What is, why you need and who cares fractional calculus?

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Nov. 8, 2013. Friday 3:30PM-4:30PM
SIAM @ UC Merced
Room COB 265 3:30-4:30 PM
The MESA Lab

- Mechatronics, Embedded Systems, Automation Lab
- [http://mechatronics.ucmerced.edu](http://mechatronics.ucmerced.edu)
- Lab Manager: Brandon Stark
  - Lab Manager: Brandon Stark
  - 4 Ph.D. Students
  - 2 MSc Students
  - 20+ Undergrads
  - 2 Visiting Ph.D. Students
  - 2 Visiting Professors

- Unmanned Aerial Systems
- Cyber-Physical Systems
- Renewable Energy Systems
- Mechatronic Systems
- Applied Fractional Calculus

AFC Talk @ SIAM @ UC Merced  11/8/2013
MESA LAB
Mechatronics, Embedded Systems and Automation Lab

Real solutions for sustainability!
Established August 2012 @ Castle of 1500 sq ft
4 Ph.D/2 MS/ 20+ undergrad members, visiting scholars || Sponsored 2
capstone projects and mentored 4+1 capstone teams (F’13)

Education and Outreach Activities:
- AfterShock
- Academic Excellence Night
- Robotics Club tutorials/workshops
- Preview Day in Merced Mall
- “The Drone Age” @ Castle Air Museum
- Robots-n-Ribs| MESABox! ASME tutorials
- 6 capstone teams (24 seniors) …
- ME142 Mechatronics (take-home labs)
- ME280 Fractional Order Mechanics

Research Areas of Excellence:
(ISI H-index=29, Google H-index=49; i10-index=217)
- Unmanned Aerial Systems & UAV-based Personal Remote Sensing (PRS)
- Cyber-Physical Systems (CPS)
  - Mechatronics
  - Applied Fractional Calculus
  - Modeling and Control of Renewable Energy Systems

Projects Related to San Joaquin Valley:
- Energy [Solar energy, CPV, Building efficiency (HVAC lighting), smart grids integration, NG pipelines]
- Water (Water/soil salinity management, water sampling UAVs)
- Precision Ag/ Environment (Crop dynamics, optimal harvest, pest …)
UCCE + MESALAB = ?

http://cemerced.ucanr.edu/

+ http://mechatronics.ucmerced.edu/

= Ag Drone Valley (fractional)
MESA Lab Philosophy and Ambition

- "We make real systems that work and others want them."
- **MESA** Lab: Staying on top and working for sustainability.
- Nationally and internationally visible and prominent!
MESA Research Areas/Strengths

• Unmanned Aerial Systems and UAV-based Personal Remote Sensing (PRS)
• Cyber-Physical Systems (CPS)
• Modeling and Control of Renewable Energy Systems
• Mechatronics

• Applied Fractional Calculus (AFC)
My courses at UC Merced

• Spring 2013
  – ME142 Mechatronics (4cr) (48 seats)

• Fall 2013
  – ME280 Fractional Order Mechanics (3cr) (10 seats) (why/what/when?)
Mechatronics has good prospects for the future because knowledge economy demands to speed up development, improve quality, reduce cost and increase energy efficiency.

Kevin Craig, Marquette Univ.

11/8/2013
Outline of this talk

• Fractional Calculus – What?
• Fractional Calculus – Why?
• Fractional Calculus – Who Cares?
• Take-Home Messages
... from integer to non-integer ...

\[ x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_n \]

\[ x^n = e^{n \ln x} \]

\[ n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n - 1) \cdot n, \]

\[ \Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \quad x > 0, \]

\[ \Gamma(n + 1) = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n = n! \]
... from integer to non-integer ...
“Fractional Order Thinking”
or, “In Between Thinking”

• For example
  – Between integers there are non-integers;
  – Between logic 0 and logic 1, there is the “fuzzy logic”;
  – Between integer order splines, there are “fractional order splines”
  – Between integer high order moments, there are noninteger order moments (e.g. FLOS)
  – Between “integer dimensions”, there are fractal dimensions
  – Fractional Fourier transform (FrFT) – in-between time-n-freq.
  – Non-Integer order calculus (fractional order calculus – abuse of terminology.) (FOC)
Conclusion of Talk

Integer-Order Calculus

Fractional-Order Calculus

Slide credit: Richard L. Magin, ICCC12
• Integer-Order Calculus

• Fractional-Order Calculus

Discrete gears vs. constantly-variable transmission


Slide credit: Calvin Coopmans, 2/28/2013 email comment

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Riemann–Liouville definition

\[ aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \left( \frac{d}{dt} \right)^n \int_a^t f(\tau) \frac{d\tau}{(t-\tau)^{\alpha-n+1}} \]

\[(n - 1 \leq \alpha < n)\]

G.F.B. Riemann  J. Liouville
(1826–1866)  (1809–1882)
Grünwald–Letnikov definition

\[ a D_t^\alpha f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left\lfloor \frac{t-a}{h} \right\rfloor} (-1)^j \binom{\alpha}{j} f(t - jh) \]

\([x]\) – integer part of \(x\)

A.K. Grünwald  A.V. Letnikov

Slide credit: Igor Podlubny
G. W. Leibniz (1695–1697)

In the letters to J. Wallis and J. Bernulli (in 1697) Leibniz mentioned the possible approach to fractional-order differentiation in that sense, that for non-integer values of $n$ the definition could be the following:

$$\frac{d^n e^{mx}}{dx^n} = m^n e^{mx},$$

L. Euler (1730)

$$\frac{d^n x^m}{dx^n} = m(m-1) \ldots (m-n+1)x^{m-n}$$

$$\Gamma(m+1) = m(m-1) \ldots (m-n+1) \Gamma(m-n+1)$$

$$\frac{d^n x^m}{dx^n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)} x^{m-n}.$$

Euler suggested to use this relationship also for negative or non-integer (rational) values of $n$. Taking $m = 1$ and $n = \frac{1}{2}$, Euler obtained:

$$\frac{d^{1/2} x}{dx^{1/2}} = \sqrt{\frac{4x}{\pi}} \left( = \frac{2}{\sqrt{\pi}} x^{1/2} \right)$$

Slide credit: Igor Podlubny
Operator $aD_t^\alpha$

A generalization of differential and integral operators:

$$aD_t^\alpha = \begin{cases} 
\frac{d^\alpha}{dt^\alpha} & \Re(\alpha) > 0, \\
1 & \Re(\alpha) = 0, \\
\int_a^t (d\tau)^{-\alpha} & \Re(\alpha) < 0.
\end{cases}$$

(7)

There are two commonly used definitions for the general fractional order differentiation and integral, i.e., the Grünwald-Letnikov definition and the Riemann-Liouville definition.
Example: Heaviside’s unit step

Example: $\sin(t)$

Fractional derivatives of function $y = H(t)$

Fractional derivatives of function $y = \sin(t)$

Slide credit: Igor Podlubny

11/8/2013
Fractional derivatives of ramp function.
Fractional Calculus was born in 1695

\[ \frac{d^n f}{dt^n} \]

What if the order will be \( n = \frac{1}{2} \)?

It will lead to a paradox, from which one day useful consequences will be drawn.

G.F.A. de L’Hôpital (1661–1704)

G.W. Leibniz (1646–1716)
Useful consequences
Useful consequences
Outline of this talk

• Fractional Calculus – What?
• Fractional Calculus – Why?
• Fractional Calculus – Who Cares?
• Take-Home Messages
More Optimal Image Processing by Fractional Order Differentiation and Fractional Order Partial Differential Equations

Dali Chen, Dingyu Xue, YangQuan Chen

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Who cares?

- Minimal dose biomedical imaging
- More optimal

Strategies for Reducing Radiation Dose in CT (McCollough 2009)

FC for what?

- Better than the best
- New sciences
- Need killer apps.
Take home message:

More optimal image processing can be made possible by using fractional order differentiation and fractional order partial differential equations.

Want to be more optimal? Go fractional calculus!

More info:
http://mechatronics.ucmerced.edu/research/applied-fractional-calculus
Optimal filtering in fractional order Fourier domain

- Original signal
- Distorted
- Estimate by filtering in fractional domain
- Estimate by filtering in Fourier domain
Optimal filtering in fractional Fourier domain
FOMs and Fractional Order Controls

- IO Controller + IO Plant
- FO Controller + IO Plant
- FO Controller + FO Plant
- IO Controller + FO Plant


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Rule of thumb for Fractional Order Thinking

- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- $1/f^a$ noise

- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (biox) …
Fractional Order Mechanics: WHY?

- Softmatter / hardmatter
- Softbody / Rigidbody
- Lumped / distributed
- Granular, particulate, porous, disordered … materials
- …
Soft matter?

• Soft matters, also known as *complex fluids*, behave unlike ideal solids and fluids.

• *Mesoscopic* macromolecule rather than microscopic elementary particles play a more important role.
Typical soft matters

- Granular materials
- Colloids, liquid crystals, emulsions, foams,
- Polymers, textiles, rubber, glass
- Rock layers, sediments, oil, soil, DNA
- Multiphase fluids
- Biopolymers and biological materials

highly deformable, porous, thermal fluctuations play major role, highly unstable
Fractional Order Mechanics!

Hooke’s law: \[ F = kx \]

Newton’s fluid: \[ F = kx' \]

Newton’s 2nd law: \[ F = kx'' \]

Going in-between: interpolation of operators:

\[ \ldots, \frac{d^{-2}f}{dt^{-2}}, \frac{d^{-1}f}{dt^{-1}}, f, \frac{df}{dt}, \frac{d^2f}{dt^2}, \ldots \]
G.W. Scott Blair (1950)

• “We may express our concepts in Newtonian terms if we find this convenient but, if we do so, we must realize that we have made a translation into a language which is foreign to the system which we are studying.”
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• Take-Home Messages
Fractional Calculus – Who Cares?

Answer: Everyone should.
**Fractional Calculus: a response to more advanced characterization of our more complex world at smaller scale**

<table>
<thead>
<tr>
<th>1695</th>
<th>1960s</th>
<th><strong>We are here</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>static models</td>
<td>dynamical models</td>
<td>fractional order modeling</td>
</tr>
<tr>
<td>geometry, algebra</td>
<td>differential and integral calculus</td>
<td>fractional calculus</td>
</tr>
</tbody>
</table>

Slide credit: Igor Podlubny
Rapid development and numerous applications

Number of papers in WoS

\[ y = 5.6345e^{0.1822x} \]
\[ y(t) = 6.1583e^{0.1727t} \]
\[ y(t) = 4.2072 E_{0.1559, 2.6082}(0.7987 t^{0.1559}) \]

Cumulative number of different WoS Subject areas

\[ y = 5.5537x - 0.0114 \]

11/8/2013
Fractional Calculus in WoK: 136 subject areas (applications)
The current map of the fractional calculus

Slide credit: Igor Podlubny

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Conclusions

• 7/13/1865 - “Go west, young man. Go West and grow up with the country.” – Horace Greeley (1811-1872)

http://upload.wikimedia.org/wikipedia/commons/1/12/American_progress.JPG

• “Go Fractional. It’s urgent!” – YangQuan Chen
Stop consuming too much information from others!
Spare some time to think ... fractionally & achieve something
Thank you for your attention!

Questions?

http://www.hub.sciverse.com/action/search/results?st=%22fractional%20order%22
Want more insights?

• Dr. Chen’s MTS (Mind, Technology Society) Seminar (view at https://vimeo.com/61141696)

Talk Title:

All Connected via Fractional Calculus: Power Law, Scale-Free, Heavy-Tailedness, Long Range Dependence, Long Memory, and Complexity due to Fractional Dynamics
Fractional Calculus for High Schoolers?

- Working on that
  - Led by Igor Podlubny
From CO to VO to DO

**CO**
\[ a D_t^\alpha f(t) = \frac{1}{\Gamma(n - \alpha)} \left( \frac{d}{dt} \right)^n \int_t^\infty \frac{f(\tau) d\tau}{(t-\tau)^{\alpha-n+1}}, \quad (n - 1 \leq \alpha < n) \]

**VO**
\[ C_0 D_t^{\alpha(t)} f(t) = \frac{1}{\Gamma(n - \alpha(t))} \int_0^t \frac{f^{(n)}(\tau) d\tau}{(t-\tau)^{\alpha(t)+1-n}}, \quad (n - 1 \leq \alpha(t) < n) \]

**DO**
\[ a D_t^{\varphi(\alpha)} f(t) = \int_c^d \varphi(\alpha) a D_t^\alpha f(t) d\alpha \quad \int_c^d \varphi(\alpha) d\alpha = 1 \]

For characterizing scale-rich dynamics?

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