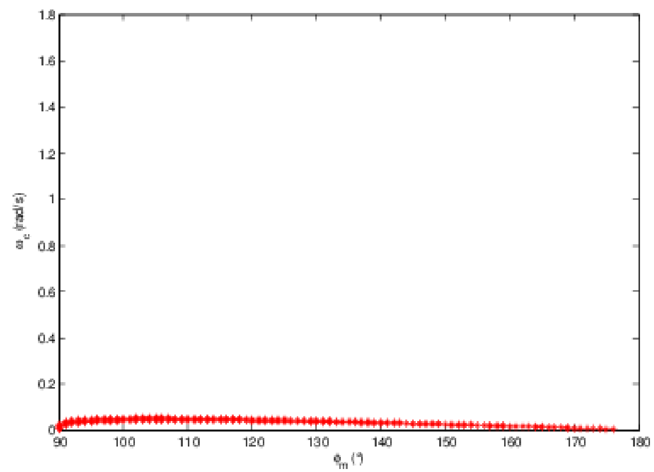
$T = 1s$ and $L = 0.1s$



$T = 10s$ and $L = 1s$



$T = 1s$ and $L = 1s$



$T = 1s$ and $L = 10s$

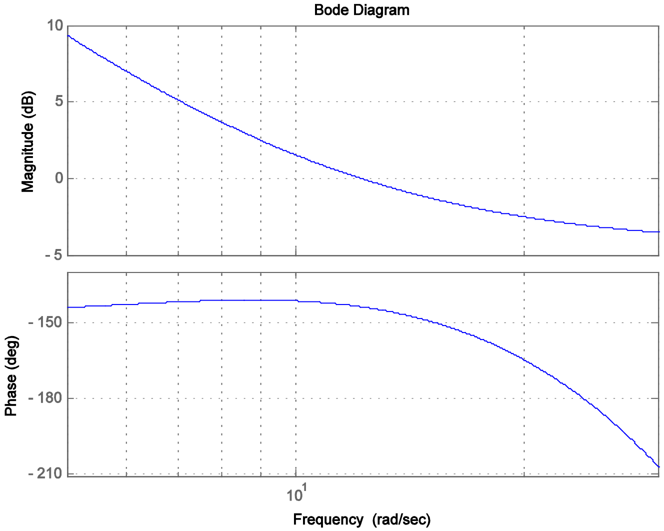
Simulation

The plant

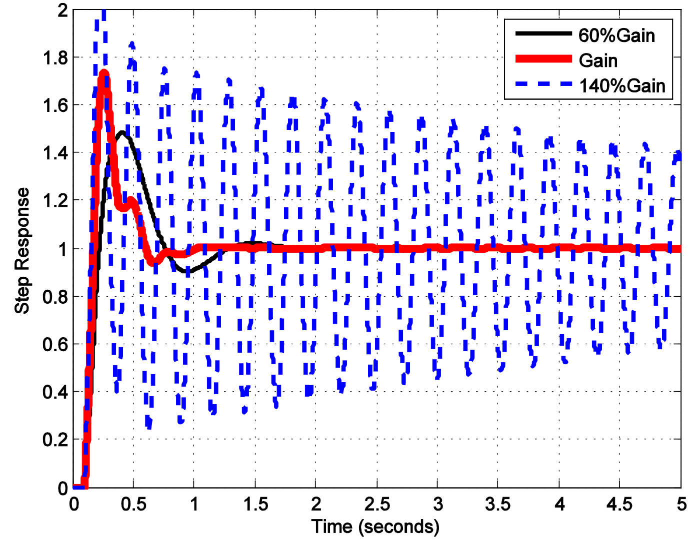
$$P(s) = \frac{1}{s+1} e^{-0.1s}$$

The PID controllers designed by different tuning methods	
Ziegler-Nichols	The flat phase constraint
$K_p = \frac{1.2T}{KL}, K_i = \frac{K_p}{2L}, K_d \frac{K_p L}{L}$ $C(s) = 12 + \frac{60}{s} + 0.6s$	$C(s) = 4.71 + \frac{14.48}{s} + 0.19s$

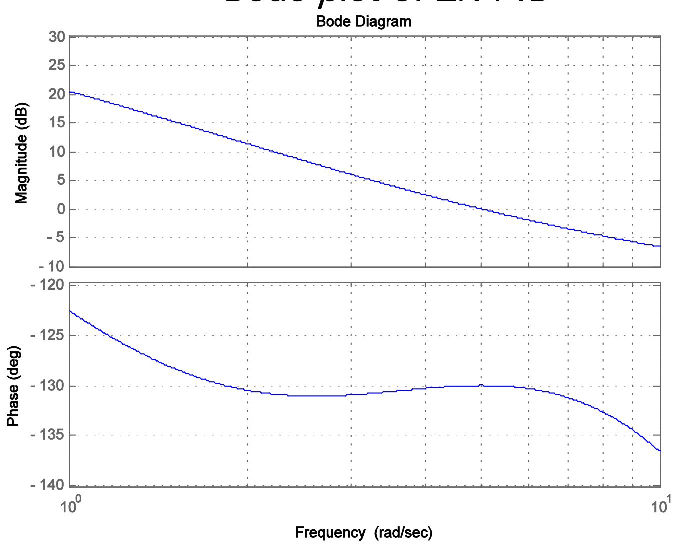
Simulation



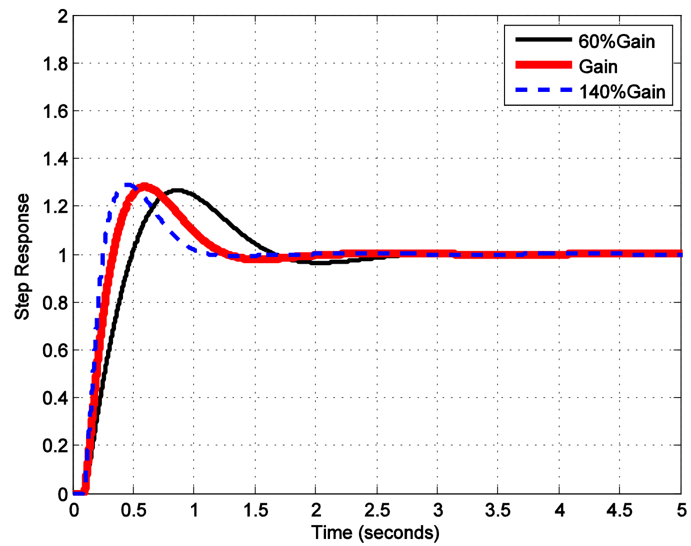
Bode plot of ZN PID



Step response of ZN PID



Bode plot of flat phase PID



Step response of flat phase PID

Stability region for FO PI

The controller

$$C(s) = K_p + \frac{K_i}{s^r}$$

the characteristic equation of the closed-loop system

$$D(K_p, K_i, A, \phi; s) = s^r(Ts+1) + Ae^{-j\phi}Ke^{-Ls}(K_p s^r + K_i)$$

RRB

$$D\left(K_p, K_i, K_d, A, \phi; s = 0\right) = 0$$

$$\Rightarrow K_i = 0$$

CRB

$$D\left(K_p, K_i, K_d, A, \phi; s = j\omega\right) = 0$$

$$\Rightarrow \begin{aligned} D(K_p, K_i, r, A, \phi; j\omega) &= (j\omega)^r(jT\omega + 1) + Ae^{-j\phi}e^{-j\omega L}K(K_p(j\omega)^r + K_i) \\ &= \omega^r \cos\frac{r\pi}{2} - T\omega^{1+r}\sin\frac{r\pi}{2} \\ &\quad + AK\cos(\phi + \omega L)(K_i + K_p\omega^r \cos r\pi 2) + AK\sin(\phi + \omega L)K_p\omega^r \sin\frac{r\pi}{2} \\ &\quad + j(T\omega^{1+r}\cos\frac{r\pi}{2} + \omega^r \sin\frac{r\pi}{2} + AK\cos(\phi + \omega L)K_p\omega^r \sin\frac{r\pi}{2} \\ &\quad - AK\sin(\phi + \omega L)(K_i + K_p\omega^r \cos r\pi 2)) \\ &= 0, \end{aligned} \quad (12$$

CRB – cont.

$$\Rightarrow \quad K_p = \frac{-(B_1 S_1 + B_2 C_1)}{AKS_2\omega^r},$$

$$K_i = \frac{B - B_1 S_1 C_1 - B_2 C_1^2}{AKS_1} + \frac{B_1 S_1 C_2 + B_2 C_1 C_2}{AKS_2}.$$

With plat phase constraint

$$\frac{\mathrm{d}\phi}{\mathrm{d}\omega} = \frac{\left(B_1^2 + B_2^2\right)\left(EF' - E'F\right) + \left(B_1' B_2 - B_1 B_2'\right)\left(E^2 + F^2\right)}{\left(B_1 E + B_2 F\right)^2 + \left(B_1 F - B_2 E\right)^2} - L$$
$$= 0,$$

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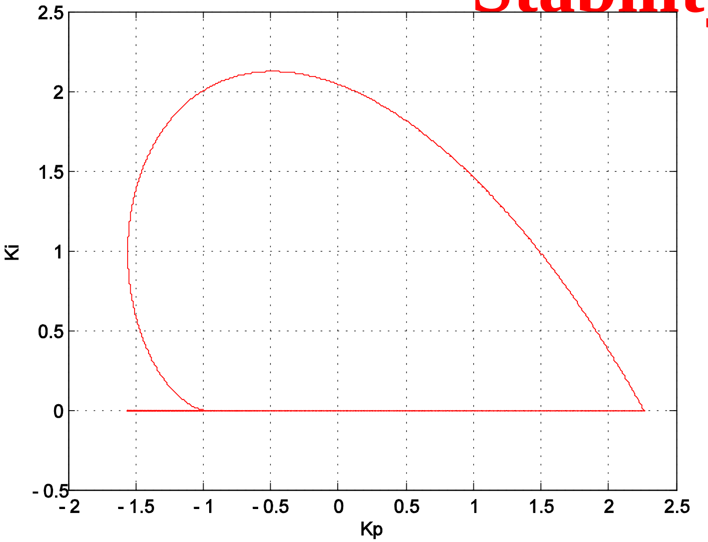
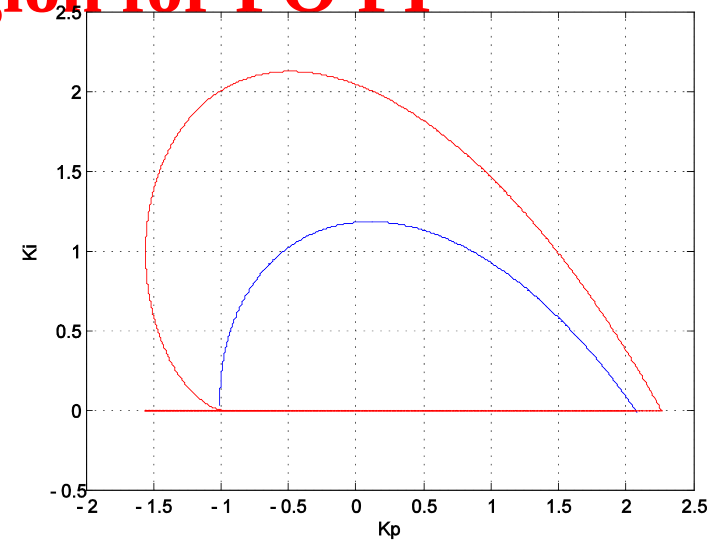
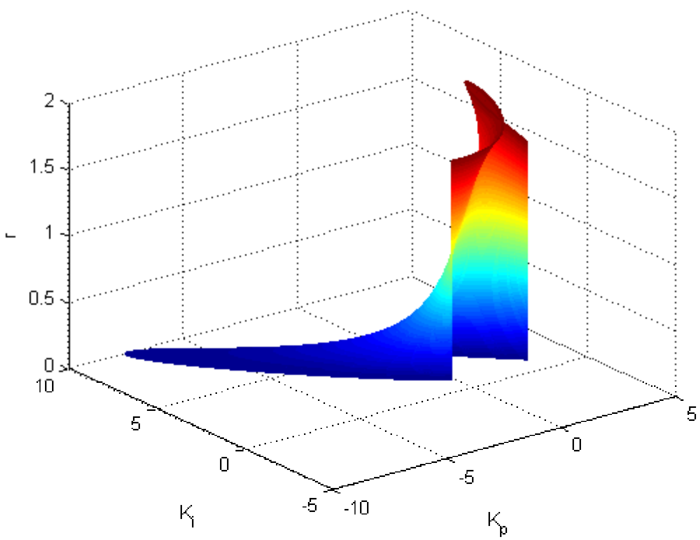
Stability region for FO PI



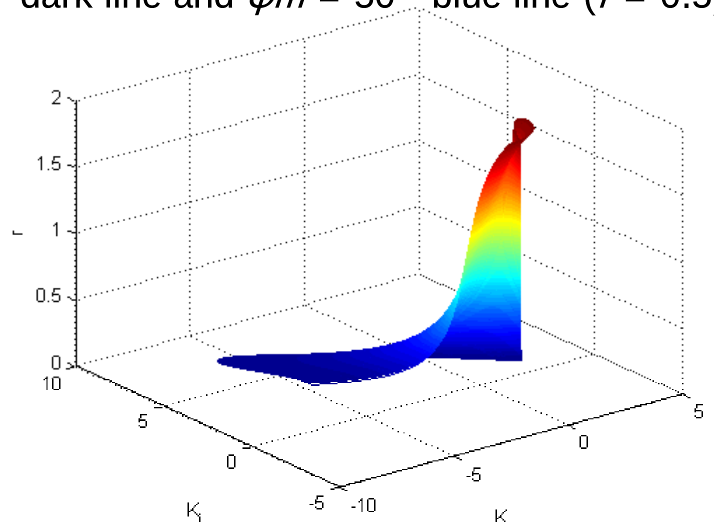
Stability boundary of *Ki* vs. *Kp* with *r* = 0.5



Stability region comparison of *Ki* vs. *Kp* with *φm* = 0 ◦ dark line and *φm* = 50◦ blue line (*r* = 0.5)

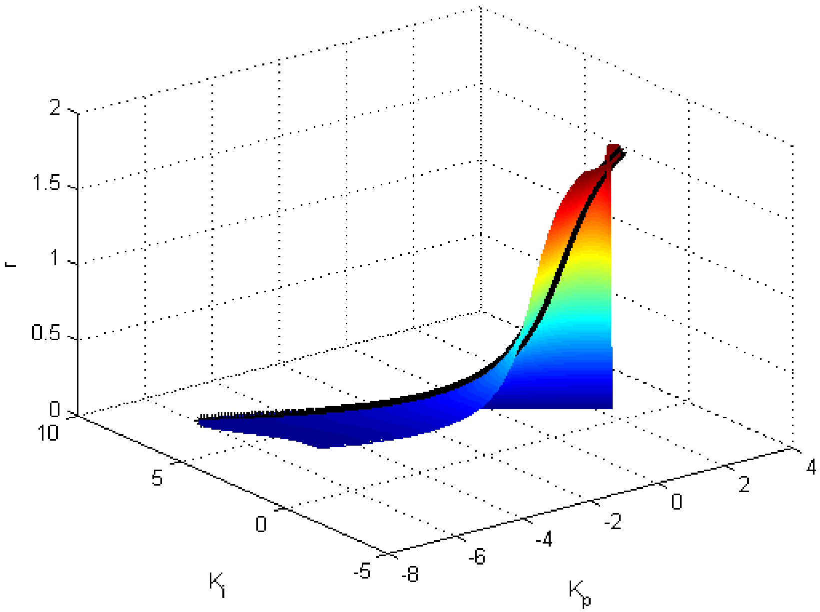


Complete stability region of Ki, Kp and r

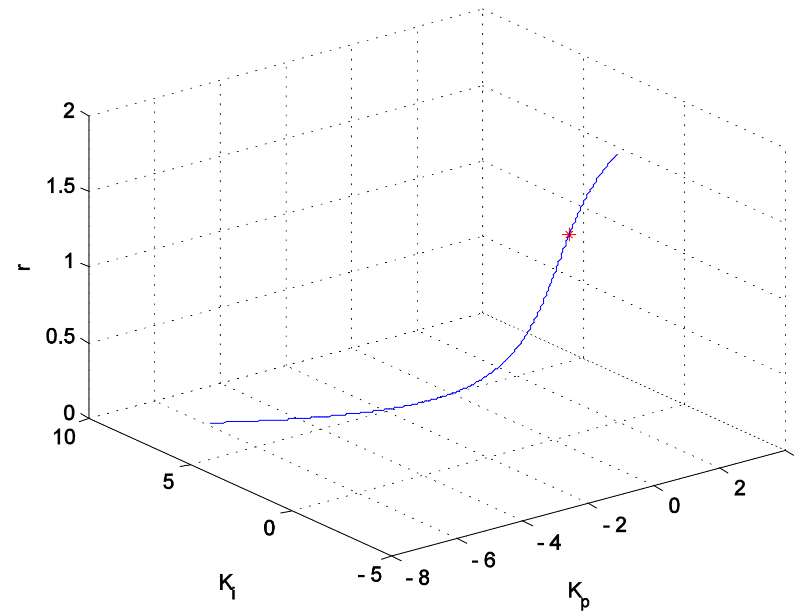


Three dimensional relative stability surface of Ki, Kp and r with φm = 50◦

Stability region for FO PI



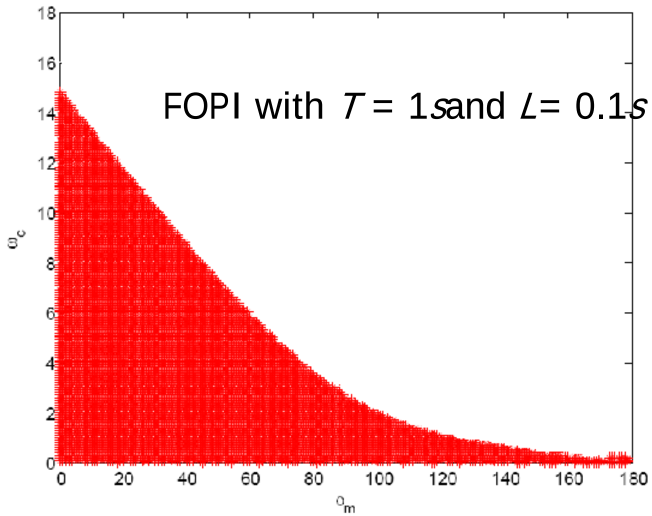
The relative stability curve on the relative stability surface

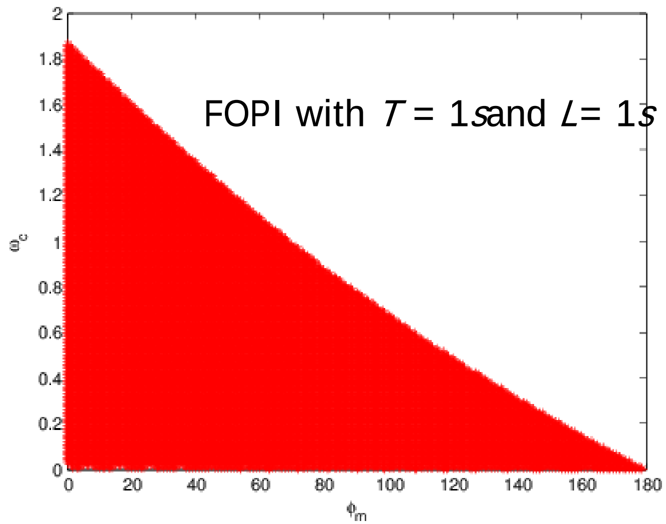


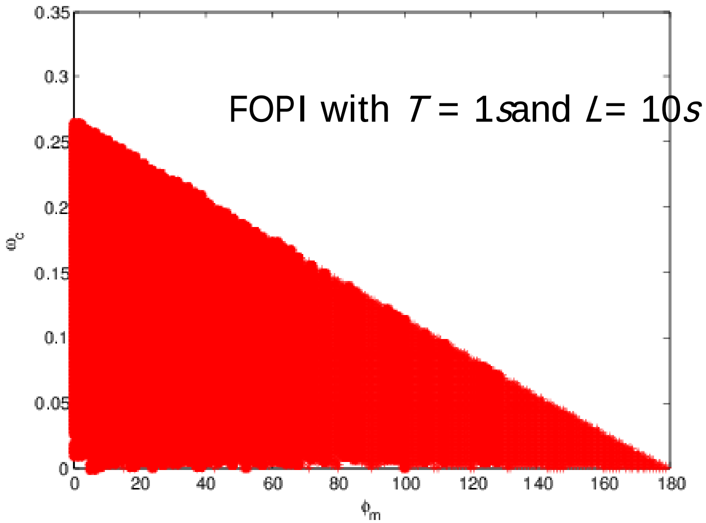
The flat phase stable point on the relative stability curve

Figure: The relative stability curve and the flat phase stable point in the 3D parameter space

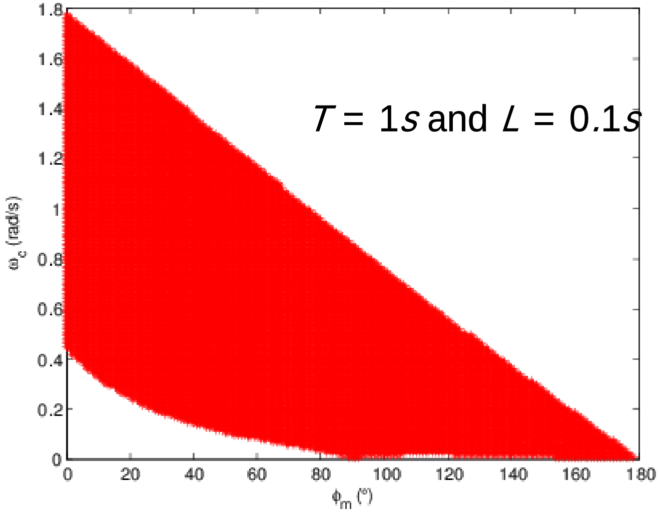
Example: Achievable region for FOPI

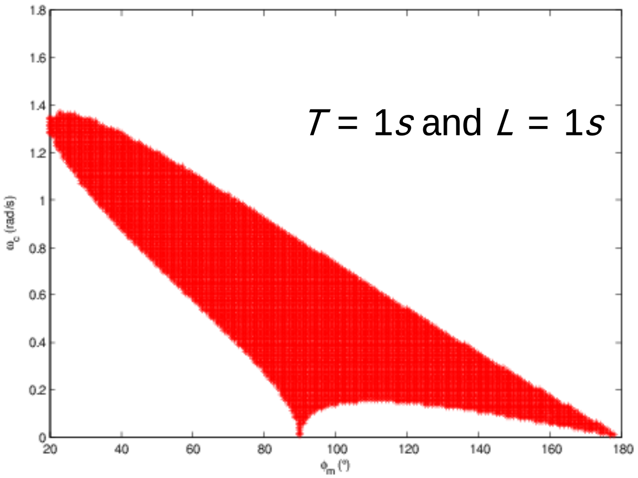


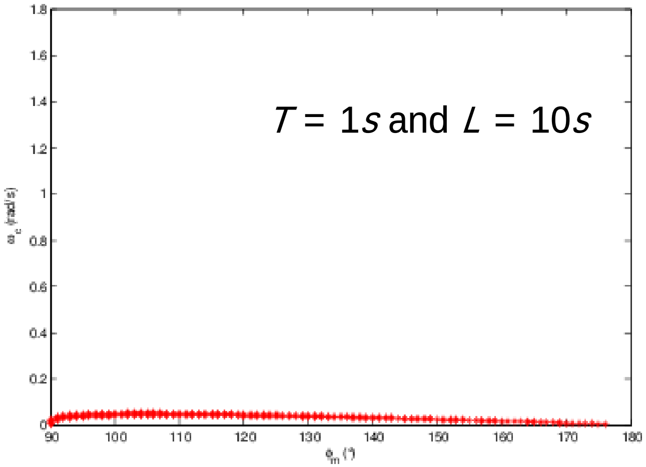




Recall: Achievable region for IOPID







Observations

- Flat phase ideal is useful – close to Bode Step idea.
- FO PI has larger achievable feasible region than IOPID
- When the relative delay is larger, fractional order control can help more

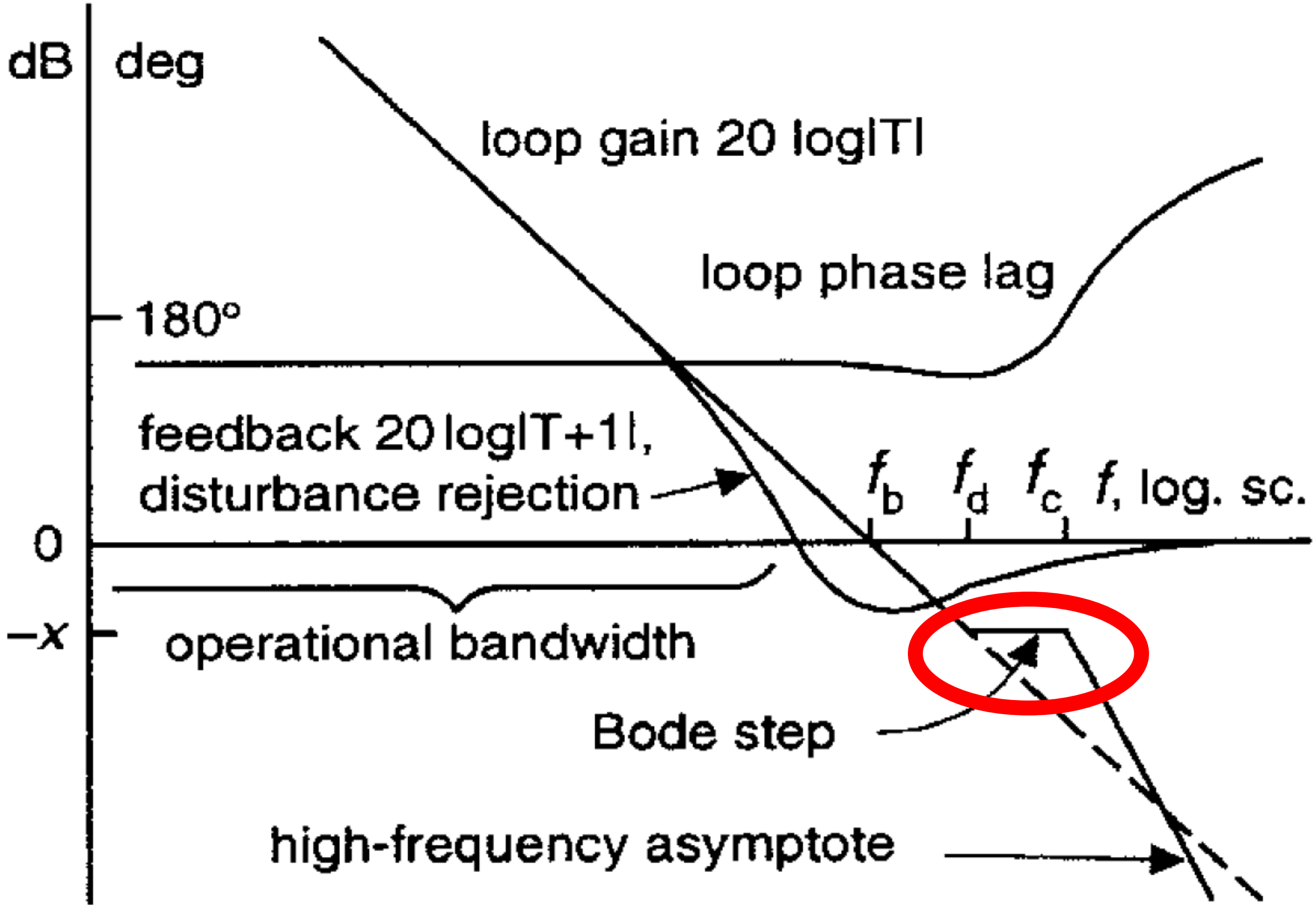


Fig. 7 Asymptotic Bode diagram with Bode step.

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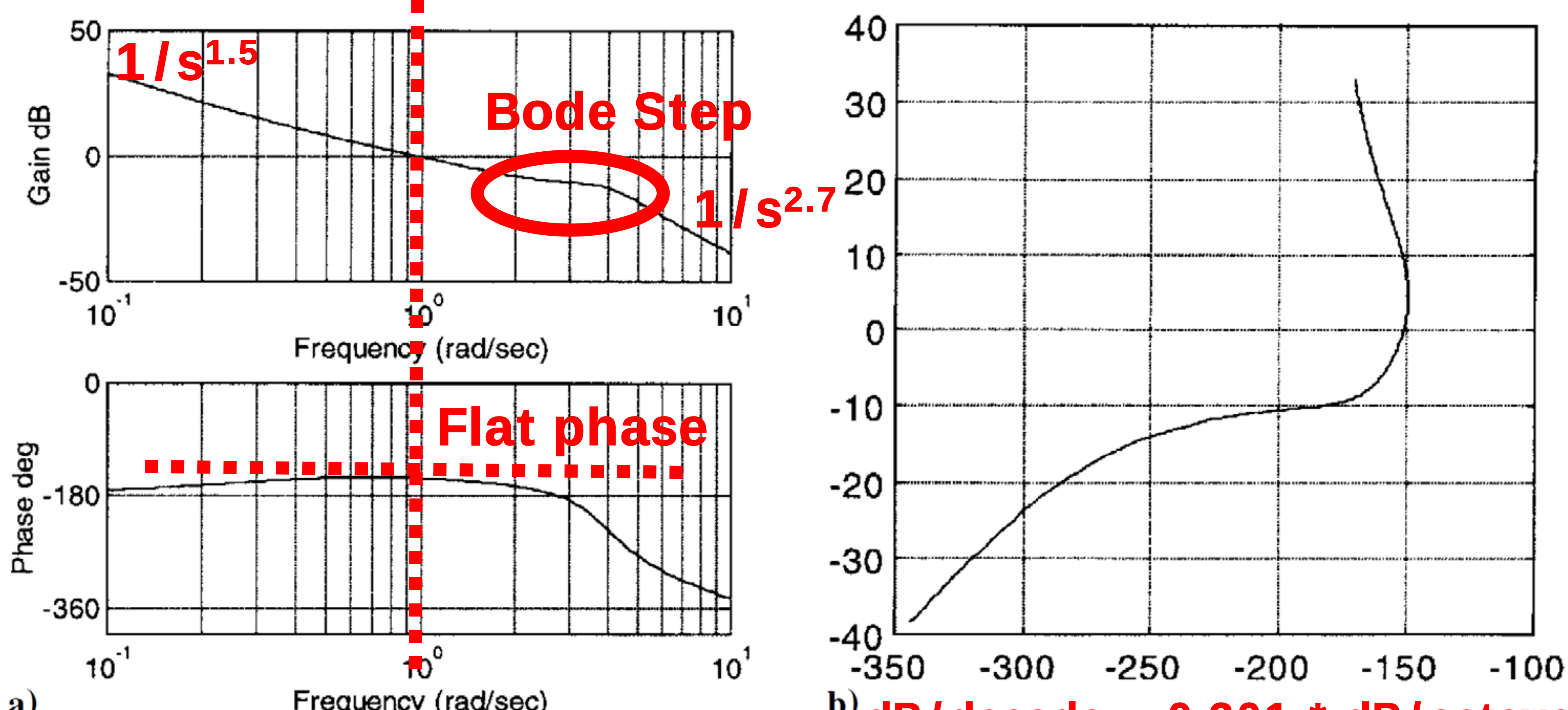
Vol. 25, No. 2, March–April 2002

System Architecture Trades Using Bode-Step Control Design

05/11/2018

Boris J. Lurie,* Ali Ghavimi,† Fred Y. Hadaegh,‡ and Edward Mettler*
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109-8099



dB/decade = 0.301 * dB/octave

0.301 = 1/log$_2$(10)

Fig. 13 Normalized open-loop response.

$$\frac{1}{s^2}\,\frac{11s^3 + 55s^2 + 110s + 36}{s^4 + 7.7s^3 + 34s^2 + 97s + 83}$$

with the crossover frequency of 1 rad/s, a Bode step of approximately 1 octave long, a low-frequency asymptotic slope of -10 dB per octave, a high-frequency asymptotic slope of -18 dB per octave, a gain margin of 10 dB, and a phase margin of 30 deg (Ref. 4). Figure 13b shows the associated Nyquist diagram in the L plane. It is then an easy task to extract a controller transfer function from the earlier loop design.

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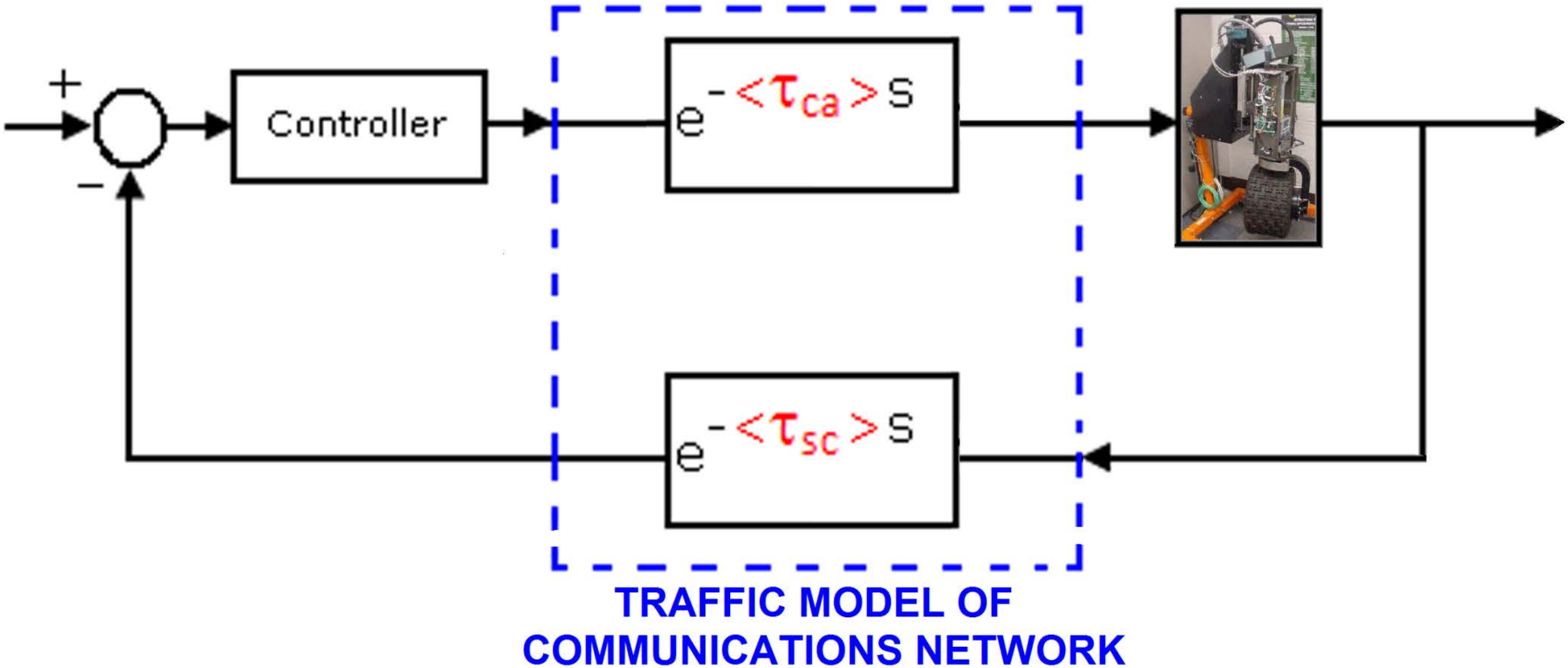
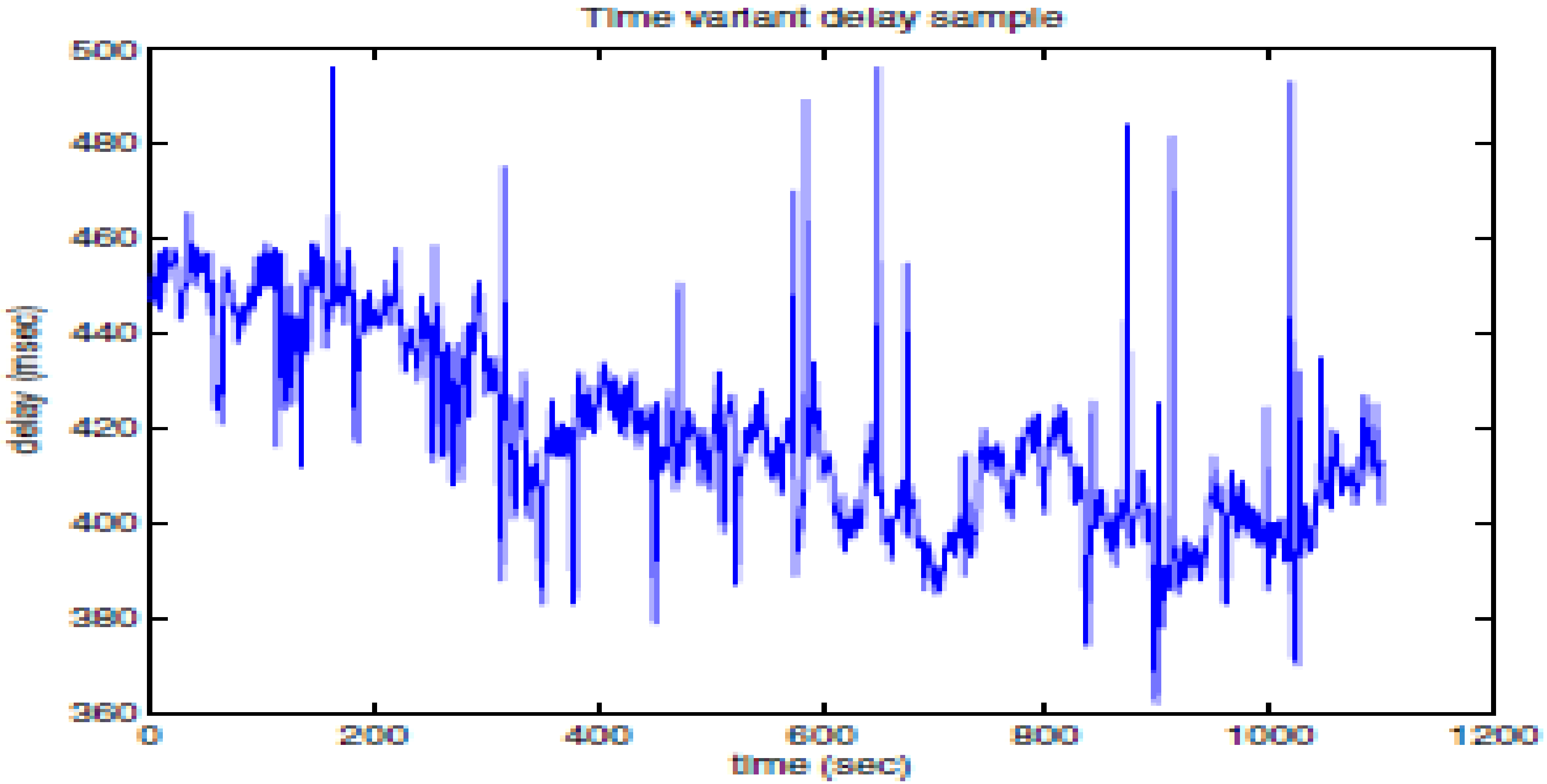
NCS – delay is random, time-varying

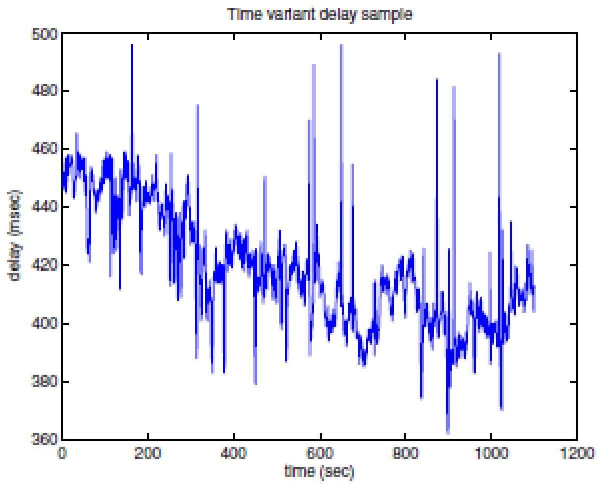


… and spiky

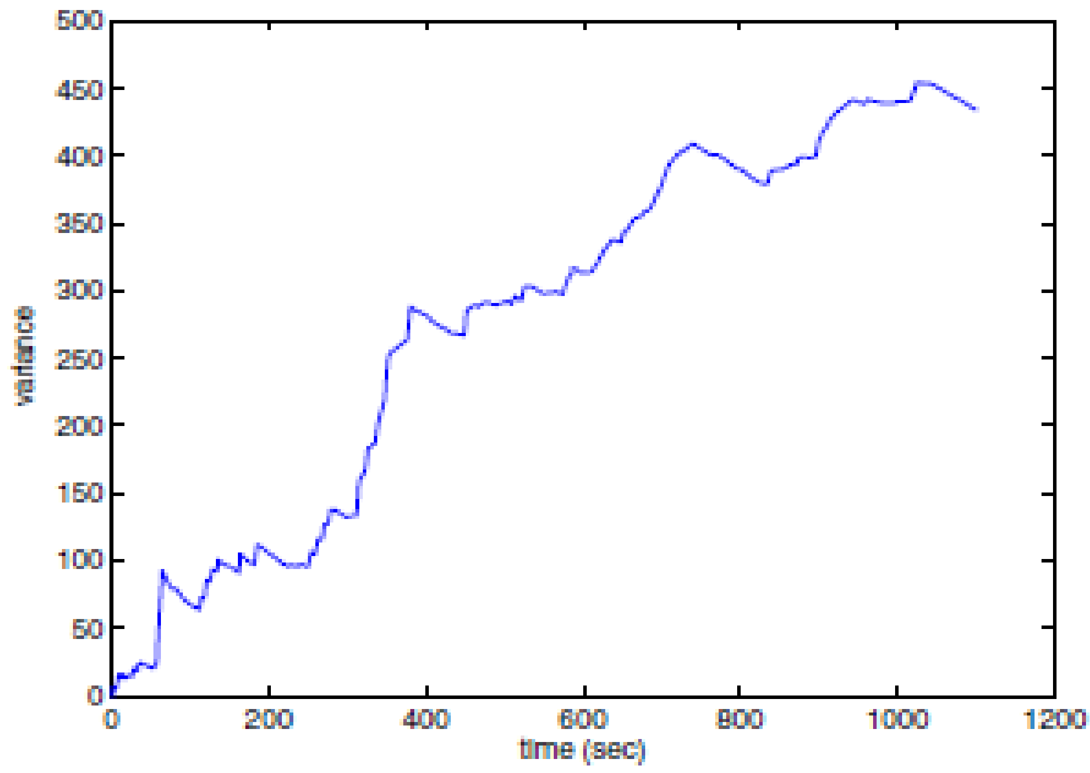


(a) Network delay samples

PROBLEM? running variance estimate is not convergent



(a) Network delay samples



(b) infinite or divergent variance

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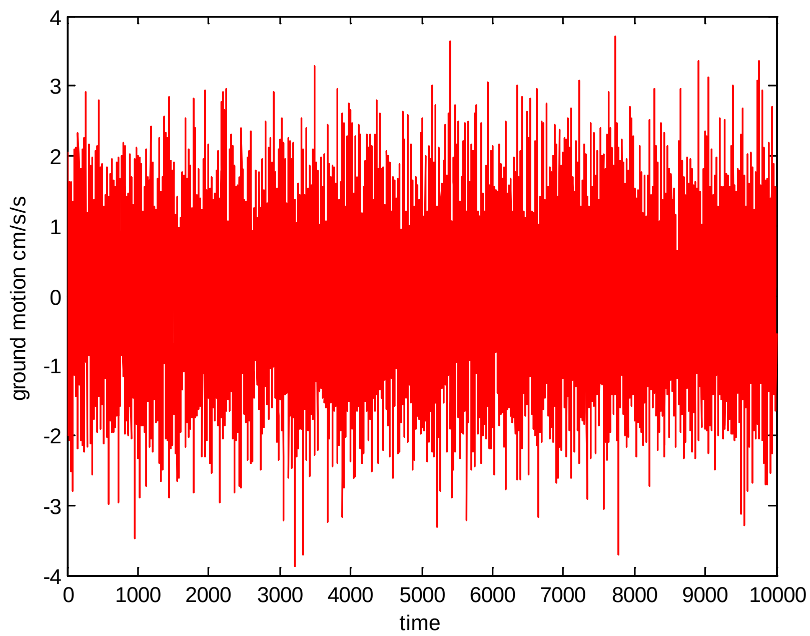
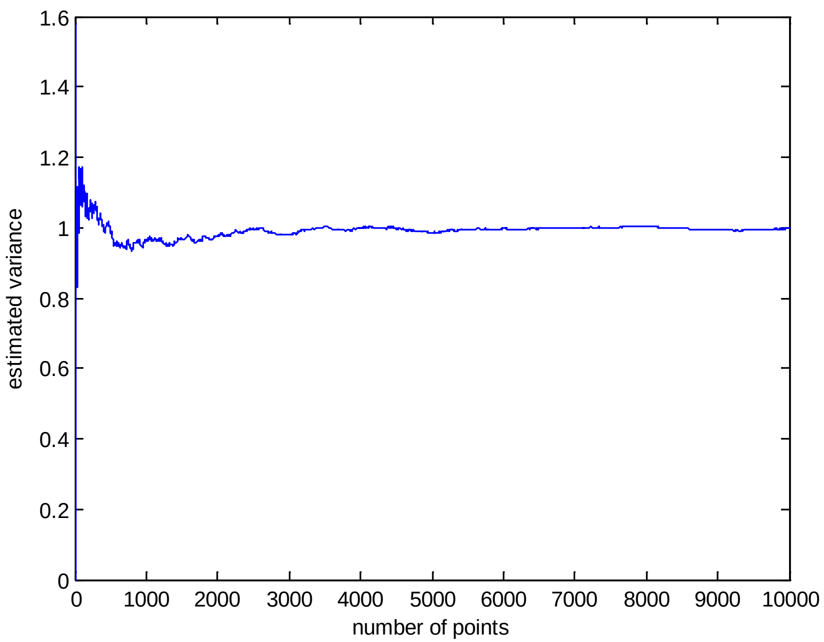
Noise - 1





Normal distribution N(0,1)　Sample Variance

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Long-range dependence

Self-similar

Hurst parameter

ARFIMA

α-stable distributions

Network Traffic, Smart Grids, HRV, Outliers in time series, biox signals …

Fractional Gaussian noise (FGn)

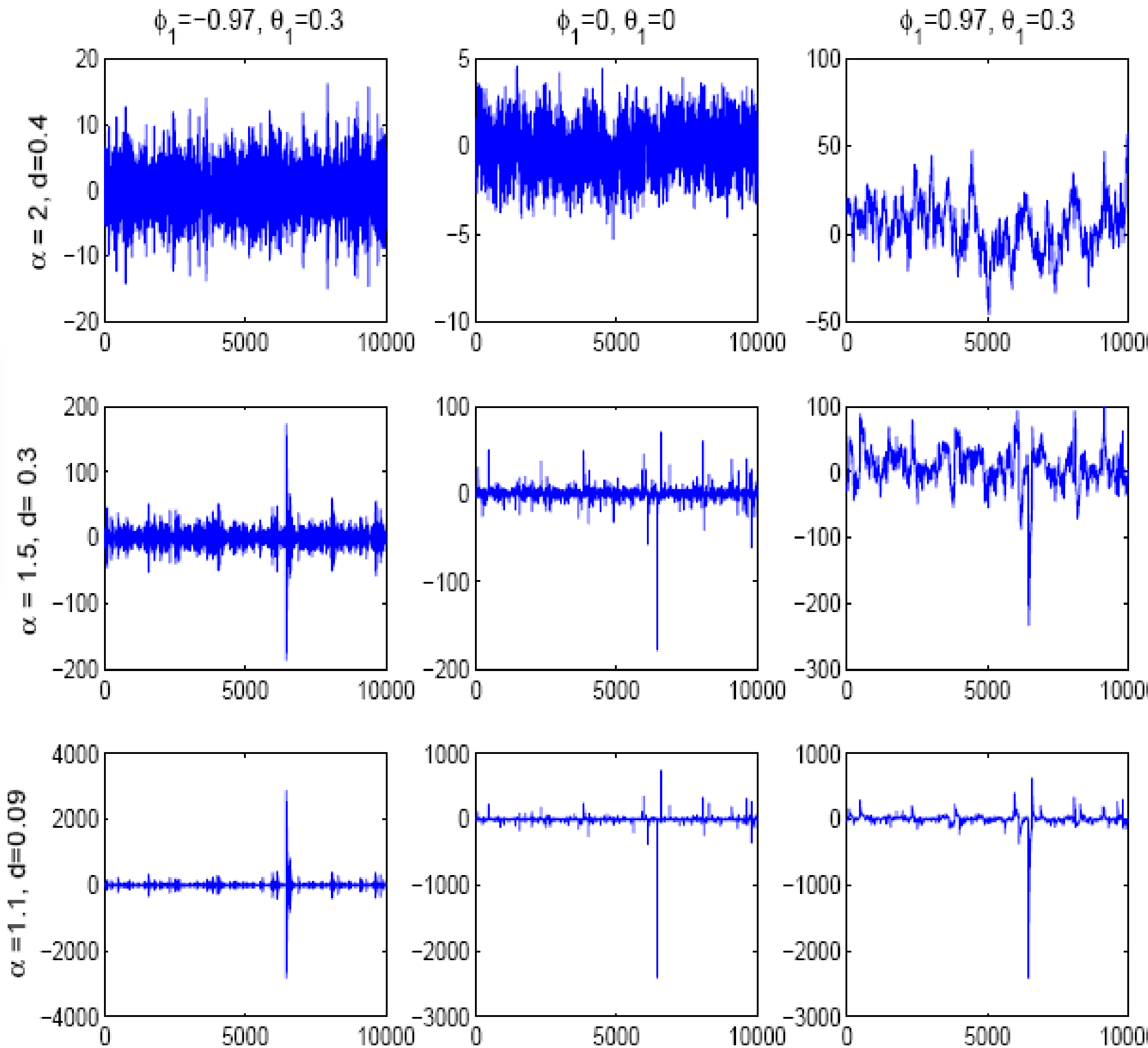
“Spikiness”

“Heavy tails”

Fractional Brownian motion (FBm)

MODELS IN LITERATURE (&IV)

[7] S. Stoev, and M.S. Taqqu, "Simulation Methods for Linear Fractional Stable Motion and FARIMA Using the Fast Fourier Transform". *Fractals*, 2004.



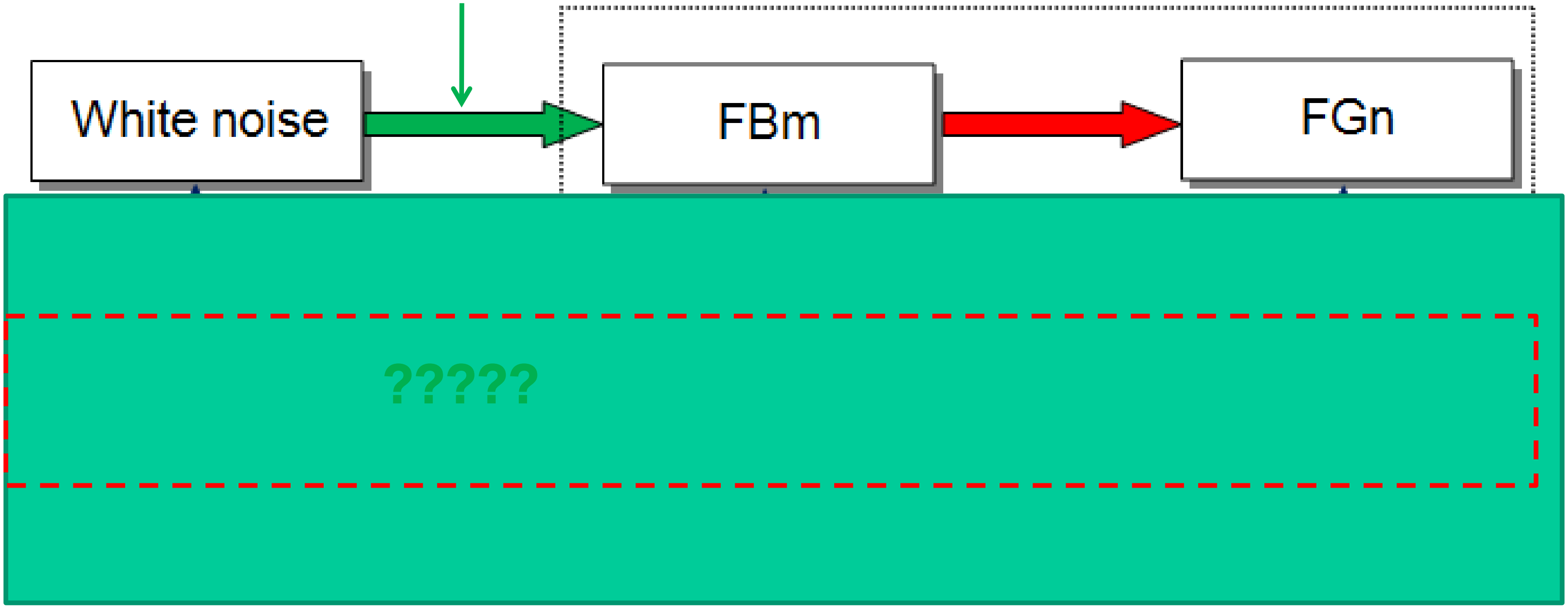
FARIMA$(1, d, 1)$

FO NETWORK DELAY DYNAMICS

$$D^{\beta}\tau(t) = B(t)$$

β the fractional-order,
$\tau(t)$ the network-induced delay,
$B(t)$ white noise.

[18] V. Pipiras, and M.S Taqqu, "Fractional calculus and its connections to fractional Brownian motion". *Theory and Applications of Long-range Dependence*, 2003.



- Self-similarity => Hurst parameter
- "Spikiness".

Operation: $D^{-\beta}$

Operation: *increments*

Self-similar processes

- **Fractional calculus, delay dynamics and networked control systems**
- YangQuan Chen
- 2010 3rd International Symposium on Resilient Control Systems Year: 2010
- Pages: 58 - 63

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  - **Human-in-the-loop model**
  - **Cyber Physical Human Systems**

Fractional Order Control

Better and Ubiquitous?

- **u·biq·ui·tous** /yuˈbɪkwɪtəs/
- –adjective existing or being everywhere, esp. at the same time; omnipresent:

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hysteresis

Biomimetic Materials and Biomimetic Actuators

- EAP (electroactive polymers), a.k.a. artificial muscle
- ferroelectric and relaxor materials
- piezoceramic and piezopolymetric materials
- liquid crystal elastomers
- electro and magnetostrictive materials
- shape memory alloys/polymers
- intelligent gels etc.

However, little has been reported on the controls of actuators made with these biomimetic materials.

Compensation of nonlinearity with memory

- e.g., hysteresis, backlash.
- My Assertion: Fractional calculus may better help us.

A Hidden Evidence

IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, VOL. 9, NO. 1, JANUARY 2001 17

Phase Control Approach to Hysteresis Reduction

Juan Manuel Cruz-Hernández, *Member, IEEE,* and Vincent Hayward, *Member, IEEE,*

***Abstract*—This paper describes a method for the design of compensators able to reduce hysteresis in transducers, as well as two measures to quantify and compare controller performance. Rate independent hysteresis, as represented by the Preisach model of hysteresis, is seen as an input–output phase lag. The compensation is based on controllers derived from the "phaser," a unitary gain operator that shifts a periodic signal by a single phase angle. A "variable phaser" is shown to be able to handle minor hysteresis loops. Practical implementations of these controllers are given and discussed. Experimental results exemplify the use of these techniques.**

***Index Terms*—Compensation, hysteresis, intelligent materials, phase control, piezoelectric transducers, smart materials, transducers.**

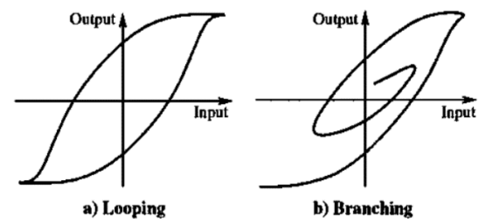

Fig. 1. Hysteresis loop and branching.

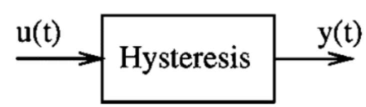

Fig. 2. A black box representation of hysteresis.

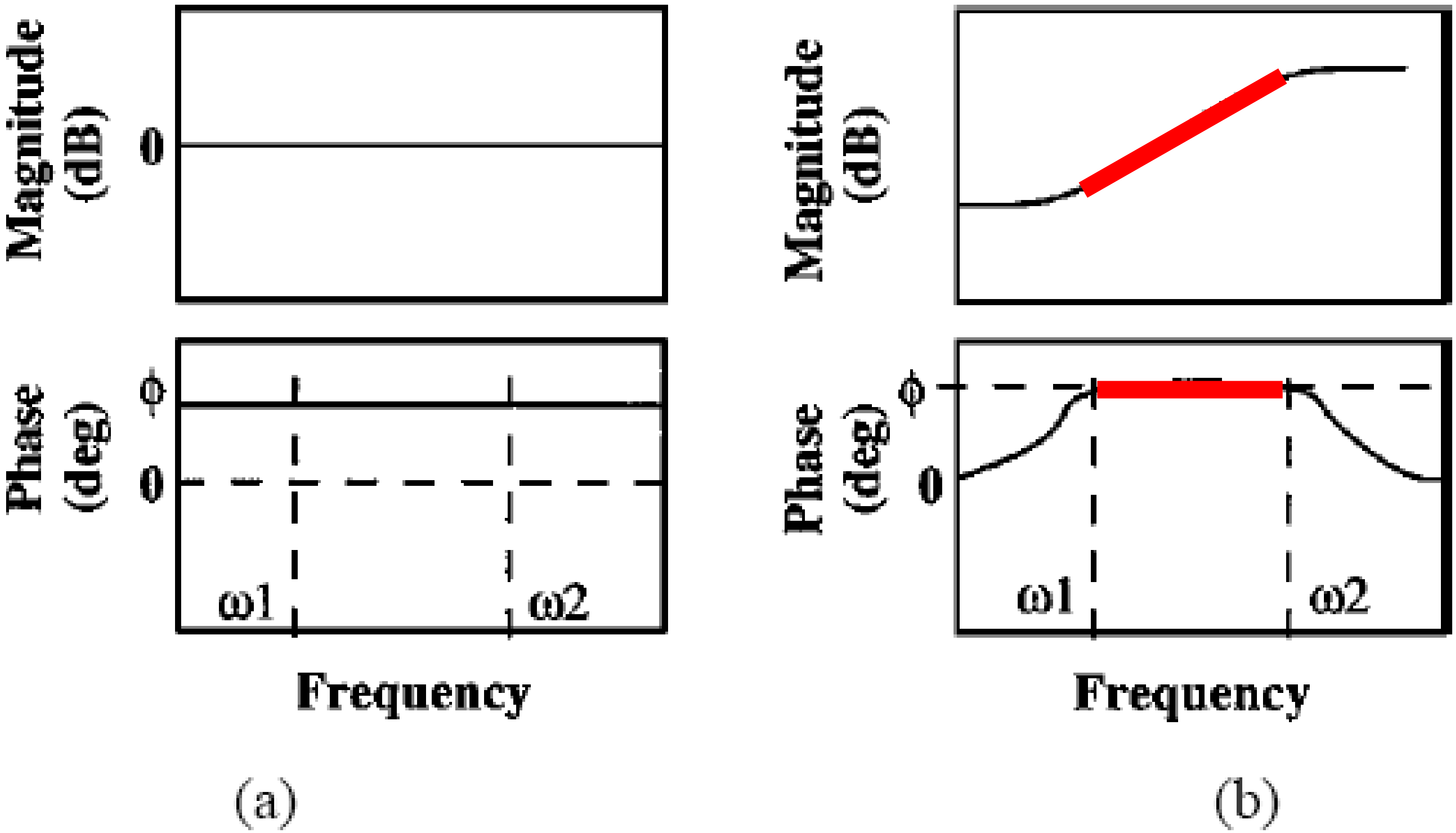

Fig. 10. Frequency response. (a) Ideal phaser. (b) Approximation.

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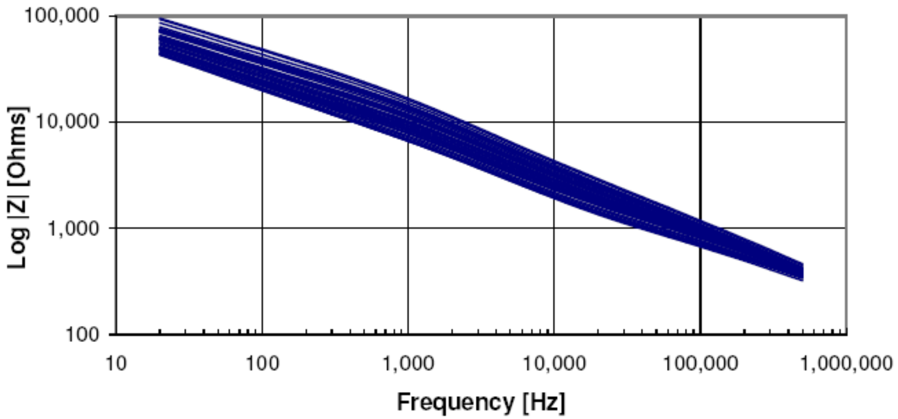
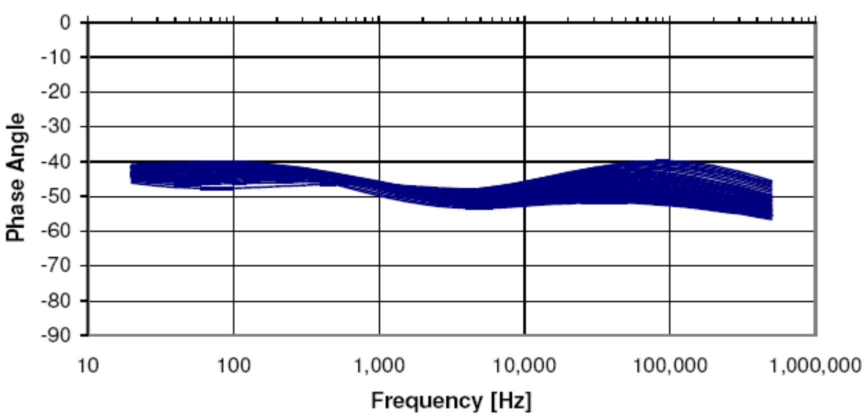
"smart material" based Fractor^{TM}



(a)



(b)

Fig. 1. Spectral response of the Fractor^{TM} used in this demonstration project; (a) the impedance magnitude and (b) impedance phase. The multiple lines show the variation over 26 impedance measurement scans.

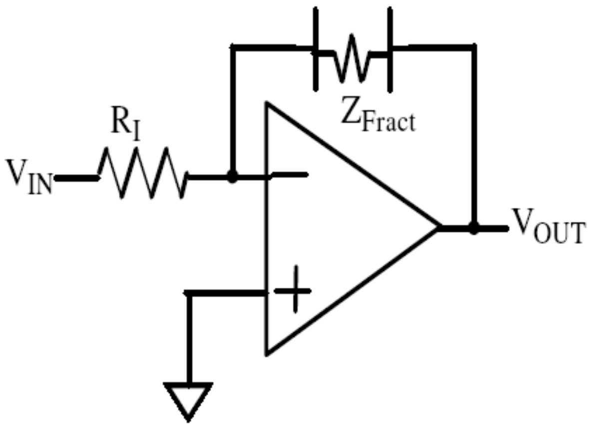


Fig. 2. Schematic for a fractional order integrator. Z_F represents the Fractor^{TM} element. The schematic symbol for the Fractor^{TM} was designed to give the impression of a generalized Warburg impedance; a mixture of resistive and capacitive characteristics.

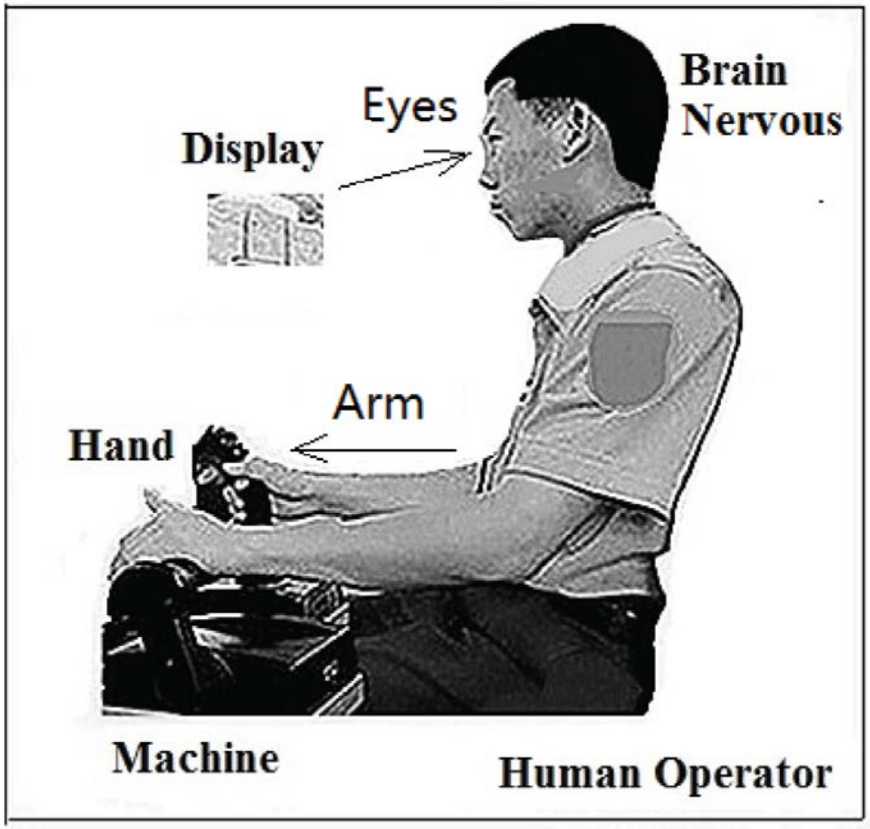
Gary W. Bohannan "**Analog Fractional Order Controller in a Temperature Control Application**". Proc. of the 2nd IFAC FDA06, July 19-21, 2006, Porto, Portugal.

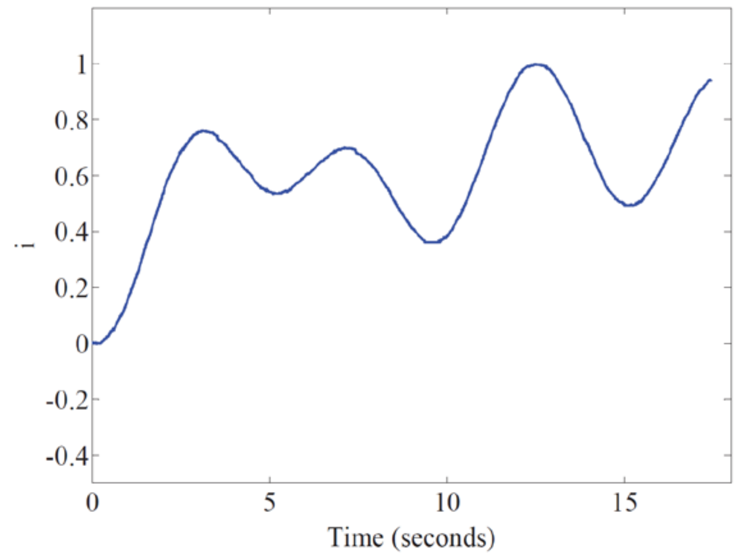
Big Picture, or, *the message for you to take home*

- the big picture for the future is the intelligent control of biomimetic system using biomimetic materials with fractional order calculus embedded. In other words, it is definitely worth to have a look of the notion of ``*intelligent control of intelligent materials using intelligent materials*."

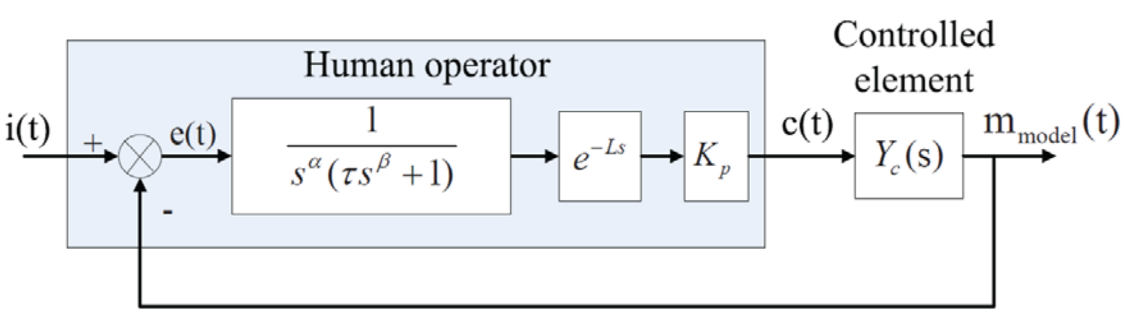
Outline

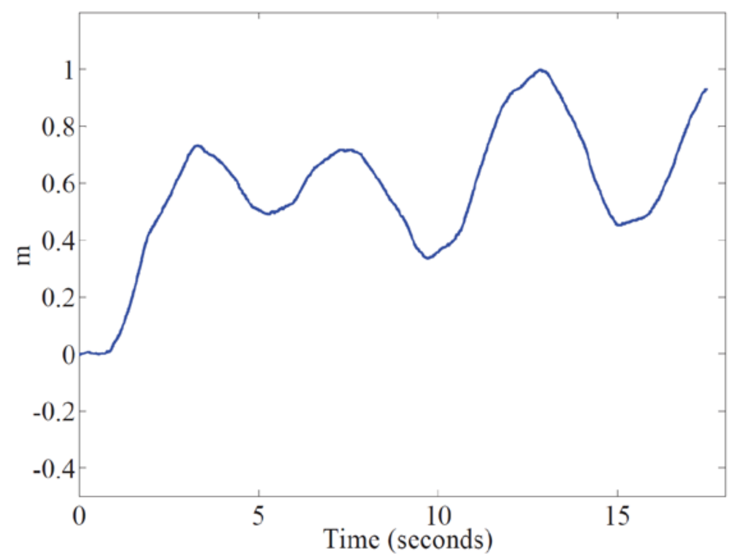
- **Fractional calculus: What, Why and When**
- **Better than the best: Example 1 - Modeling**
- **Better than the best: Example 2 – Control**
- **Sample future chances:**
  - **Networked control systems**
  - **Nonlinearities with memory**
  - **Human-in-the-loop model**
  - **Cyber Physical Human Systems**





(a) System forcing function $i(t)$





(b) System output $m(t)$

Pioneering work

- Tustin, A., The nature of the operator's response in manual control, and its implications for controller design, Journal of the Institute of Electrical Engineers-Part IIA: Automatic Regulators and Servo Mechanisms, vol. 94, no. 2, pp. 190-206, 1947.
- Craik, K. J. W., Theory of the human operator in control systems, British Journal of Psychology, General Section, vol. 38, no. 3, pp. 142-148, 1948.
- McRuer, D. T., Krendel, E. S., The human operator as a servo system element, Journal of the Franklin Institute, vol. 267, no. 6, pp. 511-536, 1959.

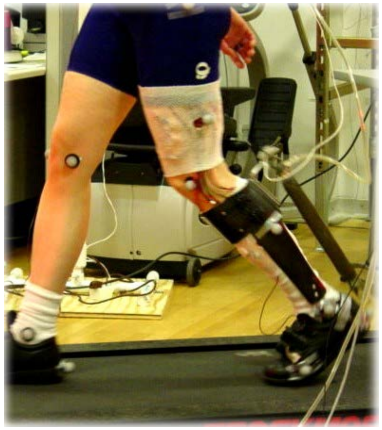
Human Operator Modeling

- **Human operator modeling based on fractional order calculus in the manual control system with second-order controlled element**
- Jiacai Huang; Yangquan Chen; Zhuo Li
- The 27th Chinese Control and Decision Conference (2015 CCDC) Year: 2015
- Pages: 4902 - 4906

- **Fractional order modeling of human operator behavior with second order controlled plant and experiment research**
- Jiacai Huang; Yangquan Chen; Haibin Li; Xinxin Shi
- IEEE/CAA Journal of Automatica Sinica Year: 2016, Volume: 3, Issue: 3
- Pages: 271 - 280

https://www.youtube.com/watch?v=o8XoMMFdLyE&t=9s

Biomechatronics



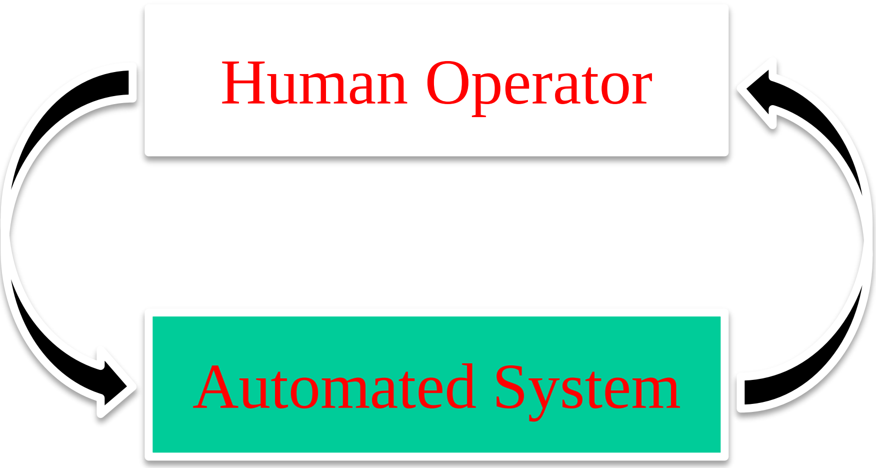
Electronically controlled leg and hand prosthesis, neural prosthesis, retinal implants, assistive and rehabilitative robots ...

-- "Emerging Trends and Innovations in Biomechatronics" by Frost & Sullivan

Source: http://www-personal.umich.edu/~ferrisdp/NSF/research.htm


STORIE IDEE E PERSONE CHE CAMBIANO IL MONDO
WIRED
EVOLUZIONE IN CORSO
HUGH HERR HA CAMBIATO LA VITA DI ATLETI COME PISTORIUS E AIMEE MULLINS
DARWIN AVEVA RAGIONE LA TECNOLOGIA INTORNO A NOI CI CAMBIERÀ

The Human-Automation Interaction Cycle

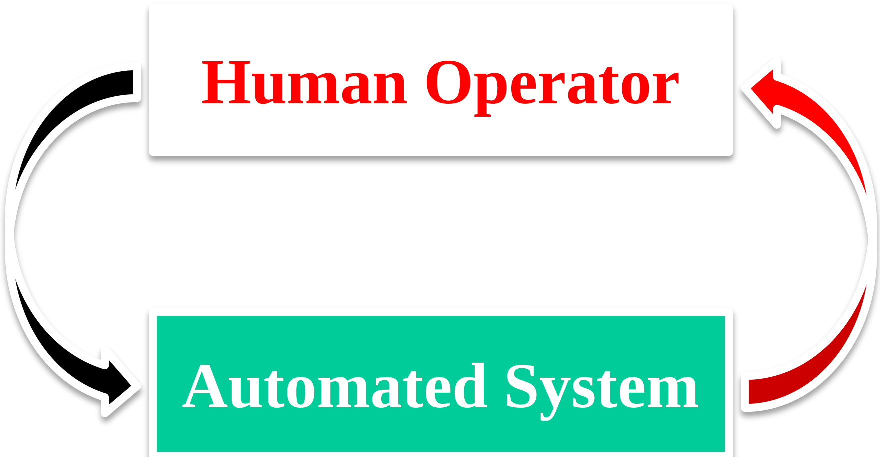


The Human-Automation Interaction Cycle

The Human Control over Automation

- Human monitors automated systems
- Human operates as the ‘sensor’ for correct behavior
- Safety
- Human-in-the-Loop

The Human-Automation Interaction Cycle



Automation for Human Assistance

- System monitors human
- Human health monitors for analysis
- Bio-feedback
- Emotional-feedback

Human-Automation Interaction

- Human Over Automation
  - Metrics
    - Situational Awareness
    - Cognitive Load
  - Legislative History
    - Humans act as the final safety level
- Automation Systems for Humans
  - Physiological Monitors
- State of Human Observer (SOHO)
- Next Level of Technology

Outline

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- **Better than the best: Example 2 – Control**
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  - **Networked control systems**
  - **Nonlinearities with memory**
  - **Human-in-the-loop model**
  - **Cyber Physical Human Systems**

What is driver's role in driverless car?

Optimal role/function assignment?

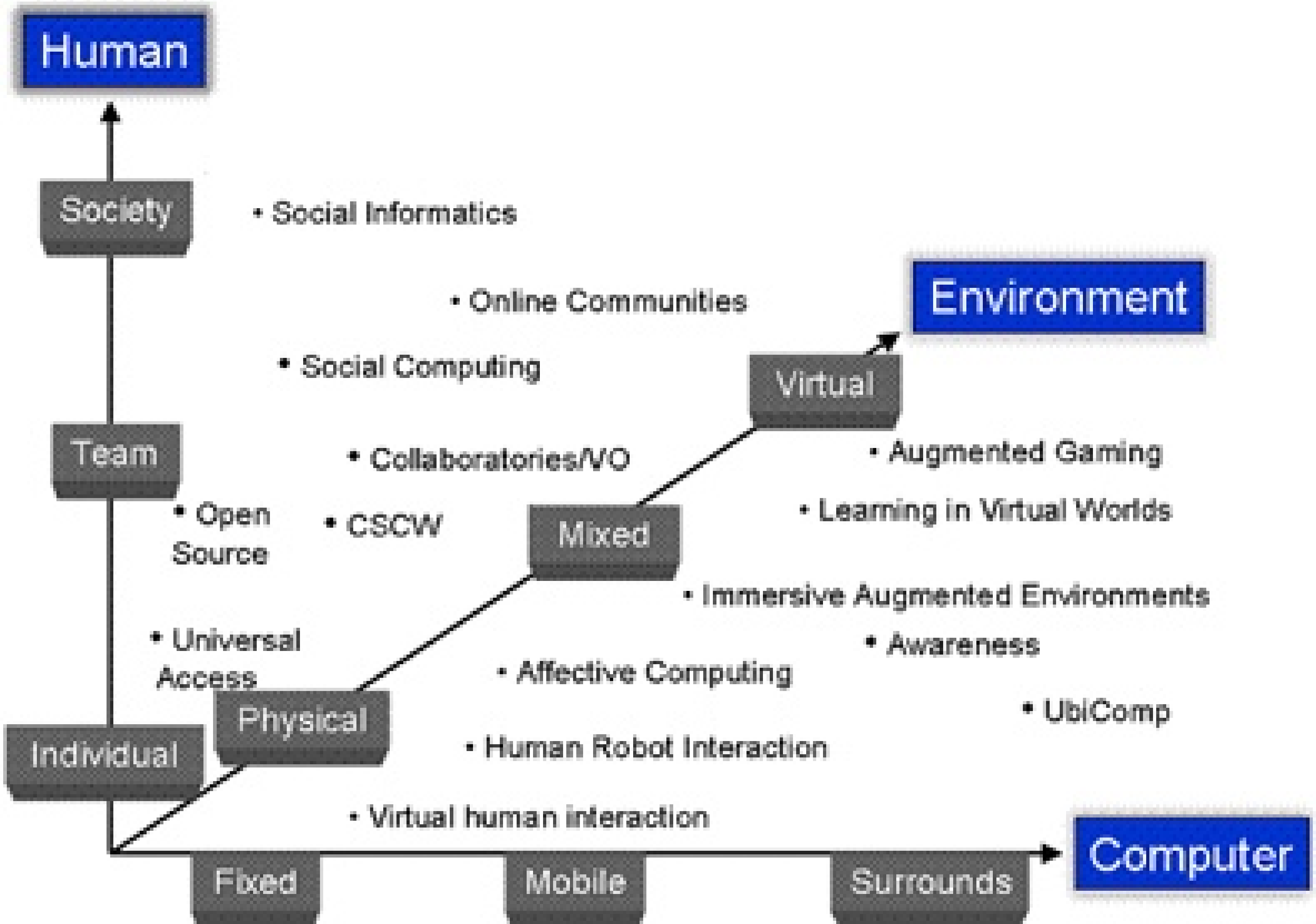
Physiology-aware; Psychology-aware

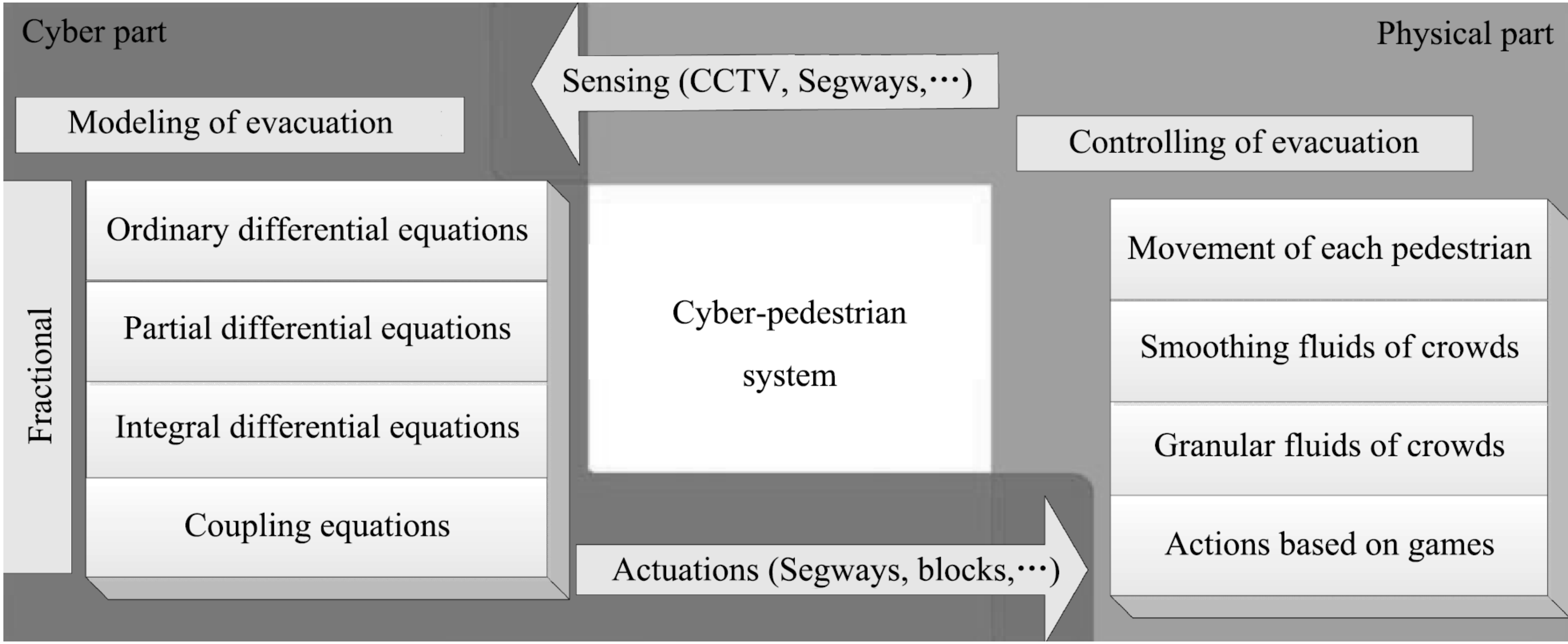
Situational Awareness

Your car reminded you: *You forgot head check when you change lane*

http://h-cps-i.sciencesconf.org/

NSF CHS (cyber-human system)

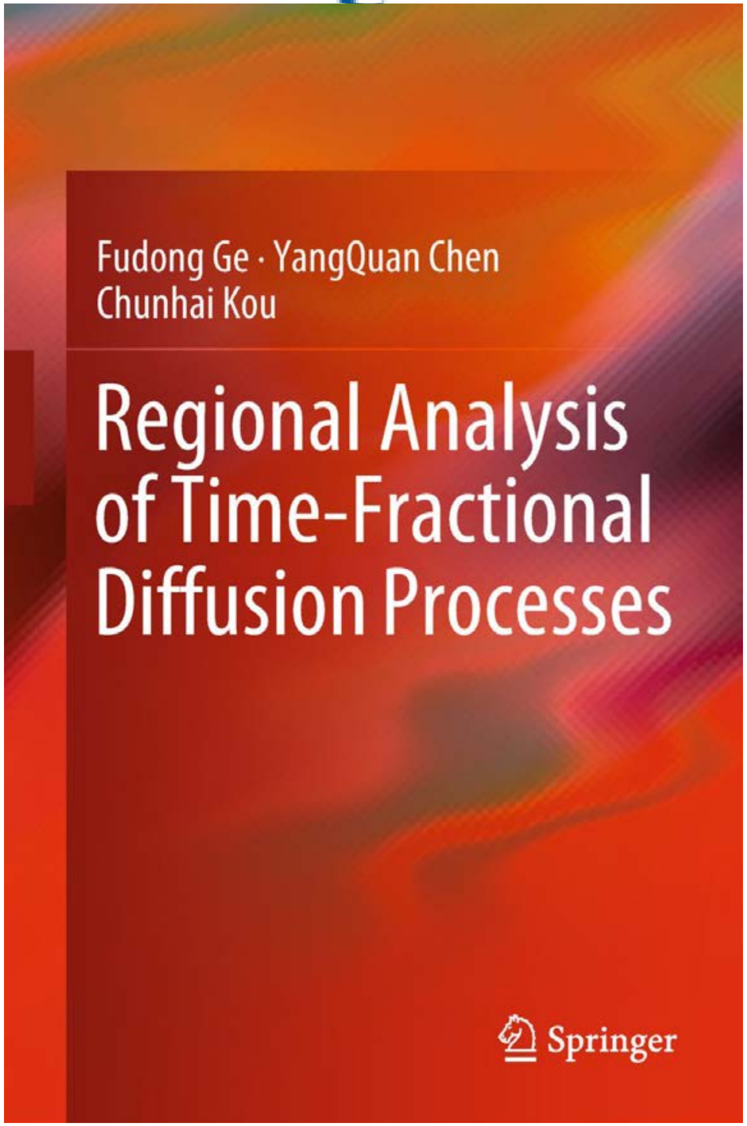




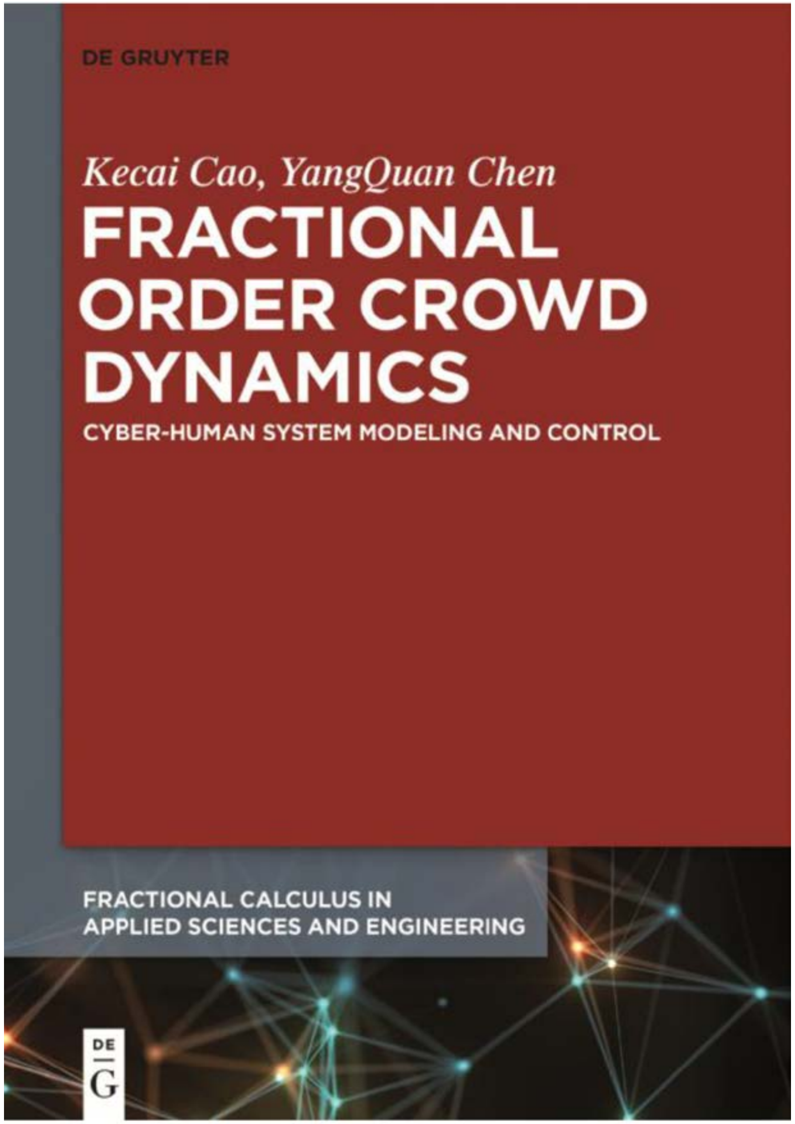
Kecai Cao, Yangquan Chen, Dan Stuart, and Dong Yue. **Cyber-physical modeling and control of crowd of pedestrians: a review and new framework**. *Automatica Sinica, IEEE/CAA Journal of*, 2(3):334–344, 2015. http://arxiv.org/abs/1506.05340.

New research monographs

- Kecai Cao and YangQuan Chen. "*Fractional Order Crowd Dynamics: Cyber Human System Modeling, and Control*" (**De Gryuter Monograph Series "Fractional Calculus in Applied Sciences and Engineering"** https://www.degruyter.com/view/product/469813)
- Fudong Ge, YangQuan Chen and Chunhai Kou. "*Regional Analysis of Time-Fractional Order Diffusion Processes*" **Springer** https://link.springer.com/book/10.1007%2F978-3-319-72896-4


Fudong Ge · YangQuan Chen
Chunhai Kou
Regional Analysis of Time-Fractional Diffusion Processes
Springer

2018


DE GRUYTER
Kecai Cao, YangQuan Chen
FRACTIONAL ORDER CROWD DYNAMICS
CYBER-HUMAN SYSTEM MODELING AND CONTROL
FRACTIONAL CALCULUS IN APPLIED SCIENCES AND ENGINEERING
DE G

2019

"cyber-human systems" 4/9/16

- ieeeXplore: 2
- Sciencedirect: 1
- Google: 3750

"cyber-physical systems" 4/9/16

- 2041
- 1318
- 450000

Dr. Chen's submission: "Cyber-Human Systems" (CHS) will be a hot topic in the next 10-20 years as human (individual, team, society/community), computer (fixed, mobile and surrounds), and environment (physical, mixed and virtual) fuse.

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Cyber-Physical Systems (CPS)

- New buzzword. New NSF thematic funding thrust after ITR (info technology research)
  - MAS-net was supported by ITR DDDAS program.
    - http://mechatronics.ece.usu.edu/mas-net/
    - http://mechatronics.ece.usu.edu/mas-net/dddas
- My Definition of CPS: Computational thinking and integration of computation around the physical dynamic systems form the Cyber-Physical Systems (CPS) where sensing, decision, actuation, computation, networking, and physical processes are mixed.
- Status: 9/11/08 Google = 5180 items; ieeeXplore= 21 items; umi.com=1 item; Amazon books=0
- Status: 5/27/09: Google = **15,700** items; ieeeXplore= **44** items; umi.com=**3** item; Amazon books=**1** item
- **Fact:** CSOIS has been doing Cyber-Physical Systems research since 2002.

Déjà vu

In the next a few years ...



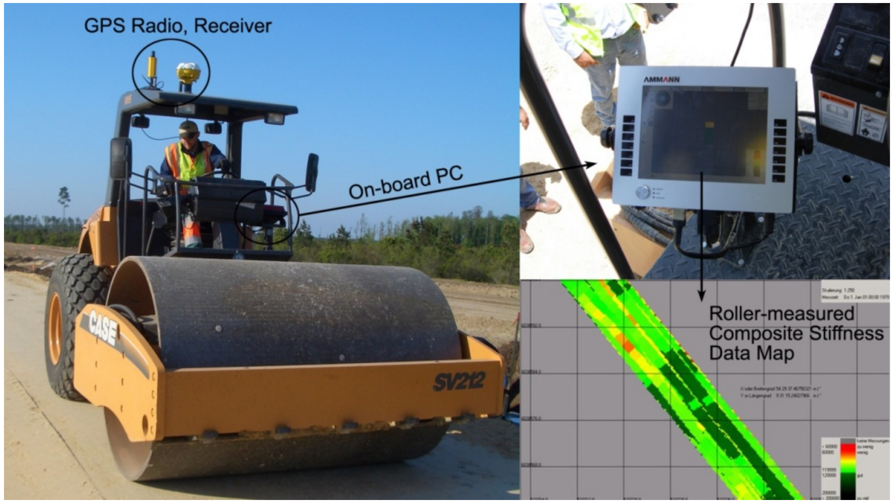

New robotics research
from controlling crop dynamics to crowd dynamics ...

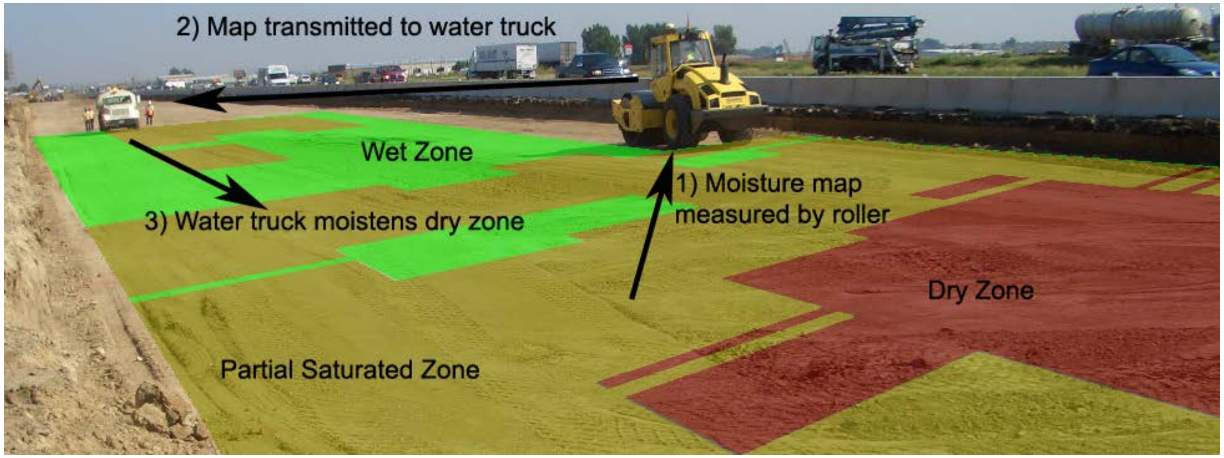
MAS-net to CPS to CHS: Robots as Sensors and Actuators

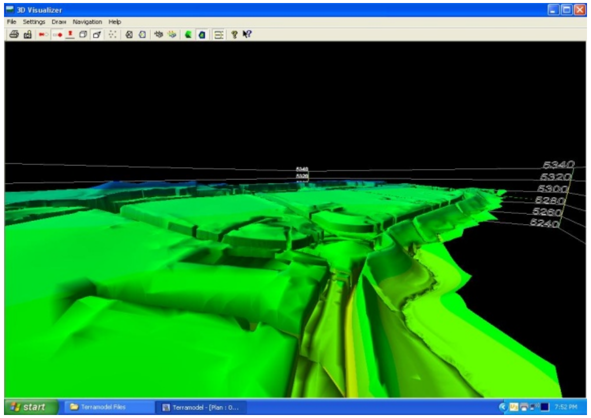
Figure: Stampede in Nigeria

Figure: Stampede in Shanghai

UAV for Earthwork Construction







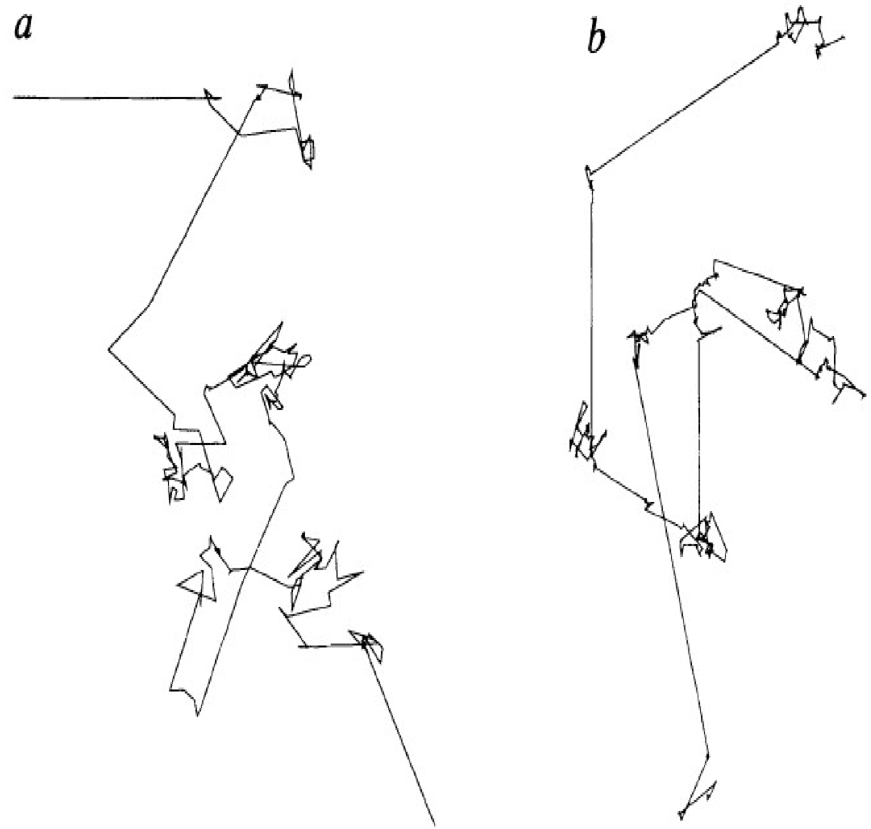
Conclusions

Want to do better than the best?

Want to be more optimal?

Go Fractional!

Backup slides

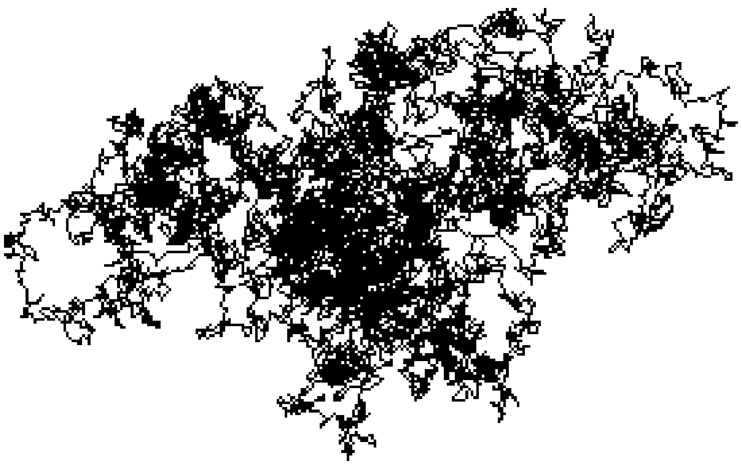


Wandering albatrosses

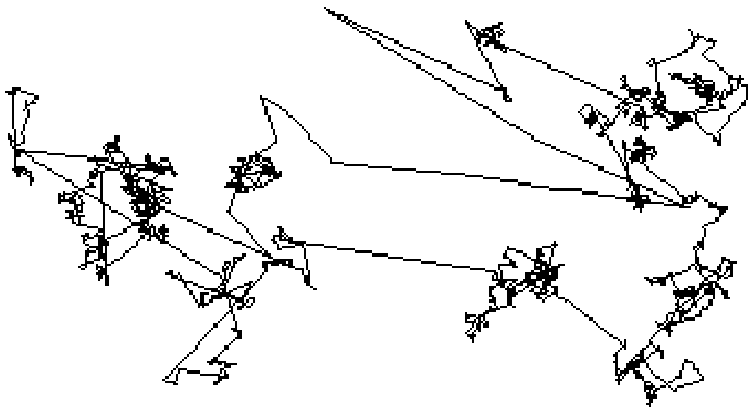
flight search patterns

G.M. Viswanathan, et al. ***Nature*** 381 (1996) 413–415.

Long jumps, intermittence



Brownian motion



Levy flights

Connection to FC via PDF

- "*Fractional Calculus and Stable Probability Distributions*" (1998) by Rudolf Gorenflo , Francesco Mainardi http://arxiv.org/pdf/0704.0320.pdf

$$\frac{\partial u}{\partial t} = D(\alpha)\,\frac{\partial^\alpha u}{\partial |x|^\alpha}\,, \quad -\infty < x < +\infty\,, \quad t \geq 0\,,$$

$$\text{with} \quad u(x,0) = \delta(x) \quad 0 < \alpha \leq 2$$

$$\frac{\partial^{2\beta} u}{\partial t^{2\beta}} = D(\beta)\,\frac{\partial^2 u}{\partial x^2}\,, \quad x \geq 0\,, \quad t \geq 0\,,$$

$$\text{with} \quad u(0,t) = \delta(t) \quad 0 < \beta < 1$$

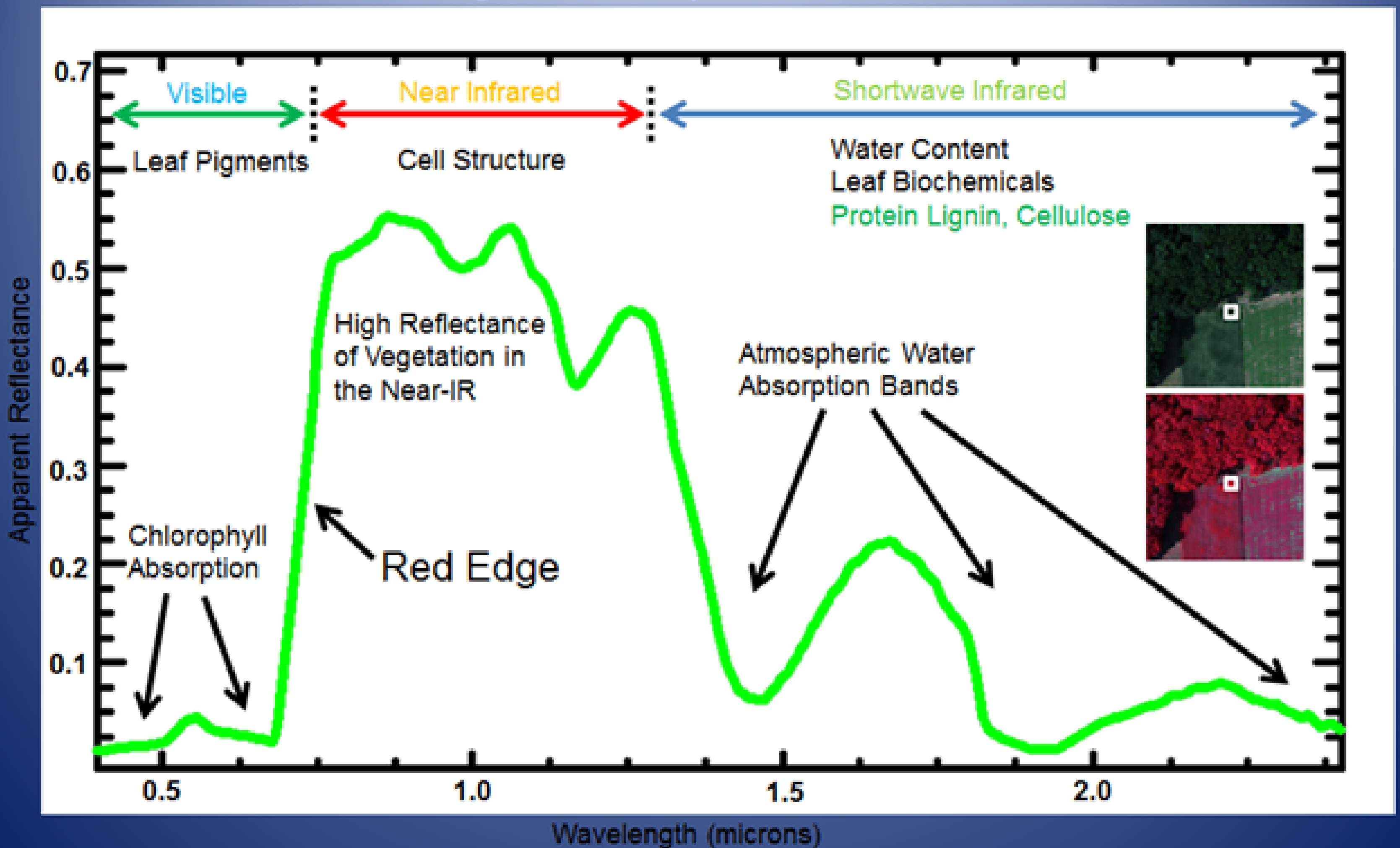
Decision and Control in the Era of Big Data ?

- Yes, we must use fractional calculus!
  - Fractional order signals, systems, controls.
  - **Fractional order data analytics**

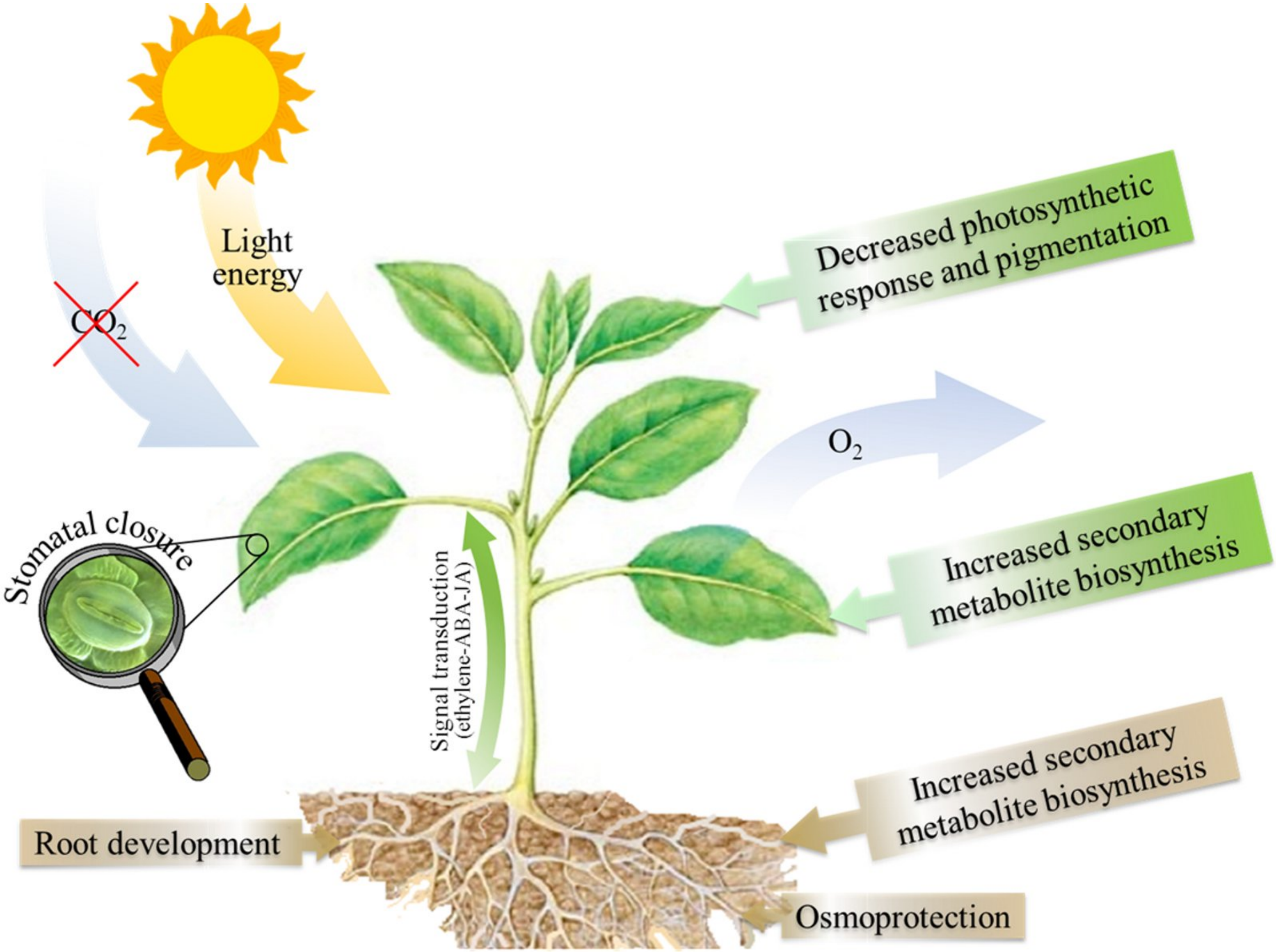
FODA: Fractional Order Data Analytics

- First proposed by Prof. YangQuan Chen @Spring15.
- *Metrics based on using fractional order signal processing techniques for quantifying the generating dynamics of observed or perceived variabilities.*
  - Hurst parameter, fGn, fBm, …
  - Fractional order integral, differentiation
  - FLOM/FLOS (fractional order lower order moments/statistics)
  - Alpha stable processes, Levy flights
  - ARFIMA, GARMA (Gegenbauer), CTRW …

The Vegetation Spectrum in Detail

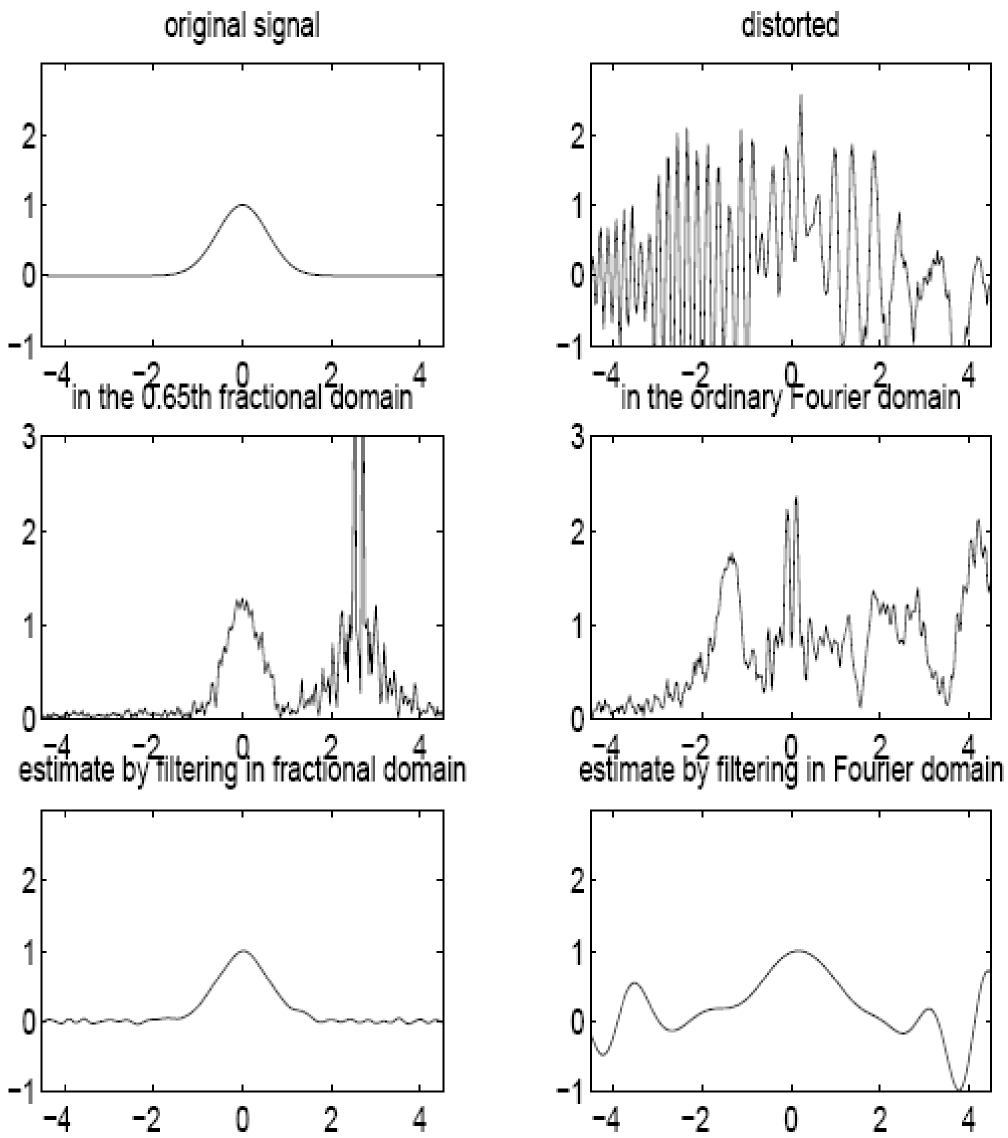


https://www.exelisvis.com/Learn/WhitepapersDetail/TabId/802/ArtMID/2627/ArticleID/13742/Vegetation-Analysis-Using-Vegetation-Indices-in-ENVI.aspx



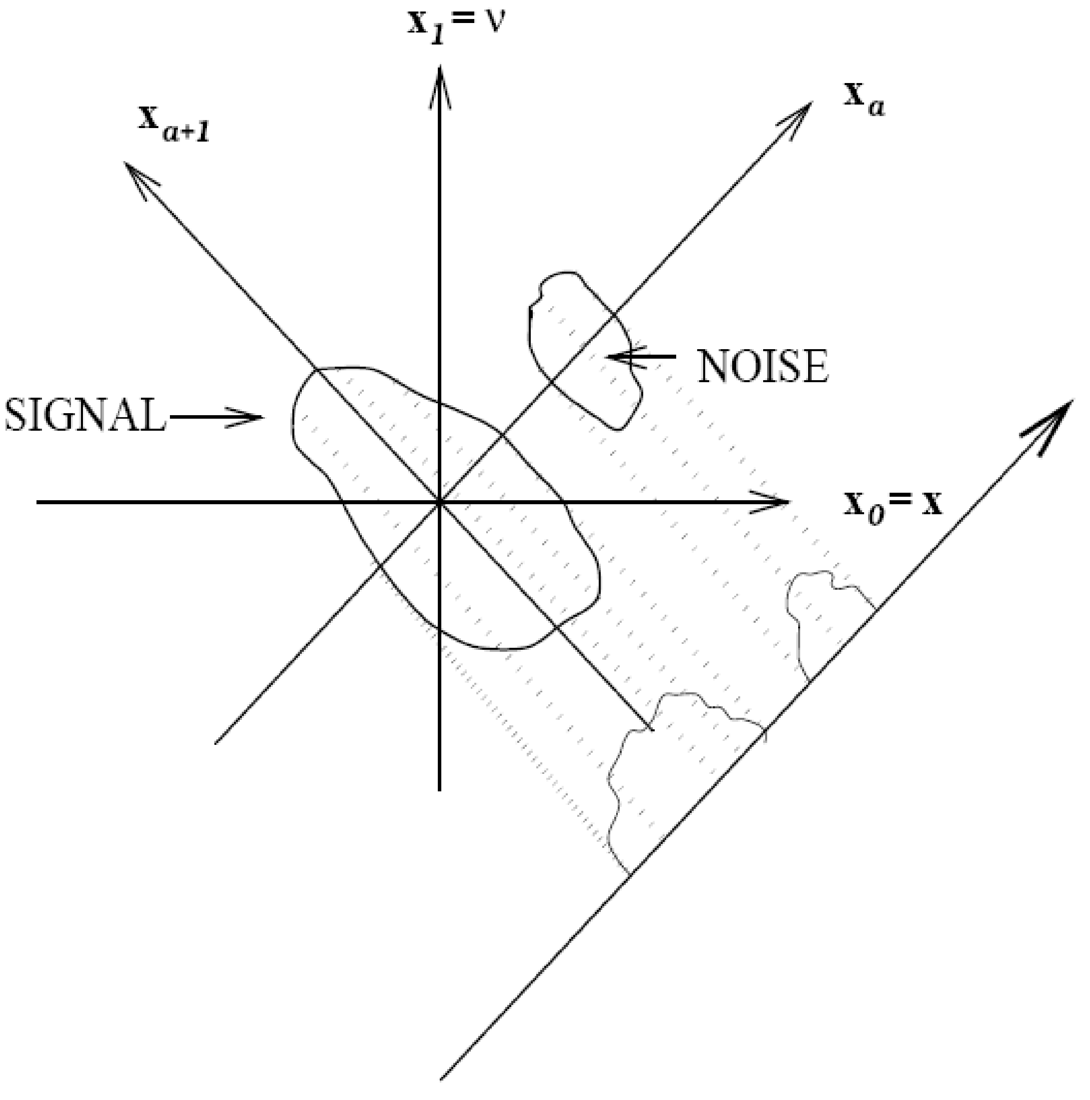
http://www.intechopen.com/books/responses-of-organisms-to-water-stress/water-stress-and-agriculture

Optimal filtering in fractional order Fourier domain



Slide credit: HALDUN M. OZAKTAS

Optimal filtering in fractional Fourier domain



[IEEE/CAA Journal of Automatica Sinica]

Special Issues on Fractional Order Systems and Controls guest coedited by Profs. YangQuan Chen, Dingyu Xue and Antonio Visioli are published and available here.

http://mechatronics.ucmerced.edu/jas-si-fosc