Systems with power-law memory and fractional attractors

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A consistent investigation of a simple general form of a deterministic system with power-law memory whose state can be described by one variable and evolution by a generating function yields surprisingly rich results. In this model a new value of the system's variable is a total (a convolution) of the generating functions of all previous values of the variable with weights, which are powers of the time passed. In a discrete case this system can be described by a difference equation in which a fractional difference on the left hand side is equal to a total (also a convolution) of the generating functions of all previous values of the system's variable with fractional Eulerian number weights on the right hand side. In the continuous limit the considered system can be described by a Grünvald-Letnikov fractional differential equation, which is equivalent to the Volterra integral equation of the second kind. This consideration yields new properties of fractional Eulerian numbers and allows to investigate general properties of nonlinear fractional dynamics.

Fractional Standard and Logistic maps are two particular implementations of systems with power-law memory whose properties depend on the value of two parameters: the nonlinearity parameter, which arises from the corresponding regular dynamical systems; and the memory parameter which is the order of the fractional derivative in the corresponding non-linear fractional differential equations. The examples of the fractional Standard and Logistic maps demonstrate that phase space of non-linear fractional dynamical systems may contain periodic sinks, attracting slow diverging trajectories, attracting accelerator mode trajectories, chaotic attractors, and cascade of bifurcations type trajectories whose properties are different from properties of attractors in regular dynamical systems. The discovered properties should be evident in the natural (biological, psychological, physical, etc.) systems with power-law memory and in engineering they may provide additional means of control by the variation of the memory parameter of corresponding systems and processes.

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Dr. Mark Edelman received his PhD in Astrophysics from the Odessa State University, former USSR. For a long time he worked as a Research Scientist and a Senior Research Scientist at the Rostov State University and Rostov Pedagogical Institute, Rostov-on-Don, USSR. From 1993 to 2010 he worked at the Courant Institute of Mathematical Sciences, New York University, New York City, USA, as a research scientist. His scientific interests are very broad; they include cosmic gas dynamics, corrugation instability of shock waves, nonlinear dynamics, chaos theory, fractional calculus. His present scientific activity is concentrated on the fractional dynamical systems and systems with power-law memory. He is a member of the editorial boards of "Fractional Calculus and Applied Analysis" and "Journal of Applied Nonlinear Dynamics" and an invited editor for the special issue "Fractional Dynamics and Systems with Power-Law Memory" of the journal Discontinuity, Nonlinearity, and Complexity (to appear in 2015). His recent publications include three book chapters and numerous journal articles. He was Invited Speaker at numerous international conferences.

Since 2009 Dr. Edelman has been teaching various physics and mathematics courses and supervising student's research at Stern College for Women, Yeshiva University, simultaneously continuing his research at the Courant Institute.