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Fractional Order Proportional-Resonant Controller

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***Abstract*—In this paper, a fractional order synchronous fra controller and its equivalent model, stationary frame control named fractional order proportional resonant controller, h been introduced using fractional order theory. Compared conventional proportional resonant controller, the proposed c troller not only ensures a zero steady-state error for AC cont systems, but also provides more robustness against fluctuati of resonant (center) frequency. Having a wider bandwidth a higher gain in the neighborhood of resonant frequency enhan the capability of this controller to deal with resonant freque variations and furthermore, improve the time domain transi performance of these systems. Analog and digital implemen tions of this novel controller have been discussed. In additi stability of the proposed controller has been investigated. Fina the proposed fractional order control strategy is successfu applied to a grid-connected system as current regulator.**

***Index Terms*—Proportional resonant controller, Fractional order calculus, Stationary frame, Synchronous frame**

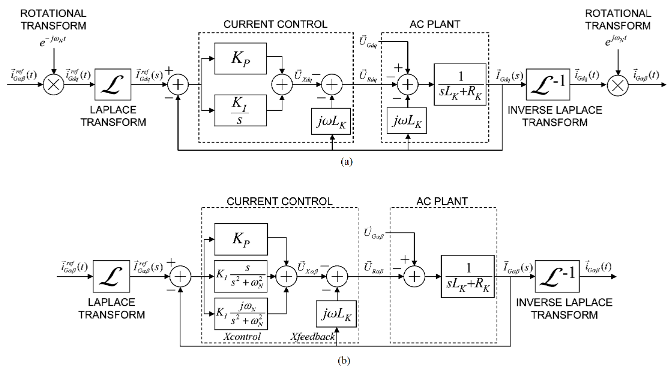

Fig. 1. (a) Synchronous frame current control loop with rotational transform operators at the input and output, (b) Stationary frame current control loop equivalent to the synchronous frame model [5]

I. Introduction

Generally, in DC control systems (i.e. servo control systems), where the control quantity under steady state condition is DC, the steady state error is determined by their open loop gain at frequencies close to zero (i.e. DC), while their transient characteristics is determined by the frequency response at the system cross-over frequency.

However, for AC control systems, such as current regulated voltage source inverter (VSI), the reference signal and control quantities are AC. Grid-connected systems, uninterruptible power supplies and three-phase active filters are some examples for AC control systems. Due to the inability of DC control system to provide infinite gain at non-zero frequencies, employing a DC regulator (e.g. a PI) to control an AC control system in the same manner as DC control systems (i.e. stationary frame) is subject to steady state error. [1], [2].

One approach to obtain zero steady state error for AC control systems is to employ a DC controller (e.g. a PI) in a synchronous frame. Synchronous frame transfers the AC reference signal to a DC quantity which provides the capability for performing DC Controller on both AC and DC control systems [2]. The synchronous frame with a PI controller is depicted in Fig. 1 (a). Another method to obtain zero steady state error in the AC control systems is to derive the equivalent regulator of aforementioned synchronous frame controller in stationary frame. This control strategy is illustrated in Fig. 1 (b). In the case of synchronous frame with a PI controller, the equivalent controller is derived in [2] and called Proportional Resonant (PR) controller.

Comparing both proposed solutions in Fig. 1, PR controller offers a fast dynamic response while it does not require the Phase Lock Loop (PLL) which eliminates its associated error and failure problems. Moreover, PR controller in stationary frame eliminates rotational transformations, thus significantly reducing the overall computation burden. In addition, this controller enhances the performance of the system in the presence of component uncertainty and unbalanced faults. This would result in diminishing the DC ripple current which leads to reduced DC capacitor sizing. Detailed comparison of these two control approaches is discussed in [3], [4].

Although there exist an extensive amount of research on the synchronous fame with PI controller and its equivalent PR controller, the scope of these studies are limited to only these two types of controller. Recently, a new generations of the PID controller family, called fractional order (FO) controllers, have been introduced and applied to DC control systems. These controllers outperform the integer order ones in a wide range of applications from renewable energy to temperature control applications [6], [7], [8], [9], [10], [11], [12]. However, application of FO controllers in the AC control systems has been narrowly studied in a very limited number of researches. One of the early studies in this field was presented in [13], [14] where authors applied a FO-PI and FO-[PI] schemes to a thee-phase synchronous frame current regulator for a grid-connected photo-voltaic system in order to improve transient response and robustness of the system. Other researchers followed similar path by replacing the PI current regulator of a grid-connected system with FO-PI and confirmed the advantages of employing these controllers [15], [16]. Although these literature demonstrate the superiority of fractional order controllers in the synchronous frame, the equivalent FO-PI controller for stationary frame has not been studied.

In a different study, authors tackled this problem by applying a nonlinear control methodology (FO sliding mode control) to the current regulator of grid-connected wind turbine

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to achieve zero steady state error [17]. Despite the promising features of applying FO nonlinear control, it intensifies complexity to the implementation stage. Furthermore, sliding mode control adds undesirable chattering by its nature. Due to these issues, this approach raises concerns for aforementioned applications. Most importantly, in proposed hybrid structures, obtaining an equivalent stationary frame (similar to PR controller which is derived for integer order PI in synchronous frame) is theoretically impossible, since the developed theories in the field of hybrid systems are very limited.

To address all the aforementioned open questions and concerns with respect to other proposed approaches, in this paper, initially, a new fractional order synchronous frame using Mittag-Leffler function is proposed. Following that, an equivalent stationary frame for introduced fractional order synchronous frame is offered. As will be discussed, this equivalent structure offers various benefits including enhancement of computational time by eliminating PLL and transformation blocks compared to fractional order synchronous frame. Additionally, this new controller shows the overall performance improvement compare to its older generation of integer order PR (IO-PR). Furthermore, Stability analysis and realization of this controller as well as its performance are discussed and compared vs. integer order PR.

The rest of the paper is organized as follows: Section II briefly introduces the fractional order operators. Section III presents an overview of synchronous and stationary control strategies. In Section IV, the proposed FO structure is presented. Section VI, the performance of new proposed controller is compared to integer order PR controller. Concluding remarks are presented in section VII.

II. Fractional Order Derivatives and Integral Definitions

The idea of fractional calculus has been known since the development of the regular calculus, with the first reference probably being associated with letter between Leibniz and L'Hospital in 1695 [6]. One of the most popular continuous definitions used for fractional differintegral is Riemann-Liouville (RL) which is defined as [18]

$$D^{\mu} f(t) = \frac{1}{\Gamma(n-\mu)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\mu-n+1}} d\tau, \tag{1}$$

where $(n-1<\mu<n)$, $n \in \mathbb{N}$ and $\Gamma(.)$ is the *Gamma* function.

Since introduction of fractional calculus, modeling of physical phenomena using fractional order operators and controlling various plants using fractional order controllers have been widely investigated among researchers and scientists in this field. All these researches imply the superiority of the fractional order operators from robustness and performance point of view [19], [20], [21]. Furthermore, wide range of applications and properties of fractional operators have been investigated in many literature including in [22], [23], [24], [25], [26], [23], [27].

A. Generalized Exponential Function

Special functions such as Mittag-Leffler plays a significant role in solving fractional order differential equations. This function is as important as regular exponential function in the case of integer order differential equations. Two general definitions for Mittag-Leffler function are

$$E_{\mu}(z) = \Sigma_{k=0}^{\infty} \frac{z^k}{\Gamma(\mu k+1)} \quad \mu, \nu \in \mathbb{R}, \quad z \in \mathbb{C}, \tag{2}$$

Development of fractional differential equations has urged scientists to define new trigonometric functions, e.g. generalized cosine and sine functions $(\cos_{\mu}(x), \sin_{\mu}(x))$ [28], [29], [30].

Before going into more detailed discussions, one important relation between generalized exponential and fractional order trigonometric functions needs to be introduced. As will be shown later, the following corollary plays a key role in the definition of fractional order synchronous and stationary frames.

Corollary: For $0<\mu<1$ and any $x \in \mathbb{R}$, the relationship between Mittag-Leffler function and trigonometric functions can be defined as

$$E_{\mu}(\pm j^{\mu} x^{\mu}) = \cos_{\mu} x^{\mu} \pm j^{\mu} \sin_{\mu} x^{\mu}, \tag{3}$$

Proof: Replacing trigonometric functions of (3) with their equivalent series gives

$$\begin{aligned} &\cos_{\mu} x^{\mu} \pm j^{\mu} \sin_{\mu} x^{\mu} = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k\mu}}{\Gamma(2k\mu+1)} \pm \\ &j^{\mu} \sum_{k=0}^{\infty} (-1)^k \frac{x^{(2k+1)\mu}}{\Gamma((2k+1)\mu+1)} \\ &= \sum_{k=0}^{\infty} (j)^{\mu k} \frac{(\pm x)^{k\mu}}{\Gamma(k\mu)+1} = \sum_{k=0}^{\infty} \frac{(\pm jx)^{k\mu}}{\Gamma(k\mu)+1} = E_{\mu}(\pm j^{\mu} x^{\mu}) \diamond \end{aligned} \tag{4}$$

B. Laplace Transform Of Fractional Order

In the case of fractional order differential equations, if $f(x)$ has a derivative from order of μ but no integer order derivative, [31] and [32] propose Laplace transform of fractional order, which is defined as

$$\begin{aligned} F_{\mu}(s) \doteq \mathscr{L}_{\mu}\{f(x)\} \doteq \int_0^{\infty} E_{\mu}(-s^{\mu} x^{\mu}) f(x) (dx)^{\mu} \\ = \lim_{M\to\infty} \int_0^M (M-x)^{\mu-1} E_{\mu}(-s^{\mu} x^{\mu}) f(x) dx, \end{aligned} \tag{5}$$

where $\mathscr{L}_{\mu}\{.\}$ is the fractional order Laplace transform and $E_{\mu}(.)$ is Mittag-Leffler function.

According to the same references, "shifting property" holds for fractional order Laplace transform and is defined as

$$\mathscr{L}_{\mu}[f(t) E_{\mu}(-c^{\mu} t^{\mu})] = \mathscr{L}_{\mu}[f(t)]_{s+c} = F_{\mu}(s+c), \tag{6}$$

where $f(t)$ is an arbitrary function in time domain, $c \in \mathbb{R}$ and F_{μ} is the fractional order Laplace transform of $f(t)$ from the

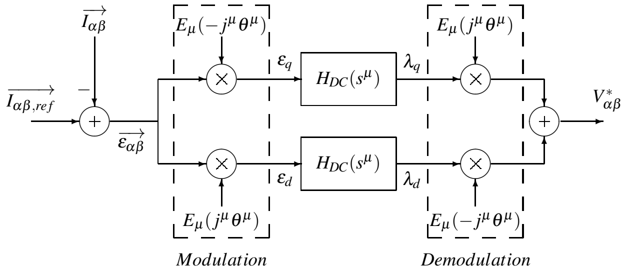

Fig. 2. Block diagram of modulation/demodulation of FO-PR controller using Mittag-Leffler function

order of $\mu \in (0,1]$. In next section, to establish FO stationary and synchronous frame, a quick review of conventional frame is done.

III. Overview of Synchronous and Stationary Frames

In [2], a stationary AC regulator, $H_{AC}(s)$, has been derived from a DC regulator, $H_{DC}(s)$, in synchronous frame, so that it has the same frequency response characteristic in the bandwidth of concern. The performance of this stationary regulator is identical to the DC regulator in synchronous frame.

In the case of having PI controller in synchronous frame, $H_{DC} = K_p + \frac{K_i}{s}$, the equivalent stationary AC compensator is

$$H_{AC} = K_p + \frac{2K_i s}{s^2 + \omega_0^2}. \tag{7}$$

In the practical world, due to the component tolerances and finite precision in digital systems, the ideal PI controller is not implementable and it is approximated by $H_{DC} = K_p + \frac{K_i \omega_c}{s+\omega_c}$. The equivalent AC regulator for this practical controller is

$$H_{AC} = K_p + \frac{2K_i \omega_c s}{s^2 + 2\omega_c s + \omega_0^2}, \tag{8}$$

where ω_c is the bandwidth of the AC regulator around the AC frequency ω_0.

IV. Fractional Order Proportional Resonant Converter

As mentioned before, a very limited efforts to adopt fractional order controllers into synchronous frame have been started in the last couple of years. Various literature have reported improvement in time domain responses of current regulated VSI by employing fractional order PI controller in the synchronous frame [17], [33]. The main stream in these research works is to replace the DC regulator of synchronous frame, $H_{DC}(s) = K_p + Ki/s$ of Fig. 2 with a fractional order PI ($H_{DC}(s) = K_p + Ki/s^\mu, 0 < \mu \leq 1$). This replacement shows some improvement in the system performance. However, the $\alpha\beta$ and dq frame rotational speed (synchronous speed), ω_0 is assumed to be the same as integer order case. Therefore, the required resonance behavior in this structure is not achievable.

To address this issue and utilize the maximum advantages of employing fractional order operators, one needs to redefine the relationship between the synchronous and stationary frames. The proposed structure in Fig. 2 is believed to be a better way of implementing fractional order operators for AC control systems. As shown in this figure, exponential functions in integer order synchronous frame is replaced by Mittag-Leffler functions.

Using (3), fractional order synchronous frame can be rewritten as

$$\begin{aligned} \varepsilon_d &= (x_\alpha + jx_\beta)(\cos_\mu(\theta^\mu) - j^\mu \sin_\mu(\theta^\mu)) \\ \varepsilon_q &= (x_\alpha + jx_\beta)(\cos_\mu(\theta^\mu) + j^\mu \sin_\mu(\theta^\mu)) \end{aligned} \tag{9}$$

Assuming a balanced three-phase system, synchronous frame regulator of Fig. 2 can be described as

$$\begin{aligned} \lambda_q(t) &= [\varepsilon_{\alpha\beta}(t) E_\mu(-j^\mu\theta^\mu)] * h_{DC}(t) \\ \lambda_d(t) &= [\varepsilon_{\alpha\beta}(t) E_\mu(j^\mu\theta^\mu)] * h_{DC}(t) \end{aligned} \tag{10}$$

where '*' denoted the convolution operator. Knowing $\theta = \omega_0 t$, and going through a lengthy mathematical calculations including applying fractional order Laplace transform of (10) and using shifting property gives

$$H_{AC}(s) = H_{DC}(s^\mu + j^\mu\theta^\mu) + H_{DC}(s^\mu - j^\mu\theta^\mu). \tag{11}$$

Thus, if the DC regulator in the proposed synchronous frame is a common fractional order PI controller, $H_{DC}(s) = K_p + K_i/s^\mu$, the equivalent AC regulator in the stationary frame, $H_{AC}(s)$ is obtained as

$$\begin{aligned} H_{AC}(s) =& K_p + \frac{K_i}{s^\mu + j^\mu \omega_0^\mu} + \frac{K_i}{s^\mu - j^\mu \omega_0^\mu} \\ =& K_p + \frac{2K_i s^\mu}{s^{2\mu} - (j\omega_0)^{2\mu}}. \end{aligned} \tag{12}$$

Using Hankel integral introduced in [34], one can conclude that

$$\frac{s^\mu}{s^{2\mu} - (j\omega_0)^{2\mu}} = \frac{s^\mu \sin(\mu\pi/2)}{s^{2\mu} - 2\cos(\mu\pi/2) s^\mu \omega_0^\mu + \omega_0^{2\mu}} \tag{13}$$

Substituting second term of (12) with (13) gives

$$H_{AC}(s) = K_p + \frac{2K_i s^\mu \sin(\mu\pi/2)}{s^{2\mu} - 2\cos(\mu\pi/2) s^\mu \omega_0^\mu + \omega_0^{2\mu}} \tag{14}$$

In the next sections, implementation of this new born controller will be discussed.

V. Implementation of FO-PR Controller

After derivation of FO-PR controller, the implementation of this controller becomes the next question. One way to look at this problem is to rewrite the second term of (14) as of the following format

$$\frac{Y_1(s)}{U(s)} = 2K_i \frac{\frac{1}{s^\mu}\sin(\mu\pi/2)}{1 - \frac{2\cos(\mu\pi/2)\omega_0^\mu}{s^\mu} + \frac{\omega_0^{2\mu}}{s^{2\mu}}} \tag{15}$$

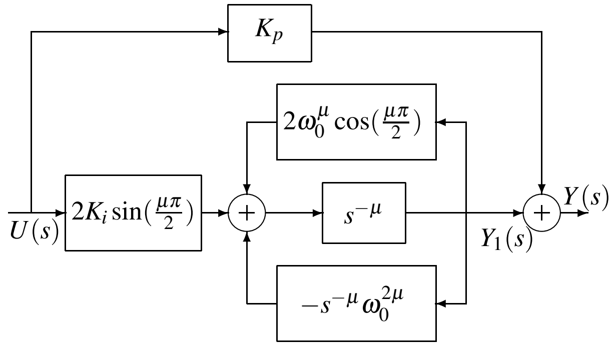

Fig. 3. Block diagram of FO-PR controller implementation

Rearranging equation gives

$$Y_1(s) = \frac{2\cos(\mu\pi/2)\omega_0^{\mu}}{s^{\mu}}Y(s) - \frac{\omega_0^{2\mu}}{s^{2\mu}}Y_1(s) + U(s)\frac{2K_i}{s^{\mu}}\sin(\mu\pi/2), \tag{16}$$

which shows that $Y_1(s)$ can be resulted as the summation of three terms. Two of them are dependent on $Y_1(s)$ which can be created by employing a feedback loop and one is dependent on $U(s)$ which is made by a feedforward loop. Complete block diagram of FO-PR controller of (14), including K_p term, can be concluded as shown in Fig. 3.

By looking at this block diagram, after tuning the controller, parameters such as μ and ω_0 are determined and known.

To conclude this section, implementation of FO-PR controller, introduced in Fig. 3, can be achieved by employing analog or digital implementation of $s^{-\mu}$ and obtaining μ, K_i and K_p through the tuning process which will be discussed later.

VI. Simulation Results and Discussion

In this section, before applying the proposed controller to a real-world system and comparing the results vs. integer order version, lets investigate the conceptual differences between integer order and fractional order controllers. Figure (4) illustrates the frequency responses of IO-PR and FO-PR controllers of (12) and (14). $\omega_0 = 2*\pi*60rad/s$, $\omega_c = 0.1rad/s$, $K_p = 0.1$, $K_i = 1$ and $\mu = 0.8$ are the considered parameters for these frequency responses. As shown in this figure, (12) and (14) introduce identical gain to the system and this gain at desired frequency (ω_0) is significantly higher than IO-PR even when the damping factor for this controller is as low as $\omega_c = 0.1rad/s$.

The other advantages of FO-PR is the wider bandwidth of this controller compare to integer order one. The wider bandwidth helps controller to tolerate more variations around center frequency, ω_0. In the real-world applications, for instance, grid-connected systems, the center frequency of the grid always varies around ω_0. For this reason, having a controller which introduces higher gain for frequencies other than center frequency is highly desirable. As Fig. 4 demonstrates, FO-PR introduces larger gain compare to IO-PR for all frequencies. Although, IO-PR bandwidth can be widened by increasing the value of ω_c, this will largely reduce the gain at center frequency, ω_0, which is undesirable. On the other hand, reducing ω_c to compensate for the gain will decrease the bandwidth. The main advantages of FO-PR are providing wider bandwidth and larger gain simultaneously.

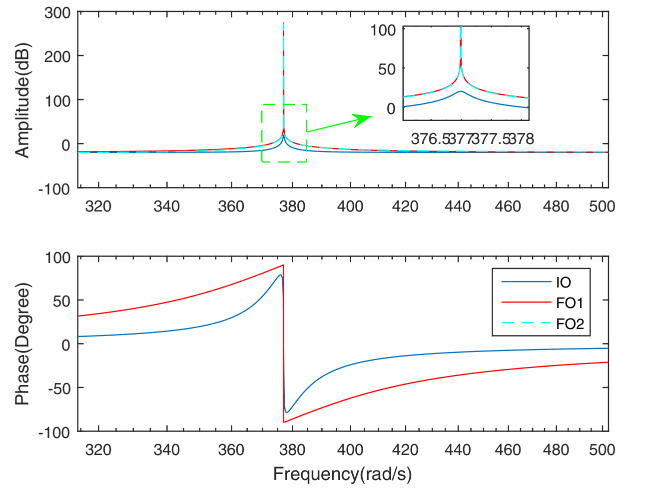

Fig. 4. Frequency responses of FO-PR and IO-PR controllers with $\omega_0 = 2*pi*60rad/s$ and $\omega_c = 0.1rad/s$

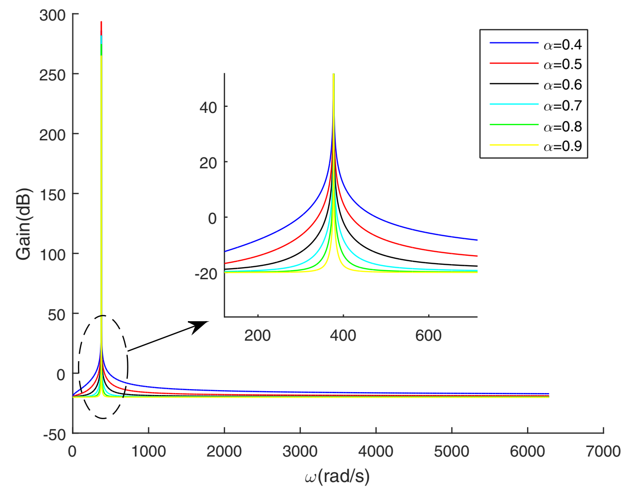

Fig. 5. Frequency responses of FO-PR controller for different values of μ with $\omega_0 = 2*pi*60rad/s$ and $\omega_c = 0.1rad/s$

Figure 5 illustrates the effect of μ on the frequency response of FO-PR controller. Clearly, lower order for FO-PR results in wider bandwidth for controller. Having a tune-able order in the structure of controller provides one more degree of freedom for design engineer to adjust K_i, K_p and μ according to the requirement of the system.

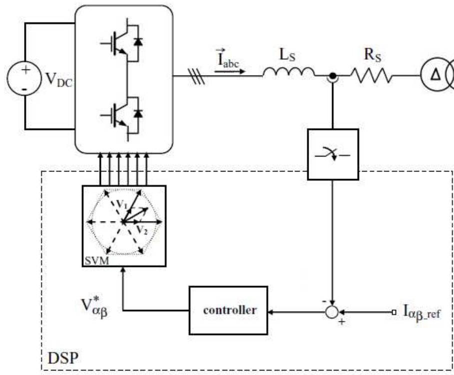

Fig. 6. Block diagrams of design example current regulator [35]

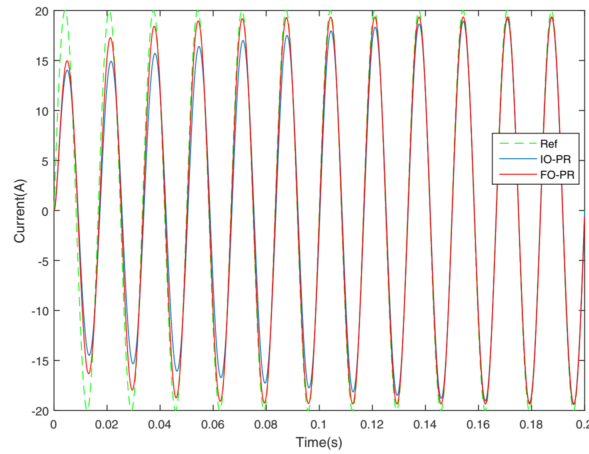

Fig. 7. Comparison of FO-PR (red line) vs. IO-PR (blue line). Reference signal is depicted in green dash line

A. Design Example:Comparison of FO-PR and IO-PR Current Regulators

In this section, a case study of current regulator using stationary frame from [35] is considered. Block diagram for this example is depicted in Fig. 6. This block diagram is applicable to grid-connected systems or motor drive applications (in this case the transformer on the AC side is not required).

In this system, the R-L load admittance is usually modeled using a first order transfer function $\frac{1}{L_s s+R_s}$. Since inverter switching frequency is generally order of magnitude higher than E_s frequency, inverter is modeled using a constant gain. Current transducer is considered as a gain, G_{TI}, as well.

The current control system with following parameter values is considered; DC voltage, $Vdc = 250V$, desired frequency (central frequency), $f_0 = 60Hz$, load inductance, $L_s = 3.5mH$, load resistance, $R_s = 1\Omega$, switching frequency, $f_s = 10kHz$, current transducer gain, $G_{TI} = 0.1V/A$. The current reference signal is $I_{ref} = 20\sin(2\pi * 60t)$. In this simulation, the controller parameters are set to be $K_p = 0.1$, $K_i = 1.2$ and $\mu = 0.8$.

Figure 7 represents the performance of fractional order proportional resonant controller (red line) vs. integer order proportional resonant controller (blue line). As can be seen from this figure, FO-PR converges to the reference signal (green dash line) faster than IO-PR. This is the direct consequence of significant higher gain of FO-PR at $f_0 = 60Hz$ compare to IO-PR gain.

For the next set of simulation, it is assumed that the frequency of the reference signal deviates $1Hz$ from its nominal value at $f_0 = 60Hz$. The effect of frequency fluctuation is presented in Fig. 8. FO-PR can still follow the reference with a phase shift which is caused by frequency fluctuation but IO-PR is not able to follow this new reference signal and the output of the system exceeds the peak value of the reference signal and as time goes, the peak value increases. This difference in behaviors of IO-PR and FO-PR is due to the difference in the bandwidth of these two controllers. As it discussed before, having a wider bandwidth in FO-PR provides a more robust

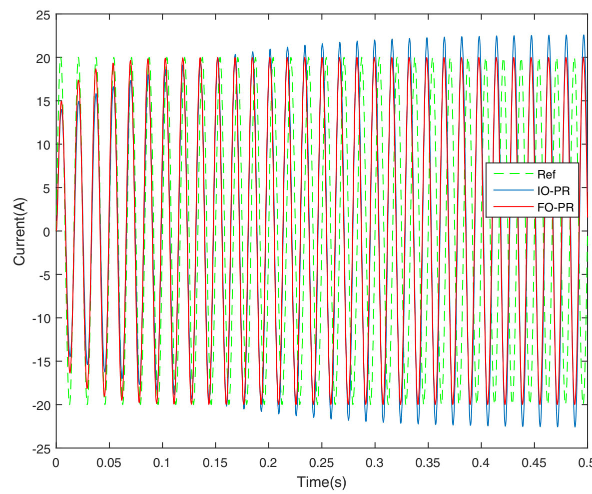

Fig. 8. Comparison of FO-PR (red line) vs. IO-PR (blue line) in the presence of frequency fluctuation. Reference signal is depicted in green dash line.

behavior against system parameter value fluctuations including frequency variation. It is worth mentioning that frequency variation is very common in grid-connected systems.

VII. Conclusion

Conventional PR controller has been proposed with the ability to alleviate the previously known shortcomings associated with conventional PI controller. Despite offering structural simplification, the PR controller provides high-gain at a very narrow range of frequencies, the resonant frequency can be easily missed and captured in the no-gain or low-gain zone in the presence of any noise or uncertainty in the system.

In this paper, employing fractional order theory, firstly, a fractional order synchronous frame structure was introduced and then a FO stationary frame controller (i.e. FO-PR) was derived. Although these two structures have identical performance, FO-PR has less implementation complexity compared to FO-synchronous frame, similar to the integer order case.

By comparing the frequency response of FO-PR with conventional PR, the main advantage of FO-PR is revealed. The proposed controller provides significantly higher gain at resonant frequency while its gain in the acceptable neighborhood of this frequency is higher than IO-PR. It is worth mentioning that high-gain at resonant frequency results in zero steady state error.

This higher gain can accommodate more fluctuations around resonant frequency which makes FO-PR a more desirable controller since in real world applications, resonant frequency variation is inevitable. Furthermore, having a wider bandwidth in FO-PR, compared to IO-PR, helps this controller to achieve a better transient response feature. These benefits of FO-PR were confirmed through an AC current regulator application.

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