UCMERCED MESA LAB

What is, why you need and who cares fractional calculus?

YangQuan Chen, Ph.D., Director,

MESA (Mechatronics, Embedded Systems and Automation) LAB

ME/EECS/SNRI/UCSolar, School of Engineering,

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T: (209)228-4672; **O**: SE1-254; **Lab**: Castle #22 (**T**: 228-4398)

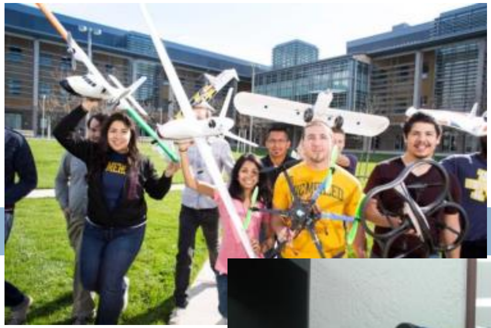
Nov. 8, 2013. Friday 3:30PM-4:30PM

SIAM @ UC Merced

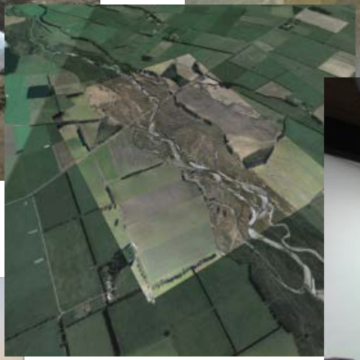
Room COB 265 3:30-4:30 PM

Skip Ad in 5 minutes

The MESA Lab



- **Mechatronics, Embedded Systems, Automation Lab**
- http://mechatronics.ucmerced.edu
- Lab Manager: Brandon Stark
  - Lab Manager: Brandon Stark
  - 4 Ph.D. Students
  - 2 MSc Students
  - 20+ Undergrads
  - 2 Visiting Ph.D. Students
  - 2 Visiting Professors



- Unmanned Aerial Systems
- Cyber-Physical Systems
- Renewable Energy Systems
- Mechatronic Systems
- Applied Fractional Calculus

UCMERCED mechatronics.ucmerced.edu MESA LAB

Dr. YangQuan Chen, yqchen@ieee.org

Mechatronics, Embedded Systems and Automation Lab

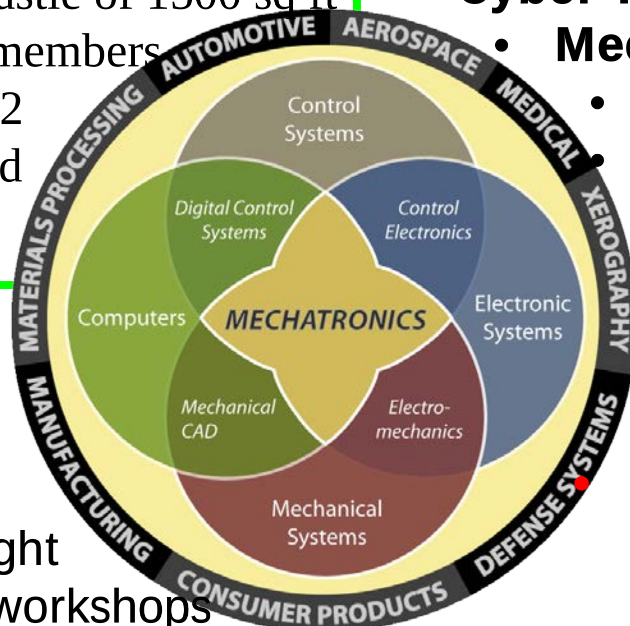
Real solutions for sustainability!

Established August 2012 @ Castle of 1500 sq ft
4 Ph.D/2 MS/ 20+ undergrad members
visiting scholars || Sponsored 2 capstone projects and mentored 4+1 capstone teams (F'13)

Research Areas of Excellence:

(ISI H-index=29, Google H-index=49; i10-index=217)

- Unmanned Aerial Systems & UAV-based Personal Remote Sensing (PRS)
- **Cyber-Physical Systems (CPS)**
  - **Mechatronics**
    - **Applied Fractional Calculus**
    - **Modeling and Control of Renewable Energy Systems**

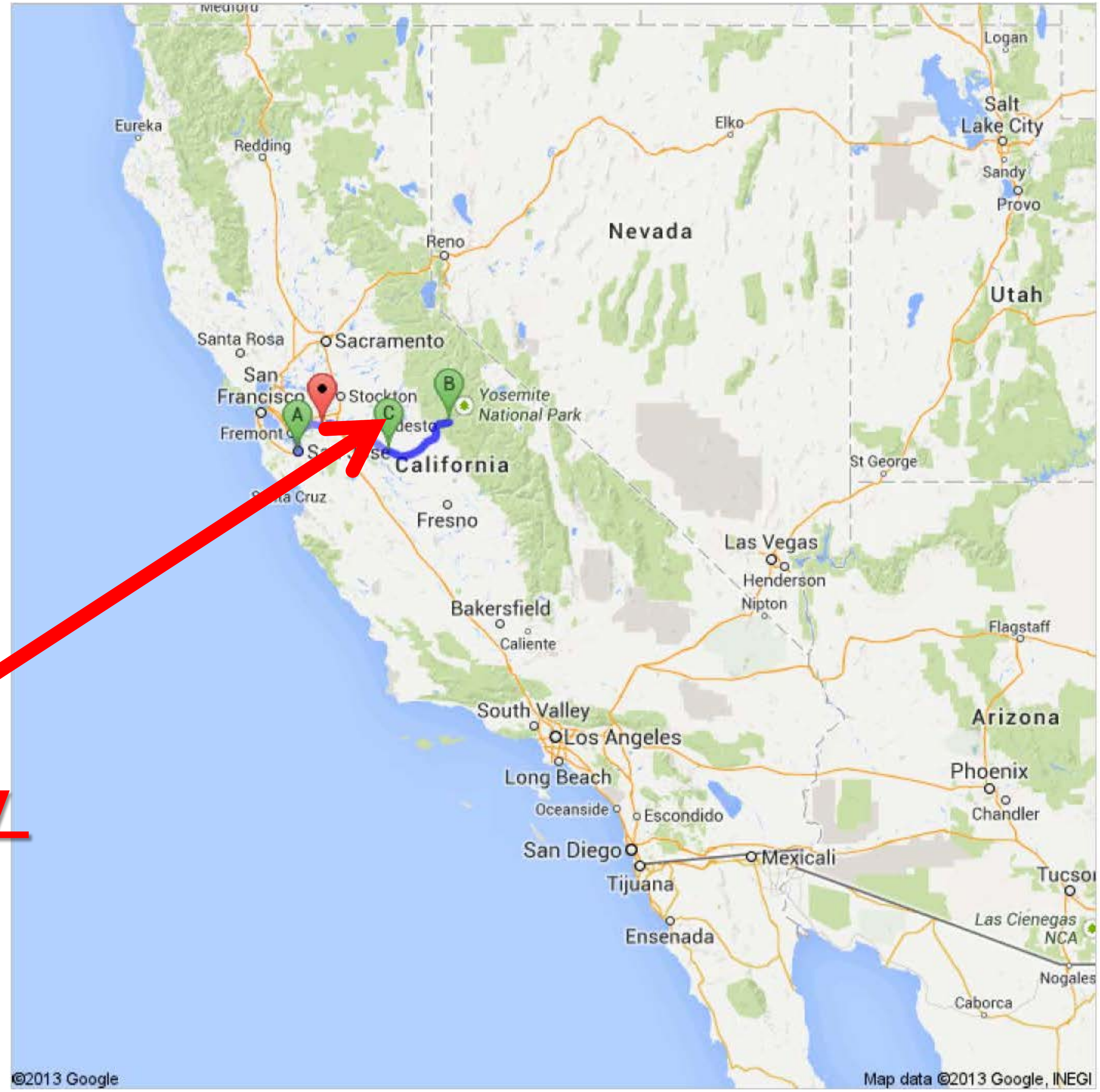


Education and Outreach Activities:

- AfterShock
- Academic Excellence Night
- Robotics Club tutorials/workshops
- Preview Day in Merced Mall
- "*The Drone Age*" @ Castle Air Museum
- Robots-n-Ribs| MESABox! ASME tutorials
- 6 capstone teams (24 seniors) ...
- **ME142 Mechatronics (**take-home labs**)**
- **ME280 Fractional Order Mechanics**

Projects Related to San Joaquin Valley:

- **Energy** [Solar energy, CPV, Building efficiency (HVAC lighting), smart grids integration, NG pipelines]
- **Water** (Water/soil salinity management, water sampling UAVs)
- **Precision Ag / Environment** (Crop dynamics, optimal harvest, pest ...)



Ag Drone Valley

UCCE + MESALAB = ?

http://cemerced.ucanr.edu/

\+ http://mechatronics.ucmerced.edu/

= Ag Drone Valley (fractional)

MESA Lab Philosophy and Ambition

- "***We make real systems that work and others want them*.**"
- **MESA Lab: Staying on top and working for sustainability.**
- **Nationally and internationally visible and prominent!**

MESA Research Areas/Strengths

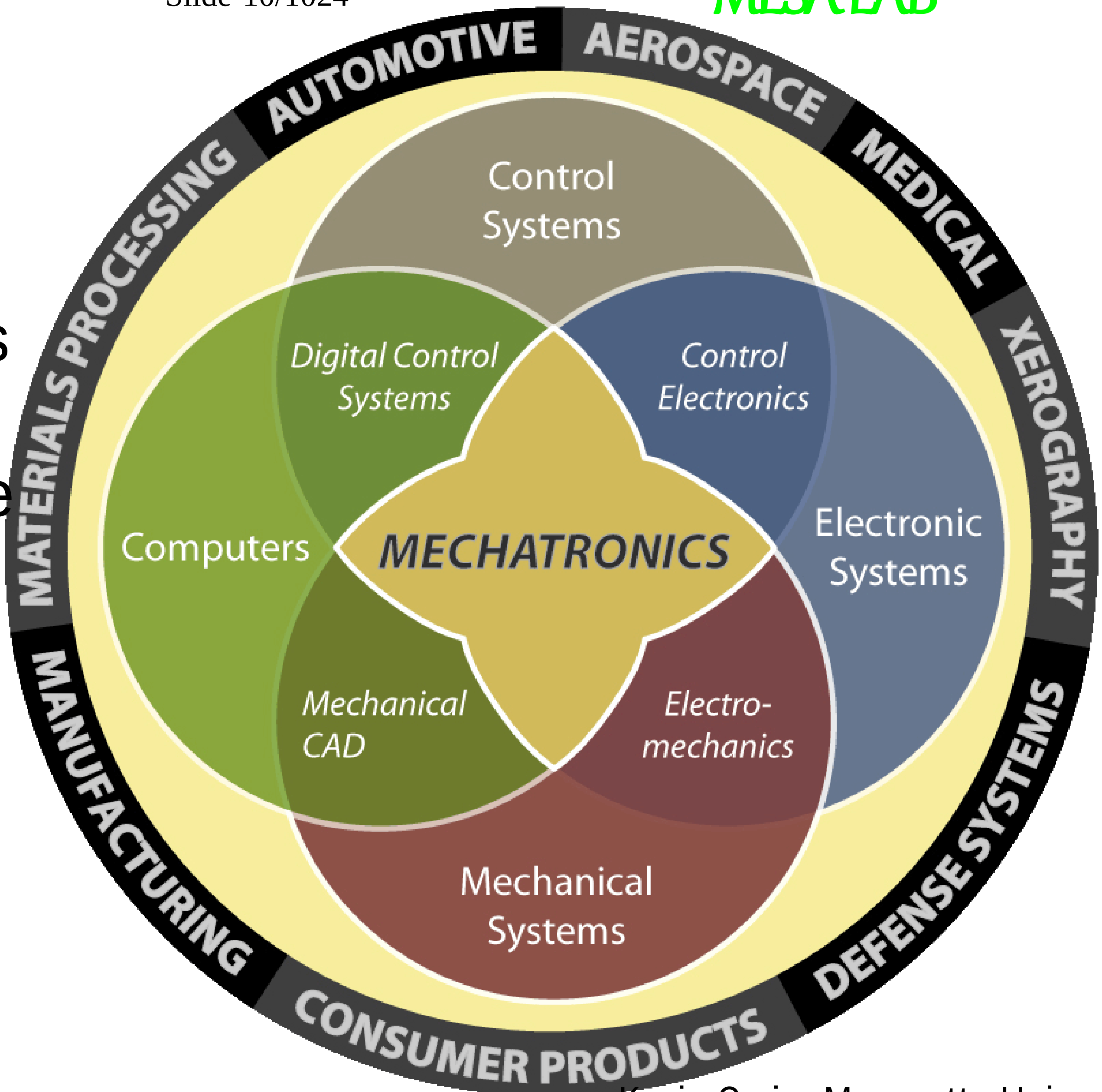
- Unmanned Aerial Systems and UAV-based Personal Remote Sensing (PRS)
- Cyber-Physical Systems (CPS)
- Modeling and Control of Renewable Energy Systems
- Mechatronics
- **Applied Fractional Calculus (AFC)**

My courses at UC Merced

- Spring 2013
  - ME142 Mechatronics (4cr) (48 seats)
- Fall 2013
  - ME280 Fractional Order Mechanics (3cr) (10 seats) (why/what/when?)

Mechatronics

has good prospects for the future because knowledge economy demands to speed up development, improve quality, reduce cost and increase energy efficiency.

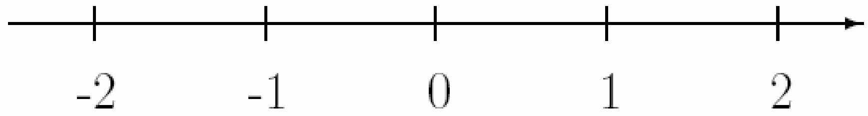


Kevin Craig, Marquette Univ.

Outline of this talk

- **Fractional Calculus – What?**
- **Fractional Calculus – Why?**
- **Fractional Calculus – Who Cares?**
- **Take-Home Messages**

… from integer to non-integer …



$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n}$$

$$x^n = e^{n \ln x}$$

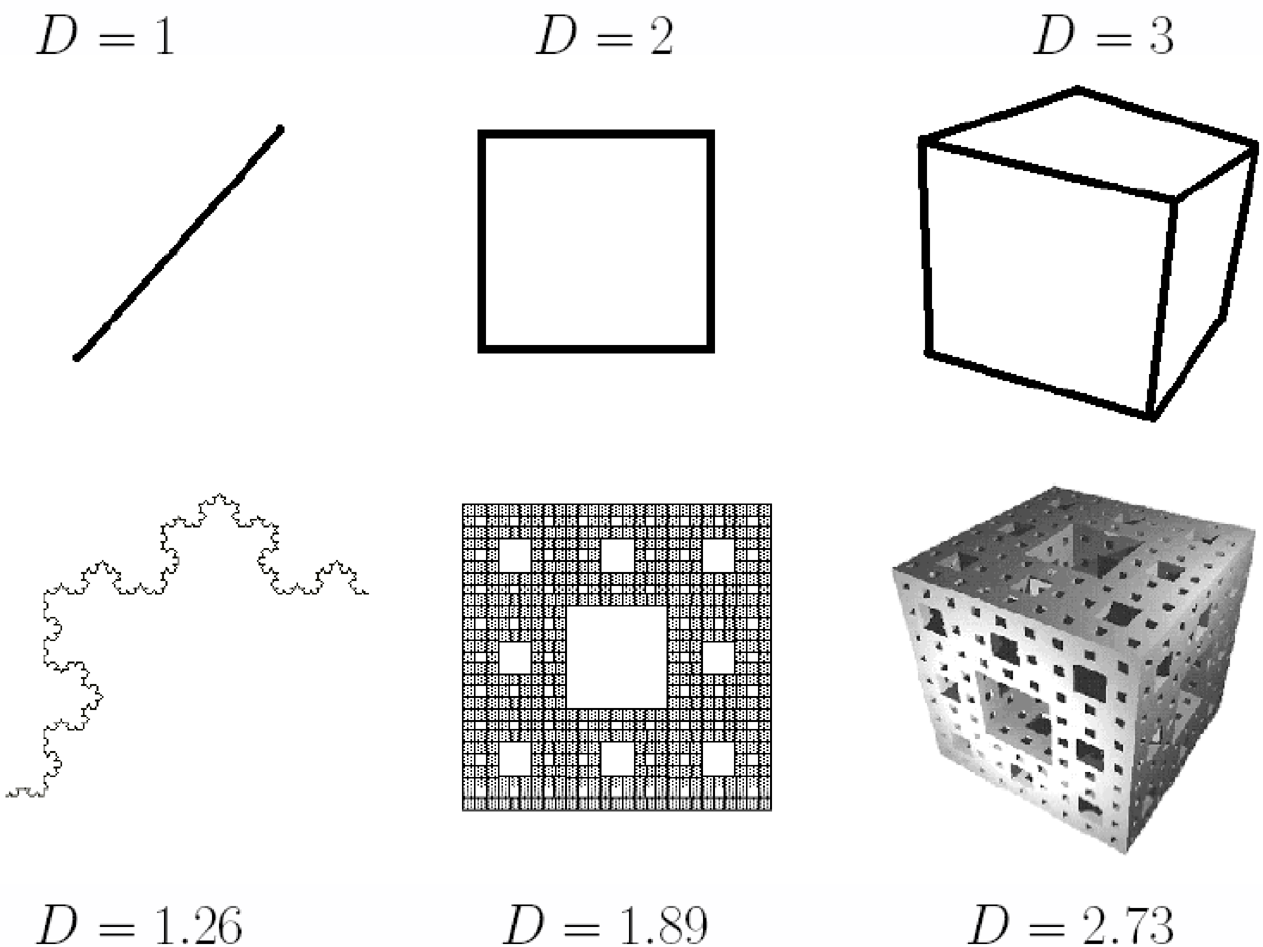
$$n! = 1 \cdot 2 \cdot 3 \cdot \ \ldots \cdot (n-1) \cdot n,$$

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \qquad x > 0,$$

$$\Gamma(n+1) = 1 \cdot 2 \cdot 3 \cdot \ \ldots \cdot n = n!$$

Slide credit: Igor Podlubny

... from integer to non-integer ...



Slide credit: Igor Podlubny

"Fractional Order Thinking" or, "In Between Thinking"

- For example
  - Between integers there are non-integers;
  - Between logic 0 and logic 1, there is the "**fuzzy logic**";
  - Between integer order splines, there are "**fractional order splines**"
  - Between integer high order moments, there are **noninteger order moments (e.g. FLOS)**
  - Between "integer dimensions", there are **fractal dimensions**
  - Fractional Fourier transform (FrFT) – in-between time-n-freq.
  - Non-Integer order calculus (fractional order calculus – abuse of terminology.) (FOC)


0
1
NEURO FUZZY
Made in Japan
KEEP WARM
EXTENDED KEEP WARM
NS-ZCC10
WHITE RICE
REG/SUSHI
SOFTER
HARDER
QUICK COOKING
18:34
MIXED
PORRIDGE
SWEET
SEMIBROWN
BROWN
COOKING REHEAT
RESET
MENU
TIMER
PRE-WASHED
TIME SETTING
ZOJIRUSHI

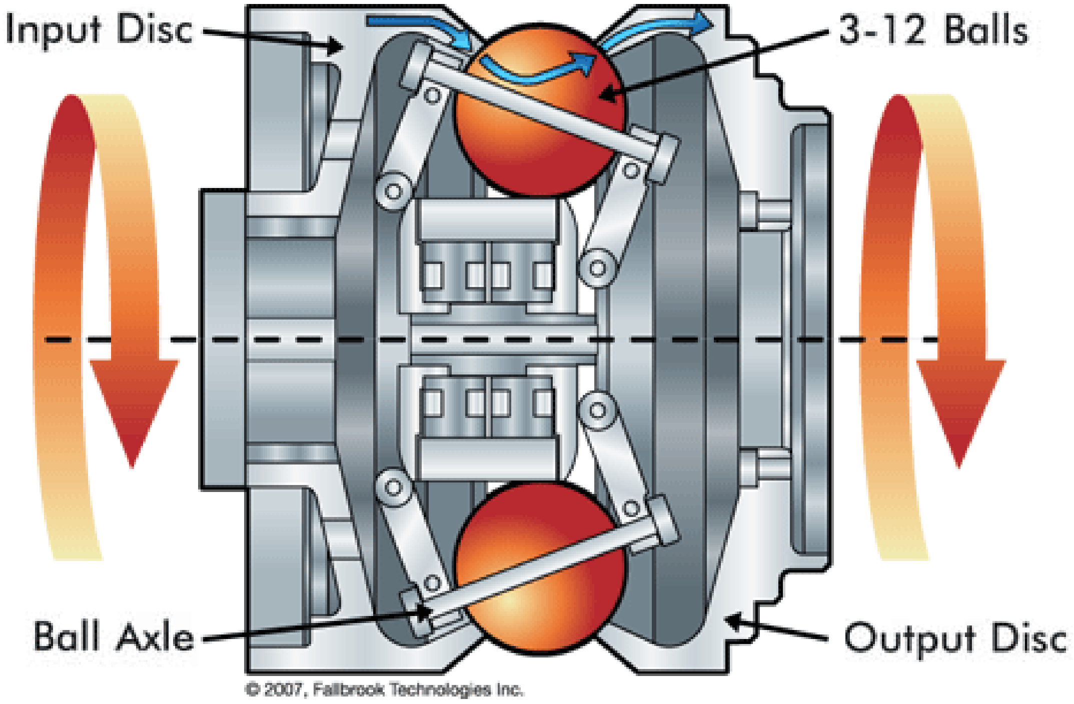
Conclusion of Talk

Integer-Order Calculus

Fractional-Order Calculus

Slide credit: Richard L. Magin, ICCC12

- Integer-Order Calculus
- Fractional-Order Calculus



Discrete gears vs. constantly-variable transmission

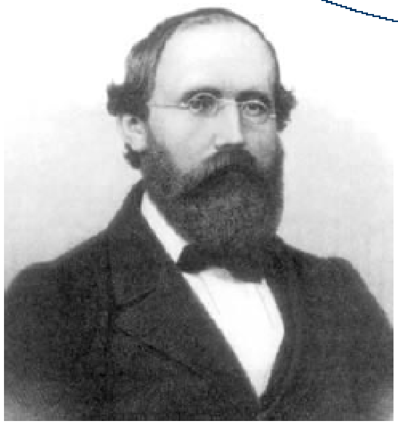
http://spectrum.ieee.org/energywise/energy/renewables/could-mechanics-best-power-electronics-in-evs

Slide credit: Calvin Coopmans, 2/28/2013 email comment

Riemann–Liouville definition

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt} \right)^n \int_a^t \frac{f(\tau)\, d\tau}{(t-\tau)^{\alpha-n+1}} $$

$$ (n-1 \le \alpha < n) $$



G.F.B. Riemann
(1826–1866)

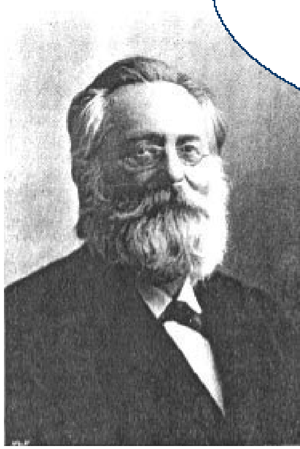
J. Liouville
(1809–1882)

Slide credit: Igor Podlubny

Grünwald–Letnikov definition

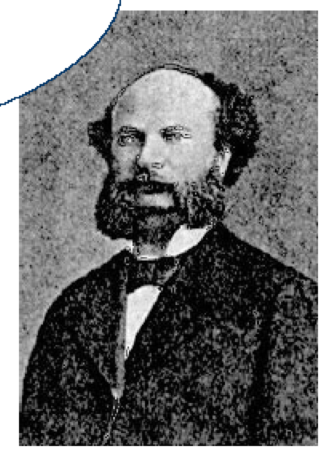
$$ {}_aD_t^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t - jh) $$

$[x]$ – integer part of x



A.K. Grünwald

A.V. Letnikov



Slide credit: Igor Podlubny

G. W. Leibniz (1695–1697)

In the letters to J. Wallis and J. Bernulli (in 1697) Leibniz mentioned the possible approach to fractional-order differentiation in that sense, that for non-integer values of n the definition could be the following:

$$\frac{d^n e^{mx}}{dx^n} = m^n e^{mx},$$

L. Euler (1730)

$$\frac{d^n x^m}{dx^n} = m(m-1)\ldots(m-n+1)x^{m-n}$$

$$\Gamma(m+1) = m(m-1)\ldots(m-n+1)\,\Gamma(m-n+1)$$

$$\frac{d^n x^m}{dx^n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)}x^{m-n}.$$

Euler suggested to use this relationship also for negative or non-integer (rational) values of n. Taking $m=1$ and $n=\frac{1}{2}$, Euler obtained:

$$\frac{d^{1/2}x}{dx^{1/2}} = \sqrt{\frac{4x}{\pi}} \qquad \left(= \frac{2}{\sqrt{\pi}}x^{1/2}\right)$$

Slide credit: Igor Podlubny

Operator ${}_a\mathrm{D}_t^\alpha$

A generalization of differential and integral operators:

$$
{}_a\mathrm{D}_t^\alpha = \begin{cases} \mathrm{d}^\alpha/\mathrm{d}t^\alpha & \mathbb{R}(\alpha) > 0, \\ 1 & \mathbb{R}(\alpha) = 0, \\ \int_a^t (\mathrm{d}\tau)^{-\alpha} & \mathbb{R}(\alpha) < 0. \end{cases} \tag{7}
$$

There are two commonly used definitions for the general fractional order differentiation and integral, i.e., the **Grünwald-Letnikov definition** and the **Riemann-Liouville definition**.

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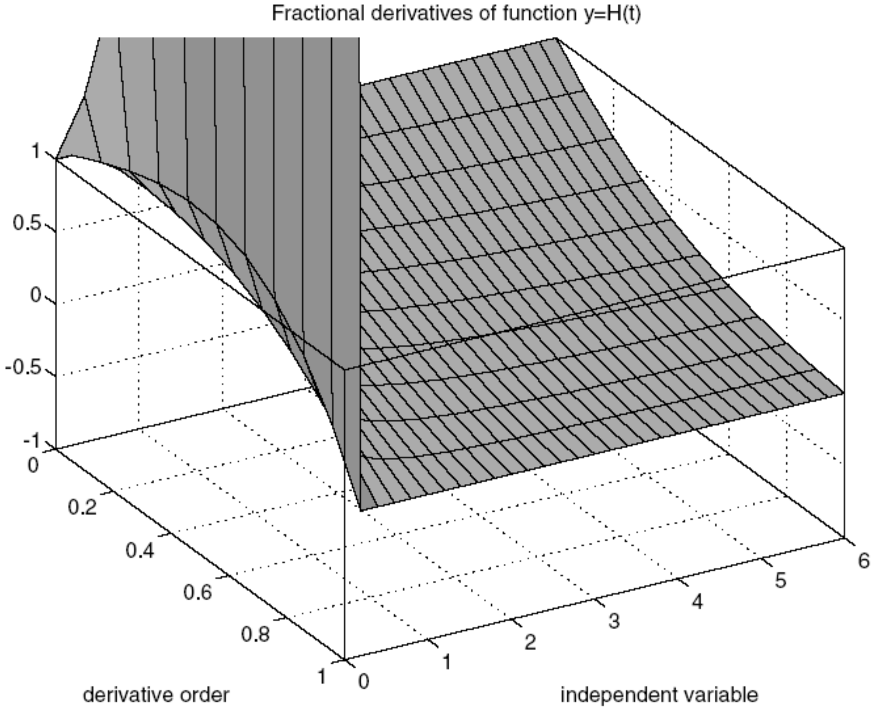
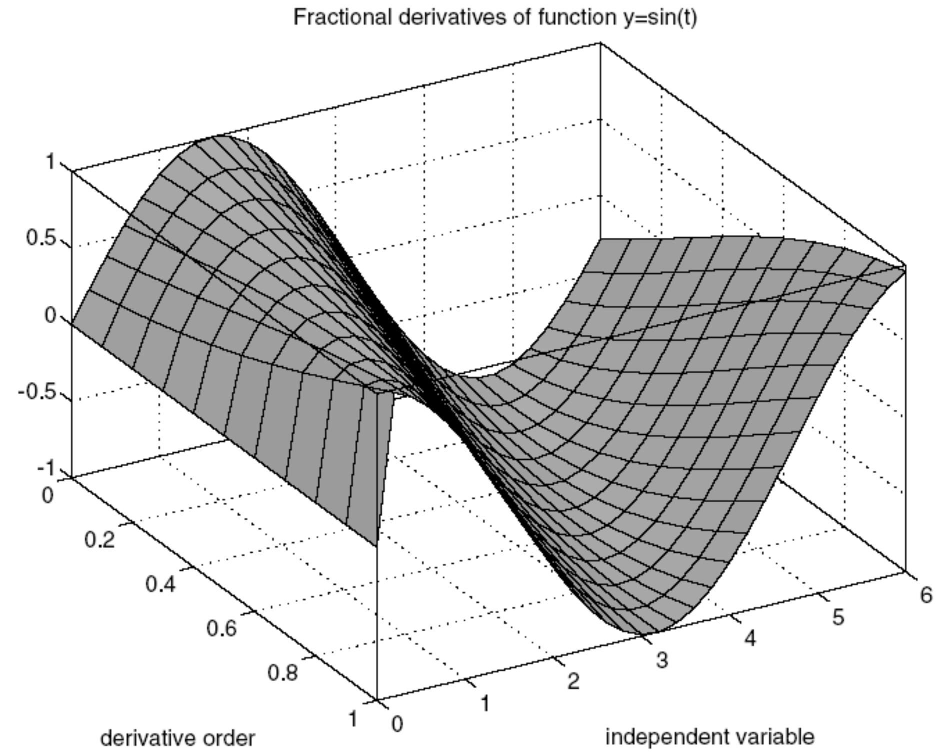
Example: Heaviside's unit step



Example: $\sin(t)$



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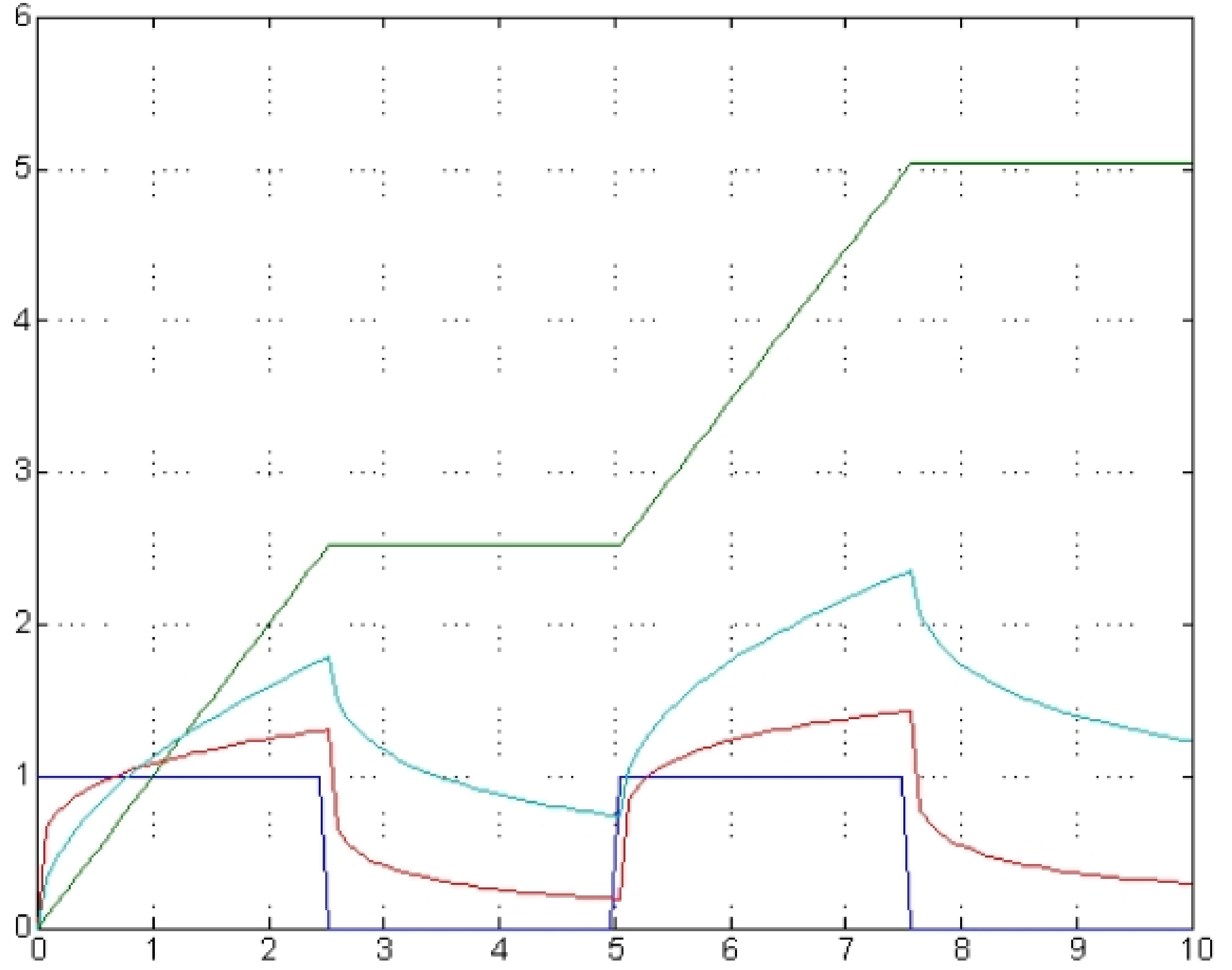
Fractional derivatives of ramp function.



Fractional Calculus was born in 1695

$$\frac{d^n f}{dt^n}$$

What if the order will be $n = ½$?

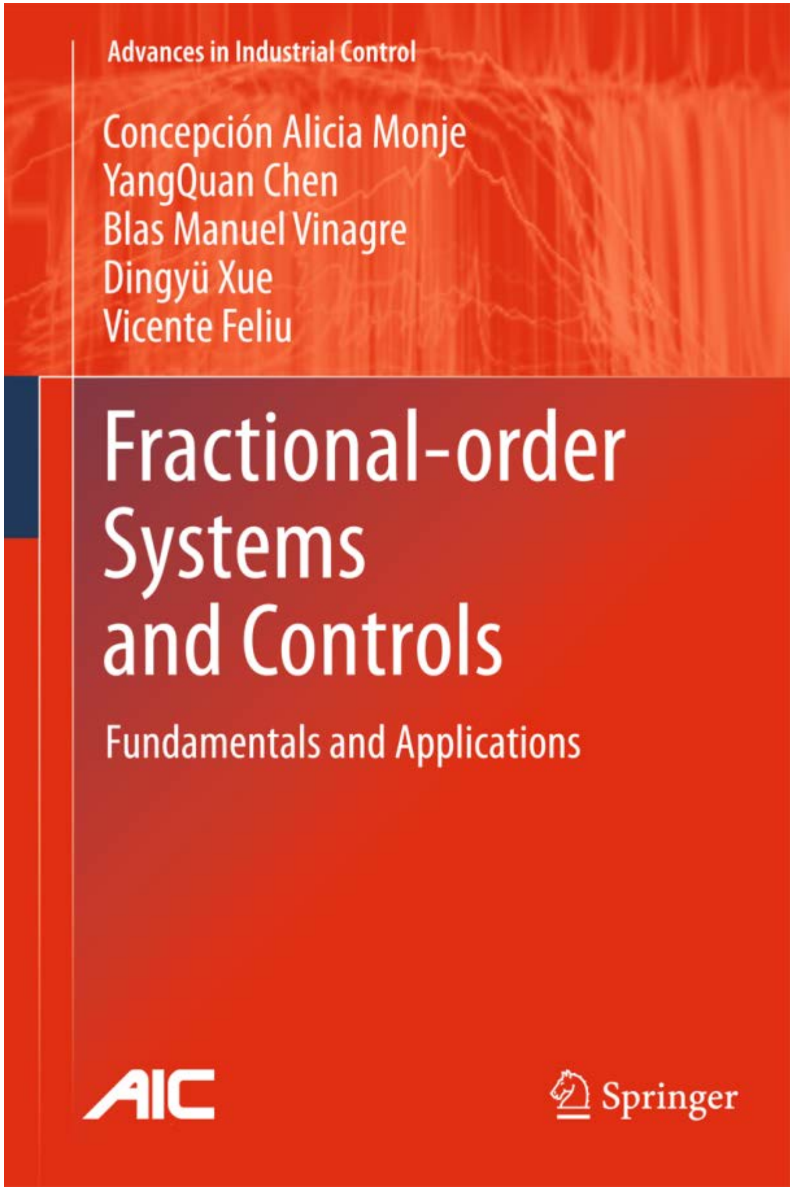
G.F.A. de L'Hôpital
(1661–1704)

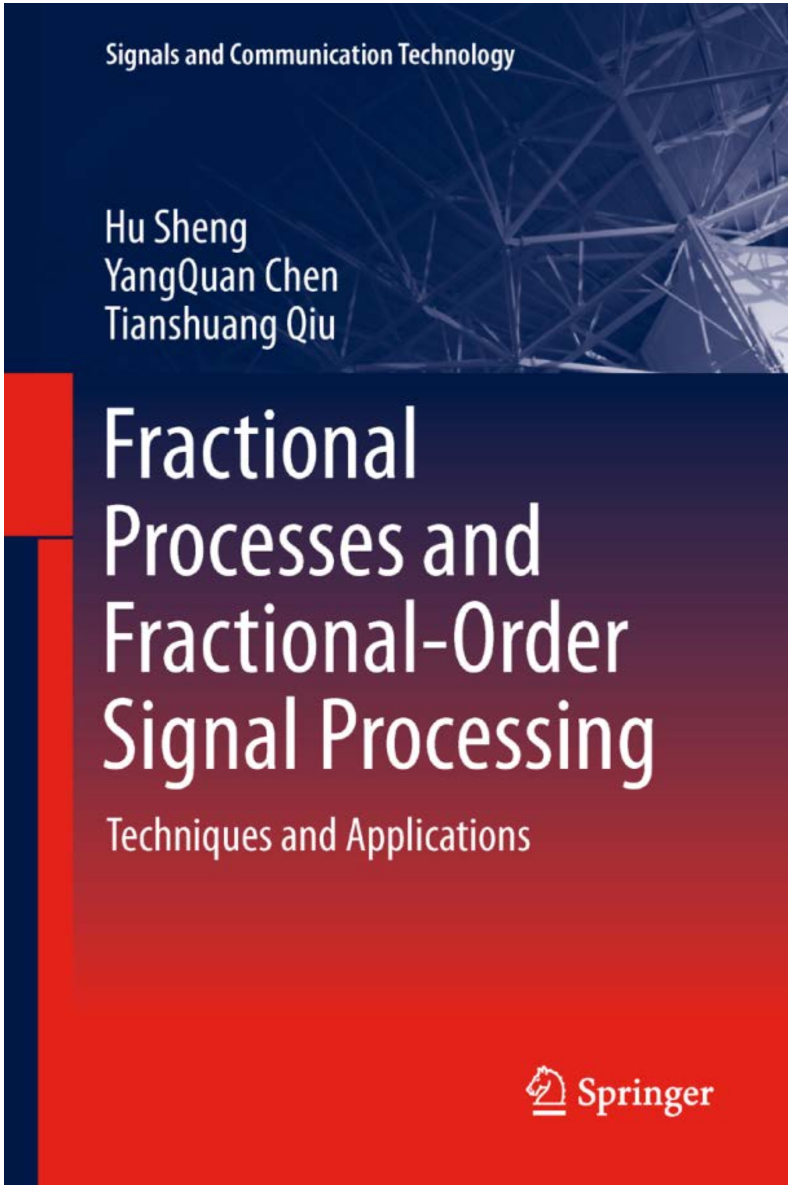
It will lead to a paradox, from which one day useful consequences will be drawn.

G.W. Leibniz
(1646–1716)

Slide credit: Igor Podlubny

Useful consequences

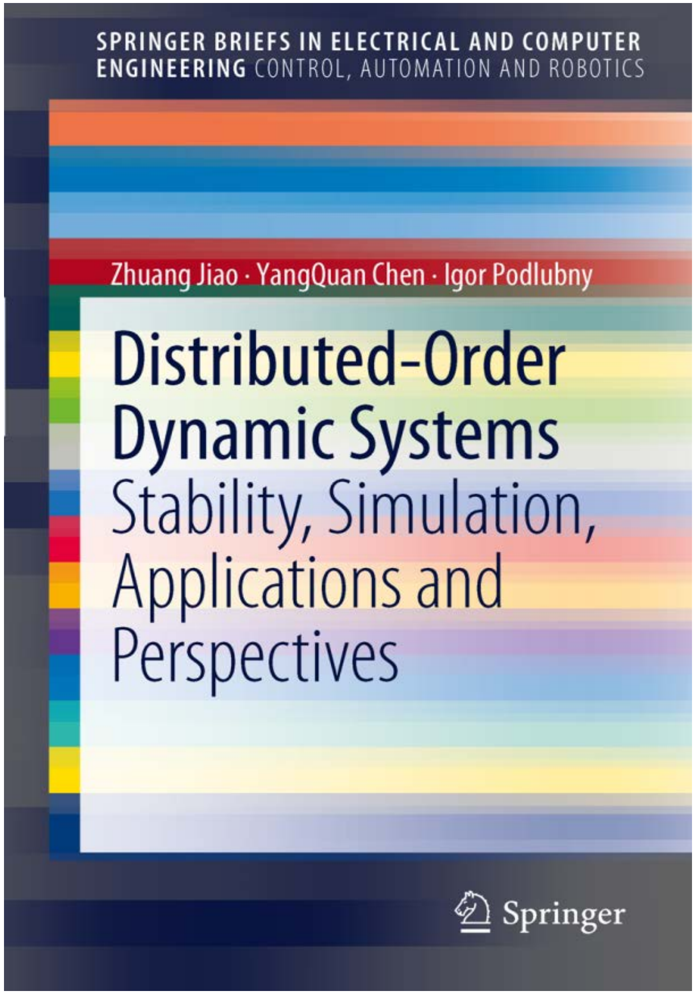


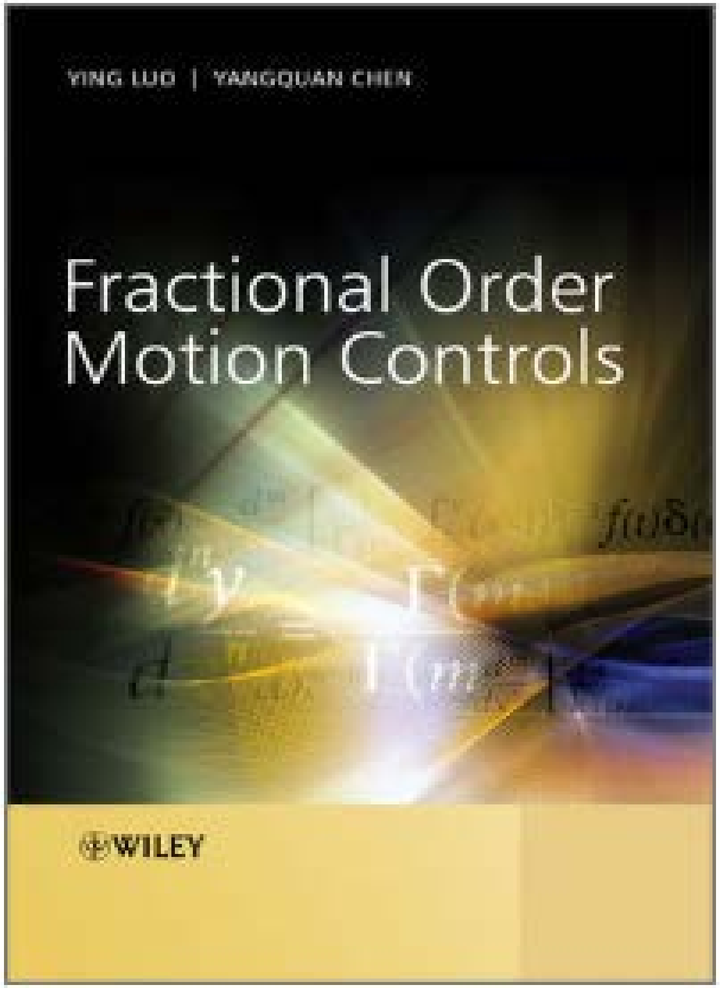


2001-2010

2005-2011

Useful consequences





Outline of this talk

- **Fractional Calculus – What?**
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- **Take-Home Messages**

International Symposium on Fractional PDEs: Theory, Numerics and Applications, June 3-5, 2013, Salve Regina University, 100 Ochre Point Avenue, Newport RI 02840

More Optimal Image Processing by Fractional Order Differentiation and Fractional Order Partial Differential Equations

Dali Chen, Dingyu Xue, YangQuan Chen

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Who cares?

- Minimal dose biomedical imaging
- More optimal

Strategies for Reducing Radiation Dose in CT (McCollough 2009)
Radiol Clin North Am. 2009 January ; 47(1): 27–40. doi:10.1016/j.rcl.2008.10.006

http://www.eurekalert.org/pub_releases/2013-05/aaft-mdc050113.php

FC for what?

- Better than the best
- New sciences

- Need killer apps.

Take home message:

More optimal image processing can be made possible by using fractional order differentiation and fractional order partial differential equations.

Want to be more optimal? Go fractional calculus!

Q & A

More info:

http://mechatronics.ucmerced.edu/research/applied-fractional-calculus

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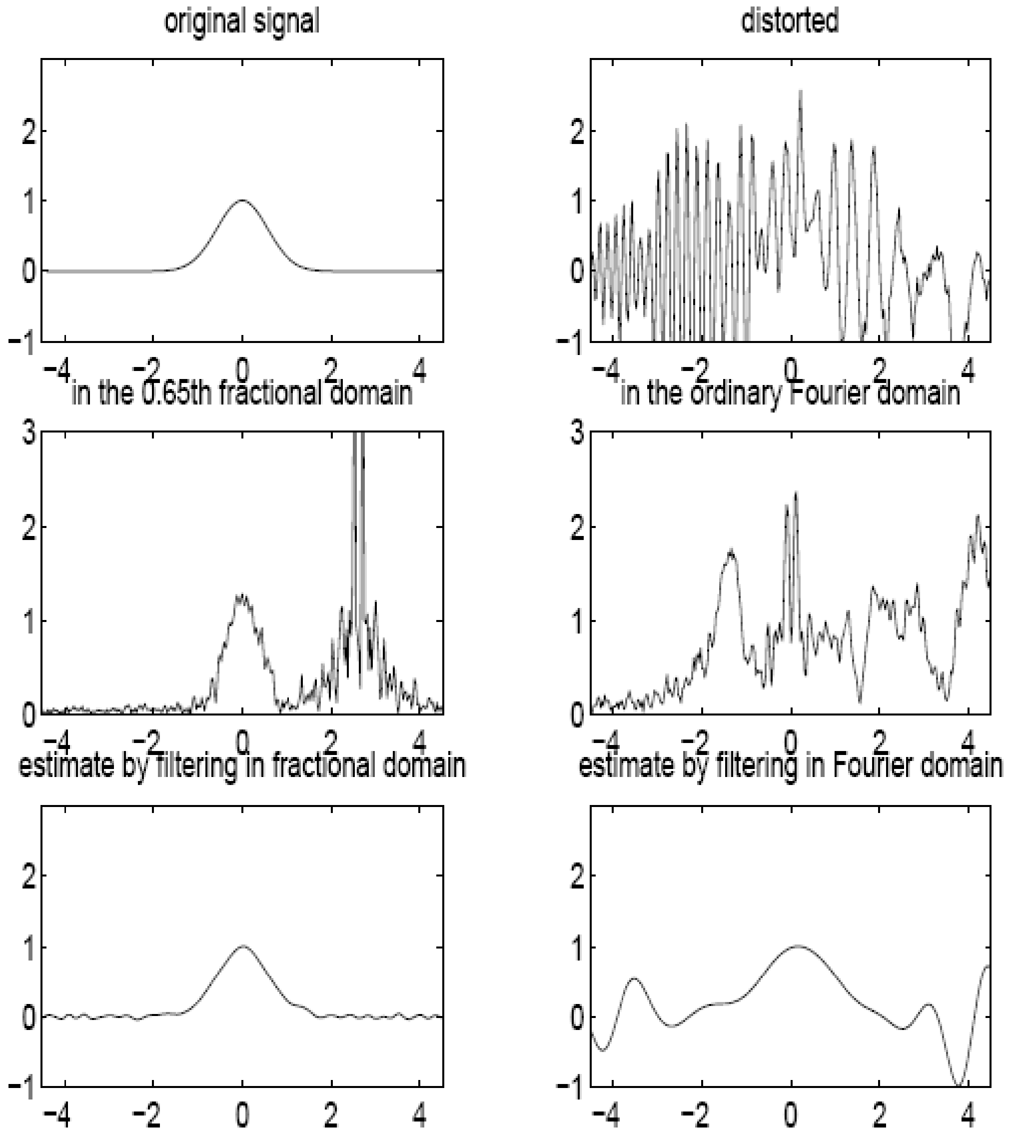
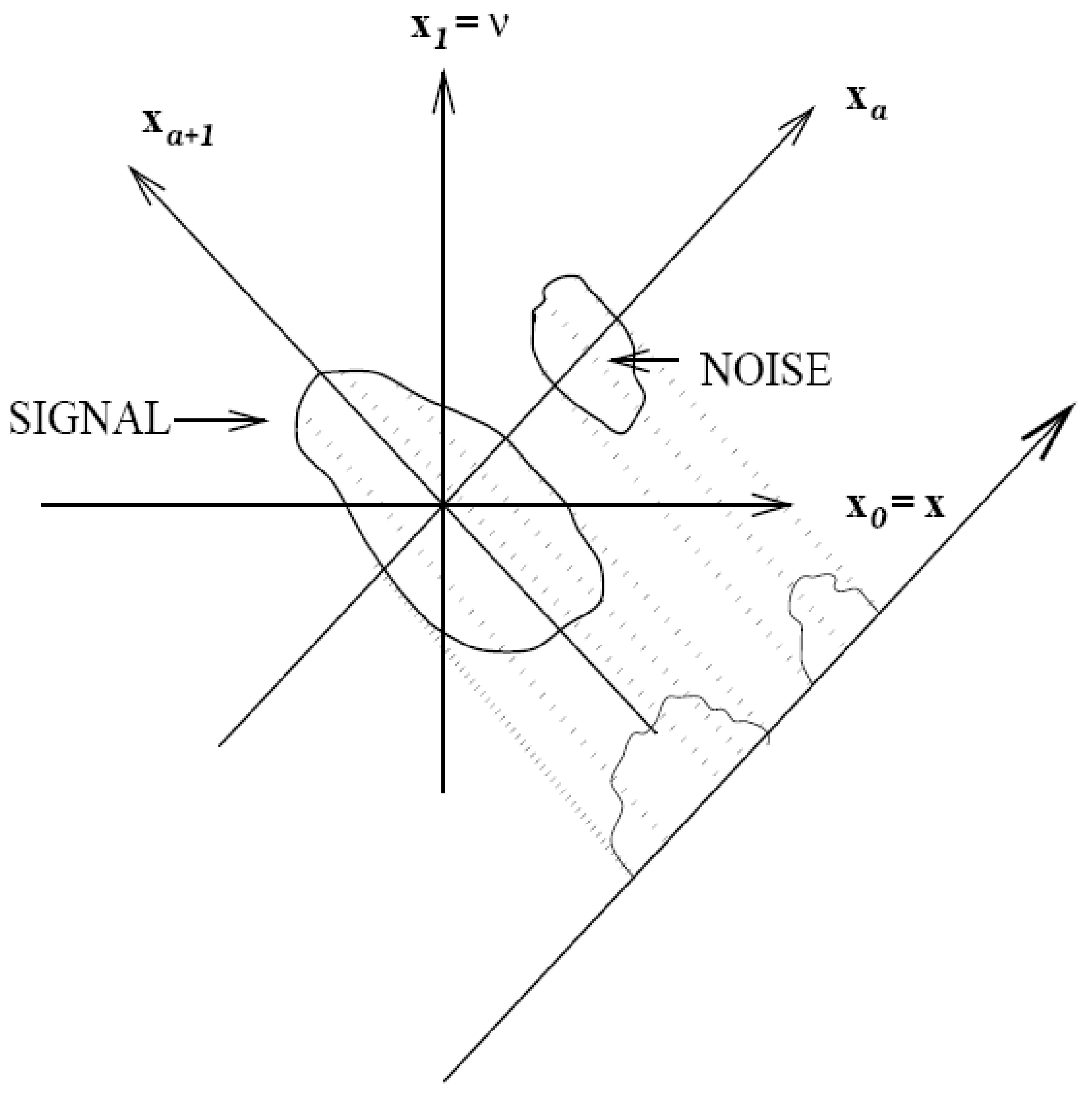
Optimal filtering in fractional order Fourier domain



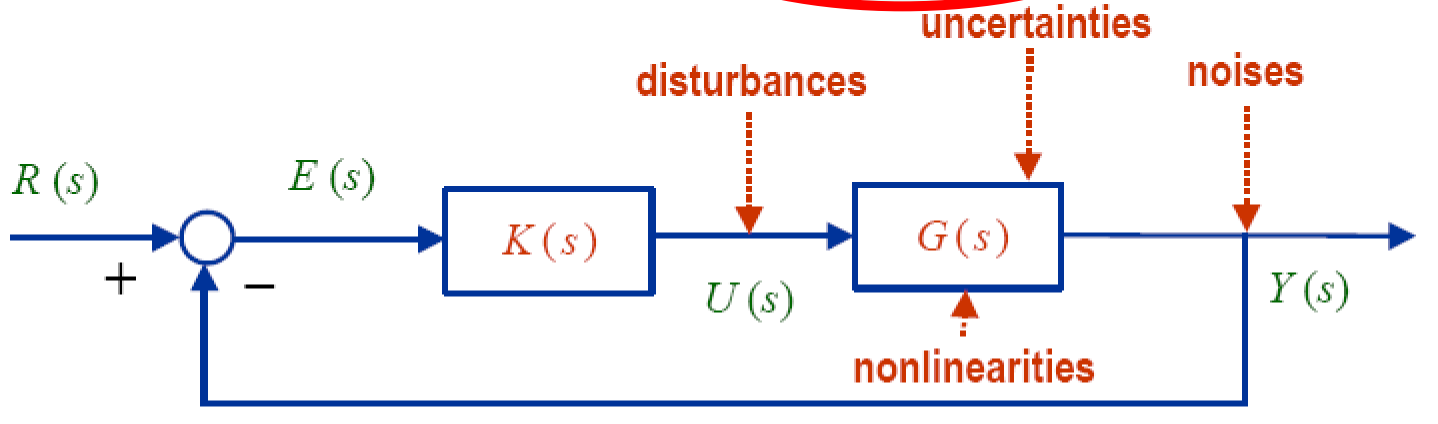
Slide credit: HALDUN M. OZAKTAS

Optimal filtering in fractional Fourier domain



FOMs and Fractional Order Controls

- **IO Controller + IO Plant**
- **FO Controller + IO Plant**
- **FO Controller + FO Plant**
- **IO Controller + FO Plant**



Concepcin A. Monje, YangQuan Chen, Blas Vinagre, Dingyu Xue and Vicente Feliu (2010). “Fractional Order Systems and Controls - Fundamentals and Applications.” Advanced Industrial Control Series, Springer-Verlag, www.springer.com/engineering/book/978-1-84996-334-3 (2010), 415 p. 223 ill.19 in color.

Rule of thumb for Fractional Order Thinking

- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- $1/f^{a}$ noise
- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …

Fractional Order Mechanics: WHY?

- Softmatter / hardmatter
- Softbody / Rigidbody
- Lumped / distributed
- Granular, particulate, porous, disordered … materials
- …

Soft matter?

- Soft matters, also known as ***complex fluids***, behave unlike ideal solids and fluids.
- ***Mesoscopic*** macromolecule rather than microscopic elementary particles play a more important role.

Typical soft matters

- Granular materials
- Colloids, liquid crystals, emulsions, foams,
- Polymers, textiles, rubber, glass
- Rock layers, sediments, oil, soil, DNA
- Multiphase fluids
- Biopolymers and biological materials

highly deformable, porous, thermal fluctuations play major role, highly unstable

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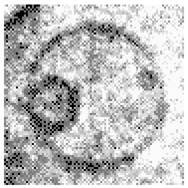
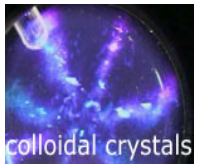
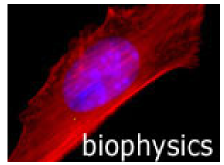
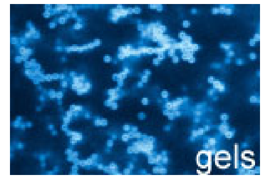
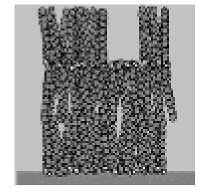
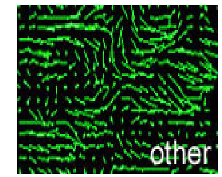
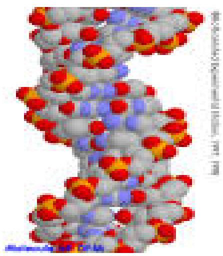



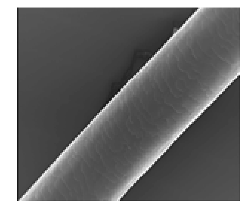
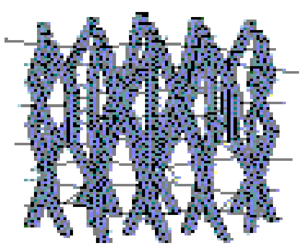
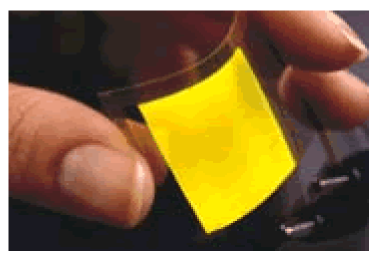
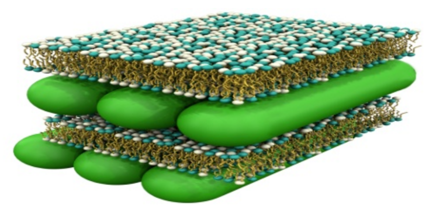








Slide credit: Wen Chen of HHU

Fractional Order Mechanics!

Hooke's law: $F = kx$

Newton's fluid: $F = kx'$

Newton's 2nd law: $F = kx''$

$\rightarrow F(t) = kx^{(\alpha)}(t)$

Going in-between: interpolation of operators:

$$\ldots, \quad \frac{d^{-2}f}{dt^{-2}}, \quad \frac{d^{-1}f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \ldots$$

G.W. Scott Blair (1950)

- "We may express our concepts in Newtonian terms if we find this convenient but, if we do so, we must realize that we have made a translation into a language which is foreign to the system which we are studying."

Outline of this talk

- **Fractional Calculus – What?**
- **Fractional Calculus – Why?**
- **Fractional Calculus – Who Cares?**
- **Take-Home Messages**

Fractional Calculus – Who Cares?

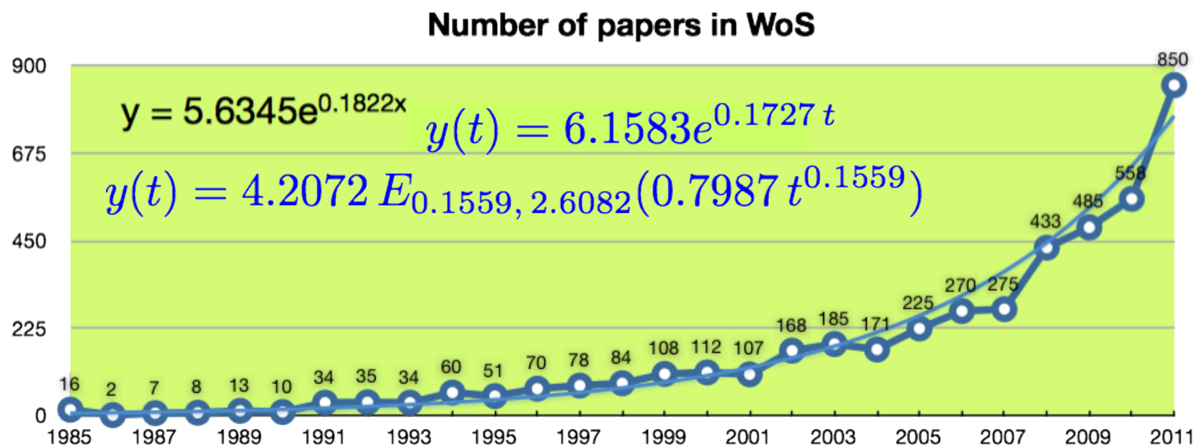
Answer: Everyone should.

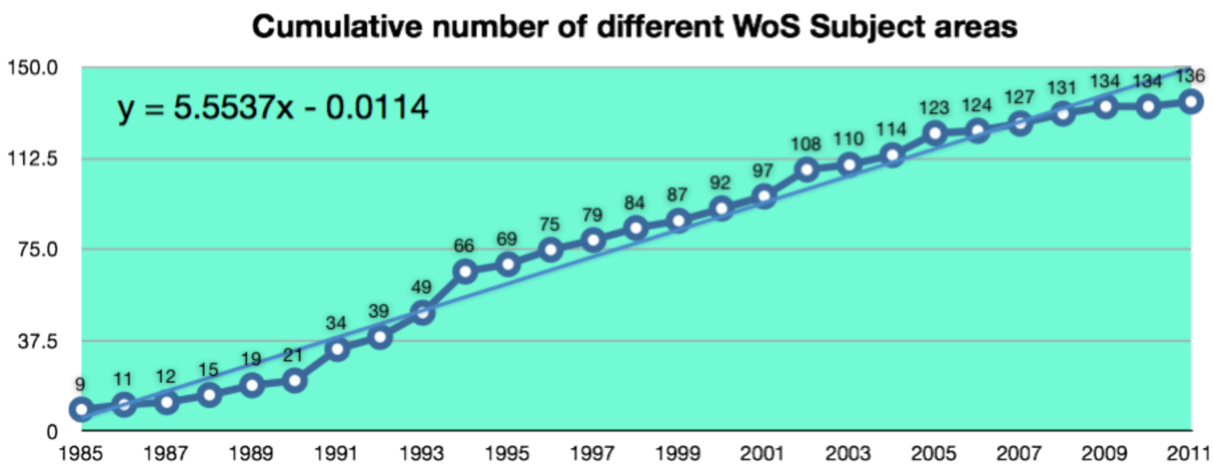
Fractional Calculus: a response to more advanced characterization of our more complex world at smaller scale

	1695	1960s — We are here
static models	dynamical models	fractional order modeling
geometry, algebra	differential and integral calculus	fractional calculus

Slide credit: Igor Podlubny

Rapid development and numerous applications





Slide credit: Igor Podlubny

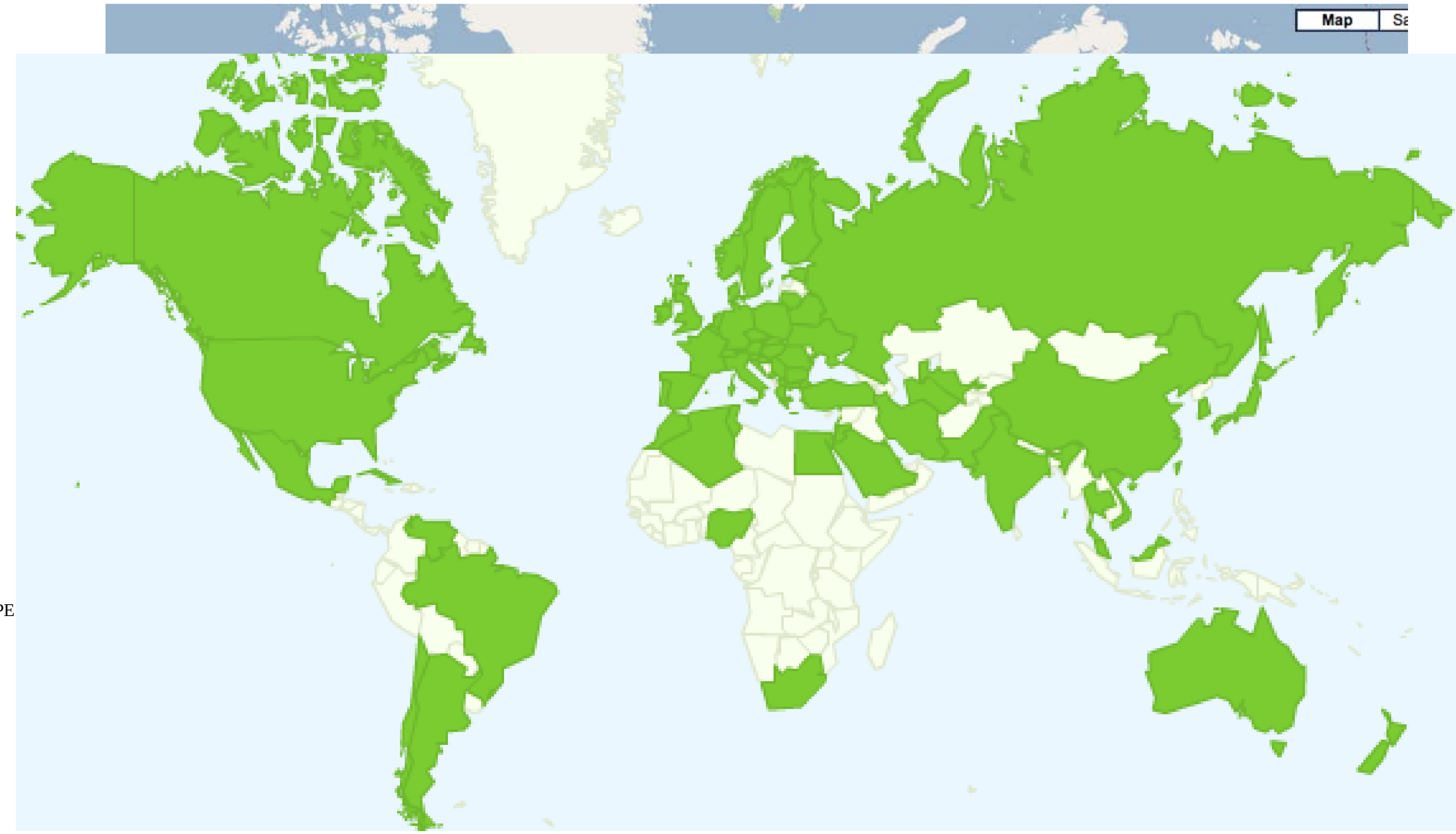
Fractional Calculus in WoK: 136 subject areas (applications)

ACOUSTICS
AGRICULTURAL ECONOMICS & POLICY
AGRICULTURAL ENGINEERING
AGRONOMY
ANESTHESIOLOGY
ASTRONOMY & ASTROPHYSICS
AUTOMATION & CONTROL SYSTEMS
BIOCHEMICAL RESEARCH METHODS
BIOCHEMISTRY & MOLECULAR BIOLOGY
BIOLOGY
BIOPHYSICS
BIOTECHNOLOGY & APPLIED MICROBIOLOGY
BUSINESS
BUSINESS, FINANCE
CARDIAC & CARDIOVASCULAR SYSTEMS
CELL BIOLOGY
CHEMISTRY, ANALYTICAL
CHEMISTRY, APPLIED
CHEMISTRY, INORGANIC & NUCLEAR
CHEMISTRY, MULTIDISCIPLINARY
CHEMISTRY, ORGANIC
CHEMISTRY, PHYSICAL
COMPUTER SCIENCE, ARTIFICIAL INTELLIGENCE
COMPUTER SCIENCE, CYBERNETICS
COMPUTER SCIENCE, HARDWARE & ARCHITECTURE
COMPUTER SCIENCE, INFORMATION SYSTEMS
COMPUTER SCIENCE, INTERDISCIPLINARY APPLICATIONS
COMPUTER SCIENCE, SOFTWARE ENGINEERING
COMPUTER SCIENCE, THEORY & METHODS
CONSTRUCTION & BUILDING TECHNOLOGY
CRIMINOLOGY & PENOLOGY
CRYSTALLOGRAPHY
DENTISTRY, ORAL SURGERY & MEDICINE
ECOLOGY
ECONOMICS
EDUCATION & EDUCATIONAL RESEARCH
EDUCATION, SCIENTIFIC DISCIPLINES
ELECTROCHEMISTRY
ENERGY & FUELS
ENGINEERING, AEROSPACE
ENGINEERING, BIOMEDICAL
ENGINEERING, CHEMICAL
ENGINEERING, CIVIL
ENGINEERING, ELECTRICAL & ELECTRONIC
ENGINEERING, ENVIRONMENTAL

Slide credit: Igor Podlubny

The current map of the fractional calculus

ALGERIA
ARGENTINA
AUSTRALIA
AUSTRIA
BELGIUM
BRAZIL
BULGARIA
BYELARUS
CANADA
CHILE
CROATIA
CUBA
CZECH REP
DENMARK
EGYPT
ENGLAND
ESTONIA
FINLAND
FRANCE
GERMANY
GREECE
GUADELOUPE
HUNGARY
INDIA
IRAN
IRELAND
ISRAEL
ITALY
JAPAN
JORDAN
KUWAIT
LEBANON
LITHUANIA
MACEDONIA
MALAYSIA
MEXICO
MOROCCO



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YUGOSLAVIA

Slide credit: Igor Podlubny

Data Source: Web of Knowledge

Rule of thumb for Fractional Order Thinking

WHEN?

- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- $1/f^{a}$ noise

- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …

Outline of this talk

- **Fractional Calculus – What?**
- **Fractional Calculus – Why?**
- **Fractional Calculus – Who Cares?**
- **Take-Home Messages**

Conclusions

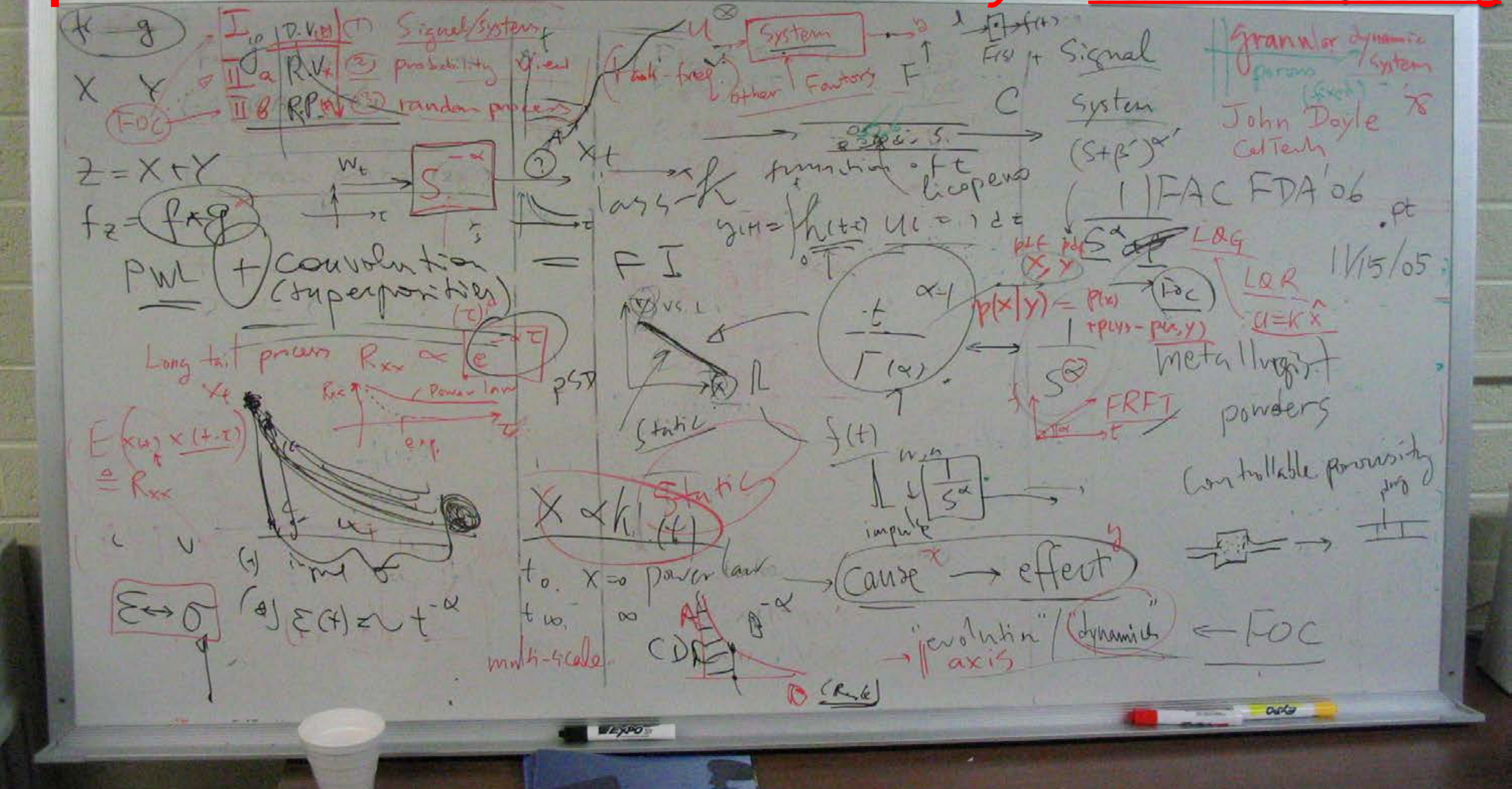
- 7/13/1865 - “Go west, young man. Go West and grow up with the country. ” – Horace Greeley (1811-1872)

http://upload.wikimedia.org/wikipedia/commons/1/12/American_progress.JPG

- “**Go Fractional. It’s urgent**!” – YangQuan Chen

Stop consuming too much information from others!

Spare some time to think … *fractionally* & achieve something



A snap shot of discussion board of Igor Podlubny and YangQuan Chen in Sept. 2005

Thank you for your attention!

Questions?

http://www.hub.sciverse.com/action/search/results?st=%22fractional%20order%22

Want more insights?

- Dr. Chen’s MTS (Mind, Technology Society) Seminar (view at https://vimeo.com/61141696)

Talk Title:

All Connected via Fractional Calculus:

Power Law, Scale-Free, Heavy-Tailedness, Long Range Dependence, Long Memory, and Complexity due to Fractional Dynamics

Fractional Calculus for High Schoolers?

- Working on that
  - Led by Igor Podlubny

From CO to VO to DO

CO

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)\, d\tau}{(t-\tau)^{\alpha-n+1}}, (n-1 \le \alpha < n) $$

VO

$$ {}_0^C D_t^{\alpha(t)} f(t) = \frac{1}{\Gamma(n-\alpha(t))} \int_0^t \frac{f^{(n)}(\tau)\, d\tau}{(t-\tau)^{\alpha(t)+1-n}}, \quad (n-1 \le \alpha(t) < n) $$

DO

$$ {}_aD_t^{\varphi(\alpha)} f(t) = \int_c^d \varphi(\alpha)\, {}_aD_t^{\alpha} f(t) d\alpha \qquad \int_c^d \varphi(\alpha)\, d\alpha = 1 $$

For characterizing scale-rich dynamics?