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Research article

A solid criterion based on strict LMI without invoking equality constraint for stabilization of continuous singular systems

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ABSTRACT

The paper considers the stabilization issue of linear continuous singular systems by dealing with strict linear matrix inequalities (LMIs) without invoking equality constraint and proposes a complete and effective solved LMIs formulation. The criterion is necessary and sufficient condition and can be directly solved the feasible solutions with LMI toolbox and is much more tractable and reliable in numerical simulation than existing results, which involve positive semi-definite LMIs with equality constraints. The most important property of the criterion proposed in the paper is that it can overcome the drawbacks of the invalidity caused by the singularity of $\Omega = PE^T + SQ$ for stabilization of singular systems. Two counterexamples are presented to avoid the disadvantages of the existing condition of stabilization of continuous singular systems.

1. Introduction

Singular systems are governed by the so-called singular differential equations, which endow the systems with many special features that are not found in normal systems. Singular systems have comprehensive practical background and their applications can be found in various fields such as electrical circuit networks, robotics, and social economic systems, and so on [1]. Stability is an important and fundamental property. An unstable system does not work in practice. Singular system are fundamentally different form normal system. The following features in singular system are not usually found in normal system. The transfer function of a singular system may not be proper. Regularity means that for a considered system there exist Laplace transformation of $sE - A$ and there exist a unique solution. Impulse-freeness guarantees that the system there does not exist impulse solution which can reduce the cost index of a system. For an arbitrary finite initial condition, the time response of a singular system may exhibit undesired impulsive behavior which can be generated by infinite dynamic modes. Even if a singular system is impulse-free, it can be still have initial finite discontinuities due to inconsistent initial conditions. Admissibility is the least constraint for a controlled singular system. In recent years, considerable effort has been devoted to the analysis of controllability and observability, pole assignment [2], stability and stabilization [3–7], admissibility [8,9], robustness [10–15], H_∞ control [16,17], state feedback stabilization of impulsive systems [18], $D-$ stability and $D-$ stabilization [19], dissipativity [20], stability and stabilization of T-S fuzzy systems [21,22] and admissibility analysis with time-varying delay of singular systems [23]. Most recently, there has been an increasing interest in the stability and stabilization issues, e.g. approximate discretization of singular systems with impulsive mode [24], mixed H_∞ and passive control for singular systems with time delay via static output feedback [25], fuzzy normalization and stabilization for rectangular descriptor systems [26], extension of diagonal stability and stabilization for fractional positive systems [27] and impulsive stabilization of singular systems with time-delays [28].

However, the most commonly proposed LMI-type conditions contain equality constraints, which may be of little problem theoretically, but may cause a big trouble in checking the conditions numerically. For the criterion of stabilization of continuous singular systems, the latest feasible results is Lemma 3.1 [10] which gives an LMI-based necessary and sufficient condition for stabilization issues of singular systems without any additional equality constraints. The same as strict LMIs conditions presented in other literatures, Lemma 3.1 need to introduce the basis of null space of singular matrix of system [10,14]. Besides Lemma 3.1 asserts that without loss of generality the involved matrix $\Omega = PE^T + SQ$ is assumed to be nonsingular, but in some cases the solution of Ω solved by (13) is probably singular. When the case of singularity of Ω occurs, some $\theta \in (0, 1)$ needs introducing such that $\hat{\Omega} = \Omega + \theta\bar{P}$ is nonsingular and holds (13), in which $\bar{P}$ is an arbitrary

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nonsingular matrix satisfying LMI $E\bar{P} = \bar{P}^T E^T \geq 0$. In this case, it means the criterion needs to append a new equality constraints and introduces positive semi-definite and non-strict inequality. Examples 4.2 and 4.3 presented in this paper are two counter-examples whose $\Omega = PE^T + SQ$ solved from LMI (13) is actually singular so that Lemma 3.1 is invalid to deal with the stabilization issues of singular systems.

Paper [7] presents strict LMI criteria without invoking equality constraint for stability and stabilization of continuous singular systems by introducing a simple assumption. While the two strict LMI conditions proposed in [7] for stability and stabilization of singular systems are only sufficient conditions but not necessary conditions which are more restrictive and are incomplete to stabilization issues of continuous singular systems. Up to now, to the best of our knowledge, a complete and effective LMI-based necessary and sufficient condition which is strict and does not involve equality constraint and may avoid the invalidity caused by the singularity of $\Omega = PE^T + SQ$ (see in Examples 4.2 and 4.3) remains an open problem on the stabilization issues of singular systems. The criterion proposed in the paper does not involve the basis of null space of the singular matrix E. They just consider the simple full rank decomposition for the singular matrix E, without the decompositions of system matrix A and input matrix B. The conditions can be easily extended to other singular systems e.g. singular fractional order systems.

2. Strict LMIs condition for stability of singular systems

Consider a continuous singular system

$$E\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the physical state of the system, $u(t) \in \mathbf{R}^l$, is the control input, and $E \in \mathbf{R}^{n\times n}$ is the system singular matrix. It can be singular, and we assume $0 \leq$ rank $(E) = m \leq n$. $A \in \mathbf{R}^{n\times n}$ is the system matrix, and $B \in \mathbf{R}^{n\times l}$ is the system input matrix.

Definition 2.1. [1,2] System (1) is regular if det $(sE - A)$ is not identically zero. System (1) is impulse-free if deg(det $(sE - A)$) = rank(E). System (1) is stable if all the roots of det $(sE - A) = 0$ have negative real parts. System (1) is admissible if it is regular, impulse-free and stable.

For System (1), we define its generalized spectral abscissa [10] as

$$\alpha(E, A) \triangleq \max_{\lambda \in \{s|\det(sE-A)=0\}} \mathrm{Re}(\lambda).$$

For simplicity, we note $\alpha(A) = \alpha(I, A)$ which is the usual spectral abscissa.

Lemma 2.1. *[10] System (1) is admissible if and only if there exists a matrix P such that*

$$E^T P = P^T E \geq 0, \tag{2}$$

$$P^T A + A^T P < 0. \tag{3}$$

It is pointed out that the conditions in (2) and (3) are non-strict LMIs, which contain equality constraints. It may result in numerical troubles when checking such non-strict LMI conditions since equality constraints are fragile and usually do not hold effectively. In most cases the non-strict LMIs do not have any feasible solution, and the equality constraints can not be directly dealt with LMIs. So strict LMI conditions are more desirable than non-strict ones from the numerical simulation point of view. To overcome the equality constraint in (2), papers [10,14] utility matrices $S \in \mathbf{R}^{n\times(n-m)}$ which are of full column rank and are composed of basis of null space of E^T. The main result in paper [10,14] is introduced as in the following lemma.

Lemma 2.2. *[10,14] System (1) is admissible if and only if there exist a positive definite matrix P and a matrix Q which satisfy the LMI*

$$A^T(PE + SQ) + (PE + SQ)^T A < 0.$$

where $S \in \mathbf{R}^{n\times(n-m)}$ is any matrix with full column rank and satisfies $E^T S = 0$.

When the regularity of the pair (E, A) in System (1) is not given. it is always possible to obtain two nonsingular matrices M and N satisfying

$$MEN = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}, MAN = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}.$$

For the above decomposition, we have the following result.

Lemma 2.3. *[1,2](a) System (1) is impulse-free if and only if A_4 is nonsingular.*

(b) System (1) is admissible if and only if A_4 is nonsingular and $\alpha(A_1 - A_2 A_4^{-1} A_3) < 0$.

Many results concerning LMI-based control use LMIs, such as Lemma 2.1, but with additional equality constraints, which, unfortunately, often cause numerical problems in computation. Although using the results of the strict LMIs conditions without equality constraint in Lemma 2.2, we can analyze and design singular systems in almost the same way as what we do in the case of state-space models, while sophisticated manipulations are needed to derive the results in Lemma 2.2. That is, the adjective and non-unique parameter S need introducing. It is necessary and essential to present a new strict LMI condition for admissibility of System (1) to avoid the troubles caused by the singularity of $\Omega = PE^T + SQ$ mentioned in Introduction. It is caused by just introducing a variable P that conditions (2) and (3) in Lemma 2.1 are non-strict LMIs, and contain equality.

If we denote the P in Lemma 2.1 as $\bar{P}$ in the following theorems, then we have that $\bar{P} = M^T P N^{-1}$ is restricted equivalent to P. Where M and N are chosen to satisfying Theorem 2.1. By using the restricted equivalent transformation and the block matrix, Lemma 2.1 can be improved as Theorem 2.1 whose format is standard LMIs. Now, we are in a position to give the new strict LMI condition without equality constraint. Although Equation (2) with non-strict inequality is used throughout the proof almost everywhere, the novelty of the proof lies in it addresses an approach on how P can be constructed without involving Eq. (2) of Lemma 2.1.

Theorem 2.1. *System (1) is admissible if and only if for arbitrary two chosen nonsingular matrices M and N satisfying $MEN = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}$, $M, N \in \mathbf{R}^{n\times n}$, there exist matrices P_1, P_2, and nonsingular P_3, $P_1 \in \mathbf{R}^{m\times m}$, $P_2 \in \mathbf{R}^{(n-m)\times m}$, $P_3 \in \mathbf{R}^{(n-m)\times(n-m)}$ such that*

$$P_1 > 0, \tag{4}$$

$$P^T MAN + N^T A^T M^T P < 0. \tag{5}$$

where m= rank (E), $P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$.

Proof. (**Sufficiency**) For System (1), it is easy to choose two nonsingular matrices M and N such that

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$$MEN = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}.$$

Suppose (4) and (5) hold. Let

$$\overline{P} = M^T P N^{-1} = M^T \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} N^{-1}.$$

Then, considering (2) and (3), we have

$$\begin{aligned} E^T\overline{P} &= N^{-T}\begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix} M^{-T}M^T P N^{-1} \\ &= N^{-T}\begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix} N^{-1} \geq 0, \\ \overline{P}^T E &= N^{-T}P^T M M^{-1}\begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix} N^{-1} \\ &= N^{-T}\begin{bmatrix} P_1^T & 0 \\ 0 & 0 \end{bmatrix} N^{-1} \geq 0, \\ & P^T MAN + N^T A^T M^T P \\ &= N^T \overline{P}^T M^{-1} MAN + N^T A^T M^T M^{-T} \overline{P} N \\ &= N^T(\overline{P}^T A + A^T \overline{P})N < 0. \end{aligned}$$

That is, the matrix $\overline{P}$ satisfies (2) and (3). By Lemma 2.1, we have System (1) is admissible.

(**Necessity**) Suppose System (1) is admissible. According to Lemma 2.3, from the full rank decomposition of singular matrix E, for the two chosen nonsingular matrices M and N satisfying

$$MEN = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}, \; MAN = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix},$$

A_4 is nonsingular, and

$$\alpha(A_1 - A_2A_4^{-1}A_3) < 0.$$

Noting the above inequality and for the following slow subsystem $\dot{x}_1(t) = (A_1 - A_2A_4^{-1}A_3)x_1(t)$, applying Lyapunov stability theory, it can be seen that there exists a matrix $P_1 > 0$ such that

$$(A_1 - A_2A_4^{-1}A_3)^T P_1 + P_1(A_1 - A_2A_4^{-1}A_3) < 0.$$

Let

$$M_1 = \begin{bmatrix} I_m & -A_2A_4^{-1} \\ 0 & A_4^{-1} \end{bmatrix}, \quad N_1 = \begin{bmatrix} I_m & 0 \\ -A_4^{-1}A_3 & I_{n-m} \end{bmatrix}, \quad \bar{P} = \begin{bmatrix} P_1 & 0 \\ 0 & -I_{n-m} \end{bmatrix}.$$

Then, M_1 and N_1 are nonsingular and we have

$$\begin{aligned} M_1MENN_1 &= M_1\begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix} N_1 = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}, \\ M_1MANN_1 &= M_1\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} N_1 \\ &= \begin{bmatrix} A_1 - A_2A_4^{-1}A_3 & 0 \\ 0 & I_{n-m} \end{bmatrix}. \end{aligned}$$

Therefore, it is easy to see

$$\bar{P}M_1 \, MAN \, N_1 + N_1^T N^T A^T M^T M_1^T \bar{P} < 0. \tag{6}$$

Pre-multiplying and post-multiplying (6) by N_1^{-T} and N_1^{-1}, respectively, we have

$$N_1^{-T}\bar{P}M_1 \, MAN + N^T A^T M^T M_1^T \bar{P} N_1^{-1} < 0.$$

Let

$$P = M_1^T \bar{P} N_1^{-1}.$$

Therefore,

$$\begin{aligned} P &= M_1^T \bar{P} N_1^{-1} \\ &= \begin{bmatrix} I_m & 0 \\ -A_4^{-T}A_2^T & A_4^{-T} \end{bmatrix}\begin{bmatrix} P_1 & 0 \\ 0 & -I_{n-m} \end{bmatrix} \\ &\times \begin{bmatrix} I_m & 0 \\ A_4^{-1}A_3 & I_{n-m} \end{bmatrix} = \begin{bmatrix} P_1 & 0 \\ -A_4^{-T}A_2^T P_1 - A_4^{-T}A_4^{-1}A_3 & -A_4^{-T} \end{bmatrix}. \end{aligned}$$

Therefore the matrices P_1 and nonsingular P satisfies (4) and (5), respectively. This completes the proof.

By duality of System (1), the following corollary is introduced as a lemma.

Lemma 2.4. *[5] System (1) is admissible if and only if there exists a matrix P such that*

$$EP = P^T E^T \geq 0, \tag{7}$$

$$P^T A^T + AP < 0. \tag{8}$$

Taking into account Lemma 2.4 and Theorem 2.1 and simply invoking duality, we can obtain the following corollary.

Corollary 2.1. System (1) is admissible if and only if there exist matrices P_1, P_2, and nonsingular P_3, $P_1 \in \mathbf{R}^{m\times m}$, $P_2 \in \mathbf{R}^{(n-m)\times m}$, $P_3 \in \mathbf{R}^{(n-m)\times(n-m)}$ such that

$$P_1 > 0, \tag{9}$$

$$P^T N^T A^T M^T + MANP < 0. \tag{10}$$

where P, M, N and m are the same specification as those of Theorem 2.1.

Remark 2.1. The decision variables P_1 and P in (4), (5), (9) and (10) of Theorem 2.1 and Corollary 2.1 are restrictive to the different chosen matrices M and N. The given different M and N lead to different pairs (MEN, MAN) which are restrictively equivalent to each other. The matrices M and N are constructed by the matlab command 'rref'. Command rref (A) produces the reduced row echelon form of A which is listed in the lines 2–4 of Appendix.

3. Conditions of stabilization of singular systems

For singular systems, it is natural to consider the criteria that guarantee that the considered singular systems are not only stable but also impulse-free and regular and then admissible.

For System (1), with the following state feedback control law:

$$u(t) = Kx(t), \; K \in \mathbf{R}^{l\times n}, \tag{11}$$

exerting control law (11) to System (1), the closed-loop system is obtained as follows:

$$E\dot{x}(t) = (A + BK)x(t). \tag{12}$$

In [10] there exists the following stabilization result.

Lemma 3.1. *[10,11] Consider the continuous singular System (1). There exists a state feedback controller (11) such that the closed-loop*

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System (12) is admissible if and only if there exist matrices $P > 0$, Q and Y such that

$$\Omega^T A^T + A\Omega + BY + Y^T B^T < 0 \tag{13}$$

where $\Omega = PE^T + SQ$, and $S \in R^{n\times(n-m)}$ is any matrix with full column rank and satisfies $ES = 0$. In this case we can assume that Ω is nonsingular (if this is not the case, then we can choose some $\theta \in (0, 1)$ such that $\hat{\Omega} = \Omega + \theta\bar{P}$ is nonsingular and satisfies (13), in which $\bar{P}$ is any nonsingular matrix satisfying $E\bar{P} = \bar{P}^T E^T \geq 0$), then a stabilizing state feedback controller can be chosen as

$$u(t) = Y\Omega^{-1}x(t).$$

Lemma 3.1 is a important tool to deal with the stabilization of singular systems. Although Lemma 3.1 proposes a strict LMIs criterion which implies that it is definite LMIs without equality constraint, it is not always effectively tractable and reliable in solving LMIs with MATLAB. By using Lemma 3.1, we have the following illustrative counterexample which does not satisfy the singularity condition of $\Omega = PE^T + SQ$. So do Examples 4.2 and 4.3.

Example 3.1. Consider an unstable singular System (1) with parameters as follows:

$$E = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, A = \begin{bmatrix} 2 & 5 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 \\ -2 \end{bmatrix}.$$

Where we can obtain

$$S = \text{null}(E) = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

which is with full column rank and satisfies $ES = 0$. Then, it can be found that the LMI in (13) is feasible. By Lemma 3.1, we can obtain

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Q = \begin{bmatrix} -1 & -1 \end{bmatrix}, Y = \begin{bmatrix} -0.5 & 0 \end{bmatrix},$$

$$\Omega^T A^T + A\Omega + BY + Y^T B^T = \begin{bmatrix} -5 & -4 \\ -4 & -4 \end{bmatrix} < 0,$$

where

$$\Omega = PE^T + SQ = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}.$$

In this example, from Lemma 3.1, as Ω is singular, in order to find stabilizing state feedback controller for System (1), we need to involve another matrix $\bar{P}$ which satisfies the following non-strict LMI with equality constraint

$$E\bar{P} = \bar{P}^T E^T \geq 0. \tag{14}$$

It is a difficult task to solve this kind of LMI. In fact, $\bar{P}$ can not be directly obtained by LMI (14) and what only can be done is for a given $\bar{P}$ to be tried to verify if it satisfies LMI (14). Unfortunately, most given $\bar{P}$ s do not satisfy LMI (14). It is noted the solution of (13) is not unique. If we choose

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, Q = \begin{bmatrix} -3 & -2 \end{bmatrix}, Y = \begin{bmatrix} -1 & 3 \end{bmatrix},$$

then it is found that these matrices are solutions to (13), but the matrix Ω is calculated as

$$\Omega = \begin{bmatrix} 5 & 5 \\ -3 & -2 \end{bmatrix},$$

which is obviously non-singular. In this example, Eq. (13) fails to conclude whether closed-loop system of (1) is admissible or not by the criteria in Lemma 3.1. In the example, the matrix A is a second order matrix whose solved matrices P, Q, Y, Ω can be chosen easily. But for high order matrix, the corresponding solved matrices P, Q, Y, Ω can only be obtained from a computer. If Eq. (13) fails to conclude a solid solution, it is difficult to draw another valid solution manually. Now, we are in a position to give the strict LMI criterion without equality condition of stabilization of continuous singular systems.

Considering the closed-loop singular System (12) follows from Corollary 2.1, we have

$$P^T N^T K^T B^T M^T + P^T N^T A^T M^T + MANP + MBKNP < 0.$$

Since from the proof of Theorem 2.1, it follows the solved matrix P is nonsingular. If denote $Q = KNP$, we have the following results desired immediately.

Theorem 3.1. *For the continuous singular System (1), there exists a state feedback controller (11) such that the closed-loop System (12) is admissible if and only if there exist appropriate dimensional matrices $P_1 > 0$, nonsingular P and Q such that*

$$Q^T B^T M^T + P^T N^T A^T M^T + MANP + MBQ < 0. \tag{15}$$

where P, M, N and m have the same specification as those of Theorem 2.1. Then a stabilizing state feedback controller can be chosen as

$$u(t) = Kx(t), K = QP^{-1}N^{-1}. \tag{16}$$

The inequality in (15) is a kind of standard LMI which can be directly solved with the MATLAB LMI toolbox. The detailed explanations on how to construct the matrices P and Q in Theorem 3.1 can be found on the lines 7–13 of the formulation in the Appendix. Different from Lemma 3.1 which may cause invalidity such as in the case of Examples 4.2 and 4.3, Theorem 3.1 works well all the time. With the program in the Appendix, the control gain matrix is easy to obtain $K = \begin{bmatrix} 0.7412 & 1.3647 \end{bmatrix}$.

Remark 3.1. The M and N introduced in Theorem 3.1 are easy to be obtained in the steps of deducing the singular matrix E as its diagonal normalized form. The conditions in Theorem 3.1 does not need to introduce the bases of null space of singular matrix E. The results in Theorem 3.1 are strict LMIs conditions, the matrix P in (15) is easy to be calculated with LMI box.

Remark 3.2. The conditions in LMI (15) do not involve equality constraint, such as $E^T P = P^T E$ and include the less solved variables of LMIs than that in Lemma 3.1.

Remark 3.3. In the case of $E = I$ the singular System (1) reduces to a normal system. So we have $m = n, M = N = I, P_1 = \bar{P}$. Then Theorem 3.1 coincides with the Lyapunov stabilization theory in [11]. Therefore, Theorem 3.1 can be regarded as an extension of Lyapunov stabilization theory for continuous normal systems to continuous singular systems.

Remark 3.4. Compared with [10–11], the contribution of the paper is to facilitate the design of stabilizing controllers via state feedback based on strict LMI without invoking equality constraint.

The criterion ensures much more tractable and reliable in numerical simulation compared to the existing results [10–11]. The most important merit of the criterion proposed in the paper is that it can overcome the drawbacks of the invalidity caused by the singularity of $\Omega = PE^T + SQ$ for stabilization of singular systems.

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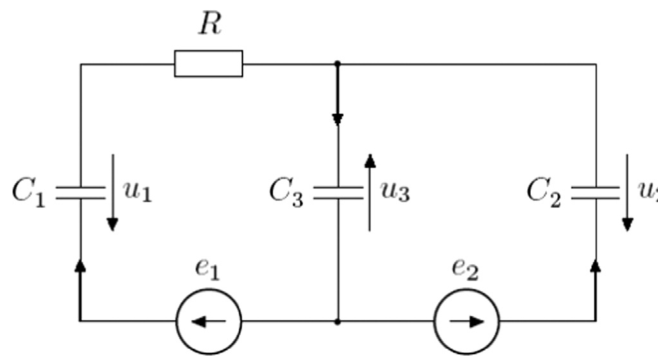

Fig. 1. Electrical circuit illustration to Example 4.1.

4. Simulation examples

4.1. Singular electrical circuit systems

The two following practical examples are given to illustrate the practicability of singular systems discussed in the paper.

Example 4.1. Consider electrical circuit shown on Fig. 1 with given resistance R, capacitances C_1, C_2, C_3 and source voltages e_1 and e_2. Using the Kirchhoff's laws, we can write for the electrical circuit the equations

$$\begin{bmatrix} RC_1 & 0 & 0 \\ C_1 & C_2 & -C_3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}.$$

Setting $R = 0.5$, $C_1 = 2$, $C_2 = 1$, $C_3 = 3$, with Lemma 2.3, it is easy to judge that $\det\ (sE - A) = 2\,s(2\,s + 3) \not\equiv 0$ which means that System (1) is regular. It is easy to verify that deg(det $(sE - A)$)= rank $(E) = 2$ which implies that System (1) is impulse-free. The all eigenvalues of the polynomial det $(sE - A) = 0$ are -1.5 and 0. Their eigenvalues have negative real parts or equal to 0. It shows that System (1) is stable.

Example 4.2. Consider electrical circuit shown on Fig. 2 with given resistances R_1, R_2, R_3, inductances L_1, L_2, L_3 and source voltages e_1 and e_2. Using the Kirchhoff's laws, we can write for the electrical circuit the equations

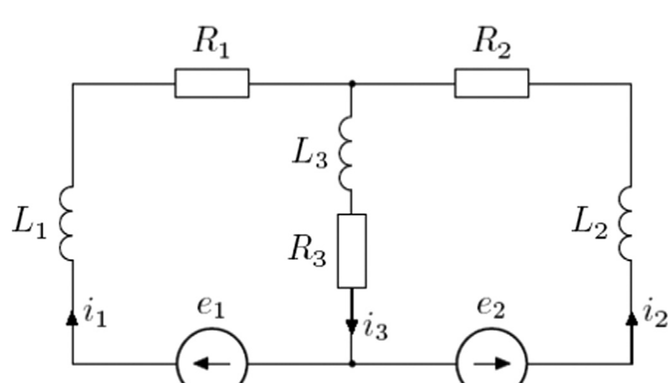

Fig. 2. Electrical circuit illustration to Example 4.2.

$$\begin{bmatrix} L_1 & 0 & L_3 \\ 0 & L_2 & L_3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{i}_3 \end{bmatrix} = \begin{bmatrix} -R_1 & 0 & -R_3 \\ 0 & -R_2 & -R_3 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}.$$

Setting $R_1 = 2$, $R_2 = 1$, $R_3 = 3$, $L_1 = 1$, $L_2 = 3$, $L_3 = 2$, it is easy to verify that det $(sE - A) = 11\,s^2 + 25\,s + 11 \not\equiv 0$ implying that System (1) is regular and deg(det $(sE - A)$)= rank $(E) = 2$ implying that System (1) is impulse-free. The all eigenvalues of the polynomial det $(sE - A) = 0$ are $\frac{-25 \pm \sqrt{141}}{22}$. All of their eigenvalues have negative real parts. It shows that System (1) is stable. From Lemma 2.3, it follows that System (1) is admissible. The matrices M and N in Theorem 2.1 can be calculated by the MATLAB commands appended on the lines 2–4 of the formulation in the Appendix. i.e.

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}, N = \begin{bmatrix} 0 & 0 & 1 \\ -\frac{1}{3} & 1 & \frac{1}{3} \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

With MATLAB to solve the LMIs (4) and (5) in Theorem 2.1, we have the feasible results:

$$P = \begin{bmatrix} 0.8451 & 0.1082 & 0 \\ 0.1082 & 2.1589 & 0 \\ 0.0964 & -0.9894 & -0.6173 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 0.8451 & 0.1082 \\ 0.1082 & 2.1589 \end{bmatrix}.$$

It is easy to check that matrix P_1 is symmetric and the two eigenvalues of matrix P_1 are 0.8363 and 2.1677, respectively. All of them are greater than zero. So we have $P_1 > 0$. According to Theorem 2.1, let

$$\begin{aligned} \overline{P} =&\ M^T P N^{-1} \\ =&\ \begin{bmatrix} 0.8451 & 0.1082 & 1.7623 \\ 0.0361 & 0.7196 & 0.5519 \\ -0.5209 & -0.9894 & -0.4667 \end{bmatrix}, \end{aligned}$$

then, from (2) and (3), it is easy to verify that

$$\begin{aligned} E^T\overline{P} =&\ \overline{P}^T E \\ =&\ \begin{bmatrix} 0.8451 & 0.1082 & 1.7623 \\ 0.1082 & 2.1589 & 1.6556 \\ 1.7623 & 1.6556 & 4.6284 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} & A^T\overline{P} + \overline{P}^T A \\ =& \begin{bmatrix} -4.4222 & -1.7627 & -6.1140 \\ -1.7627 & -3.4181 & -2.5125 \\ -6.1140 & -2.5125 & -12.9517 \end{bmatrix}. \end{aligned}$$

From Example 4.2, we can see that both the matrices $E^T\overline{P}$ and $A^T\overline{P} + \overline{P}^T A$ are symmetric. The eigenvalues $E^T\overline{P}$ are 5.9827, 1.6497 and 0, respectively. The eigenvalues of $A^T\overline{P} + \overline{P}^T A$ are -16.8322, -2.8237 and -1.1361, respectively. So we have $E^T\overline{P} \geq 0$ and $A^T\overline{P} + \overline{P}^T A < 0$. From the simulation result it follows that matrix $\overline{P}$ satisfies with Eqs. (2) and (3).

The following a numerical simulation example (Example 4.3) and a counterexample (Example 4.4) with given parameters are presented to illustrate the availability of criteria obtained in the paper.

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4.2. Stability of singular systems

Example 4.3. Consider System (1) with parameters as follows:

$$E=\begin{bmatrix}0&-1&0&-1\\1&2&0&1\\-2&-2&1&-2\\-1&-1&0&0\end{bmatrix}, A=\begin{bmatrix}-2&-2&0&-1\\-2&-4&-1&-2\\0&-1&-7&-1\\-1&-2&-1&-5\end{bmatrix}.$$

With Lemma 2.3, it is easy to judge that det $(sE-A)=9\,s^3+65\,s^2+142\,s+103\not\equiv 0$ which means that System (1) is regular. It is easy to verify that deg(det $(sE-A)$)= rank $(E)=3$ which implies that System (1) is impulse-free. The all eigenvalues of the polynomial det $(sE-A)=0$ are $-1.6205\pm 0.4986i$ and -3.9812. All of their eigenvalues have negative real parts. It shows that System (1) is stable. So we can obtain that System (1) is admissible. The matrices M and N in Theorem 2.1 can be obtained by using the simple MATLAB commands which are listed on the lines 2–4 of the formulation in the Appendix. Therefore, it follows

$$M=\begin{bmatrix}0&-1&0&-2\\0&1&0&1\\0&0&1&-2\\1&1&0&1\end{bmatrix}, N=\begin{bmatrix}0&0&0&1\\1&1&0&-1\\-2&0&1&2\\-1&0&0&1\end{bmatrix}$$

With MATLAB to solve the LMIs (4) and (5) in Theorem 2.1, we have the feasible results:

$$P=\begin{bmatrix}0.3592&0.3805&0.072&0\\0.3805&1.3581&-0.0412&0\\0.072&-0.0412&0.1666&0\\0.19&-0.5919&0.1222&0.0418\end{bmatrix},$$
$$P_1=\begin{bmatrix}0.3592&0.3805&0.072\\0.3805&1.3581&-0.0412\\0.072&-0.0412&0.1666\end{bmatrix}.$$

It is easy to see that matrix P_1 is symmetric and the three eigenvalues of matrix P_1 are 0.1111, 0.2861 and 1.4867, respectively. All of them are greater than zero. So we have $P_1>0$. According to Theorem 2.1, let

$$\begin{aligned}\overline{P}&=M^TPN^{-1}\\&=\begin{bmatrix}0.2318&-0.5919&0.1222&-1.0263\\0.2531&0.3857&0.009&0.1564\\0.072&-0.0412&0.1666&-0.4464\\-0.2501&0.0876&-0.3962&1.1719\end{bmatrix},\end{aligned}$$

then, from (2) and (3), we have

$$\begin{aligned}E^T\overline{P}&=\overline{P}^TE\\&=\begin{bmatrix}0.3592&0.3805&0.072&-0.1227\\0.3805&1.3581&-0.0412&1.06\\0.072&-0.0412&0.1666&-0.4464\\-0.1227&1.06&-0.4464&2.0755\end{bmatrix},\end{aligned}$$

$$\begin{aligned}&A^T\overline{P}+\overline{P}^TA\\&=\begin{bmatrix}-1.4394&-0.723&-0.3732&1.0084\\-0.723&-0.986&0.1605&-1.0467\\-0.3732&0.1605&-1.558&3.4707\\1.0084&-1.0467&3.4707&-9.3992\end{bmatrix}.\end{aligned}$$

From Example 4.3, we can see that both the matrices $E^T\overline{P}$ and $A^T\overline{P}+\overline{P}^TA$ are symmetric. The eigenvalues $E^T\overline{P}$ are 2.8957, 0.9425, 0.1212 and 0, respectively. The eigenvalues of $A^T\overline{P}+\overline{P}^TA$ are -10.9286, -1.9737, -0.4185 and -0.0619, respectively. So we have $E^T\overline{P}\geq 0$ and $A^T\overline{P}+\overline{P}^TA<0$. From the simulation result it means matrix $\overline{P}$ also satisfies with Eqs. (2) and (3).

4.3. Stabilization of singular systems

The following example is an alternative counterexample like Example 3.1 which conflicts with Lemma 3.1.

Example 4.4. Consider System (1) with parameters as follows:

$$E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},$$
$$A=\begin{bmatrix}2&0&-3\\0&-1&0\\0&1&0\end{bmatrix}, B=\begin{bmatrix}-2&0\\0&-1\\0&3\end{bmatrix}$$

It is easy to verify the system is not regular, has impulse mode and is unstable. In order to use Lemma 3.1, we first use matlab command

$$S=\mathrm{null}(E)=\begin{bmatrix}0\\0\\1\end{bmatrix},$$

which is with full column rank and satisfies $ES=0$. By solving the LMIs (13) in Lemma 3.1, we can obtain

$$P=\begin{bmatrix}444.8246&0&0\\0&401.0714&0\\0&0&444.8246\end{bmatrix}>0,$$
$$Q=10^4\begin{bmatrix}-3.5451&0&0\end{bmatrix},$$
$$Y=\begin{bmatrix}5.3732&0&0\\0&-0.0162&-0.0075\end{bmatrix},$$
$$\Omega=PE^T+SQ=10^4\begin{bmatrix}0.0445&0&0\\0&0.0401&0\\-3.5451&0&0\end{bmatrix}.$$

From Lemma 3.1, as Ω is singular, in order to find stabilizing state feedback controller for System (1), we need to involve another matrix $\bar{P}$ which satisfies the following non-strict LMI with equality constraint (14). In this example, Eq. (13) does not work and fails to conclude whether closed-loop System (12) is admissible or not by the criteria in Lemma 3.1. Although the feasible solution of Equation (13) is not unique, but whenever singularity of Ω arises in the computer simulation, the approach in Lemma 3.1 is incapable of action. Lemma 3.1 assumes that Ω is nonsingular (if this is not the case, then one can choose some $\theta\in(0,1)$ such that $\hat{\Omega}=\Omega+\theta\bar{P}$ is nonsingular and satisfies (13), in which $\bar{P}$ is any nonsingular matrix satisfying $E\bar{P}=\bar{P}^TE^T\geq 0$, in this case a non-strict LMI with equality constraint is involved).

However this example can be perfectly and completely solved by the approaches presented in Theorem 3.1. Using the standard command 'feasp' in MATLAB LMI control toolbox to solve for LMI feasibility problems of LMI (15) in Theorem 3.1, the feasible solution can be highly and effectively obtained. Detailed algorithm refers to in the program of the Appendix. Therefore we get the feasible solution in detail as follows:

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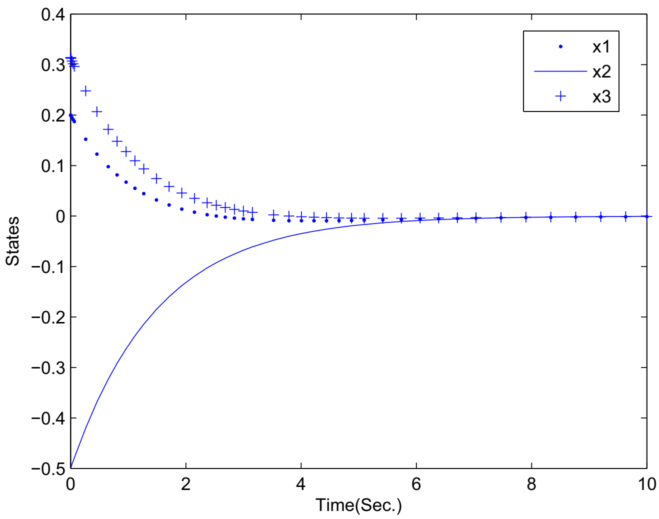

Fig. 3. The closed-loop singular system in Example 4.4.

$$P=\begin{bmatrix}444.8246 & 0 & 0\\ 0 & 401.0714 & 0\\ 346.1933 & -7.4544 & 222.4123\end{bmatrix},$$

$$Q=\begin{bmatrix}36.7408 & -4.9696 & -285.1651\\ 32.3022 & -162.2516 & -74.7451\end{bmatrix},$$

$$K=\begin{bmatrix}1.0805 & -0.0362 & -1.2821\\ 0.3342 & -0.4108 & -0.3361\end{bmatrix}.$$

With the control gain matrix K obtained in Example 4.4, we can draw the state curves of the closed-loop system in Example 4.4 shown as in Fig. 3. It is easy to see that although the open loop singular system is unstable and is not impulse free, by Fig. 3, the closed-loop singular system is admissible and it can be stabilized by the control law (16) in about 5 s. Compared with the result in Lemma 3.1 (Theorem 3.1 in [10] and Theorem 2 in [11]), from Examples 3.1 and 4.1 it is shown that Theorem 3.1 works effectively even when Lemma 3.1 does not work.

From Example 4.4, it is easy to conclude that the stabilization criterion (Theorem 3.1) of strict LMI algorithm without invoking equality constraint for singular systems is effective and easy to operate.

5. Conclusions

This paper considers new criteria for stability and stabilization of continuous singular systems in light of strict LMIs, that is, definite and adjective parameter free criteria without equality constraint. The P_i matrices constituted with the conditions in Theorems 2.1 and 3.1 depend only on the matrix E not on A and B. The LMIs in the criteria are much more effective and reliable, specially in the case that the other approaches become invalid. The similar fruits of robust stabilization, H_2 control, H_∞ control and stabilization with time delay of linear time-invariant uncertain singular systems with strict LMIs can also be expanded by the same approaches as in Theorem 3.1. The conditions can also be extended to singular fractional order systems.

Appendix

The main LMI simulation formulation of Theorems 2.1 and 3.1 is listed as follows.

```
n = size(B, 1); m = rank(E);
op = rref([E, eye(n)]); M = op(: , n + 1: 2*n);
y = op(: , 1: n)'; z = rref([y, eye(n)]);
N = z(: , n + 1: 2*n)';
A = M*A*N; B = M*B;
setlmis([]);
[X1, n1, sX1] = lmivar(1, [m1]);
[X2, n2, sX2] = lmivar(2, [n − mm]);
[X3, n2, sX3] = lmivar(1, [n − m1]);
[X, n1, sX] = lmivar(3, [sX1, …
zeros(m(1), n − m(1)); sX2, sX3]);
Y = lmivar(2, [n(2), n(1)]);
lmiterm([ − 111X1], 1, 1);
lmiterm([211X], A, 1, 's');
lmiterm([211Y], B, 1, 's');
lmiterm([ − 311X3], 1, 1, 's')
```

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