The Interplay Between Control Theory and Optimization

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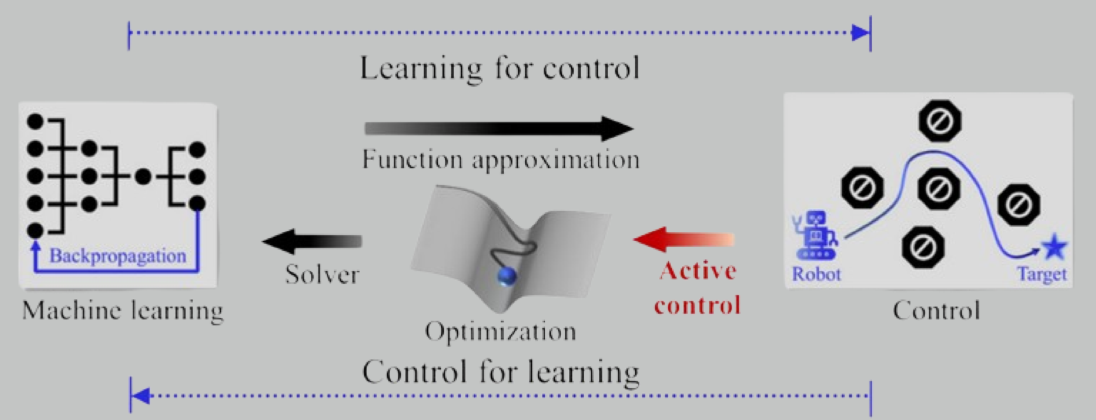
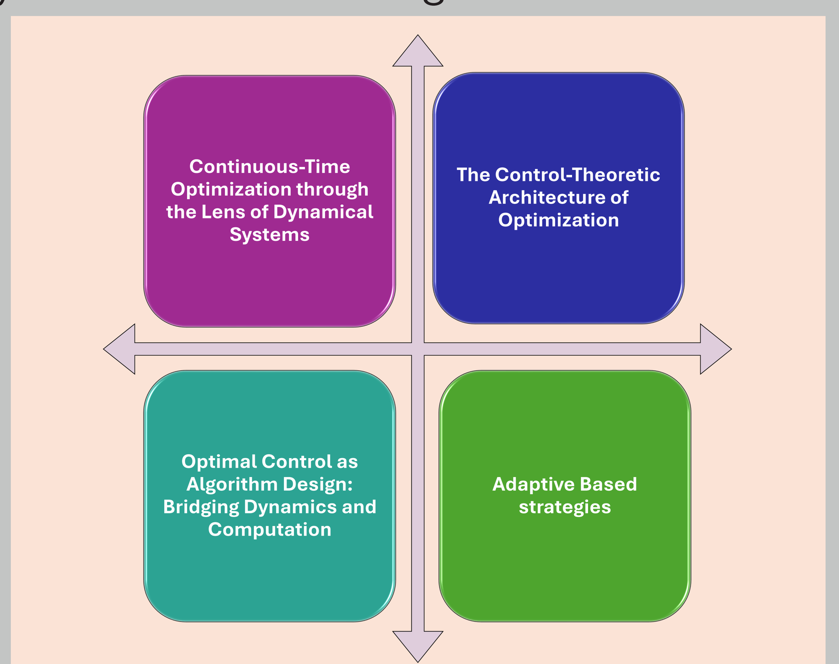
Abstract

The increasing integration of data-driven intelligence and automation is reshaping modern control and optimization, calling for frameworks that are adaptive, interpretable, and robust. This work investigates a bidirectional synthesis between control theory and optimization, unifying advances in machine learning, digital twins, and robust control analysis. Two complementary perspectives are explored: learning for control and control for learning. In learning for control, the focus is on data-driven and self-optimizing control architectures for complex and uncertain systems. Control laws and setpoints are adapted in real time using high-performance optimization, including gradient-based and derivative-free methods. Paradigms such as self-optimizing control, run-to-run control, and iterative learning control are studied within a hierarchical framework supported by digital twins, enabling real-time analytics, scenario testing, and continuous system updates. In control for learning, optimization algorithms are treated as dynamical systems. Control-theoretic tools—Lyapunov methods, dissipativity theory, and robust control analysis—are employed to characterize convergence, robustness, and acceleration, and to design optimization algorithms with finite- or fixed-time guarantees. Overall, this work bridges control and optimization to enable smart, self-optimizing engineering systems and to develop control-inspired learning algorithms with provable performance and robustness.



Control For Optimization

Optimization as a Dynamical System: Bridging Control Theory and Machine Learning



Data-Driven Control (1)

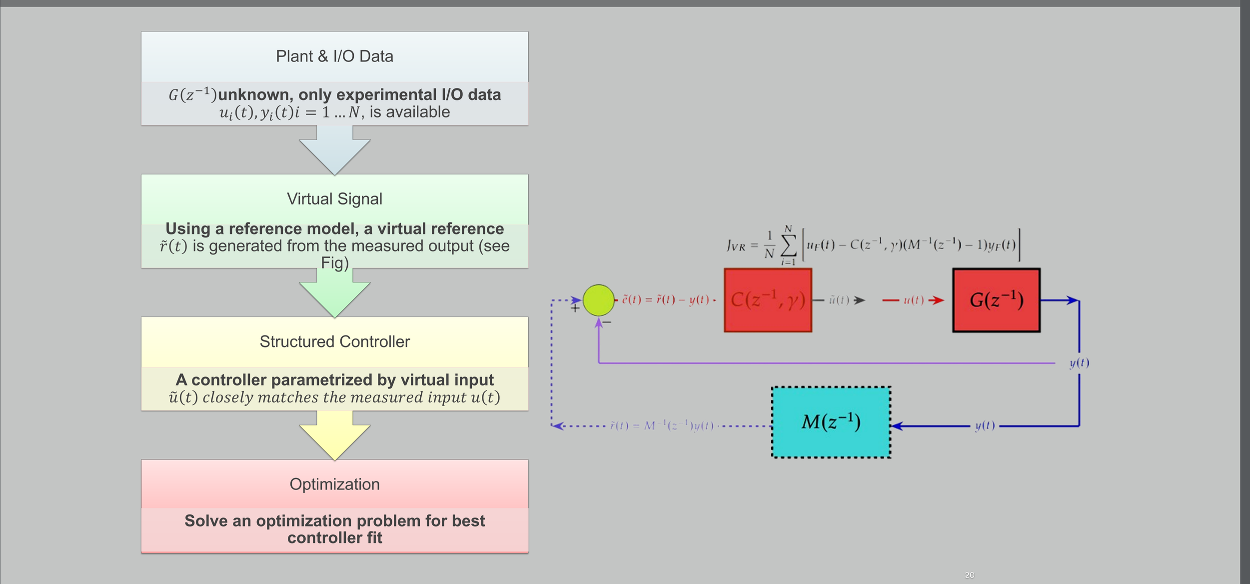


Figure: The framework of Virtual Reference Feedback Tuning.

Example: Control of a Rotary Flexible Joint

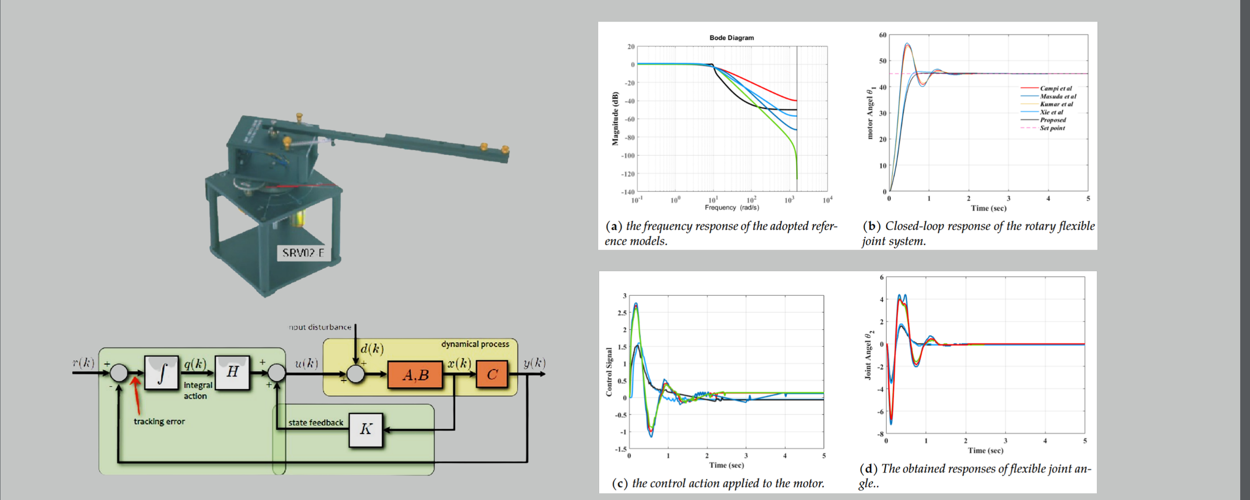


Figure: Data-Driven Based Control of rotary Flexible Joint.

Self-Optimizing Control

- The development and evaluation of a Self Optimizing Control framework based on derivative-free optimization algorithms for the performance improvement of a stable closed-loop system by adjusting the parameter of the closed-loop controller according to a performance cost function.

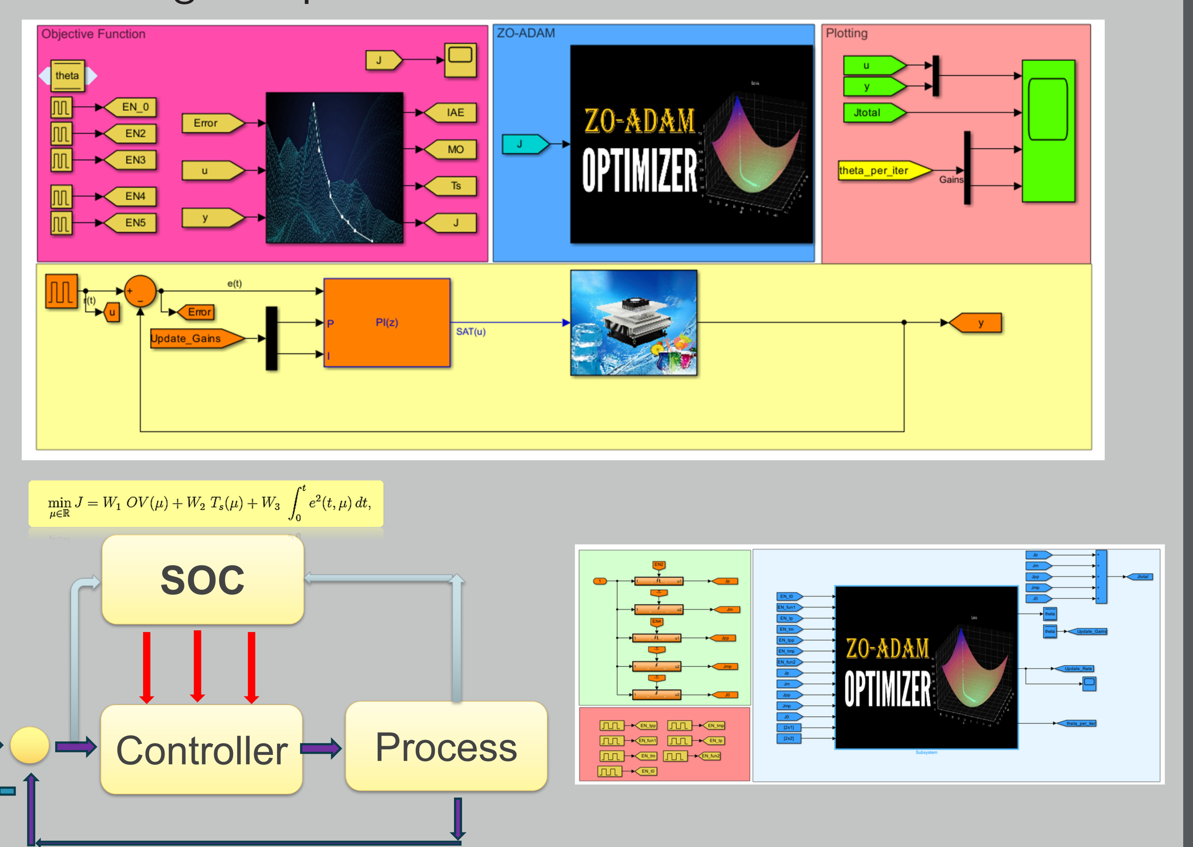
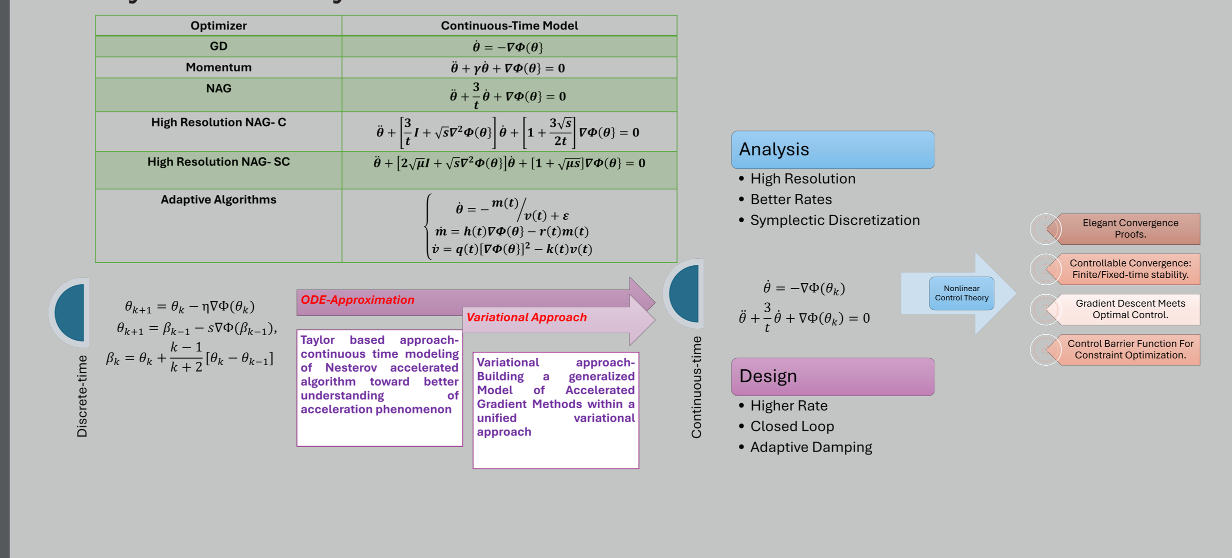


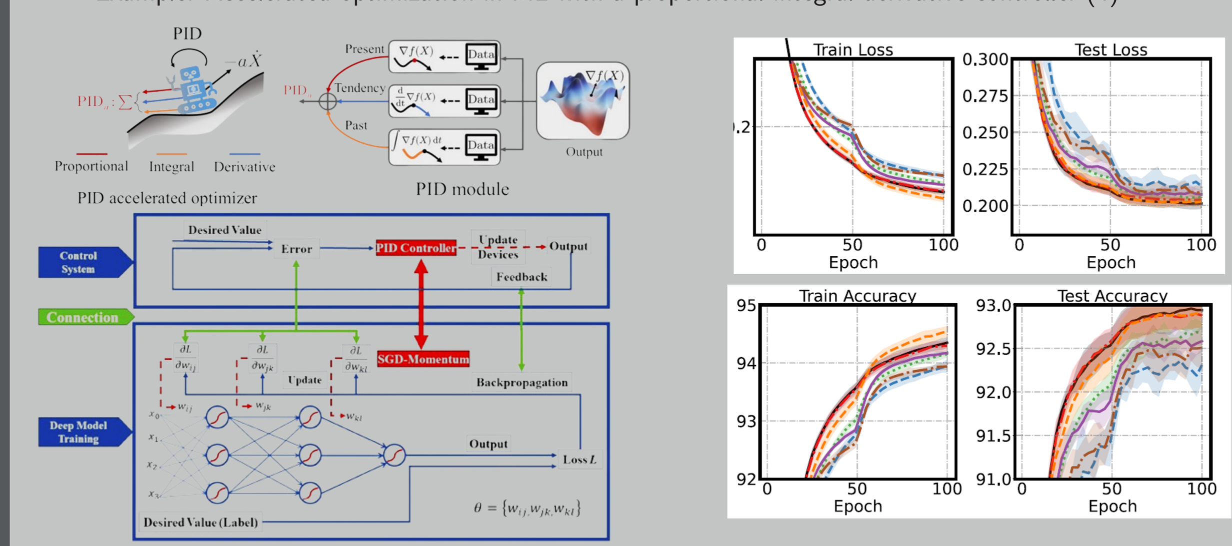
Figure: The Architecture of Self-Optimizing Control.

Continuous-Time Optimization (2; 3)

Continuous-Time Optimization through the Lens of Dynamical Systems



Example: Accelerated optimization in ML with a proportional-integral-derivative controller (4)



Finite-Time Convergent Algorithms (5; 6)

- Control-Theoretic Foundations of Finite-Time Optimization.
- Bridging Control Theory and Machine Learning.

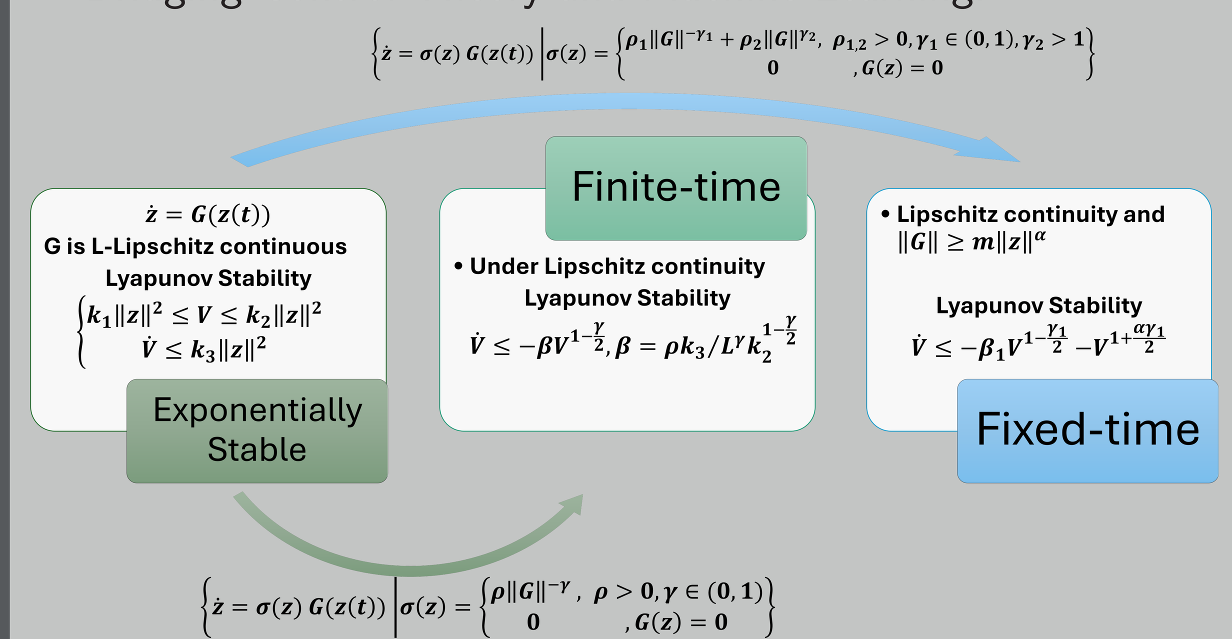


Figure: Beyond Asymptotics: Bridging Exponential and Finite/Fixed-Time Convergence of Gradient Flows.

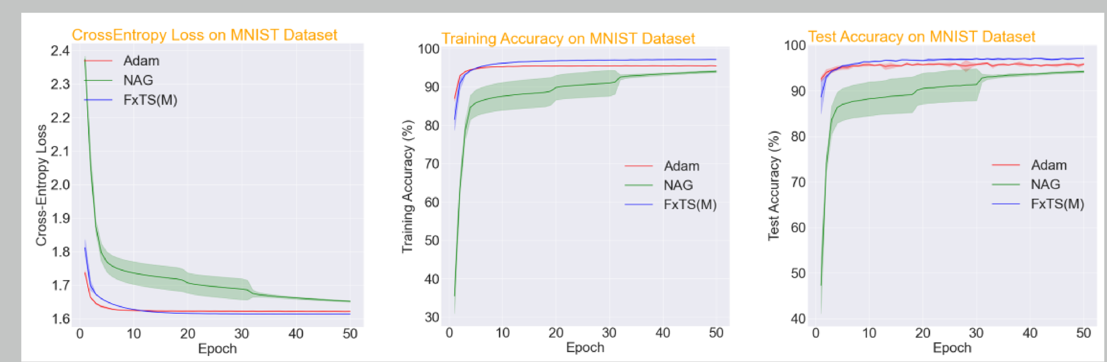
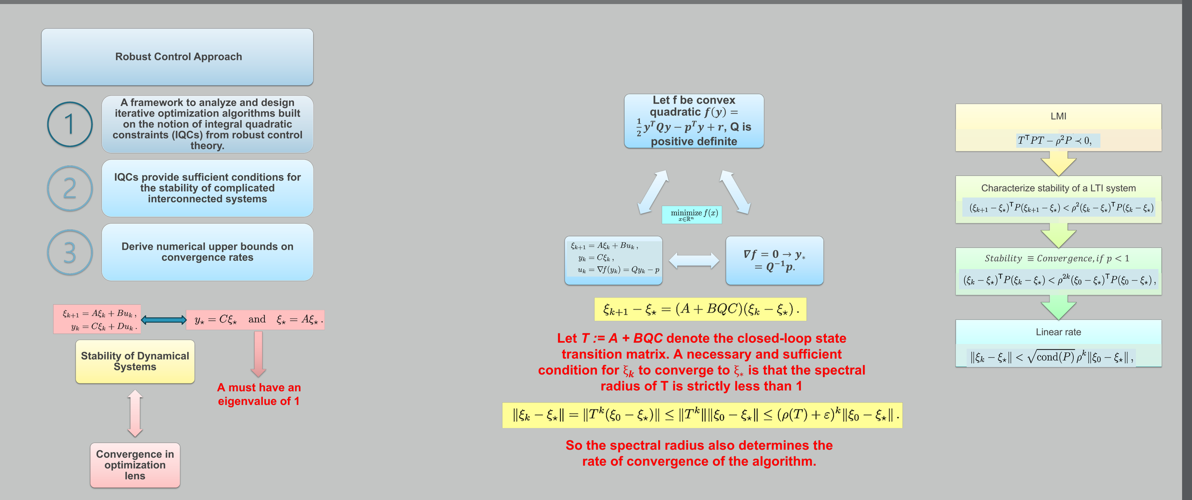


Figure: Comparison of several optimization algorithms for training deep neural networks on MNIST dataset. The Fixed-time Algorithms outperforms Adam and NAG optimizers on various performance measures.

Algorithms As Feedback Systems (7)

Algorithm	Main Formulation	Diagram
Gradient Descent	$x^{k+1} = \Pi_C(x^k - \eta\nabla\Phi(x^k))$	$x^{k+1} = u_1^k$, $y_1^k = x^k - \eta u_2^k$, $y_2^k = x^k$; feedback: $u_1^k = \Pi_C(y_1^k)$, $u_2^k = \nabla\Phi(y_2^k)$
Nesterov Accelerated Gradient	$y^k = x^k + \beta(x^k - x^{k-1})$ $x^{k+1} = y^k - \eta\nabla\Phi(y^k)$	$x_1^{k+1} = (1+\beta)x_1^k - \beta x_2^k - \eta u^k$, $x_2^{k+1} = x_1^k$; feedback: $u^k = \nabla\Phi(y^k)$
General Algorithm	$\xi^{k+1} = A\xi^k + Bu^k$ $\theta^k = C\xi^k + Du^k$ $u^k = \phi(\theta^k)$	$\xi^{k+1} = A\xi^k + Bu^k$, $\theta^k = C\xi^k + Du^k$; feedback: $u^k = \phi(\theta^k)$

Algorithm Analysis via Robust Control Theory



Optimal Control Meets Optimization

We derive gradient descent from Pontryagin's Maximum Principle (PMP), revealing a unifying view between optimal control and iterative optimization.

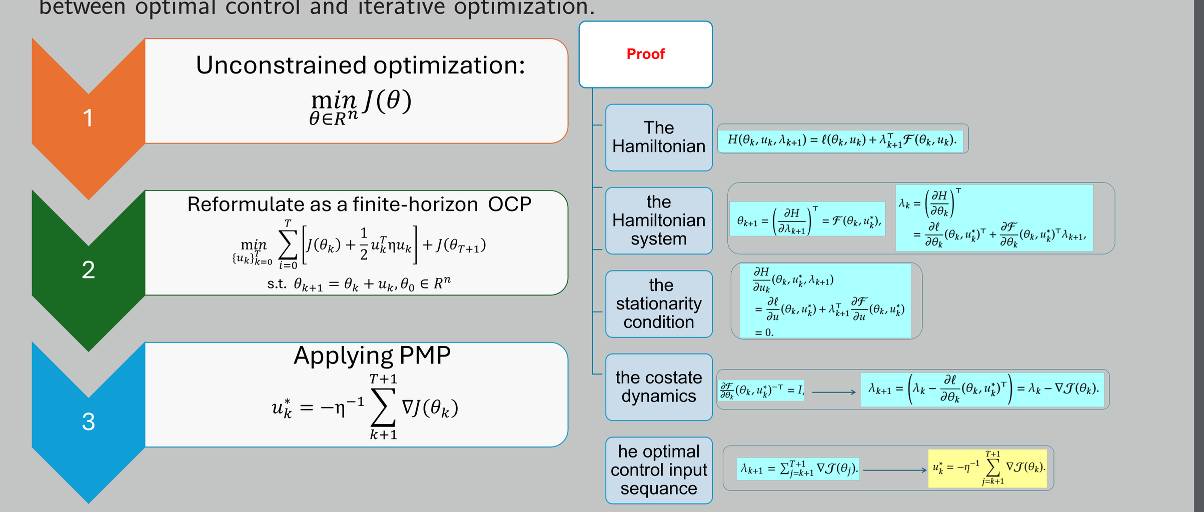


Figure: Deriving Gradient Descent via PMP.

References

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