I. INTRODUCTION

The research of refrigeration systems has received much attention for over a century due to its cooling property, which was introduced by industry and research institutes. Refrigeration is a cooling generation to attain and maintain a temperature of some product or space below that of the surroundings. It has many possible uses in food preservation, chemical and process industries, manufacturing process, cold treatment of metal, drug manufacturing, ice manufacturing and so on. With rapid advances in modern technology, vapor compression refrigeration systems are now the most common means for commercial and residential space cooling, which lead to fast growth in energy consumption, negatively energy and economic balances effects [1].

Recently, there has been extensive research adopted linear techniques regarding the control of vapor compression refrigeration systems. For example, decentralized PID control [2], decoupling multivariable control [3], optimal control [4], LQG control [5], [6], model predictive control (MPC) [7], [8], and robust control [9], [10]. However, there are many challenges associated with refrigeration systems control stemming from the components themselves to the fundamental characteristics of a heat transfer process, which cause high thermal inertia, dead times, high coupling between variables, and strong nonlinearity. Therefore, a less accurate model of the plant will result in a controller with an unsatisfied performance. When the model is not available or when many parameters cannot be determined, learning feedforward control (LFFC) may be considered. As shown in Fig. 1, LFFC can be implemented by using a learning controller that is comprised of a feedback component (FBC) and a separate learning component (FFC). The FBC part is designed on basis of the prior available process model with the aim of delivering a robust controlled system. Meanwhile, the FFC part is equipped to compensate reproducible disturbances and optimize the system performance with process knowledge.

Being a variant of iterative learning control (ILC) [11]–[13], learning feedforward control (LFFC) [14] shares basic ideas with ILC. Differ from most existing control methods, ILC exploits every possibility to incorporate past control information into the construction of the present control action which can also be treated as a reverse solution of system [15]. Due to its simplicity and effectiveness, ILC has received considerable attention and applications in many areas, such as piezoelectric actuator [11], multi-agent systems consensus tracking [16], permanent magnet linear motor [17]. A
comprehensive review of iterative learning control and its applications can be found in the coming monographs [18].

First proposed for motion systems subjected to reproducible disturbances, LFFC is designed to compensate the reproducible disturbances as value-added blocks [19]. As an extra degree of freedom, LFFC generates steering signals that enhance the feedback control performance [20] and make the output of the process $y$ follow the reference signal $y_d$, perfectly. Thus feedforward part can be adapted as a function approximator that creates a mapping from the reference signal $y_d$ to the steering signal $u_{ff}$. The mapping can be implemented as follow

$$u_{ff}^{j+1}(t) = u_{ff}^j(t) + \gamma u_{fb}^j(t), \quad (1)$$

where $u_{fb}^j(t)$ is the output of feedback part, and $u_{ff}^j(t)$ is the output of feedforward part at the $j$-th iteration, $\gamma$ is the learning rate, $0 < \gamma \leq 1$. In previous research, the LFFC has been widely applied in many areas, such as robotics [20], linear motor [21], piezoelectric Actuator [22], and UPS Inverter [23]. These previous research had shown that learning feedforward control can improve system performance and acquire enhanced extrapolation capabilities for repetitive tracking control tasks with little modeling information. The main contribution of this paper is to apply two-parameter tunable LFFC schemes for the control of refrigeration systems introduced in PID2018 benchmark problem [1]. The detailed contributions of the paper include the following.

1) Combined with feedback control, learning feedforward control is utilized to the control of vapor compression systems which is simple and implementable.

2) No model identification is needed in LFFC design, and the convergence analysis ensures the convergence of the proposed strategy.

3) The performance of the proposed controller has been verified by simulation for reference trajectories of benchmark problems.

In the rest of this paper, the one-stage vapour-compression refrigeration system is first discussed in more details in Section 2. Next, the design of the learning feedforward control system is discussed (Section 3). Simulation results are presented in Section 4. We end with conclusions in Section 5.

II. ONE-STAGE REFRIGERATION SYSTEM

As shown in Fig. 2, a simplified schematic diagram of one-stage refrigeration system includes electronic expansion valve, variable-speed compressor, evaporator, and condenser. In this thermodynamic refrigeration cycle, refrigerant works as a circulating fluid enters the compressor as a vapor. Being compressed at constant entropy, the superheated vapor goes through the condenser where heat is first exchanged with the secondary flux and then the vapor is condensed into liquid. Traveling through the expansion valve, heat is absorbed at the evaporator by evaporating the liquid refrigerant at low pressure and temperature. The main control objective is to provide the desired cooling power, which can be reflected in a reference for the outlet temperature of evaporator secondary flux ($T_{out,sec,e}$). Furthermore, a low but constant set point on the degree of superheating ($T_{SH}$) is introduced to ensure a high Coefficient of Performance ($COP$). Therefore, control scheme is designed to get these two variables to track their references as efficient as possible by operating two manipulated variable (the compressor speed $N$ and the valve opening $A_v$). Hence, the whole control system would be a two-input, two-output system.

Concerning this type of process, high thermal inertia, dead times, high coupling between variables and strong nonlinearities give rise to the control difficulty. Hence, it is difficult to obtain an accurate model of the process. In previous work, model-based feedback controller has been used to control the complex one-stage refrigeration process. To cope with the coupling and uncertainties neglected in modelling, these controllers have to make a compromise between performance and robust stability. Different from the model-based feedback controller, the feedback component used in the LFFC does not need an accurate process model.

III. DESIGN OF LEARNING FEEDFORWARD COMPONENT

One-stage refrigeration systems are mostly controlled by PID or robust control strategies based on process dynamic model. A compromise has to be made between performance and robust stability to cope with the coupling and uncertainties
that may exist when modelling systems linearly. Herein, the main disturbances that remain unknown in advance cannot be compensated for properly. It makes sense to utilize feedforward controller as a value-added block for improving the feedback control performance by taking advantage of the repetitiveness of these systems’ operation. Therefore, the design of the learning controller can be divided into two steps: (1) feedback component design and (2) feedforward component design. Since many algorithms have been proposed considering the former part [2] [3], we will mainly elaborate upon the design of the learning feedforward controller.

A. LEARNING FEEDFORWARD COMPONENT DESIGN

The learning feedforward component is an ideal function approximator that can create a mapping between the reference input and the desired output. The mapping can be realized by most neural networks [24], [25], such as a multilayer perceptron [26], a radial basis function network [27], and a B-spline network (BSN) [28]. The approach we take in this manuscript is a B-spline network which features a relatively short evaluation time for learning and computationally attractive. Generally, such network that consists of one-hidden-layer networks with adaptable weights is a function approximator that can create a mapping between the reference input and the desired output. The mapping can be realized.

The B-spline weights change according to error results in minimizing the summed squared error of the network. Whenever the learning feedforward component mapping imperfectly, tracking errors will occur and can be compensated for by feedback component. Therefore, the feedback controller can be interpreted as an error measure for the feedforward steering. Hence, it is reasonable to utilize feedback control signals as the output error measure for feedforward controller. The value of the learning weights changes according to the following updating rule

\[
\Delta \omega_i^j = \gamma u_{ff}^{j-1}(t) \mu_i(t),
\]

where \(\omega_i^j\) is the B-spline weight in iteration \(j\), \(\mu_i\) is the membership function of the \(i\)th B-spline \((i = 1, \cdots, N)\). The output of the BSN at input \(t\), \(u_{ff}(t)\), is the learning feedforward control signal in iteration \(j\).

The B-spline network weights change according to error results in minimizing the summed squared error of the network. Whenever the learning feedforward component mapping imperfectly, tracking errors will occur and can be compensated for by feedback component. Therefore, the feedback controller can be interpreted as an error measure for the feedforward steering. Hence, it is reasonable to utilize feedback control signals as the output error measure for feedforward controller. The value of the learning weights changes according to the following updating rule

\[
\Delta \omega_i^j = \gamma u_{ff}^{j-1}(t) \mu_i(t).
\]

Apply equation (3) to equation (2) yields

\[
u_{ff}(t) = u_{ff}^{j-1}(t) + \gamma u_{ff}^{j-1}(t) \mu_i(t).
\]

Clearly, the linear manipulation of the signal \(u_{ff}^{j-1}\) can be in any filter form. Hence, the above equation can be written as the following filter form:

\[
u_{ff}^j = u_{ff}^{j-1} + \gamma H(z, z^{-1}) u_{ff}^{j-1}.
\]

The membership functions of \(n\)-th order BSN are piecewise polynomial functions of order \(n - 1\). As shown in Fig. 3, we use a second-order BSN to obtain continuous control signals with bounded time derivatives. The support of the B-spline \(d\) corresponding to the input space that \(\mu_i(t)\) is not equal to zero. For this LFFC problem, suppose there are \(N\) equally-spaced B-splines within \([0, T]\) time interval. Then the B-spline support can be gotten as \(d = 2T/(N - 1)\). Hence, for the learning feedforward controller, there are mainly two parameters remaining to be determined. That is the learning rate \(\gamma\) and the support of splines \(d\). The support of splines need to be determined according to the input signal to ensure the mapping with a certain smoothness. The learning rate \(\gamma\) is chosen as a compromise between fast learning and assuring stability. These two parameters are chosen by the rule of the thumb.

B. CONVERGENCE ANALYSIS

For convergence analysis, we first assume that the transfer function of the plant \(P\) be linear. The plant may be nonlinear with uncertainty, but it can certainly be approximated by a linear model for frequencies below a frequency of interest. The convergence analysis of the LFFC scheme is in the sense that \(u_{ff}\) approaches to a fixed signal and the output approaches to reference \(r\) from trial to trial. The convergence of learning feedforward control is given in Theorem 1 and proved as follows, where \(\mathcal{F}\) denotes the standard Fourier transform.

Theorem 1: For the linear system as shown in Fig. 1 with an existing feedback controller performs a given task repeatedly. A learning feedforward scheme is added to utilize process knowledge to optimize system performance. Furthermore, there exist a real constant \(\gamma\) and learning feedforward approximator \(H(z, z^{-1})\) such that the learning process is convergent.

\[
\lim_{j \to \infty} U_{ff}(j\omega) \rightarrow R(j\omega)/P(j\omega),
\]

where \(U_{ff}(j\omega) = \mathcal{F}[u_{ff}(t)]\) and \(R(j\omega) = \mathcal{F}[r(t)]\). The convergence rate can be derived as

\[
|\rho(\omega, \gamma)| \triangleq |1 - \gamma H(j\omega)G(j\omega)| \leq 1,
\]

where \(G(j\omega)\) is the closed-loop transfer function, let \(C(j\omega)\) denotes feedback controller, herein \(G(j\omega) = \frac{1}{\mathcal{F}[P(j\omega)/R(j\omega)]}\).
\[ C(j\omega)P(j\omega)/(1 + C(j\omega))P(j\omega) \]. Considering frequency domain notion, the mapping (5) becomes
\[ U_{ff}^j(j\omega) = U_{ff}^{-1}(j\omega) + \gamma H(j\omega)U_{fb}^{-1}(j\omega), \quad (8) \]
where \( U_{fb}^{-1}(j\omega) = \mathcal{F}[u_{fb}^{-1}(j\omega)] \).

Proof: According to Fig.1, the feedback signal can be denoted as
\[ U_{fb}(j\omega) = -G(j\omega)U_{ff}(j\omega) + G(j\omega)R(j\omega)/P(j\omega). \quad (9) \]
Substituting equation (9) into equation (8) yields
\[ U_{ff}^j(j\omega) = [1 - \gamma H(j\omega)G(j\omega)]U_{ff}^{-1}(j\omega) \]
\[ + \gamma H(j\omega)G(j\omega)R(j\omega)/P(j\omega). \quad (10) \]

Iterating equation (10) yields
\[ U_{ff}^j(j\omega) = [1 - \gamma H(j\omega)G(j\omega)]^2U_{ff}^{-1}(j\omega) \]
\[ + [1 - \gamma H(j\omega)G(j\omega)]^2R(j\omega)/P(j\omega). \quad (11) \]

Since \( G(j\omega) \) has a low pass filter characteristics, hence, we can conclude that it is possible to choose a suitable \( \gamma \) and \( H(j\omega) \) such that equation (6) is true. That is, the output \( y(t) \) converge to the reference \( r(t) \) for all \( t \in [0, T] \) as \( j \to \infty \).

Remark: According to the above proof, the learning convergence is independent on the initial feedforward control \( u_{ff}^0 \). Therefore, \( u_{ff}^0 \) can be chosen arbitrarily. In practice, as no prior knowledge available, \( u_{ff}^0 \) is usually set to be 0.

**IV. SYSTEM SETUP AND NUMERICAL SIMULATIONS**

In this section, we use benchmark examples to demonstrate feasibility of learning control for one-stage refrigeration cycle.

The Benchmark PID 2018 is first introduced in brief to give the necessary information for the control system design process [1]. In the Benchmark PID 2018 a particular application of one-stage refrigeration systems as shown in Fig. 2 is considered. Working with R404a as refrigerant, the cycle is expected to provide a certain cooling power to a continuous flow entering the evaporator as secondary flux. As shown in Fig. 4, a standard simulation of the refrigeration control system has been scheduled for testing any control systems, considering also the disturbances, which are included in Table 1. As shown in Fig. 5-6, the simulation includes step changes in the references on \( T_{e,sec,\text{out}} \) and \( T_{SH} \) and in the most important disturbances: the inlet temperature of the evaporator secondary flux \( T_{e,sec,in} \), inlet temperature of the condenser secondary flux \( T_{c,sec,in} \). It is important to note that the manipulated variables, \( A_v \) and \( N \), are subjected to limits, \( A_v \in [10, 100] \) and \( N \in [30, 50] \), and are saturated within the system block.

The Benchmark PID 2018 provides a default discrete decentralized feedback controller, where the outlet temperature of the evaporator secondary flux is controlled by means of the expansion valve, meanwhile the compressor speed controls the degree of superheating. Simulations are performed with the MATLAB program to demonstrate feasibility of learning control. The sampling time is 1s and the simulation time is 1200s. The qualitative and quantitative comparisons are explored between our LFSC scheme with controller provided in the Benchmark PID 2018. In the quantitative comparison, the discrete decentralized PID controller plays the role of controller of reference (labelled as Controller 1 in Fig. 7-9) and our proposed controller plays the role of controller to evaluate (labelled as Controller 2 in Fig. 7-9). Moreover, eight individual performance indices and one combined index are applied to further evaluate in comparison which are listed as follows

\[
\text{IAE}_i = \int_0^\text{time} |e_i(t)| \, dt, \quad (12)
\]
\[
\text{IAVU}_i = \int_0^\text{time} \frac{du_i(t)}{dt} \, dt, \quad (13)
\]
\[
\text{RIAE}_i(C_2, C_1) = \frac{\text{IAE}_i(C_2)}{\text{IAE}_i(C_1)}, \quad (14)
\]
\[
\text{RITAE}_i(C_2, C_1, t_c, t_s) = \frac{\text{ITAE}_i(C_2, t_c, t_s)}{\text{ITAE}_i(C_1, t_c, t_s)}, \quad (15)
\]
Guidelines for Tuning The learning feedforward scheme offers considerable flexibility with the number of tuning knobs that it provides. To fit a mapping accurately, B-splines need to be implemented with a small support $d$. In terms of the learning gain $r$, a cautiously small learning gain can ensure the convergence of this scheme. The width of the B-splines and the learning gain were tuned by starting from value 0 and changing the values by try and error in the direction that reduces the combined index $J$.

TABLE 1: Disturbance vector

<table>
<thead>
<tr>
<th>Disturbance</th>
<th>Mathematical symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet temperature of the condenser secondary flux</td>
<td>$T_{c, sec, ini}$</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>Mass flow of the condenser secondary flux</td>
<td>$\bar{m}_{c, sec}$</td>
<td>g/s</td>
</tr>
<tr>
<td>Inlet pressure of the condenser secondary flux</td>
<td>$P_{c, sec, ini}$</td>
<td>bar</td>
</tr>
<tr>
<td>Inlet temperature of the evaporator secondary flux</td>
<td>$T_{e, sec, ini}$</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>Mass flow of the evaporator secondary flux</td>
<td>$\bar{m}_{e, sec}$</td>
<td>g/s</td>
</tr>
<tr>
<td>Inlet pressure of the evaporator secondary flux</td>
<td>$P_{e, sec, in}$</td>
<td>bar</td>
</tr>
<tr>
<td>Compressor surroundings temperature</td>
<td>$T_{surr}$</td>
<td>$^\circ C$</td>
</tr>
</tbody>
</table>

$RIAVU_i(C_2, C_1) = \frac{IAVU_i(C_2)}{IAVU_i(C_1)}$, \hspace{1cm} (16)

$J(C_2, C_1) = \sum_{i=1}^{s} w_i RIAE_i(C_2, C_1) + w_3 RIAE_i(C_2, C_1)$

+ $w_4 RIAE_i(C_2, C_1, t_{c1}, t_{s1})$

+ $w_5 RIAE_i(C_2, C_1, t_{c2}, t_{x2})$

+ $w_6 RIAE_i(C_2, C_1, t_{c3}, t_{x3})$

+ $w_7 RIAE_i(C_2, C_1, t_{c4}, t_{x4})$

+ $w_8 RIAE_i(C_2, C_1)$. \hspace{1cm} (17)

TABLE 2: Quantitative Comparisons of Controller 2 with Controller 1

<table>
<thead>
<tr>
<th>Index</th>
<th>Controller 2 vs Controller 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RIAE_1(C_2, C_1)$</td>
<td>0.5389</td>
</tr>
<tr>
<td>$RIAE_2(C_2, C_1)$</td>
<td>0.6068</td>
</tr>
<tr>
<td>$RIAE_1(C_2, C_1, t_{c1}, t_{s1})$</td>
<td>0.6915</td>
</tr>
<tr>
<td>$RIAE_2(C_2, C_1, t_{c2}, t_{x2})$</td>
<td>0.9157</td>
</tr>
<tr>
<td>$RIAE_2(C_2, C_1, t_{c3}, t_{x3})$</td>
<td>0.5753</td>
</tr>
<tr>
<td>$RIAE_2(C_2, C_1, t_{c4}, t_{x4})$</td>
<td>0.6583</td>
</tr>
<tr>
<td>$RIAVU_1(C_2, C_1)$</td>
<td>1.0383</td>
</tr>
<tr>
<td>$RIAVU_2(C_2, C_1, t_{c1}, t_{x1})$</td>
<td>1.0514</td>
</tr>
<tr>
<td>$J(C_2, C_1, t_{c1}, t_{x1})$</td>
<td>0.6536</td>
</tr>
</tbody>
</table>

For a BSN LFFC with $m=9$ and $\gamma=0.1$, labelled as Controller 2, Fig. 7-Fig. 10 show its tracking performance after 10th learning iterations compared with the discrete decentralized PID controller. As shown in Fig. 7, Controller 2 achieves better tracking performance on the outlet temperature of the evaporator secondary flux and the degree of superheating than Controller 1, specially regarding the disturbance rejection, which is demonstrated in almost all indices. Fig. 10 depicts combined index $J$ versus learning iteration number where the monotonic convergence is obvious. The eight performance indices shown in Table 2 further testify the control effort in BSN LFFC. Although the relative indices
RIA VU(C2,C1) and RIAVU2(C2,C1) are greater than one which indicate that control effort in Controller 2 is higher. Considering the index weighting, Controller 2 significantly outperforms Controller 1 as it improves the overall index J by 35%. The improvement in results can be explained by the comparison results as shown in Fig. 8-9. As the blue lines go smaller, the feedforward controller undertakes more work with iterations.

V. CONCLUSIONS
In this paper, we demonstrated the effectiveness of a learning feedforward control participated scheme for vapour-compression refrigeration system. Combined with feedback PID controller, the BSN based learning feedforward controllers are applied. The learning controller is able to improve system performance drastically with only two parameters to adjust: the support of B-spline and learning gain. Simulation results suggests that the proposed LFFC with B-spline network scheme can achieve satisfied tracking control performance on difficult dynamical systems without extensive and time-consuming modelling. It is noteworthy that the network we applied during simulations was obtained by the rule of the thumb. Hence, an optimized tuning of parameters can help to improve the performance of the discussed control system. Our future work includes systematic design methods for learning feedforward control.

REFERENCES
FIGURE 9: Compressor efficiency and Coefficient of Performance comparison between decentralized PID control system and BSN LFFC.

FIGURE 10: The change of combined index with iterations under BSN LFFC.


**YANG ZHAO** received her Ph.D. degree in Pattern Recognition and intelligent system from Shandong University in 2019. She was a visiting scholar in MESA lab at University of California Merced during 2017-2018. She received her M.S. degree in Control Engineering from Shandong University in 2011. She is currently a lecturer of School of Electrical Engineering and Automation at Qilu University of Technology, Jinan 250353, PR China. Her research interests include applied fractional calculus in robust control, iterative learning control, system identification and robotics.

**YAN LI** received his Ph.D. degree in Applied Mathematics from Shandong University in 2008. He was a visiting scholar in CSOIS at Utah State University during 2007-2010. Since 2010, He has been a faculty member of School of Control Science and Engineering at Shandong University, Jinan 250061, PR China where he teaches complex analysis and equations of mathematical physics for undergraduates, and optimal control and fractional order control systems for graduates. His research interests include applied fractional calculus in cybernetics; test, modeling and simulation of power batteries; big data analytics in power batteries and microbes; biomechanics; iterative learning control; high gain adaptive control; optimal control; complex systems and networks, etc.

**SINA DEHGHAN** earned his B.Sc. in Mechanical Engineering from Isfahan University of Technology (IUT), Iran, 2013. He is a Ph.D. candidate in Mechatronics, Embedded Systems and Automation (MESA) Lab at University of California Merced. He has been working on a USDA funded project serving as Process Control specialist where he helps development of process control for a conventional and a novel system for carbon activation using California Local Biochar. Currently, he works on a project in collaboration with Lam Research Corporation leveraging data analytics and machine learning for Advanced Process Control (APC). His research interests include mechatronics and automation, data analytics and machine learning, advanced process control, precision temperature control, and fractional order control.

**YANGQUAN CHEN** received his Ph.D. degree in Advanced Control and Instrumentation from Nanyang Technological University, Singapore, in 1998. He was on the Faculty of Electrical and Computer Engineering at Utah State University before he joined the School of Engineering, University of California, Merced in 2012 where he teaches mechatronics for juniors and fractional order mechanics for graduates. His research interests include mechatronics for sustainability, cognitive process control and hybrid lighting control, multi-UAV based cooperative multi-spectral personal remote sensing and applications, applied fractional calculus in controls, signal processing and energy informatics; distributed measurement and distributed control of distributed parameter systems using mobile actuator and sensor networks.