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Modulated Wideband Converter for α-Bandlimited Signals in Fractional Fourier Domain

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***Abstract*—The classical bandlimited sampling theorem has been extended to fractional Fourier bandlimited signals with the fractional Fourier transform (FrFT). However, the implementation of those existing extensions are inefficient because of the high sampling rate which is related to the signal fractional Fourier rate, and the situation is same with the Fourier domain. This paper introduces a practical multichannel compressed sampling method for the α-order multi bandlimited signal in the FrFT domain. The theory which combines Modulated Wideband Converter with the fractional Fourier sampling method can reduce the sampling rate for the α-bandlimited signals. First we multiply the analog signal by a bank of periodic waveform, and the product is sampled by the fractional domain method, the sampling rate is much lower than the Nyquist rate. We can easily get the signal information with the orthogonal matching pursuit algorithm. The simulation verifies the sampling proposal.**

***Index Terms*—Fractional Fourier Transform; α-Bandlimited Signal; Modulated Wideband Converter; Compressed Sensing**

I. Introduction

Fractional Fourier transform (FrFT) is a new signal processing tool, and it has received much attention due to its numerous applications in recent years. The fractional Fourier transform uses a transform kernel which essentially allows the signal in the time-frequency domain to be projected onto a line of arbitrary angle. The definition is given as follows [1] :

$$F_\alpha(u) = \mathcal{F}^\alpha\{f(t)\} = \int_{-\infty}^{+\infty} K_\alpha(u,t)f(t)\mathrm{d}t, \tag{1}$$

where $\mathcal{F}^\alpha$ denotes FrFT operator, and the kernel function $K_\alpha(u,t)$ is given by :

$$K_\alpha(u,t) = \begin{cases} A_\alpha e^{j\pi((u^2+t^2)\cot\alpha - 2ut\csc\alpha)}, & \alpha \neq k\pi \\ \delta(u-t), & \alpha = 2k\pi \\ \delta(u+t), & \alpha = (2k+1)\pi, \end{cases} \tag{2}$$

where $A_\alpha = \sqrt{(1-j\cot\alpha)}$, $k \in \mathbb{Z}$.

The inverse FRFT operator is $\mathcal{F}^{-\alpha}$ which is the FrFT at angle $-\alpha$, expressed as following:

$$K_{-\alpha}(u,t) = A_\alpha e^{-j\pi(u^2+t^2)\cot\alpha + 2j\pi ut\csc\alpha}. \tag{3}$$

FrFT can be interpreted as a rotation in time-frequency plane with an angle α, the FrFT reduces to Fourier transform (FT) when $\alpha = \frac{\pi}{2}$. The operator $\mathcal{F}^\alpha$ is additive in angle, for example $\mathcal{F}^{\alpha_1}\mathcal{F}^{\alpha_2} = \mathcal{F}^{\alpha_1+\alpha_2}$. FrFT may offers some advantages in filtering, sampling, reconstruction and image recovery and so on.

Due to the importance of the FrFT in signal processing, many classic Fourier signal processing methods have been extended to the fractional Fourier domain, just like Shannon-Nyquist sampling law in [2, 3]. According to the Shannon-Nyquist, analog-digital converter (ADC) must work at twice higher rate than the signal rate in the Fourier domain. Sometimes Nyquist sampling method may not be practical because the Nyquist rate may exceed the specifications of the best analog-to-digital converters (ADCs) by orders of magnitude. But most situation can be described as effectiveness of the communication channel is not high at some time. In other words, the information rate is much lower than the signal rate. The Nyquist rate is the sufficient condition to reconstruction the sampled signal, but it is not the necessary. If we know the carrier frequency, the demodulation is a good practical method to reduce the sampling rate and the sampling rate is equal to the actual frequency occupancy. In most scenarios, the carrier frequency is unknown, so designing a receiver at a sub-Nyquist rate is a challenging task. We know the Fourier transform is a special case for the fractional Fourier transform when $\alpha = \frac{\pi}{2}$, so there is the same problem for sampling in fractional Fourier domain [4].

Compressed Sensing (CS) is a new good way to collect the information directly. The Compressed Sensing combines the compression and the sampling to make the sampling and compressing at the same time. It can greatly reduce the sampling rate, signal processing rate and the storage [5, 6]. Sometimes it could not be realized since the probability of the recovery for the CS is too small and the calculation is very complexity. Recently a new architecture called Modulated wideban converter (MWC) is studied by Mishali [7]. In this strategy, the signal is demodulated by a high-rate pseudo-random number generator, after the filter, sampled at a low rate. As the MWC systems develop, the real systems have been shown [7, 8]. As we know the classic MWC only work in Fourier domain, sometimes traditional non-band-limited signals in the Fourier domain may be band-limited in the fractional Fourier domain [9]. Most research efforts were on the uniform sampling theorem expansions for the band-limited signal in the fractional Fourier domain from different perspectives in [10, 11], but few make research effort in the compressed sampling for α-bandlimited signal sampling, so it is necessary to make a progress.

As we know the lower mitigated interference power and wider the nonzero part of the available spectrum is, the better property a time-domain fundamental modulation waveform has. The purpose of this paper is to find out the sparse base for α-bandlimited signal, and propose a practical compressed sampling theorem for the fractional bandlimited signals. The outline of this paper is organized as follows. In section II, some basic theorems are introduced, including the definition of α-bandlimited signal and fractional convolution, and the Modulated Wideband converter. In section III, a compressed sampling method for α-bandlimited signals based on Fourier modulated wideband converter is proposed, and the results of the simulation and the potential applications are both discussed in the section IV.

II. Preliminaries

A. Definition of and Sampling for α-Bandlimited Signal

A Ω_α fractional bandlimited signal $f(t)$ must satisfying that its energy is finite and the FrFT $F_\alpha(u)$ is zero outside the region $(-\Omega_\alpha, +\Omega_\alpha)$. That is

$$F_\alpha(u) = 0, \text{ for } |u| \geq \Omega_\alpha, \tag{4}$$

$$\int_{-\infty}^{+\infty} |f(t)|^2 \mathrm{d}t = \int_{-\Omega_\alpha}^{+\Omega_\alpha} |F_\alpha(u)|^2 \mathrm{d}u < \infty. \tag{5}$$

Based on the well-known sampling theorem for the FrFT reported by Xia [2], the signal $f(t)$ can be restored as follows:

$$\begin{aligned} f(t) = A_\alpha e^{j\pi t^2 \cot\alpha} \sum_{n=-\infty}^{+\infty} f(nT_s) e^{j\pi (nT_s)^2 \cot\alpha} \\ \cdot \frac{\sin[(t - nT_s)\Omega_\alpha \csc\alpha]}{(t - nT_s)\Omega_\alpha \csc\alpha}, \end{aligned} \tag{6}$$

where $T_s = \pi \sin\alpha / \Omega_\alpha$. The Nyquist sampling rate is $f_{NYQ} = \Omega_\alpha / (\pi \sin\alpha)$.

From the (6), it is obvious that the sampling rate is dependent on the signal fractional Fourier rate Ω_α. In other words, the analog-digital converter must work at high rate when the signal fractional Fourier frequency is high. We can think about the situation that the signal is bandlimited within the region $(\Omega_\alpha, \Omega_\alpha + B_\alpha)$, so the highest fractional frequency rate is $\Omega_\alpha + B_\alpha$ and the bandwidth is B_α. so the sampling rate must be $T_s = \pi \sin\alpha / (\Omega_\alpha + B_\alpha)$, if the Ω_α is very big, the ADC must work at very high rate which is difficult to be realized in practice.

B. Fractional Convolution

The relationship between the FrFT and FT as:

$$\begin{aligned} &\mathcal{F}_\alpha[f(t)](u) \\ &\quad = A_\alpha e^{j\pi u^2 \cot\alpha} \cdot \mathsf{F}\big[f(t) e^{j\pi t^2 \cot\alpha}\big](u \csc\alpha), \end{aligned} \tag{7}$$

where F is the traditional integral order Fourier transform operator.

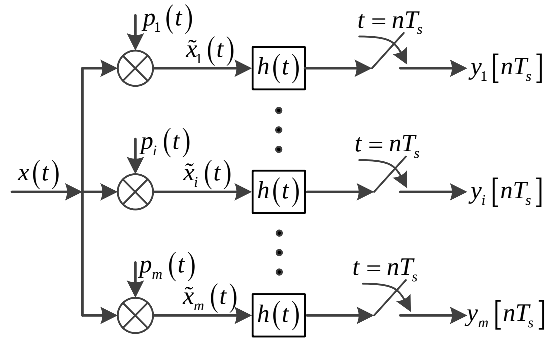

Fig. 1. Structure for Modulated wideband converter

We denote $z(t)$ as

$$z(t) = e^{-j\pi t^2 \cot\alpha} \big[\big(x(t) e^{j\pi t^2 \cot\alpha}\big) * h(t)\big], \tag{8}$$

where $*$ denotes the convolution operator.

We can easily prove that the fractional Fourier transform can be expressed as [12]:

$$Z_\alpha(u) = \sqrt{2\pi} X_\alpha(u) H(u \csc\alpha). \tag{9}$$

The proof is as follows:

$$\begin{aligned} &Z_\alpha(u) \\ =&\int_{-\infty}^{+\infty} e^{-j\pi t^2 \cot\alpha} \big[\big(x(t) e^{j\pi t^2 \cot\alpha}\big) * h(t)\big] \\ &\times K_p(u,t) \mathrm{d}t \\ =&\mathsf{F}\big[\big(x(t) e^{j\pi t^2 \cot\alpha}\big) * h(t)\big](u \csc\alpha) A_\alpha e^{j\pi u^2 \cot\alpha} \\ =&\mathsf{F}\big[x(t) e^{j\pi t^2 \cot\alpha}\big](u \csc\alpha) \\ &\times \sqrt{2\pi} A_\alpha e^{j\pi u^2 \cot\alpha} \mathsf{F}[h(t)](u \csc\alpha) \\ =&\sqrt{2\pi} X_\alpha(u) H(u \csc\alpha). \end{aligned} \tag{10}$$

C. Modulated Wideband Converter (MWC)

In 2008, Elder [7] proposed a multi-channel parallel modulated and sampled architecture called Modulated Wideband Converter (MWC), which is comprised of a bank of modulators and low-pass filters. In the MWC system, MWC sampling system is shown in Fig. 1. The system consists some channels, firstly the signal is multiplied by a certain periodic waveform every channel, whose period is the biggest bandwidth of the signals, also the style of the waveform can be square or other periodic sign waveform. After the demodulation, the product $\tilde{x}(t)$ is then low-pass filtered, so the signal is alias to the baseband which can be sampled at a low rate. With this digital architecture we can process various signal at the low rate. Roughly speaking, the more mixtures the easier the recovery is, so there must be sufficient number of channels to recover the sparsity signals. Specifically, the signal $x(t)$ is the input to m channels simultaneous. Taking the ith channel as an example to explain how every channel works. The signal is mixed with the sign waveform $p_i(t)$ whose period is T_p. The mixed product is filtered by a low-pass filter with cutoff $1/(2T_s)$, and the T_s is the channel sample rate. There is a tradeoff between the number of channels and every channel's sample rate. The best parameters are needed to be chosen, such as the number of channels m, the sampling rate $1/T_s$. The mixing function $p_i(t)$ and its period is T_p, $p_i(t)$ is set as a sign function for each of M equal time intervals and other forms are possible, since the system only requires $p_i(t)$ periodic expressed as follows:

$$p_i(t) = \alpha_{ik}, k\frac{T_p}{M} \leqslant t \leqslant (k+1)\frac{T_p}{M}, 0 \leqslant k \leqslant M-1, \quad (11)$$

where $\alpha_{ik} \in \{+1, -1\}$ and $p_i(t)$ is a periodic function with peroid T_p.

Frequency domain analysis can make a better understanding for the relationship between the sample sequences $y_i[n]$ and the original signal $x(t)$, also the parameters can be chosen easily in the frequency domain. For the ith channel, the Fourier expansion of T_p-periodic $p_i(t)$ is as below:

$$p_i(t) = \sum_{l=-\infty}^{+\infty} c_{il} e^{j\frac{2\pi}{T_p}lt}, \quad (12)$$

where $f_p = 1/T_p$. c_{il} is denoted by

$$c_{il} = \frac{1}{T_p}\int_0^{T_p} p_i(t) e^{-j\frac{2\pi}{T_p}lt}. \quad (13)$$

The Fourier transform of mixing signal $\tilde{x}(t) = x(t)p_i(t)$ is

$$\tilde{X}_i(f) = \sum_{l=-\infty}^{+\infty} c_{il} X(f - lf_p). \quad (14)$$

Suppose that the filter $H(f)$ is an ideal rectangular function. Consequently, the uniform sequence $y_i[n]$ has only frequencies in the $[-f_s/2, +f_s/2]$. So the discrete time Fourier transform of $y_i[n]$ is expressed as

$$Y_i(e^{j2\pi f T_s}) = \sum_{l=-L_0}^{+L_0} c_{il} X(f - lf_p), \quad (15)$$

where L_0 is selected as the smallest integer as:

$$\frac{f_s}{2} + (L_0 + 1) f_p \leqslant \frac{f_{NYQ}}{2}, \quad (16)$$

where f_s is sample rate, and f_{NYQ} is the Nyquist rate for $x(t)$.

From the above equation, c_{il} can be denoted as,

$$\begin{aligned} c_{il} &= \frac{1}{T_p}\int_0^{\frac{T_p}{M}} \Big(\sum_{k=0}^{M-1} \alpha_{ik} e^{-j\frac{2\pi}{M}lk}\Big) e^{-j\frac{2\pi}{T_p}tl} \mathrm{d}t \\ &= \frac{1}{T_p}\sum_{k=0}^{M-1} \alpha_{ik} e^{-j\frac{2\pi}{M}lk} \int_0^{\frac{T_p}{M}} e^{-j\frac{2\pi}{p}lt} \mathrm{d}t. \end{aligned} \quad (17)$$

Set d_l as:

$$d_l = \frac{1}{T_p}\int_0^{\frac{T_p}{M}} e^{-j\frac{2\pi}{p}lt}\mathrm{d}t = \begin{cases} \frac{1}{M}, & l = 0, \\ \frac{1-\theta^l}{2j\pi l}, & l \neq 0. \end{cases} \quad (18)$$

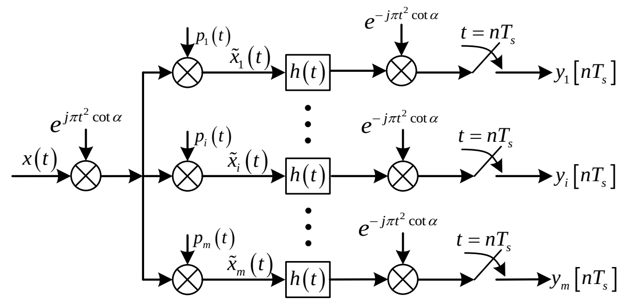

Fig. 2. Modulated wideband converter in fractional domain

Let $\bar{F}$ be the $M \times M$ discrete Fourier transform (DFT) matrix with the ith column is denoted by

$$\bar{F}_i = \left[\theta^{0i}, \theta^{1i}, \cdots, \theta^{(M-1)i}\right]^T. \quad (19)$$

Let F be subset of $\bar{F}$ with columns $[\bar{F}_{L_0}, \cdots, \bar{F}_{-L_0}]$. Let S be the $m \times M$ sign matrix, with $S_{ik} = \alpha_{ik}$, and $D = diag(d_{L_0}, \cdots, d_{-L_0})$ is an $L \times L$ diagonal matrix with d_l defined by (18). The $y(f)$ can be given by:

$$y(f) = SFDz(f), \quad (20)$$

where $z(f) = [X(f - L_0 f_p), \cdots, X(f), \cdots, X(f + L_0 f_p)]^T$.

There are some benefits, for examples, the mixer bandwidth can be defined as the input signal bandwidth. If the waveform is sign function, it can be realized by a shift register. The sampling rate $1/T_s$ is chosen on the order of bandwith B of a signal component, and is equal to the cutoff of the analog filter $H(f)$. There is no time shift in all the channel, so all the channels sampling is synchronized.

III. Modulated Wideband converter for α-bandlimited signals

A. A practical fractional compress sampling

We know that the fractional frequency shifting does not have any impact on the convergence fractional order α [1], so the α-bandlimited signal can be compressively sampled with the MWC structure in the FrFT domain. The strategy is the same with the MWC in Fourier domain. First we can mix the α-bandlimited signal with signal $e^{j\pi t^2 \cot\alpha}$ and then demodulate by a random sign which can move the chirp signal to the baseband without any information loss. The production of the mixing signal is filtered by a low pass filter. Because of the feature that the α-bandlimited signal has sparsity at fractional order α, it is easy to recover from the fractional Fourier domain.

In section II, the Fourier bandlimited signal can be reconstructed as (20) in the Fourier transform domain. As we known the Fourier transformation is just one special case of the fractional Fourier transform domain. In this section, we expand MWC from the Fourier domain to the fractional Fourier domain.

Fractional Fourier MWC system structure is proposed as in Fig. 2. Consider the ith channel. The $p_i(t)$ is also a T_p-periodic signal, and its Fourier expansion is the same with the (12), and the coefficients c_{il} are the same with the (13). The filter $h(t)$ has the rectangular fractional Fourier response which is simaliar to the ideal frequency domain low-pass filter. After the filtering the signal is denoted as following:

$$y_i(t) = e^{-j\pi t^2 \cot\alpha}\big[\big(x(t)p_i(t)e^{j\pi t^2 \cot\alpha}\big) * h(t)\big]. \tag{21}$$

From (10), the fractional Fourier transform of the analog multiplication $y_i(t)$ is

$$\begin{aligned} Y_i^{\alpha}(u) &= \mathcal{F}^{\alpha}\{y_i(t)\} \\ &= \sqrt{2\pi}\mathcal{F}^{\alpha}[p_i(t)x(t)](u)H(u\csc\alpha), \end{aligned} \tag{22}$$

where the $\mathcal{F}^{\alpha}[p_i(t)x(t)]$ is computed as:

$$\begin{aligned} &\int_{-\infty}^{+\infty} p_i(t)x(t)K_{\alpha}(\mu,t)\mathrm{d}t \\ =&\int_{-\infty}^{+\infty} x(t)\Big(\sum_{l=-\infty}^{\infty} c_{il}e^{j\frac{2\pi}{T_p}lt}\Big)A_{\alpha}K_{\alpha}(\mu,t)\mathrm{d}t \\ =&A_{\alpha}\sum_{l=-\infty}^{+\infty} c_{il}e^{-j\pi(\frac{l\sin\alpha}{T_p})^2\cot\alpha+\frac{2jul\pi}{T_p}\cos\alpha} \\ &\times \mathcal{X}_{\alpha}(u-\frac{l\sin\alpha}{T_p}), \end{aligned} \tag{23}$$

where the $\mathcal{X}_{\alpha}(u)$ is α-order FrFT of the $x(t)$, and the bandwidth of α-bandlimited signals do not exceed B_{α}. For a certain α-bandlimited signal, the mixing just moves the starting frequency to $\Omega_{\alpha} - \frac{l\sin\alpha}{T_p}$ without changing the bandwidth B_{α}. The filter $H(u\csc\alpha)$ is an ideal Fourier rectangular filter, and it can be denoted as $H(u\csc\alpha) = 1, \text{if } u \leq |B_{\alpha}|/2, \text{otherwise } H(u\csc\alpha) = 0$. Obviously, the Fourier transform of the $H(u\csc\alpha)$ is $\mathrm{sinc}(\frac{t-nT_s}{T_s})$ function. Therefore the (22) can be simplified as:

$$\begin{aligned} Y_i^{\alpha}(u) =&\sqrt{2\pi}A_{\alpha}\sum_{l=-L_0}^{+L_0} c_{il}e^{-j\pi(\frac{l\sin\alpha}{T_p})^2\cot\alpha+\frac{2jul\pi}{T_p}\cos\alpha} \\ &\times \mathcal{X}_{\alpha}(u-\frac{l\sin\alpha}{T_p}), \end{aligned} \tag{24}$$

where L_0 is chosen as the smallest integer such that it must cover all nonzero of the $\mathcal{X}_{\alpha}(u)$. The exact value of L_0 is calculated by:

$$\begin{aligned} &-\frac{u_s}{2} + (L_0+1)u_p \geq \frac{u_{\alpha,NYQ}}{2} \\ &\rightarrow L_0 = \big[\frac{u_{\alpha,NYQ}+u_s}{2u_p}\big] - 1, \end{aligned} \tag{25}$$

where $u_s = f_s\sin\alpha$ and $u_p = f_p\sin\alpha$, and the fractional Nysquist sampling rate of α-bandlimited signal is $2(\Omega_0 + B_{\alpha}/2)$, when the highest fractional Fourier frequency of the signal is $\Omega_{\alpha} + B_{\alpha}/2$, and Ω_0 is the carrier "frequency" in FrFT domain.

Consider that the coefficients c_{il} are the same as those in the Fourier transform that given by (17), (18) and (19). So $Y_{\alpha}(u)$ has the same form with the (20).

$$Y_{\alpha}(u) = S_{\alpha}F_{\alpha}D_{\alpha}GZ_{\alpha}(u). \tag{26}$$

The matrixes as S_{α}, F_{α}, and D_{α} are the same with the Fourier domain coefficient S, F and D respectively. The other parameters are given as following: $G = diag(g_{-L_0},\cdots,g_{L_0})$ and $g_l = \exp(\frac{2jul\pi}{T_p}\cos\alpha)$. $Z_{\alpha}(u) = \exp(-j\pi(\frac{l\sin\alpha}{T_p})^2\cot\alpha)[\mathcal{X}_{\alpha}(u-\frac{L_0\sin\alpha}{T_p}),\cdots,\mathcal{X}_{\alpha}(u+\frac{L_0\sin\alpha}{T_p})]$.

B. Choice of Parameters and Recovery

We choose the sign matrix signal $u_{\alpha,p} = B_{\alpha}$, that means $T_p = \frac{\sin\alpha}{B_{\alpha}}$, and $M_{min} = L$. After the choice of the parameters, the system changes to a compressed sensing system. From (26), G is related with $Z(u)$, so we can not recover the signal $x(t)$ directly. Firstly, we can use the Orthogonal Matching Pursuit (OMP) method to recover the sparsity bases $\exp(-j\pi\cot\alpha(\frac{l\sin\alpha}{T_p})^2 + \frac{2jul\pi}{T_p}\cos\alpha)\mathcal{X}_{\alpha}(u-\frac{l\sin\alpha}{T_p})$. After the inverse fractional Fourier transform, we can get the time domain signal $e^{2j\pi tl/T_p}x(t)$, where $l \in [-L_0, -L_0+1,\cdots,L_0-1,L_0]$. That means we can reconstruct the original signal $x(t)$ from $y(t)$.

IV. Numerical Simulation

It is obvious that the Fig. 2 will be reduced to the classic structure when the bandlimited angle is $\alpha = \pi/2$. The chirp signal is a typical α bandlimited signal, so we use the chirp signal as the test subject. The original signal is given by the following.

$$x_i(t) = \sqrt{E_iB_i}\sin(B_i(t-\tau_i)^2)\cos(2\pi f_i(t-\tau_i)) \tag{27}$$

where $x_i(t)$ is the ith original signal, $\sqrt{E_iB_i}$ is the signal amplitude, $B_i = 50M\text{Hz}$ is the signal modulate frequency, and τ is the time delay between different signals, and f_i is random frequency carrier. We choose three signals with a fixed time delay τ. The result is shown in the Fig. 3. The "Original signal" is $x_1(t)+x_2(t)+x_3(t)$ without white Gaussian noise. The "Original noised signal" is the original signal with white Gaussian noise, denoted by $x_1(t)+x_2(t)+x_3(t)+n(t)$. The "Reconstructed signal" is recovered signal with the white Gaussian noise. The "FrFT signal" is the FrFT for the original signal. From the figure we can see the convergent angle is close to $\pi/2$. As we know the modulated rate of the α order convergence chirp signal is equal to the $\tan(\alpha-\pi/2)$, so the big signal modulate frequency rate B_i means the experimental result matches the theory. .

We can use the normalized mean squared error (NMSE) to measure the performance of the compressed sampling by proposed signal. The computation is as following:

$$\text{NMSE} = \frac{\int_{-\infty}^{+\infty}|x(t)-\hat{x}(t)|^2\mathrm{d}t}{\int_{-\infty}^{+\infty}|x(t)|^2\mathrm{d}t}, \tag{28}$$

where $x(t)$ is the original signal and $\hat{x}(t)$ denoted the recovered signal. The NMSE of the noise-corrupted signal is 0.0943. Thus the proposed system is useful for α bandlimited signal in fractional Fourier transform domain.

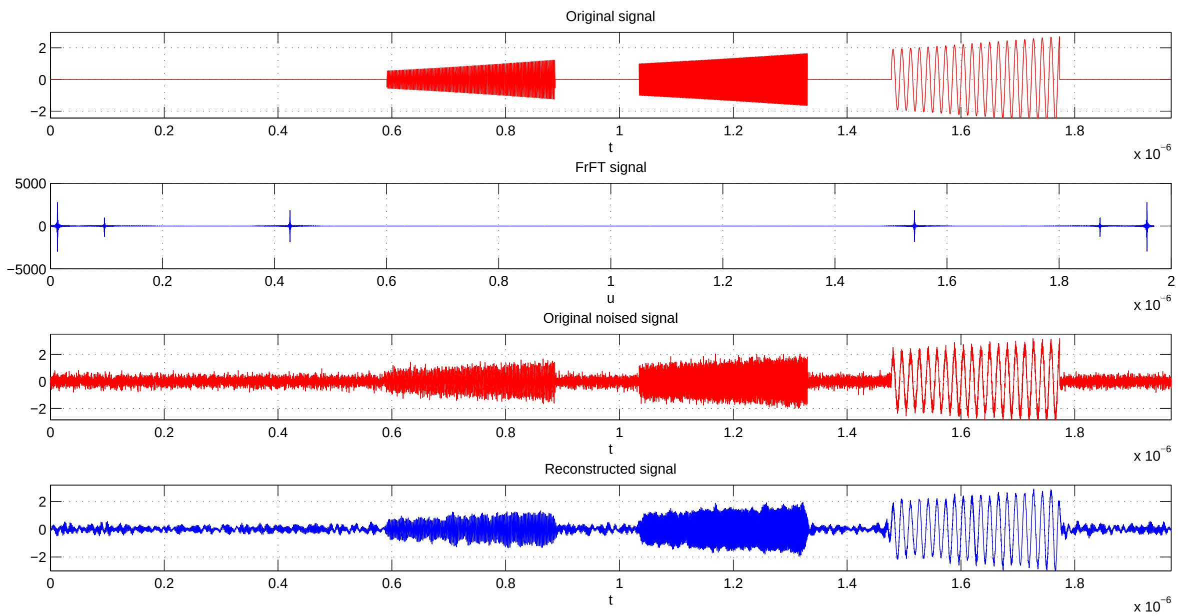

Fig. 3. compressed sampling and reconstruction for chirp signals

V. Conclusion

This paper introduces a practical multichannel compressed sampling method for the α-bandlimited signals in the FrFT domain. The theory combines Modulated Wideband Converter with the fractional Fourier domain sampling method. The simulation shows good performance for the chirp signal. There is also a future work to do before the system can be put into the practise, including the investigation into the transform angle or the bandlimit angle α.

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