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Fractional order equivalent series resistance modelling of electrolytic capacitor and fractional order failure prediction with application to predictive maintenance

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Hadi Malek[1] ✉, *Sara Dadras*[1], *Yangquan Chen*[1,2]

[1]*Electrical and Computer Engineering Department, Utah State University, Logan, UT 84321, USA*
[2]*School of Engineering, University of California, Merced, CA 95343, USA*
✉ *E-mail: h.m.malek@ieee.org*

Abstract: Being one of the most used passive components in power electronics, electrolytic capacitors have the shortest life span due to their wear-out failure which is mainly caused by vaporisation and deterioration of capacitor electrolyte. Knowing these two phenomena increase equivalent series resistance (ESR) of the capacitor, tracking ESR value over the system operating time can be a good indicator for state of health of an electrolytic capacitor. To set the maintenance schedule, various ESR monitoring algorithms have been investigated in literature. These classical real-time algorithms lead the maintenance program to be either risky if the prediction is longer than the actual life-time or more expensive if it is shorter than the actual life span. This study presents a generalised equivalent model using fractional order (FO) element for electrolytic capacitor to estimate the ESR and impedance of faultless running capacitor. Furthermore, a novel failure predictive model using Mittag-Leffler function is proposed to track the ESR increment and estimate the failure time. Hence, the predictive maintenance of the system with capacitors nearing their failure time can be set more precisely. These two FO models are compared against classical ESR and life-time prediction models to illustrate the enhanced performances of the proposed models.

1 Introduction

Low cost and large capacity have made electrolytic capacitors to be widely used as an energy storage component in power converters or in filtering applications. However, electrolytic capacitors have the highest rate of failure among electronic components. As aluminium electrolytic capacitor is known to be the weakest link in power electronics circuits and to have the highest probability of failure compared with other components, the life-time of this component has critical impact on the performance and life-time of the converters [1–5].

Detailed failure mechanism of an electrolytic capacitor has been discussed in the literature. A primary reason for wear-out in aluminium electrolytic capacitors is vaporisation and degradation of electrolyte, which leads to a drift in the two main electrical parameters of the capacitor, the equivalent series resistance (ESR) and the capacitance [6, 7]. Generally, electrolyte vaporisation and degradation are caused by temperature rise [8, 9]. Deterioration of the capacitor electrolyte content is reflected by increasing ESR value decreasing leakage current and capacitance of the components [10]. Higher ESR results in higher-power loss and increases the internal temperature (hot-spot temperature), which in turn affect the capacitor performance and reduces the capacitor life-time [11].

EIA IS-479 standard, which unifies the life-time testing among capacitor manufacturers, defines capacitor life-time as the time at which 10% of the capacitors under the test have failed due to parametric failure (including capacitance, dissipation factor and ESR) and no more than 10% due to structural failure. According to this standard, a capacitor is reached to the end of life (EOL) when its ESR rises by 200% from its initial value [12–14, 7].

Both impedance and ESR of a capacitor are function of frequency and temperature. The mathematical relationship between impedance (or ESR), frequency and temperature has been derived and implemented through different capacitor models. These models have a broad range; from a simple series *R*–*L*–*C* circuit to more complex models including series and parallel branches of *R*s, *L*s, *C*s and even non-linear components. Some of the most well-known models are presented in Fig. 1. Though many works [10, 15–17] consider the simplified equivalent model (A) in their analysis, this model does not take into account the leakage current flows between the plates. This current becomes larger as the oxide layer degrades during operation, which makes the model more inaccurate when capacitor ages. Proposed model (B) has resolved the aforementioned issue by adding shunt resistors. This model is a better representation of electrolytic capacitor behaviour, and the electrical parameters (C, L, R_a, R_b and R_c) are frequency independent [18]. However, its ESR estimation is not accurate below resonant frequency. Since capacitors are always employed to work far under this frequency, model (B) will not be a good candidate for ESR condition monitoring approach. Models (C) and (D), which have been proposed by some manufacturers [13, 19], employ a non-linear component (diode) in their structure. Having a non-linear model makes the design of monitoring system more challenging because more complex non-linear monitoring technique is required to estimate the electrical parameters of components. Model (E) carries all useful features of model (B); moreover, its frequency response is more coincident with the ESR behaviour for frequencies below resonant frequency, where the capacitive behaviour of the component is completely dominant while the inductive behaviour is negligible. This model and its variants have been extensively used by manufacturers [20, 21] and research groups [16, 14, 22–26]. In Section 3 of this paper, model (E) will be considered and improved using FO operators to follow ESR behaviour more precisely.

Developing an acceptable model which provides more accurate responses is the building block of the electrolytic capacitor's condition monitoring system. Next step is to implement an effective monitoring technique which generally depends on two essential aspects [27]:

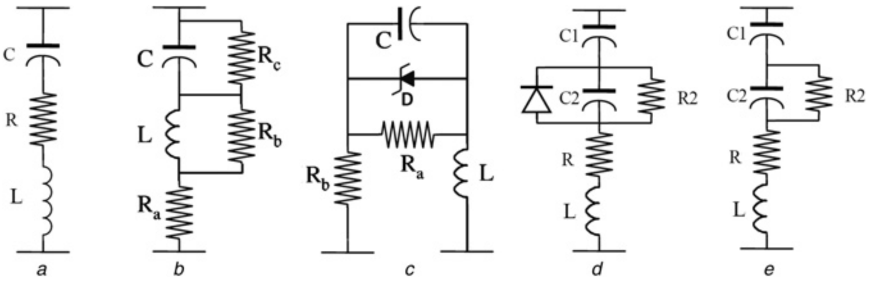

Fig. 1 *Various electrolytic capacitor models*

- Capturing and indicating the real-time electrical parameters of capacitor including ESR and/or capacitance.
- Predicting the EOL of the component for preventive maintenance.

Various innovative monitoring techniques have been proposed to capture and indicate the electrical parameters of electrolytic capacitors in real-time [7, 14, 23, 24, 28–30]. Most of these methods are based on ESR estimation which can be implemented through two main approaches [31, 32]:

- ESR = (V_{CRMS}/I_{CRMS}), where V_{CRMS} and I_{CRMS} are the RMS voltage and RMS current of capacitor, respectively.
- ESR = $\left(P_C/I_{CRMS}^2\right)$, where P_C and I_{CRMS} are the average dissipated power and RMS current of capacitor, respectively.

These two general approaches have been compared and discussed in [32]. Since the real-time ESR estimation technique is out of the scope of this research, more details regarding this topic can be found in the introduced references.

As mentioned above, by capturing the real-time ESR (first aspect of condition monitoring system), the remaining capacitor life can be worked out with an appropriate life model (second aspect of monitoring system). It is commonly agreed that the EOL of the electrolytic capacitor is dictated by the law of Arrhenius [29, 33]

$$\frac{t}{t'} = \exp\left(E\frac{T - T'}{(T + 273)(T' + 273)}\right), \tag{1}$$

where E is the activation energy/Boltzmann's constant and equals 4700. Here, t is the life-time at temperature T and t' is the life-time at temperature T'. The linear inverse model (2) is considered to be an extension of Arrhenius law which has been widely used as a life-time model as well [29, 33, 34]

$$\frac{1}{\text{ESR}_t} = \frac{1}{\text{ESR}_0}\left(1 - t.k.\exp\left(\frac{-E}{T + 273}\right)\right), \tag{2}$$

Different condition monitoring techniques utilise (1) and/or (2) as their life-time model in preventive maintenance algorithm. Though these two models are widely adopted in failure prediction algorithms, but these models can provide false alarms. To address this issue, in the second part of this paper, an improved life-time model will be proposed.

Before going into the details of introducing novel FO models for electrolytic capacitor and its life-time, FO operators are briefly introduced in the next section. Following, to tackle the inaccuracy problem in the parameter estimation of the commonly used integer order (IO) equivalent model, a new FO model for electrolytic capacitors is proposed. Furthermore, a new life-time prediction model has been introduced to more precisely estimate the expected failure time and set the predictive maintenance of the system. It ought to be mentioned that the new models greatly reduce the sensitivity of the model-based maintenance assessment.

2 FO derivative and definitions

Exhibiting fractional behaviour alludes to the fact that many physical phenomena in nature can be modelled by FO differential equations [35, 36]. The idea of fractional calculus has been known since the development of the regular calculus, with the first reference probably being associated with letter between Leibniz and L'Hospital in 1695 [37].

Grunwald–Letnikov (GL) and Riemann–Liouville (RL) are two of the most popular definitions which are widely used in various literature [38]. The GL which is the discrete definition is defined as

$$_aD_t^\alpha f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{[(t-a/h)]} (-1)^j \binom{\alpha}{j} f(t - jh), \tag{3}$$

where $[\cdot]$ means the integer part of the number. The RL which is the continuous definition is defined as

$$_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}\,\mathrm{d}\tau, \tag{4}$$

for $(n - 1 < \alpha < n)$ and $\Gamma(\cdot)$ is the *gamma* function. When $a = 0$ sometime authors use D^α notation which is equal to $_0D_t^\alpha$.

In some special cases, (3) and (4) are not equivalent. More details regarding these two definitions and their differences have been discussed in [38]. Since all the analysis in this paper is in continuous domain, RL definition of FO differintegral is considered in this paper. Some other important properties and applications of the fractional derivatives and integrals can be found in [37].

Since the introduction of fractional calculus to engineering world, the modelling and control of physical phenomena using FO operators have been widely investigated among researchers and scientists. All previous researches on the applications of FO operators in the engineering field imply the superiority of the FO operators compared to the classical IO ones from robustness and performance point of view [37, 38].

3 Capacitor modelling

3.1 *Classical model for electrolytic capacitor*

As discussed before, there are several mathematical models for capacitors and amongst all, the model depicted in Fig. 1 (E) has been widely adopted in academia and industry. Considering the negligible amount of series inductance in the operational frequency range ($L \simeq 0$) and separating electrolyte resistance and foil, tube and terminal resistance as shown in Fig. 2, the

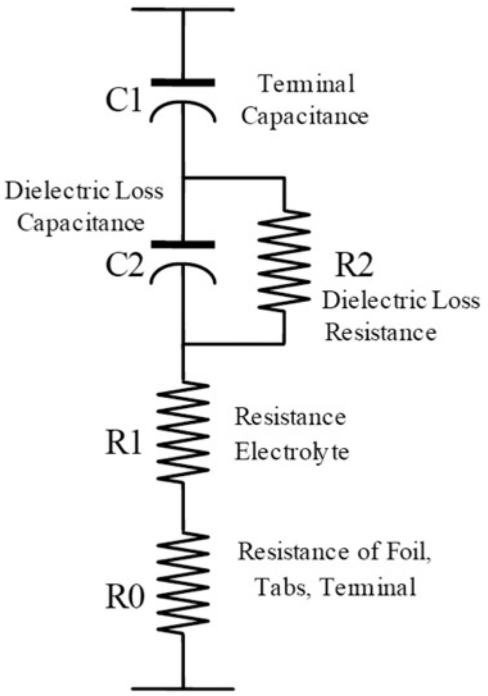

Fig. 2 *Classical equivalent model for electrolytic capacitor [14]*

impedance of the capacitor is [14, 26]

$$Z_{\mathrm{CAP}} = \frac{1}{(1/R_2) + \mathrm{j}\omega C_2} + R_1 + R_0 + \frac{1}{\mathrm{j}\omega C_1}. \qquad (5)$$

From (5), the ESR of capacitor which is the real part of the impedance, Z_{CAP}, is

$$\mathrm{ESR} = \Re\{Z_{\mathrm{CAP}}\} = \frac{R_2}{1 + (\omega C_2 R_2)^2} + R_1(T) + R_0, \qquad (6)$$

where in this equation R_2 is the dielectric loss resistance, C_2 is the dielectric loss capacitance, R_0 is the summation of foil, tabs and terminal resistances and

$$R_1(T) = R_{1T}.\exp\left(\frac{T_{\mathrm{Base}} - T_{\mathrm{Core}}}{E_c}\right) \qquad (7)$$

is the electrolyte resistance which reduces with increment. To estimate ESR parameters, one can sweep the frequency and measure the resistance of the capacitor from its terminals. At higher frequencies, ESR is almost constant and can be approximated by

$$\mathrm{ESR} = R_1(T) + R_0, \qquad (8)$$

while in lower frequencies, the approximated ESR is equal to (6). As can be seen in (6), terminal capacitance, C_1, has no direct effect on ESR calculation. Therefore, using ESR to predict the failure time, this capacitor has no effect on the time span prediction algorithm. Though this model gives an acceptable estimation for some applications, it is not able to describe the behaviour of the capacitor in all frequencies or various operating temperatures precisely. To tackle this problem, a very sophisticated RLC-based model (as shown in Fig. 3) has been proposed to overcome the aforementioned issues [39].

Among FO scientists, this type of ladder structure is known to behave as an FO element [40]. As a matter of fact, the properties of an FO element can be approximately realised with classical electrical network containing a finite number of RC branches. In the following section, by combining model (E) and ladder structure of Fig. 3, an FO model for electrolytic capacitors with reduced number of components will be proposed.

3.2 *Fractional model for electrolytic capacitor*

The ability of FO models to yield a more accurate description and give a deeper insight into the physical processes underlying a long range memory behaviour brought them into attention in the field of capacitor modelling to obtain better results in the identification process.

Very recently, few sophisticated models for electro-chemical capacitors have been introduced using FO differential equations [41]. Though authors agree with the concept of capacitor's fractional behaviour, but this performance is not caused by the terminal capacitor since this behaviour is not dominant in the air core capacitors. Furthermore, according to previous studies the fractional behaviour of capacitors is also caused by the electro-chemical materials which have been used as electrolyte. These materials have been applied to make an FO capacitor called *fractor* [42, 43]. Bearing all these reasons in mind, the authors believe C_2 causes the non-integer behaviour of capacitor and thus an equivalent model for the capacitor is proposed and illustrated in Fig. 4. In this model, C_2 is an FO capacitor. Current–voltage relationship and impedance of this component are $i = C_2(\mathrm{d}^\alpha V/\mathrm{d}t^\alpha)$ and $Z_{C_2} = \left(1/C(\mathrm{j}\omega)^\alpha\right)$, respectively [43].

Considering FO behaviour of C_2 in (5), mathematical model for an electrolytic capacitor can be derived as

$$Z_{\mathrm{CAP}} = \frac{1}{(1/R_2) + (\mathrm{j}\omega)^\alpha C_2} + R_1(T) + R_0 + \frac{1}{\mathrm{j}\omega C_1}. \quad 0 < \alpha \le 1, \qquad (9)$$

Knowing $\mathrm{j}^\alpha = \cos(\alpha\pi/2) + \mathrm{j}\sin(\alpha\pi/2)$, ESR for the proposed model, which is the real part of (9), can be obtained as

$$\begin{aligned}\mathrm{ESR} &= \Re\{Z_{\mathrm{CAP}}\}\\ &= \frac{R_2\left(1 + \Omega\cos\left((\alpha\pi/2)\right)\right)}{1 + 2\Omega\cos\left((\alpha\pi/2)\right) + \Omega^2} + R_1(T) + R_0, \end{aligned} \qquad (10)$$

where $\Omega = \omega^\alpha C_2 R_2$. In the case of $\alpha = 1$, ESR will be equal to (6).

4 Simulation results

To check the validity of the proposed model, a KEMET automotive rated capacitor, *A700V156M016ATE045*, is considered. The practical measurements for the impedance and ESR of capacitor are extracted from KEMET website [44] and depicted in Fig. 5.

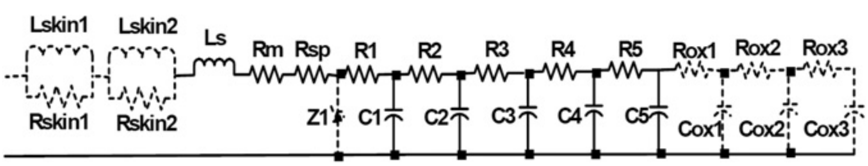

Fig. 3 *Cornell Dubilier equivalent model for electrolytic capacitor [39]*

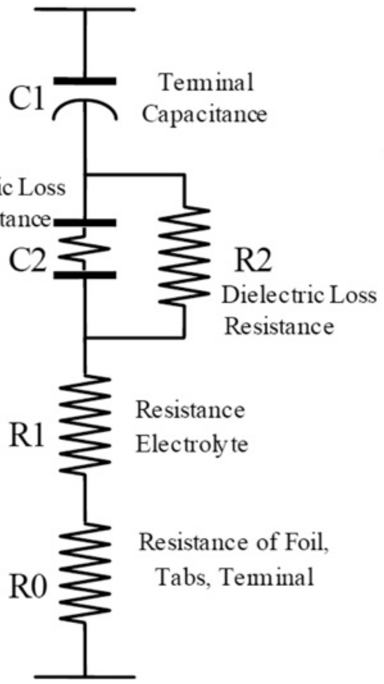

Fig. 4 *Proposed FO equivalent model for electrolytic capacitor*

Using (6) and (10), dielectric loss resistance and capacitance (R_2 and C_2) are estimated for both FO and IO models and then ESR is calculated and compared with actual values obtained from KEMET website. To estimate these parameters, least-square error method is used for both IO and FO estimations.

Estimated values for each model under different temperatures are presented in Table 1. R_0 and R_1 are extracted from ESR model response at higher frequencies, provided in the capacitor datasheet. As can be seen in Fig. 6 and Table 1, the FO model gives a more precise estimation of the capacitor behaviour over a wide range of frequencies compared to the IO one which will help to obtain more accurate responses in the circuit analysis.

As mentioned before, ESR is dependent on not only the frequency and temperature, but also the operation time of the capacitor. The more the operation time is, the more the ESR. In the next section, the relationship between the operation time of the capacitor and its ESR will be discussed and a new model will be proposed to increase the precision of the estimation.

Table 1 ESR estimated values for FO and IO models

Temperature, °C	C_{2IO}, mF	R_{2IO}, mΩ	C_{2FO}, mF	R_{2FO}, mΩ	α	% of mean estimation error (IO, FO)
25	0.50	1.59	1.60	5.0	0.835	18, 11
85	0.63	1.59	2.00	5.0	0.825	20, 9

4.1 Life-time estimation

As discussed before, in many applications, the life-time of electronic systems is directly linked to the life-time of their capacitors and life span of these capacitors is the vital factor that ensures the reliability of the system. The wear-out of aluminium electrolytic capacitors is due to vaporisation of electrolyte that leads to a drift of the main electrical parameters of the capacitor. It has been shown in [29] that the relationship between electrolyte volume, V_{ol}, and ESR can be expressed as

$$\frac{\text{ESR}}{\text{ESR}_0} = \left(\frac{V_{ol,0}}{V_{ol}}\right)^2, \tag{11}$$

where ESR_0 is the initial ESR of the capacitor and $V_{ol,0}$ is the initial volume of electrolyte. When the volume of electrolyte is reduced by 40%, which implies that ESR is increased by 2.8 times, the electrolytic capacitor is presumed to reach its EOL [29].

In [23], a model is proposed in which it is assumed that the rate of electrolyte loss is directly proportional to the vapour pressure of the electrolyte

$$\frac{\mathrm{d}V}{\mathrm{d}t} = kP. \tag{12}$$

A constant k, leak rate constant, is used to characterise the quality of the end seal and P is vapour pressure of electrolyte [23].

Diffusion in electrolyte capacitors would result in loss of electrolyte which leads to increase in capacitance and decrease in ESR [29]. In addition, this latter evolution is important since it determines the self-heating and, indirectly, the capacitor life-time. In other words, thermal degradation affects both the capacitance and ESR values of electrolytic capacitors. Hence, the electrolytic capacitor failure prediction can be done by measuring the change of ESR. The ESR is not only dependent on the service life of the capacitor, but also at any ageing time

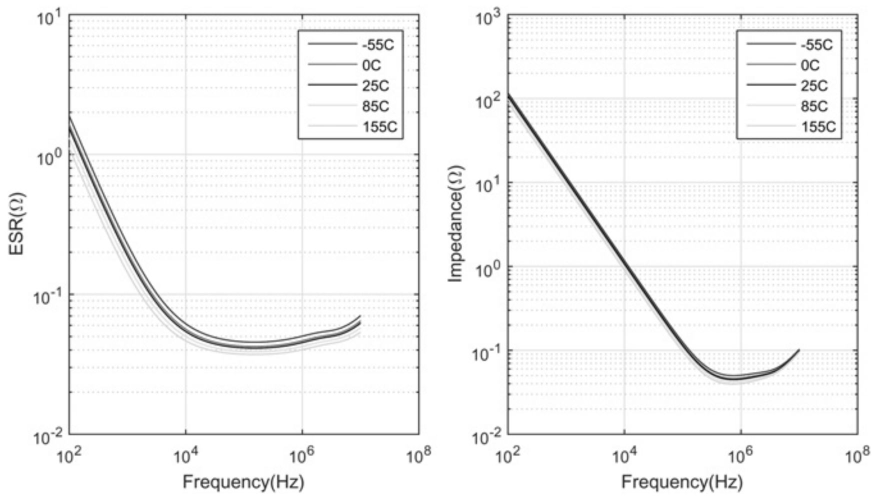

Fig. 5 *Capacitance and ESR values for an automotive rated capacitor from KEMET in different temperatures*

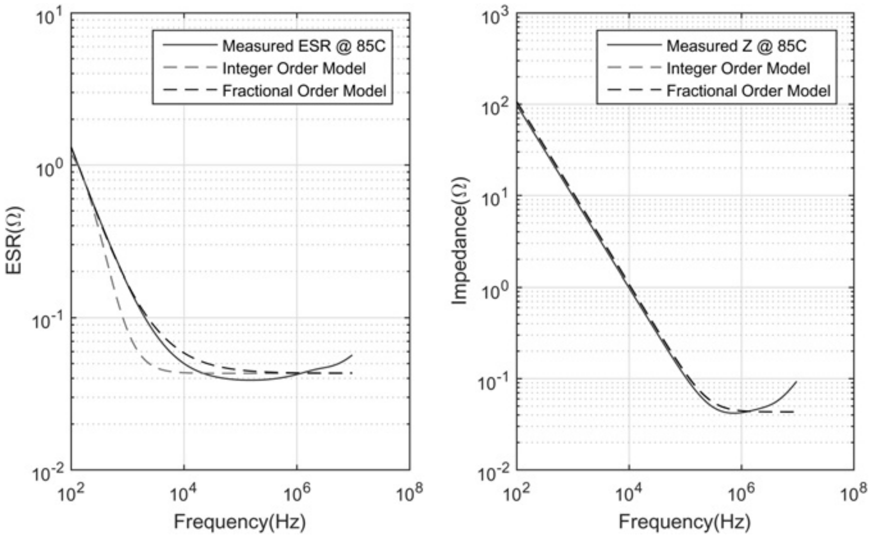

Fig. 6 *Estimated ESR and impedance at 85°C for FO and IO models of under test KEMET capacitor*

t, it is inversely affected by temperature as

$$\frac{1}{\mathrm{ESR}_t} = \frac{1}{\mathrm{ESR}_0}\left(1 - t.k.\exp\left(\frac{-E_c}{T+273}\right)\right), \tag{13}$$

where ESR_t is the ESR value at time t, T is the ageing temperature in Celsius, t is the ageing time, ESR_0 is a data representing the ESR value at time $t = 0$, k is a constant which depends on the design and the construction of the capacitor and E_c is the activation energy/Boltzmann's constant and equals 4700.

In [45], the actual ESR degradation is measured for electrolyte capacitors in a DC–DC converter circuit. Under nominal conditions, this converter has a variable input from 22 to 36 V DC and gives an output of 5 V with 1% ripple and noise. Here, 2200 μF electrolytic capacitors with a maximum rated voltage of 10 V and maximum current rating of 1 A are used in this paper for which the maximum operating temperature is 85°C. The converter hardware circuit is shown in Fig. 7.

Utilising the least-squares method, the model parameters are identified for (13) as $k = 1086$ at $T = 25^{\circ}$C. The model response is compared with real ESR values in Fig. 8.

As depicted in this figure, though the classical estimation model is able to predict the ESR degradation properly for a very short period in the beginning of the operation time, but as time goes, this model overestimates or underestimates the ESR values which will accelerate or delay the maintenance schedule improperly. Former case escalates the maintenance cost while the latter imposes a higher level of risk for the maintenance schedule.

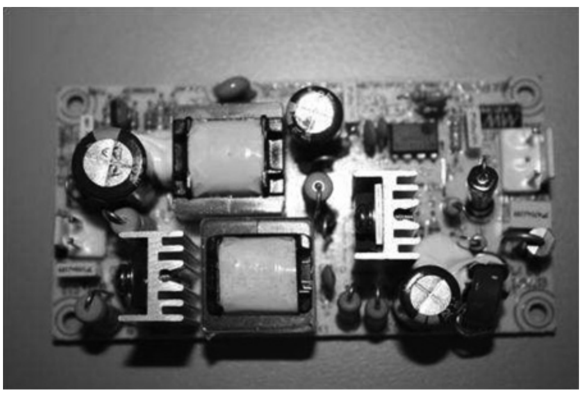

Fig. 7 *DC–DC converter hardware [45]*

To resolve this issue, using Mittag-Leffler function, a more precise model is proposed in the next section.

4.2 FO life-time estimation

The Mittag-Leffler function is originated to solve the fractional integral equations [46] and defined by a series

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{G(\beta + \alpha k)}. \tag{14}$$

The ordinary and generalised Mittag-Leffler functions interpolate between a purely exponential law and power-like behaviour of phenomena governed by ordinary kinetic equations and their fractional counterparts [47]. In the special case of $\alpha = 1$, $E_\alpha(z) = \exp(z)$.

To ameliorate the response of the IO model (13), Mittag-Leffler function is used in ESR estimation modelling. Employing this function gives more flexibility to the model to estimate the combination of exponential and power-like behaviours. Hence, the new life-time model can be written as

$$\frac{1}{\mathrm{ESR}_t} = \frac{1}{\mathrm{ESR}_0}\left(1 - t.k.E_{\alpha,\beta}(t_{\mathrm{norm}}).\exp\left(\frac{-E_c}{T+273}\right)\right), \tag{15}$$

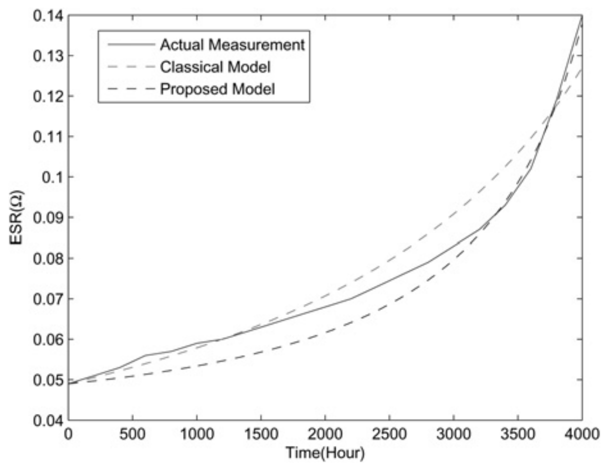

Fig. 8 *Estimated life-time using FO and IO models*

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where ESR_t is the ESR value at time t and t is the ageing time. $E_{\alpha,\beta}$ is the Mittag-Leffler function at normalised time, t_{norm}.

Similar to the previous section, using the least-squares scheme, the model parameters are estimated for (15) as $\alpha = 0.9657$, $\beta = 1.2063$, $k = 431$ at $T = 25^{\circ}$C. The new model response is compared against the practical ESR values and classical model response in Fig. 8.

As shown in this figure, the recorded ESR data from DC to DC converter was seen to be more comparable with (15) response and failure prediction according to this model gives a more precise response for the time when the capacitor reaches its EOL.

5 Conclusions

Since, state of the health can be directly affected by ESR value for electrolytic capacitor, monitoring this key factor can not only prevent detrimental maintenance programmes due to delayed schedule, but also enhance the cost efficiency of the system avoiding accelerated part replacement. This paper focuses on analysing a novel generalised equivalent ESR model for electrolytic capacitor by employing FO operators. Unlike other existing FO models, proposed model considers an FO dynamic only in the dielectric losses and the terminal capacitor remains IO as observed in the actual capacitor's behaviour. Hence, monitoring this value can provide key information to avoid excess cost or risk due to improper accelerated or delayed maintenance schedule.

Moreover, an enhanced failure predictive model is presented using Mittag-Leffler function to track the ESR increment caused by ageing of the capacitor. By means of the new model, the predictive maintenance of the system with capacitors nearing their failure time can be set more precisely due to the life-time prediction accuracy improvement. These two FO models are compared against classical ESR and life-time prediction linear inverse models to illustrate the effectiveness and superiority of our results over the existing ones.

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