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IMAGE SEGMENTATION BASED ON FRACTIONAL DIFFERENTIATION AND RSF MODEL

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ABSTRACT

Intensity inhomogeneity or weak texture region image segmentation plays an important role in computer vision and image processing. RSF(Region-Scalable Fitting) active contour model has been proved to be an effective method to segment intensity inhomogeneity. However RSF model is sensitive to the initial location of evolution curve , it tends to fall into local optimal. Aiming at the problem, this paper proposed a new method for image segmentation based on fractional differentiation and RSF model. The proposed method adds the global Grünwald-Letnikov fractional gradient into the RSF model. Thus the gradient of the intensity inhomogeneity and weak texture regions is strengthened. As a result, both the robustness of initial location of evolution curve and efficiency of image segmentation are improved. Theoretical analysis and experimental results demonstrate that the proposed algorithm is capable of segmenting the intensity inhomogeneities and weak texture images. It is robust to curve initial location, furthermore the efficiency of segmentation is improved.

1 INTRODUCTION

Image segmentation is one of the most important problems in computer vision and iamge processing. In recent years, active contour models (ACM)[1]have been widely developed and have several notable advantages over classical image segmentation methods, so they are extensively used in computer vision and medicine image analysis. Generally, the existing ACM methods can be classified into two types: edge-based models[1−3] and region-based models[4−8]. There are many advantages of region-based models when compared with edge-based models, such as robustness against initial contour and insensitivity to image noise, thus region-based models are more widely used. However, common region-based active contour models tend to rely on intensity homogeneity in each of the regions to be segmented. For example, the popular C-V model[5], which has been successfully used in binary phase segmentation with the assumption that each image region is statistically homogeneous. But, the C-V model does not work well for the images with intensity inhomogeneity. Vese and Chan extended their work and proposed the piecewise constant (PC) models[6], this model utilize multi-phase level set functions to represent multiple regions. However, both the C-V and the PC models have the drawback described above. Li et al.[7] proposed the RSF(Region-Scalable Fitting) model, which utilizes the local image information as constraints, can well segment objects with intensity inhomogeneity. But when segmenting images with weak texture and edges, RSF model is sensitive to the initial location of evolution curve during the optimization process, it tends to fall into local optimal with slow evolution rate, because the models only use local information. To solve this problem, Zhang et al.[8] improved the method to segment images with intensity inhomogeneity, but this methods is time consuming. Song et al.[9] combined Laplace zero crossing

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operator and put forward a regularized gradient flows method, which improved edges location. Zhang et al.[10]proposed Local Image Fitting (LIF) model, this method decreases computational complexity over RSF model. Wang et al.[11] designed a new nonlinear weighted item based on Bayes rule, this weighted item is capable of adaptively evolution, thus can well deal with boundary leak. However, the above methods do not solve the problem of sensitivity to initial evolution curve. Fractional differentiation have the advantage of strengthening the components of high and medium frequency, while reserving the part of very low frequency in nonlinear manner. This excellent features are widely used in keeping texture details and weak edges in image processing[12]. For example, Ren et al.[13] introduced fractional calculus to CV models and add fractional fitting term, Tian et al.[14] employed fractional divergence operator to CV model, both methods improve capacity of weak edges location and the performance of segmenting weak edges, in addition both mentioned above methods are more robust to noise. However, both models do not work well in images with intensity inhomogeneity because they only use image local information. Though RSF model has been proved to be an effective method to segment intensity inhomogeneity. It has limitation in segmenting image with weak texture and weak edge, troubled by inclining to local minimum and slow evolution speed. Aims at the problem, this paper introduced fractional differentiation to RSF model, using global fractional gradient fitting term as a new driving force to attract evolution curve to stop at object boundary. So, image gradient is calculated using fractional differentiation instead of traditional gradient function. Therefore, detail feature is strengthened while image gradient of regions with intensity inhomogeneity and weak texture is enhanced. As a result, both the robustness to initial evolution curve and efficiency of image segmentation are improved. Theoretical analysis and experimental results indicate that the proposed algorithm is capable of segmenting the intensity inhomogeneity and weak texture images. It is capable of dealing with the problem that RSF model is sensitive to initial location of evolution curve, and improving the efficiency of segmentation.

2 BACKGROUND

2.1 RSF Model

Chan and Vese proposed an ACM based on the Mumford-Shah model[4], which utilizes image's global intensity to calculate information, and assumes that the intensity of both background and foreground is homogeneous. CV model has been successfully used in binary phase segmentation with intensity uniform without image smooth processing in advance. However, it has limitation in segmenting images with intensity inhomogeneous. To solve this problem, Li et al.[7] proposed the Region-Scalable Fitting (RSF) model by embedding the local image information instead of global image information and introduced a kernel function to define an RSF energy function, moreover a new symbol distance function is defined to stable evolution of the level set function. RSF is able to segment images with intensity inhomogeneities and is much more efficient and accurate than the CV model. Level set energy equation of RSF is as follows:

$$\begin{aligned} &E_\varepsilon(f_1(x), f_2(x), \phi) \\ &= \sum_{i=1}^{2} \lambda_i \int \left(\int_{\Omega i} K_\sigma(x-y) |I(y) - f_i(x)|^2 M_i^\varepsilon(\phi(y)) dy \right) dx \\ &+ v \int |\nabla H_\varepsilon(\phi(x))| dx + \mu \int \frac{1}{2} (|\nabla \phi(x) - 1|)^2 dx, \end{aligned} \tag{1}$$

where $K_\sigma(x-y)$ is a Guassian kernal with standard deviation σ, $M_1^\varepsilon(\phi) = H_\varepsilon(\phi)$, $M_2^\varepsilon(\phi) = 1 - H_\varepsilon(\phi)$, H_ε is regularized Heaviside function, $f_i(x) (i = 1,2)$ are two smooth functions that approximate the local image intensities inside and outside the contour C, respectively. $f_i(x) (i = 1,2)$ can be expressed as following:

$$f_i(x) = \frac{K_\sigma(x) * [M_i^\varepsilon(\phi(x)) I(x)]}{K_\sigma(x) * [M_i^\varepsilon(\phi(x))]}, i = 1,2. \tag{2}$$

The first item in formula (1) is local intensity fitting value, driving evolution curve to stop at object contour; the second one is length constraint, controlling the length of evolution curve; the last one is symbol distance function, preventing the evolution curve from initializing. Minimizing the energy function can be implemented by solving the corresponding Euler-Lagrange equations. Then,we have the gradient descent flow as follows:

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= -\delta_\varepsilon(\phi)(\lambda_1 e_1 - \lambda_2 e_2) + v \delta_\varepsilon(\phi) div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) \\ &\quad + \mu\left(\Delta \phi - div \frac{\nabla \phi}{|\nabla \phi|}\right), \end{aligned} \tag{3}$$

where $\delta_\varepsilon(\phi)$ is regularized Dirac function,

$$e_i(x) = \int K_\sigma(y-x) |I(x) - f_i(y)|^2 dy, i = 1,2. \tag{4}$$

RSF model improves greatly the performance of segmenting images with intensity inhomogeneous. But it tends to fall into local optimal because it only uses image local information, furthermore, it is sensitive to initial location of evolution curve. To get desired result, initial location of curve should be properly chosen according to segmentation result, thus its application is restricted.

2.2 Fractional Differential Definition

Fractional calculus[15,16] is a branch of integer order calculus, which extends the traditional integer order calculus. Generally, the most popular definitions among them involve: Riemann Liouville(R-L)definition, Grünwald-Letnikov(G-L)definition and Caputo definition[15,16], where R-L definition and G-L definition can perform convolution operation, so they are wildly used in the field of signal processing. G-L definition is more accurate than R-L definition[15,16]thus we deduce this paper's fractional differential operator based on G-L definition. $\forall \alpha \in \mathbf{R}$(includes fraction), let the integer part of α as $[\alpha]$. If signal $s(t), t \in [a,b] (a < b, a \in \mathbf{R})$ exists $n+1 (n \in \mathbf{Z})$ order continuous derivative, when $\alpha > 0$, n at least equals to $[\alpha]$, then the G-L fractional order(α) derivative is defined as[15,16] :

$$ {}_a^G D_b^\alpha s(t) = \lim_{\substack{h \to 0 \\ mh = b-a}} h^{-\alpha} \sum_{r=0}^{m} \begin{bmatrix} -\alpha \\ r \end{bmatrix} s(t - rh), \tag{5} $$

where, $\begin{bmatrix} -\alpha \\ r \end{bmatrix} = \frac{(-\alpha)(-\alpha+1)...(-\alpha+r+1)}{r!}; h = \frac{b-a}{m}$, when $h \to 0, m \to \infty, m = \left[\frac{b-a}{h}\right]$.

2.3 Fraction Differential Characteristics

For an arbitrary square integrable energy signal $I(x) \in \mathbf{R^2}$, its Fourier transform is as following :

$$ \hat{f}(x) = \int_{\mathbf{R}} f(x) \exp(-\mathrm{i}\omega \mathrm{x}) \mathrm{dx}. \tag{6} $$

Suppose n order derivative of signal $I(x)$ is $I^n(x) (n \in \mathbf{N})$, the following equal can be obtained based on Fourier transform characteristics:

$$ D^n I(x) \overset{FT}{\Leftrightarrow} (\hat{D}I)^n(\omega) = (i\omega)^n \hat{I}(\omega) = \hat{d}^n(\omega) \hat{I}(\omega). \tag{7} $$

We extend integer order to fractional order, then α order of $I(x)$ is $I^\alpha(x) (\alpha \in \mathbf{R}_+)$.Similarly, fractional order Fourier transform is:

$$ D^\alpha I(x) \overset{FT}{\Leftrightarrow} (\hat{D}I)^\alpha(\omega) = (i\omega)^\alpha \hat{I}(\omega) = \hat{d}^\alpha(\omega) \hat{I}(\omega), \tag{8} $$

where,

$$ \begin{cases} \hat{d}^\alpha(\omega) = (i\omega)^\alpha = \hat{a}^\alpha(\omega) \exp(\mathrm{i}\theta^\alpha(\omega)), \\ \hat{a}^\alpha(\omega) = |\omega|^\alpha, \theta^\alpha(\omega) = \frac{\alpha\pi}{2} sgn(\omega). \end{cases} \tag{9} $$

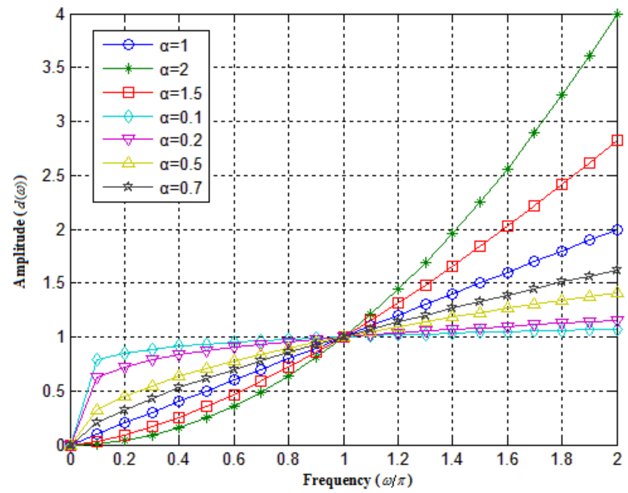


FIGURE 1. THE AMPLITUDE-FREQUENCY CURVE OF SIGNAL.

According to formula (8) and formula (9) , the amplitude-frequency curve of signal can be drawn and shown in Fig.1. It is noted that for low frequency signal such as $0 < \omega < 1$, fractional differential attenuates the signal less than the integer one, and for high frequency one such as $\omega > 1$, fractional differential enhances signal less than the integer one, thus we get the conclusion that fractional differential can enhance the high frequency signals, meanwhile reinforce the medium frequency one, and non-linear retain the low frequency one. In digital image, weak edges and texture details correspond to low frequency parts, and noise and boundaries correspond to high frequency ones. If sign is processed by integer derivative, weak edge and texture tend to be greatly weaken, meanwhile noise will be strengthen tremendously. Fortunately fractional differential is capable of solving this drawback, that is, noise will not be strengthened tremendously and weak edge and texture will be retained nonlinearity. These advantages can be introduced to preserve weak edges and texture, and resist noise to some extent. In a short, we chose different order according to our requirements. In this paper, we need to preserve texture details and weak edges, it is obvious from Fig.1 that when $0 < \alpha < 1$, low frequency signal can be reserved better than integer one, and high frequency one is suppressed better than integer one, so we suggest the reasonable order between (0,1) in this paper.

3 CONSTRUCT FRACTIONAL DIFFERENTIAL MASK BASED ON G-L DEFINITION

According to formula (5), if the interval of $s(t)$ is $t \in [a,b]$, we divide $[a,b]$ by equal interval of $h = 1$, then $m = (b-a)/h = b-a$. We can deduce the fractional order differential formula of $s(t)$, which is given by:

$$\frac{d^\alpha s(t)}{dt^(\alpha)} \approx s(t) + (-\alpha)s(t-1) + \frac{(-\alpha)(-\alpha+1)}{2}s(t-2) + \ldots + \frac{\Gamma(-\alpha+m-1)}{(m-1)!\Gamma(-\alpha)}s(t-m+1). \quad (10)$$

As the computer processes digital signal, the biggest variation of image intensity is limited, and the image intensity just varies in two adjacent pixels. So the minimum equal interval of $f(x,y)$ must be $h=1$. The backward fractional partial differential formula of $f(x,y)$ can be defined as follows:

$$\frac{\partial^\alpha f(x,y)}{\partial x^\alpha} \approx f(x,y) + (-\alpha)f(x-1,y) + \frac{(-\alpha)(-\alpha+1)}{2}f(x-2,y) + \ldots + \frac{\Gamma(-\alpha+m-1)}{(m-1)!\Gamma(-\alpha)}f(x-m+1,y) + \ldots, \quad (11)$$

$$\frac{\partial^\alpha f(x,y)}{\partial y^\alpha} \approx f(x,y) + (-\alpha)f(x,y-1) + \frac{(-\alpha)(-\alpha+1)}{2}f(x,y-2) + \ldots + \frac{\Gamma(-\alpha+m-1)}{(m-1)!\Gamma(-\alpha)}f(x,y-m+1) + \ldots, \quad (12)$$

From formula (11) and (12), it can be seen that coefficient of each item is different, the sum of each item coefficient does not equal to zero, which is much different to integer one. The general formula of coefficient is:

$$H = \frac{\Gamma(-\alpha+m-1)}{(m-1)!\Gamma(-\alpha)}, \quad (13)$$

where $\Gamma(\bullet)$ is the Gamma function. According to the gradient of integer order, fractional order gradient can be rewritten as follows:

$$\nabla^\alpha s = [G_x^\alpha \quad G_y^\alpha]^T = [\frac{\partial^\alpha s}{\partial x^\alpha} \quad \frac{\partial^\alpha s}{\partial y^\alpha}]^T. \quad (14)$$

The fractional order gradient magnitude is:

$$mag(\nabla^\alpha s) = [(G_x^\alpha)^2 + (G_y^\alpha)^2]^{\frac{1}{2}}. \quad (15)$$

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
0	0	0	$\cdots$	0
1	$-\alpha$	$\frac{(-\alpha)(-\alpha+1)}{2}$	$\cdots$	$\frac{\Gamma(-\alpha+1)}{(m-1)!\Gamma(-\alpha+m)}$
0	0	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

(a) POSITIVE DIRECTION OF X AXIS

$\cdots$	0	1	0	$\cdots$
$\cdots$	0	$-\alpha$	0	$\cdots$
$\cdots$	0	$\frac{(-\alpha)(-\alpha+1)}{2}$	0	$\cdots$
$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$
$\cdots$	0	$\frac{\Gamma(-\alpha+1)}{(m-1)!\Gamma(-\alpha+m)}$	0	$\cdots$

(b) POSITIVE DIRECTION OF Y AXIS

FIGURE 2. THE FRACTIONAL DIFFERENTIAL MASK

For simplicity, from formula (11) and (12) we select the leading several terms as the fractional partial differential approximate value of $f(x,y)$ in the positive direction of X and Y, and construct fractional order mask with dimension of $M \times M$, as shown in Fig.2.

4 FRACTIONAL ORDER RSF MODEL

Intensity inhomogeneity, weak texture and weak edge often occur in images due to various factors. For example, these cases are usually due to technical limitations or artifacts introduced by the object being imaged in medical images. For this situations, the RSF model is not suitable for image segmentation, it is sensitive to the initial location of evolution curve during the optimization process, it tends to fall into local optimal with slow evolution. Aiming at this problem, we proposed a new segmentation model combining G-L fractional order derivative and RSF model.

4.1 A New RSF Model

We introduce G-L fractional order gradient to energy function, considering fractional order gradient as another drive force. Evolution curve stops at image boundary driven by local intensity fitting term and global fractional order gradient. We combined G-L fractional order gradient and RSF model, and constructed a novel energy function as following:

$$E^{FG}(m_1,m_2,C) = \beta_1 \int_\Omega \| \nabla^\alpha I(x,y)| - m_1|^2 dydx + \beta_2 \int_\Omega \| \nabla^\alpha I(x,y)| - m_2|^2 dydx, \quad (16)$$

Where $\nabla^\alpha I(x,y)$ is fractional order gradient, C is evolution curve, m_1 and m_2 are the averages of fractional order gradient magnitude inside and outside the counter, respectively. β_1 and β_2 are constants, generally $\beta_1 = \beta_2 = 1$. We embed unknown curve C to high-dimensional level set, substitute evolution curve with level set function $\phi(x,y)$, and suppose that: if point (x,y) is inside the curve C , then $\phi(x,y) > 0$; if point (x,y) is outside the curve C, then $\phi(x,y) < 0$; if point (x,y) is on the boundary, then $\phi(x,y) = 0$. So energy equation can be rewritten as:

$$\begin{aligned} &E^{FG}(m_1,m_2,\phi) \\ &= \beta_1 \int_\Omega \| \nabla^\alpha I(x,y)| - m_1|^2 H_\varepsilon(\phi) dydx \\ &+ \beta_2 \int_{\overline{\Omega}} \| \nabla^\alpha I(x,y) - m_2|^2 (1 - H_\varepsilon(\phi)) dydx, \end{aligned} \tag{17}$$

where m_1 and m_2 can be computed according to the following formulas:

$$\begin{aligned} m_1 &= \frac{\int_\Omega |\nabla^\alpha I(x,y)| H_\varepsilon(\phi(x,y)) dydx}{\int_\Omega H_\varepsilon(\phi(x,y)) dydx} \\ m_2 &= \frac{\int_{\overline{\Omega}} |\nabla^\alpha I(x,y)| (1 - H_\varepsilon(\phi(x,y))) dydx}{\int_\Omega (1 - H_\varepsilon(\phi(x,y))) dydx}. \end{aligned} \tag{18}$$

Where $H_\varepsilon(\phi)$ and $\delta_\varepsilon(\phi)$ are the regularized versions of Heaviside function and Dirac function[10], respectively. Combining this energy equation(Formula (17)) to RSF model,the total energy equation is as following:

$$\begin{aligned} &E(f_1,f_2,m_1,m_2,\phi) \\ &= \sum_{i=1}^{2} \beta_i \int_\Omega |\nabla^\alpha I(x,y) - m_i|^2 dydx \\ &+ \nu \int_\Omega |\nabla H_\varepsilon(\phi)| dx + \mu \int \frac{1}{2} (|\nabla \phi(x)| - 1)^2 dx \\ &+ \sum_{i=1}^{2} \lambda_i \int (\int_\Omega K_\sigma(x,y) |I(y) - f_i(x)|^2 M_i^\varepsilon(\phi(y) dy) dx \end{aligned} \tag{19}$$

Minimize the energy function can be resort to solve the corresponding Euler-Lagrange equation. We can obtain level set evolution equation of curve by using the gradient descend method:

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \delta_\varepsilon(\phi)(-\beta_1(|\nabla^\alpha I| - m_1)^2 \\ &+ \beta_2(|\nabla^\alpha I| - m_2))^2 + \nu \delta_\varepsilon(\phi) div(\frac{\nabla \phi}{|\nabla \phi|}) \end{aligned} \tag{20}$$

Suppose $\frac{\partial \phi}{\partial t} = \Delta \phi$, then update equation of level set function is:

$$\phi^{n+1} = \phi^n + \Delta t \Delta \phi \tag{21}$$

4.2 Algorithm

Step 1: According to ref [7,10,12] and experiment experiences, set initial parameters: ν,μ,λ_1,λ_2,β_1,β_2 and initial level set function ϕ_0;

Step 2: According to formula(15), calculate fractional order gradient magnitude;

Step 3: According to formula(2), calculate f_1,f_2,and then compute the sum of average fractional order gradient inside and outside the curve according to formula (18);

Step 4: Keep f_1, f_2, m_1 and m_2 fixed, update level set function according to formula (20) and (21)

Step 5: Judge whether level set function is stable or not, if level set function get stable, then output final result, otherwise return to Step 3.

5 EXPERIMENT RESULTS AND ANALYSIS

Computer environment in this paper is: CPU with Intel Core i3-2130 and RAM of 4GB, 64bit Windows 7.0 operating system, programming tool is Matlab of version R2013a. The proposed method has been tested from the aspects of segmentation performance and robustness to initial curve location.

5.1 Segmentation Performance

This section focuses on testing segmentation performance of the proposed method, to ensure contrast experiment fair, we select four images, two are from ref[8] (numbered as picture I and picture II), and another two are galaxy and fingerprint pictures(numbered as picture III and picture IV), all of the four pictures are intensity inhomogeneity, and picture I and II include weak edges, picture III and picture IV involve weak texture. We take experiments using above four pictures and compare our method with LIF model, ref [13] model, RSF model. Segmentation result and segmentation time are introduced to evaluate segmentation performance of each model. Parameters of each model in our experiments are all from their references, parameters of proposed model are based on experiments experience, as follows: picture I:$\Delta t = 0.1, \lambda_1 = \lambda_2 = \beta_1 = \beta_2 = 1, \mu = 1, \sigma = 3, \nu = 0.001 \times 255 \times 255$, mask size of fractional order is 3×3, order $\alpha = 0.6$; picture II: $\nu = 0.004 \times 255 \times 255, \alpha = 0.7$, other parameters are the same as that of picture I; picture III: fractional order $\alpha = 0.2$, other parameters are the same as that of picture II; picture IV: $\alpha = 0.85$,other parameters are the same as that of picture I. Experiment results are shown in Fig.3, the first row to the fourth row are picture I,II,III and IV, respectively;

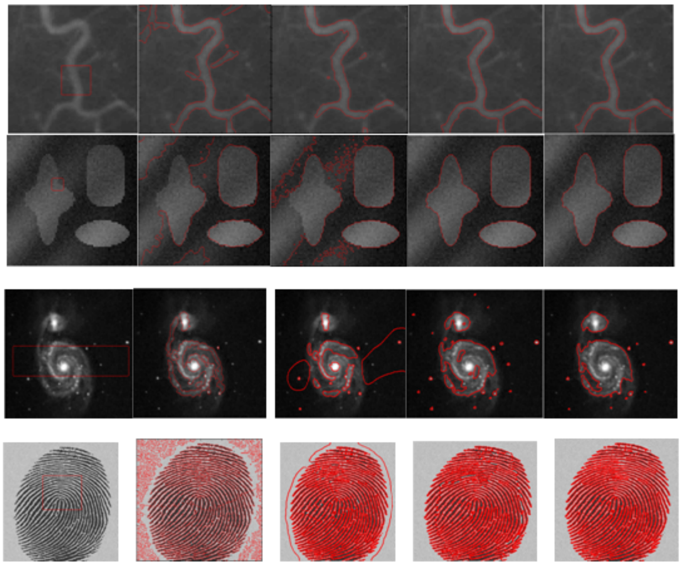


FIGURE 3. SEGMENTATION RESULTS. The first row to the fourth row are picture I, II, III and IV respectively; The first columninput images red squares denote initial location Second column: LIF modelThird column: Ref[13] model Fourth column: RSF modelFifth column: proposed method (optimal order is 0.2,0.6,0.7,0.85, respectively).

The first column is input images and the red squares denote initial contour, from the second column to the fifth are segmentation results using LIF model, ref[13]model, RSF model and proposed model. It can be seen from Fig.3 that both LIF model and ref [13] model can not segment object successfully. RSF model is capable of segmenting picture I and picture II, but limited in segmenting picture III and picture IV. However, our method segment all of the four pictures successfully. Additionally, we analyze time consuming of our method, and compare it with other methods as mentioned above. We repeat the experiments above 100 times and then compute their average time. The result is shown in Tab.1. From table 1, it is obvious that our method is faster than others, thus the segmentation efficiency is improved. This is because our method not only keep the advantage of original RSF model, but also combine new fractional order fitting term, thus driving force of curve evolution is strengthened, and then the curve evolution is accelerated.

5.2 Robustness To Initial Curve Location

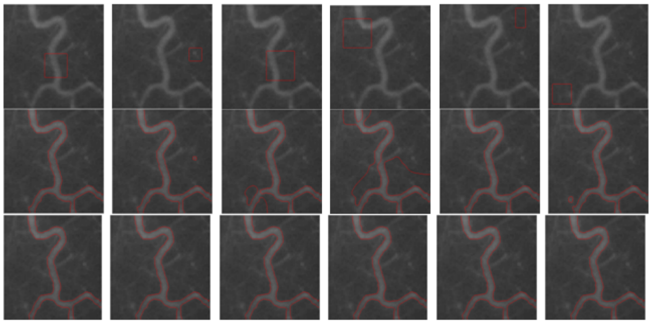
This section aims to test the robustness to initial curve location. We selected two images with weak edge and texture, and randomly select five different initial locations, as shown in Fig.4, where squares with red color denote different initial locations. We take experiments and compare our method with RSF model. All the parameters in this section are the same as those in section5.1, experiment results are shown in Fig.4. The first row of each group shows input images and six different initial locations (red square), the second row is segmentation results using RSF model, and the third one is segmentation results using our model. Experiment results indicate that RSF model is sensitive to initial location of curve. Different initial locations always result in different segmentation output, and even get mistaken segmentation when initial location is not appropriate. But our method can segment objects successfully even though selecting different initial contour. In conclusion, our model is robust to initial location of evolution curve. This is because our method adds the global G-L fractional gradient into the RSF model, which increase the driving force of evolution curve and avoid falling into local optimal in the regions with weak edge and texture. So our method can solve well the problem that RSF model is sensitive to evolution curve initial location.

TABLE 1. SEGMENTATION TIMES(UNITS: s).

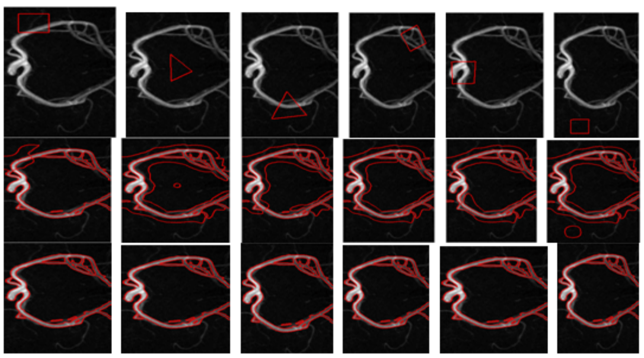
Number	LIF	Ref[13]	RSF	OURS
picture I	7.29	7.29	2.94	1.96
picture II	2.85	2.85	3.47	2.60
picture III	37.81	26.86	29.40	8.62
picture IV	9.80	5.20	8.61	3.90

6 CONCLUSION

An improved RSF model based on fractional calculus is proposed in this paper. This model combines G-L fractional global gradient with RSF model, which is capable of solving the problem that RSF is sensitive to evolution curve initial location. On one hand, global fractional order gradient fitting is added in RSF model, evolution drive force is strengthened, which can avoid curve evolution falling into local optimal in the regions with weak weak and texture, thus segmentation accuracy is improved. On the other hand, G-L fractional differentiation enhances the gradient of the intensity inhomogeneity and weak texture regions, driving force of curve evolution is amplified, so segmentation efficiency is approved too. Theoretical analysis and experimental results show that the proposed algorithm is capable of segmenting the intensity inhomogeneity and weak texture images. It can deal with the problem that RSF model is sensitive to initial location of evolution curve, and improve the efficiency of segmentation. Future works involve designing the optimization algorithm of fractional order and applying this algorithm in ultrasonic image processing system.



(a) First group



(b) Second group

FIGURE 4. Robust to initial location. The first row of each groupinput images and five different initial locations (red squares); second row: segmentation results using RSF model; third row: segmentation results using our model

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