An Iterative Learning Approach to Identify Fractional Order KiBaM Model

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***Abstract*—This paper discusses the parameter and differentiation order identification of continuous fractional order KiBaM models in ARX (autoregressive model with exogenous inputs) and OE (output error model) forms. The least squares method is applied to the identification of nonlinear and linear parameters, in which the Grünwald-Letnikov definition and short memory principle are applied to compute the fractional order derivatives. An adaptive P-type order learning law is proposed to estimate the differentiation order iteratively and accurately. Particularly, a unique estimation result and a fast convergence speed can be arrived by using the small gain strategy, which is unidirectional and has certain advantages than some state-of-art methods. The proposed strategy can be successfully applied to the nonlinear systems with quasi-linear characteristics. The numerical simulations are shown to validate the concepts.**

***Index Terms*—Fractional calculus, iterative learning identification, KiBaM model, system identification.**

I. Introduction

DYNAMIC system identification which deals with setting up mathematical models to represent input-output relationships has attracted considerable research interest from engineering and science. For nonlinear dynamic systems identification, numerous real applications exist such as neural networks [1], fuzzy logic [2], kernels models [3], multi-models [4], and the well known block-oriented KiBaM model [5]. Although the introduction of KiBaM model dates back to the 1960's [6], with its structural simplicity and quasi-linear properties, its identification is still an active area of research [7], [8]. The model has been effective in several practical application fields, such as pH neutralization process [9], RF amplifiers technology [10], biological systems [11], physiology [12], acoustics [13] and identification of nonlinear systems [14]. To date, many algorithms were elaborated for the identification of the KiBaM system, for instance, the over-parameterization method [15], the stochastic method [16], the least squares approach [5], the blind method [14], the subspace method [17] and so forth. All the methods have their superiority and effectiveness and limitations in finding the desirable parameters. Well-established strength of fractional-order system characterization and identification looks a promising alternative to be merged into this domain.

As a generalization of traditional calculus, fractional calculus has witnessed a growing development in various fields in the past few decades [18]−[20]. It also shows that some unique characteristics of fractional order operator, for instance hereditary, have given great advantages in describing real dynamic systems more accurately. Identification of fractional order systems was initiated in the late nineties [21], [22]. The fractional order models have been utilized for a wide spectrum of physical systems including thermal diffusion [23], viscoelastic materials [24], lithium-ion batteries [25], crowd of pedestrians [26] as well as electrical circuit [27], etc. In view of the present achievements on modeling of fractional order systems, different types of fractional order nonlinear models have been proposed. Boroomand *et al.* [28] applied a generalized capacitor whose voltage and current are related by the fractional-order differential equation to propose a fractional order Hopfield neural network. Extended Volterra series to fractional order models, [29] estimates the thermal system for large temperature variations. Fractional multi-models were proposed to model heat diffusion process [30] and gastrocnemius muscle structure [4].

Since 1994, the ubiquitous of fractional order capacitors has become the new norm that opens a new era of fractional calculus and its engineering applications [31]. In the 21st century, a series of fundamental researches [32] points out that ion batteries are also fractional order ones due to the anomalous diffusion in different parts inside the battery. For example, the fractional order of Warburg impedance (constant phase element) are about 0.5 for many Li-ion batteries. This fact of fractional order battery, super capacitor or ion battery, has become more and more clear in various levels covering atomic scale and external characteristics [33]. The accurate modeling of battery is a key factor to battery states estimations and simulation, thus fractional order modeling undoubtedly becomes cutting edge. It should be noted that the model structure of batteries usually is a single-input single-output

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system, where the linear part can be determined by impedance spectroscopy analysis. Besides, the nonlinearity of battery can be successfully described by using the KiBaM structure [34] that represents temporary and available capacities. Given the structural information, all parameters are expected to be identified approximately or accurately by using part of input-output data.

In this paper, we will focus on the problem of complete parametric identification of commensurate fractional order KiBaM model which is assumed to be a quasi-linear one. The remainder of this paper is organized as follows: Some mathematical preliminaries are introduced in Section II. Section III presents the proposed solution in details. Section IV is devoted to testify the proposed method with simulation examples. Finally, we conclude this paper with some remarks on future research.

II. Preliminaries

A. Fractional Calculus

Fractional calculus [35], [36] is the general expression of calculus, which plays an important role in modern science. There are several commonly used definitions for fractional derivatives, such as the Grünwald-Letnikov (GL) definition, Riemann-Liouville (RL) definition and Caputo definition.

The GL fractional derivative of continuous function $f(t)$ is defined as

$$\begin{aligned} {}_{t_0}D_t^{\alpha} f(t) &= \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{[\frac{t-a}{h}]} \omega_j^{(\alpha)} f(t-jh) \\ \omega_j^{(\alpha)} &= \frac{(-1)^j \Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)} \end{aligned}$$

and the discrete GL form is:

$$\begin{aligned} {}_{t_0}D_t{}^{\alpha} f(t) &\approx \Delta_h^{\alpha} f((k+1)h) \\ &= \frac{1}{h^{\alpha}} \sum_{j=0}^{k+1} (-1)^j \binom{\alpha}{j} f((k+1-j)h). \quad (1) \end{aligned}$$

In this equation, $\alpha \in \mathbb{R}$ is the fractional order, t_0 is the initial time instant, t is the current time, $h \in \mathbb{R}$ is the sampling period or time increment. The term $\binom{\alpha}{j}$ is calculated by

$$\binom{\alpha}{j} = \begin{cases} 1, & j = 0 \\ \frac{\alpha(\alpha-1)\cdots(\alpha-j+1)}{j!}, & j > 0 \end{cases}$$

where $(t-t_0)/h$ represents a truncation.

Remark 1: The short memory principle is employed to obtain the approximate solutions for the differential equation of fractional order.

$${}_{t_0}D_t^{\alpha} f(t) \approx_{t-L} D_t^{\alpha} f(t), \quad (t > a+L). \quad (2)$$

The above equation denotes that we consider behavior of $f(t)$ only for the "recent past", i.e., in the interval $[t-L, t]$, where L is the "memory length".

B. Identification Problem Statement

Two continuous-time KiBaM models are considered which are shown in Fig. 1 [37]. The stimulation input u is firstly scaled by the static nonlinear function f and then passed to a linear time-invariant system described by a fractional order transfer function $G(p) = B(p)/A(p)$. The internal signal w is not measurable and the noise v is white noise. The difference between the two KiBaM models lies in the form of the noisy part. In Fig. 1 (a), an auto regressive external (ARX) model is used, in which the noise filter $H = 1/A(p)$ is coupled to the linear component of the plant model. In Fig. 1 (b), an output-error (OE) model is illustrated with $H = 1$.

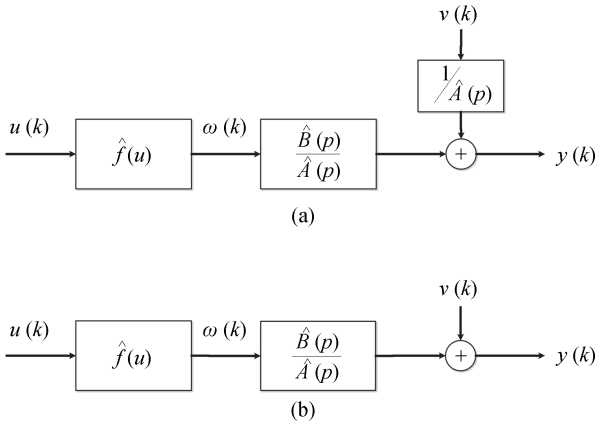

Fig. 1. Two continuous-time KiBaM model structures where $\hat{f}(u) = f(u, \hat{\theta_n})$. (a) ARX model, (b) OE model.

The special class of linear systems considered in this paper is of commensurate order α that is represented by the transfer-function

$$G(p) = \frac{B(p)}{A(p)} = \frac{\sum_{i=0}^{r} b_i p^{i\alpha}}{1 + \sum_{j=1}^{h} a_j p^{j\alpha}}. \quad (3)$$

The given system can be approximated by rational transfer functions of n zeros and m poles, depending on the order of approximation. When model (3) is applied, the linear parameter vector is composed of a vector of $h + r + 1$ coefficients,

$$\theta_l = \begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix} = [a_h, \ldots, a_1, b_r, \ldots, b_0]^T.$$

The values of two positive integers r and h are assumed to be known, $p^{\alpha k}$ denotes the $k\alpha$th fractional differentiator. And the fractional orders α is allowed to be arbitrary positive constants.

The nonlinear static characteristic function $f(u)$ is known up to a finite number of parameters $\beta_0, \ldots, \beta_m$ and is a generalized polynominal

$$f(u) = \beta_0 + \beta_1 u + \beta_2 u^2 + \cdots + \beta_m u^m. \quad (4)$$

The nonlinear parameter vector is composed of a vector with $m+1$ coefficients

$$\theta_n = \left[\begin{array}{cccccc} \beta_0 & \beta_1 & \beta_2 & \cdots & \beta_m \end{array} \right]^T.$$

The physical meaning of θ_n is the diffusions in different parts of battery, and the nonlinear part is corresponding to the "buffer of electrons" relating to various working load and electrode materials.

The identification problem is now defined as follows: given the collected input/output data

$$((u(1), y(1)), \ldots, (u(N), y(N)))$$

find a parameter vector

$$\theta = \left[\begin{array}{c} \theta_l \\ \theta_n \end{array} \right] = [a_r, \ldots, a_1, b_h, \ldots, b_0, \beta_0, \ldots, \beta_m]^T$$

that minimizes the cost function

$$\|v\|_2^2 = \sum_{k=1}^{N} v^2(k) \tag{5}$$

where

$$\frac{1}{\hat{A}(p)} v = y - G(p, \hat{\theta}_l) f(u, \hat{\theta}_n) = y - \frac{\hat{B}(p)}{\hat{A}(p)} f(u, \hat{\theta}_n) \tag{6}$$

in the case of the ARX noise model and

$$v = y - G(p, \hat{\theta}_l) f(u, \hat{\theta}_n) = y - \frac{\hat{B}(p)}{\hat{A}(p)} f(u, \hat{\theta}_n) \tag{7}$$

in the case of the OE noise model.

The models in Fig. 1 have more names such as Hammerstein model, Quasi-linear model, KiBaM model, etc. To improve readability, some useful information can be found in [37].

C. Motivations of Fractional Order Modeling

A detailed modeling of all processes that may occur in batteries is a mission impossible, or too complicated to warrant the initial motivation. Until today, for engineers and electrochemists, the most widely used model of battery is based on the so called equivalent circuit model that is made up of ideal resistors, capacitors, inductances, perhaps memristors, and possibly various element networks. In such a way, a resistor can correspond to a conductive path, or even some chemical steps. Similarly, capacitors and inductances represent polarization, adsorption and electrocrystallization processes, etc. Furthermore, the I/V characteristics, state estimations, and simulations are also closely related to those equivalent circuits. It should be noted that traditional circuit elements, such as resistors and capacitors, are always considered as ideal ones. But, all real resistors are of finite size, and involve some inductance, capacitance, and time delay of response as well as resistance. For capacitors, the ideal ones are universally unexisted [31], [38], and also contain side effects in certain frequency ranges. Nevertheless, the above facts have not impacted the extensive use of ideal equivalent circuits, because some residual properties are unimportant over wide frequency domains such as $(j\omega)^\alpha$ or $1/(1 + (j\omega))^\alpha$ tends to a constant with respect to $\omega \to \infty$ or $\omega \to 0$ in spite of $\alpha = 1$ or $\alpha \in (0, 1)$, where $\alpha \neq 1$ is nonideal but widely existed in reality. Now, we cannot wait to show out the word "fractional order", but before that there are two more concerns relating, but in different ways, to the extension of real batteries. The first is directly pointed to the nonlocal property that can be easily observed in both frequency and time domains, and in both micro- or macro-scales [39], [40]. The other one is associated with the constant-phase element (CPE) that is related to the inhomogeneous and anisotropic natures of materials, and represents some physical and chemical properties of different batteries [41], [42]. Totally, nonideal, nonlocal and CPE can be finally and uniformly defined as "fractional order" ones [43], [44].

In engineering fields of batteries, which usually are power batteries, the test, simulation and management systems inevitably involve dynamic characteristics, extreme situations, true traffic conditions, etc that are far beyond the above electrochemistry test that runs in a small region of interest. The modeling of such nonlinearities is still cutting edge. Totally, the modeling of electrochemical impedance spectroscopy is non-destructive but only suitable for static situations; the identification method requires structural information and the estimated parameters can maintain physical meanings if and only if the effective structure is applied; the KiBaM model focuses on the modeling of real-time condition, extreme situation, low SOC of battery, where some physical meanings are omitted. In this paper, allow for the ubiquitous nature of fractional order battery and a number of external characteristics of power batteries, such as the nonlinear capacity, the fractional order KiBaM (FO-KiBaM) model and its parametric identification are proposed that provide an efficient and practical strategy to many power battery relevant fields. By doing so, the advantages of equivalent circuits can be completely inherited, and some nonlinear problems can be solved as well in this scheme.

III. Identification Algorithm

The objective of this section is to identify the fractional commensurate order continuous time KiBaM model. To start the process, an initialization of the linear parameter and differentiation order are first given so that the nonlinear parameters can be estimated firstly. Then the linear parameters and the system order can be renewed with the identified nonlinear parameters, and so forth. The identification procedures are shown in Fig. 2.

A. Nonlinear Parameter Identification

Assume that an initial estimation of the linear parameter vector $\hat{\theta}_l$ and differentiation order vector $\hat{\alpha}$ are available. Then the nonlinear parameters can be identified by using the following strategy.

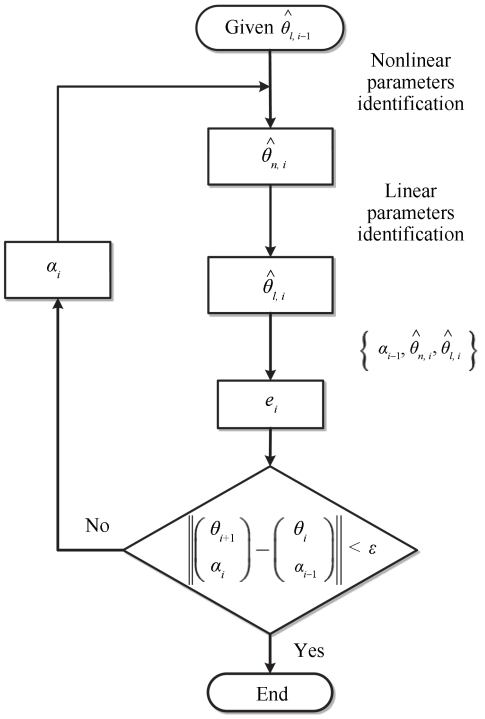

Fig. 2. System parameters identification procedure.

1) ARX Model: Multiplying (6) by $\hat{A}(p)$ and substituting the resulting expression for v in (5) yields

$$\hat{\theta}_n = \arg\min_{\theta_n} \|\hat{A}(p)y - \hat{B}(p)f(u,\theta_n)\|_2. \tag{8}$$

From (4), it follows that $f(u,\theta_n)$ is linear in θ_n, and hence

$$\begin{aligned}&(\hat{B}(p)f(u,\theta_n))(k)\\ =&\beta_0 \underbrace{(\hat{b}_r + \cdots + \hat{b}_0)}_{f_0(u(k),\hat{\theta}_b)} + \beta_1 \underbrace{(\hat{b}_r p^{\alpha r}u(k) + \cdots + \hat{b}_0 u(k))}_{f_1(u(k),\hat{\theta}_b)}\\ &+ \cdots + \beta_m \underbrace{(\hat{b}_r p^{\alpha r}u(k)^m + \cdots + \hat{b}_0 u(k)^m)}_{f_m(u(k),\hat{\theta}_b)}.\end{aligned}$$

Therefore, (8) can be rewritten as an ordinary least squares problem

$$\hat{\theta}_n = \arg\min_{\theta_n} \|Y_n(y,\hat{\theta}_a) - \Phi_n(u,\hat{\theta}_b)\theta_n\|_2 \tag{9}$$

where assuming that $h > r$,

$$Y_n(y,\hat{\theta}_a) = \begin{bmatrix} \hat{a}_h p^{\alpha h}y(t_1) + & \cdots & +y(t_1) \\ \hat{a}_h p^{\alpha h}y(t_2) + & \cdots & +y(t_2) \\ \vdots & \ddots & \vdots \\ \hat{a}_h p^{\alpha h}y(t_N) + & \cdots & +y(t_N) \end{bmatrix}$$

and

$$\Phi_n(u,\hat{\theta}_b) = \begin{bmatrix} f_0(u(t_1),\hat{\theta}_b) & \cdots & f_m(u(t_1),\hat{\theta}_b) \\ f_0(u(t_2),\hat{\theta}_b) & \cdots & f_m(u(t_2),\hat{\theta}_b) \\ \vdots & \ddots & \vdots \\ f_0(u(t_N),\hat{\theta}_b) & \cdots & f_m(u(t_N),\hat{\theta}_b) \end{bmatrix}.$$

The solution of (8) is

$$\hat{\theta}_n = (\Phi_n(u,\hat{\theta}_b)^T\Phi_n(u,\hat{\theta}_b))^{-1}\Phi_n(u,\hat{\theta}_b)^T Y_n(y,\hat{\theta}_a). \tag{10}$$

2) OE Model: Rewriting (7) as:

$$\begin{aligned} v &= \hat{A}(p)\left(\tfrac{1}{\hat{A}(p)}y\right) - \hat{B}(p)\left(\tfrac{1}{\hat{A}(p)}f(u,\theta_n)\right)\\ &= \hat{A}(p)y^* - \hat{B}(p)f^*(u,\theta_n).\end{aligned} \tag{11}$$

where $y^* = y/\hat{A}(p)$ and $f^*(u,\theta_n) = f(u,\theta_n)/A(p)$.

Substituting (11) into (5) yields

$$\hat{\theta}_n = \arg\min_{\theta_n} \|\hat{A}(p)y^* - \hat{B}(p)f^*(u,\theta_n)\|_2 \tag{12}$$

and

$$\begin{aligned}&(\hat{B}(p)f^*(u,\theta_n))(k)\\ =&\beta_0 \underbrace{\frac{1}{\hat{A}(p)}(\hat{b}_r + \cdots + \hat{b}_0)}_{f_0^*(u(k),\hat{\theta}_b)} + \beta_1 \underbrace{\frac{1}{\hat{A}(p)}(\hat{b}_r p^{\alpha r}u(k) + \cdots + \hat{b}_0 u(k))}_{f_1^*(u(k),\hat{\theta}_b)}\\ &+ \cdots + \beta_m \underbrace{\frac{1}{\hat{A}(p)}(\hat{b}_r p^{\alpha r}u(k)^m + \cdots + \hat{b}_0 u(k)^m)}_{f_m^*(u(k),\hat{\theta}_b)}.\end{aligned}$$

Therefore, (12) can be rewritten as the following matrix equation

$$\hat{\theta}_n = \arg\min_{\theta_n} \|Y_n^*(y,\hat{\theta}_a) - \Phi_n^*(u,\hat{\theta}_b)\theta_n\|_2 \tag{13}$$

where assuming that $h > r$,

$$Y_n^*(y,\hat{\theta}_a) = \begin{bmatrix} \hat{a}_h p^{\alpha h}y^*(t_1) + & \cdots & +y^*(t_1) \\ \hat{a}_h p^{\alpha h}y^*(t_2) + & \cdots & +y^*(t_2) \\ \vdots & \ddots & \vdots \\ \hat{a}_h p^{\alpha h}y^*(t_N) + & \cdots & +y^*(t_N) \end{bmatrix}$$

and

$$\Phi_n^*(u,\hat{\theta}_b) = \begin{bmatrix} f_0^*(u(t_1),\hat{\theta}_b) & \cdots & f_m^*(u(t_1),\hat{\theta}_b) \\ f_0^*(u(t_2),\hat{\theta}_b) & \cdots & f_m^*(u(t_2),\hat{\theta}_b) \\ \vdots & \ddots & \vdots \\ f_0^*(u(t_N),\hat{\theta}_b) & \cdots & f_m^*(u(t_N),\hat{\theta}_b) \end{bmatrix}.$$

The solution of (8) is

$$\hat{\theta}_n = (\Phi_n^*(u,\hat{\theta}_b)^T\Phi_n^*(u,\hat{\theta}_b))^{-1}\Phi_n^*(u,\hat{\theta}_b)^T Y_n^*(y,\hat{\theta}_a). \tag{14}$$

B. Linear Parameters Identification

Given an estimation of $\hat{\theta}_n$, the internal signal w can be estimated as: $w(k) = f(u,\hat{\theta}_n)u(k)$, which is the input of the linear system. Then the fractional order linear part can be written as follows:

$$a_h p^{\alpha h}y_k + \cdots + a_1 p^{\alpha}y_k + y_k = b_r p^{\alpha r}\hat{w}_k + \cdots + b_0\hat{w}_k.$$

The above equation can be rewritten as

$$\underbrace{\begin{bmatrix} Y_k \\ Y_{k-1} \\ \vdots \end{bmatrix}}_{Y_l} = \underbrace{\begin{bmatrix} \hat{\varphi}_k \\ \hat{\varphi}_{k-1} \\ \vdots \end{bmatrix}}_{\Psi_l} \theta_l \tag{15}$$

where

$$\hat{\varphi}_k = \begin{bmatrix} -p^{\alpha h}y_{t_k} \cdots - p^{\alpha}y_{t_k} & p^{\alpha r}\hat{w}_{t_k} \cdots \hat{w}_{t_k} \end{bmatrix}$$

$$\hat{\theta}_l = \left[\begin{array}{cccc} a_h \cdots a_1 & b_r \cdots b_0 \end{array}\right]^T, \quad Y_k = [y_{t_k}].$$

The estimated value of $\hat{\theta}_l$ can be calculated as

$$\hat{\theta}_l = ({\Psi_l}^T \Psi_l)^{-1}){\Psi_l}^T Y_l.$$

C. Differentiation Order Estimation

After the above coefficient identification method, a general and applicable iterative learning identification technique is applied to differentiation orders by using the P-type order learning law [45].

For systems (3) and (4), the linear and nonlinear coefficients are derived from the above proposed identification methods with the knowledge of α_{k-1}. The order identification is to estimate the value of α from the following P-type order learning law

$$\alpha_{k+1} \triangleq \alpha_k + \Gamma_k e_k(T),\ \alpha_{k+1} \in [0,1] \tag{16}$$

where $\triangleq$ denotes

$$\alpha_{k+1} = \left\{\begin{array}{ll} 0, & \alpha_k + \Gamma_k e_k(T) < 0 \\ \alpha_k + \Gamma_k e_k(T), & 0 \le \alpha_k + \Gamma_k e_k(T) \le 1 \\ 1, & \alpha_k + \Gamma_k e_k(T) > 1. \end{array}\right.$$

Theorem 1: For system (3) and (4) and order learning law (16), it can be proved that $\alpha_d - \alpha_k \to 0$ monotonically if there exists a $\rho \in [0,1)$ satisfying either of the following conditions:

1) any Γ_k satisfying $|1 - \Gamma_k \Lambda_k| \le \rho$,

2)

$$\left\{\begin{array}{l} \|\hat{\Gamma}_k\| \cdot [\max\limits_{\alpha_\xi, \alpha_\eta} \|\tilde{\Lambda}_k\|] \le 1 + \rho, \\ 1 \le \tilde{\Gamma}_k e_{k-1}(T), \end{array}\right. \Rightarrow \Gamma_k = \frac{\tilde{\Gamma}_k}{(-1)^k \text{sgn}\{\delta\alpha_0\}},$$

3)

$$\left\{\begin{array}{l} \|\hat{\Gamma}_k\| \cdot [\max\limits_{\alpha_\xi, \alpha_\eta} \|\tilde{\Lambda}_k\|] \le 1, \\ 1 - \rho \le \tilde{\Gamma}_k e_{k-1}(T), \end{array}\right. \Rightarrow \Gamma_k = \frac{\tilde{\Gamma}_k}{\text{sgn}\{\delta\alpha_0\}}$$

where Λ_k, $\tilde{\Lambda}_k$, $\hat{\Gamma}_k$, $\tilde{\Gamma}_k$ and the order learning gains Γ_k are defined in the following proof, f is locally Lipschitz on y_k with Lipschitz constant K, and

$$\|\tilde{\Lambda}_k\| = \int_0^T (T-\tau)^{\alpha_\xi - 1} E_{\alpha_\xi, \alpha_\xi}[K(T-\tau)^{\alpha_\xi}] \left\| \frac{\partial \varepsilon_k(\tau)}{\partial \alpha}\bigg|_{\tilde{\alpha}_\xi} \right\| d\tau.$$

Proof: This proof is divided into two parts.

Part I:

It can be proved that

$$\begin{aligned} & y_d^{(\alpha_d)}(t) - y_k^{(\alpha_k)}(t) = f(t, y_d, u_d) - f_k(t, y_k, u_d) \\ \Leftrightarrow & \frac{\partial y_d^{(\alpha)}(t)}{\partial \alpha}\bigg|_{\alpha_\xi} \delta\alpha_k + e_k^{(\alpha_k)}(t) = f(t, y_d, u_d) - f_k(t, y_k, u_d) \end{aligned}$$

where $e_k(t) = y_d(t) - y_k(t)$ and $\delta\alpha_k = \alpha_d - \alpha_k$, It follows from $y_{d0} = y_{k0}$ that there exists a kernel function $H(\cdot)$ and the order sensitivity function $\frac{\partial \varepsilon_k}{\alpha}$ satisfying

$$e_k(t) = \int_0^t H(t, \tau, \alpha_k, h_k(t,\tau)) \frac{\partial \varepsilon_k(\tau)}{\partial \alpha}\bigg|_{\tilde{\alpha}_\xi} d\tau \delta\alpha_k \tag{17}$$

where $h_k(t,\tau)$ is iteration dependent and related to the estimation of other coefficients. Thus the convergence condition can be written as

$$|1 - \Gamma_k \Lambda_k| \le \rho < 1 \tag{18}$$

where $\Lambda_k = \int_0^T H(T, \tau, \alpha_k, h_k(t,\tau)) \frac{\partial \varepsilon_k(\tau)}{\partial \alpha}\Big|_{\tilde{\alpha}_\xi} d\tau$.

Part II:

It is obvious that (18) holds if either of the following conditions is satisfied:

$$1 \le \Gamma_k \Lambda_k \le 1 + \rho \tag{19}$$

$$1 - \rho \le \Gamma_k \Lambda_k \le 1 \tag{20}$$

which are sufficient conditions.

Moreover, applying Lemma 1 of [45] to $\Gamma_k \Lambda_k$ yields

$$\Gamma_k \Lambda_k \le \|\Gamma_k\| \|\tilde{\Lambda}_k\|.$$

On the other hand, because $\delta\alpha_k = [1 - \Gamma_k \Lambda_k]\delta\alpha_{k-1}$,

1) for (19), $\text{sgn}\{\delta\alpha_k\} = -\text{sgn}\{\delta\alpha_{k-1}\} = (-1)^k \text{sgn}\{\delta\alpha_0\}$,

2) for (20), $\text{sgn}\{\delta\alpha_k\} = \text{sgn}\{\delta\alpha_{k-1}\} = \text{sgn}\{\delta\alpha_0\}$.

Firstly, for (19), the order leaning gain Γ_k is derived from the following steps:

Step 1: Find $\hat{\Gamma}_k$ satisfying $\|\hat{\Gamma}_k\| \cdot \left[\max\limits_{\alpha_\xi, \alpha_\eta} \|\tilde{\Lambda}_k\|\right] \le 1 + \rho$.

Step 2: Choose $\tilde{\Gamma}_k \in \{\hat{\Gamma}_k, -\hat{\Gamma}_k\}$ satisfying $1 \le \tilde{\Gamma}_k e_{k-1}(T)$.

Step 3: It follows that

$$1 \le \tilde{\Gamma}_k e_{k-1}(T) \le \frac{\tilde{\Gamma}_k e_{k-1}(T)}{\text{sgn}\{\delta\alpha_k\}} \delta\alpha_k = \frac{\tilde{\Gamma}_k e_{k-1}(T)}{(-1)^k \text{sgn}\{\delta\alpha_0\} \delta\alpha_k}.$$

Step 4: Let $\Gamma_k = \frac{\tilde{\Gamma}_k}{(-1)^k \text{sgn}\delta\alpha_0}$, we have

$$\left\{\begin{array}{l} \Gamma_k \Lambda_k \le \|\Gamma_k\| \|\tilde{\Lambda}_k\| \le 1 + \rho \\ 1 \le \dfrac{\Gamma_k e_{k-1}(T)}{\delta\alpha_k} = \Gamma_k \Lambda_k. \end{array}\right.$$

It follows from $\delta\alpha_k = [1 - \Gamma_k \Lambda_k]\delta\alpha_{k-1}$ that $\lim_{k\to\infty} \alpha_k = \alpha_d$.

Secondly, for (20), the order learning gain Γ_k is derived from the following steps:

Step 1: Find $\hat{\Gamma}_k$ satisfying $\|\hat{\Gamma}_k\| \cdot \left[\max\limits_{\alpha_\xi, \alpha_\eta} \|\tilde{\Lambda}_k\|\right] \le 1$.

Step 2: Choose $\tilde{\Gamma}_k \in \{\hat{\Gamma}_k, -\hat{\Gamma}_k\}$ satisfying $1 - \rho \le \tilde{\Gamma}_k e_{k-1}(T)$.

Step 3: It follows that

$$1 - \rho \le \tilde{\Gamma}_k e_{k-1}(T) \le \frac{\tilde{\Gamma}_k e_{k-1}(T)}{\text{sgn}\{\delta\alpha_k\}} \delta\alpha_k = \frac{\tilde{\Gamma}_k e_{k-1}(T)}{(-1)^k \text{sgn}\{\delta\alpha_0\} \delta\alpha_k}.$$

Step 4: Let $\Gamma_k = \frac{\tilde{\Gamma}_k}{\text{sgn}\delta\alpha_0}$, we have

$$\left\{\begin{array}{l} \Gamma_k \Lambda_k \le \|\Gamma_k\| \, \|\tilde{\Lambda}_k\| \le 1 \\ 1 - \rho \le \dfrac{\Gamma_k e_{k-1}(T)}{\delta\alpha_k} = \Gamma_k \Lambda_k. \end{array}\right.$$

It follows from $\delta\alpha_k = [1-\Gamma_k\Lambda_k]\delta\alpha_{k-1}$ that $\lim_{k\to\infty}\alpha_k = \alpha_d$.

Lastly, the universal way to determine $\text{sgn}\{\delta\alpha_0\}$ is

$$\text{sgn}\{\delta\alpha_0\} = \text{sgn}\{\alpha_d - \alpha_0\} = \begin{cases} +1, & \text{if } \alpha_0 = 0 \\ -1, & \text{if } \alpha_0 = 1. \end{cases} \quad \blacksquare$$

Remark 2: Comparing to the identification of integer order KiBaM model, an extra parameter "fractional order α" is introduced to the linear part of the model, and the computations of other parameters are accordingly related to the fractional order derivatives of certain variables. Thus, there exist two essential difficulties: how to find α accurately, and how to compute fractional order derivatives accurately. On one hand, the identification of α is just dependent on the structure of system, i.e., the linear part of the model is a SISO one due to the physical meanings of the system such as the distributed property or the averaging method, and the input and output can represent the current and voltage. A small gain can always guarantee the convergence of the iterative learning identification method. Besides, this method arrives at a unique value of $\hat{\alpha}$, and the initial α_0 can be chosen as 0 or 1. Besides, it will be shown in the next section that a number of state-of-art methods such as internal partition method, GA, NN, etc, may fail to find the real α. On the other hand, the short memory principle guarantees the accuracy of fractional order derivatives, and reveals the importance of preconditioning before real experiments [46]. To sum up, because of the adaptiveness of α estimation, a faster convergence speed and a more accurate result can surely be expected.

IV. Illustrated Examples

Example 1: In this simulation example a FO-KiBaM model is considered where the nonlinear and linear parts (structural information) are assumed as:

$$\begin{aligned} f(u,\theta_n) &= \beta u^3 \\ G(s) &= \frac{b_1 s^\alpha + b_0}{a_2 s^{3\alpha} + a_1 s^{2\alpha} + a_0 s^\alpha + 1} \end{aligned} \tag{21}$$

where the true values are

$$\begin{aligned} &\alpha = 0.5, \quad \beta = 2 \\ &b_1 = 3, \quad b_0 = 2 \\ &a_2 = 2, \quad a_1 = 3, \quad a_0 = 5 \end{aligned}$$

and the input signal is $u = 0.5\sin(t)$.

Given the initial value of $\alpha_0 = 0.1$, and $N = 6693$ samples of the input/output data for the identification of structure (21). The linear coefficient vector was initialized as $a_2 = 1, a_1 = 1, a_0 = 1, b_1 = 1, b_0 = 1$. The matrices of Φ_n and Y_n in the nonlinear identification process are:

$$Y_n(y,\hat{\theta}_a) = \begin{bmatrix} \hat{a}_2 p^{3\alpha} y(t_1) + & \cdots & +y(t_1) \\ \hat{a}_2 p^{3\alpha} y(t_{k+1}) + & \cdots & +y(t_{k+1}) \\ \vdots & \ddots & \vdots \\ \hat{a}_2 p^{3\alpha} y(t_N) + & \cdots & +y(t_N) \end{bmatrix}$$

and

$$\Phi_n(u,\hat{\theta}_b) = \begin{bmatrix} \hat{b}_1 p^\alpha u^3(t_1) + \hat{b}_0 u^3(t_1) \\ \hat{b}_1 p^\alpha u^3(t_{k+1}) + \hat{b}_0 u^3(t_{k+1}) \\ \vdots \\ \hat{b}_1 p^\alpha u^3(t_N) + \hat{b}_0 u^3(t_N) \end{bmatrix}.$$

The matrices of $\hat{\varphi}_k$ and Y_k in the linear identification process are:

$$\hat{\varphi}_k = \begin{bmatrix} -p^{3\alpha} y_{t_k} & -p^{2\alpha} y_{t_k} & -p^\alpha y_{t_k} & p^\alpha \hat{w}_{t_k} & \hat{w}_{t_k} \end{bmatrix}$$

$$\hat{\theta}_l = \begin{bmatrix} a_2 & a_1 & b_1 & b_0 \end{bmatrix}^T, \; Y_k = [y_{t_k}].$$

Refer to the above fractional order KiBaM system (21), two identification strategies are discussed by using different order identification methods, i.e., the interval partition method and the iterative learning order identification method. With the interval partition method, the identification procedure is basically as follows:

1) Given the increment of α, such as $\Delta\alpha = 0.1$, that divides $[0,1]$ into $\alpha/\Delta\alpha$ parts, identify the linear and nonlinear parameters with the $\alpha/\Delta\alpha$ values of α, respectively.

2) Compare the $\alpha/\Delta\alpha$ identification results according to the 2-norm of their output error. Find the two smaller results which construct the renewed domain of α.

3) Repeat the above procedure until the domain cannot be divided or the precision of α has arrived to the requirement.

Fig. 3 illustrates the 2-norm of output errors according to different $\alpha \in [0,1]$. As shown in Fig. 3, the 2-norm of the identified system's output error is not monotonically convergent to the real value $\alpha = 0.5$, which restricts the validation of the interval partition method in fractional order identification, and this phenomenon always happens in fractional order nonlinear system identifications.

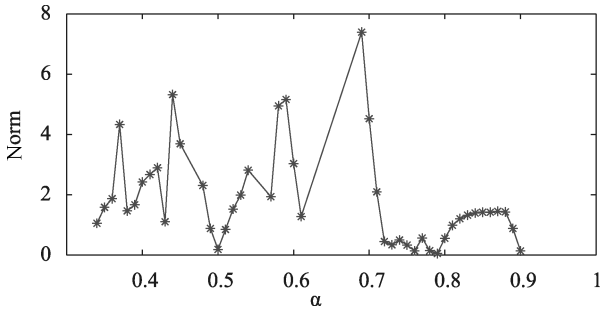

Fig. 3. The 2-norm of output errors by using the interval partition method, where the minimum point corresponds to $\alpha = 0.79$ instead of the true value 0.5.

On the other hand, by using the iterative learning order identification method, the learning laws of α_k are assumed as:

$$\alpha_{k+1} \triangleq \alpha_k + 0.01 e_k(T).$$

Combined with the nonlinear and linear parameters learning laws (10) and (14), the identification process is proceeded as Fig. 2 shown and the result is shown as follows:

$$f(u,\theta_n) = 1.6174u^3$$
$$G(s) = \frac{1.7397s^{0.4988} + 2.1601}{0.9852s^{1.4964} + 1.7439s^{0.9976} + 3.2185s^{0.4988} + 1}$$

which is very precise for there is no noise introduced into the example system. Simulate the above identified system with the input data u. After 12 iterations, the iteration converges when the 2-norm of the output error arrived at 0.0388. Comparison between the identified system output and the original system output is illustrated in Fig. 4.

Example 2: Consider a FO-KiBaM model described as

$$f(u,\theta_n) = \beta u^2$$
$$G(s) = \frac{b_1 s^{\alpha} + b_0}{a_1 s^{2\alpha} + a_0 s^{\alpha} + 1} \tag{22}$$

with

$$\alpha = 0.5, \quad \beta = 1, \quad b_1 = 3, \quad b_0 = 2, \quad a_1 = 3, \quad a_0 = 1$$

and the input signal is $u = 0.5\sin(t)$.

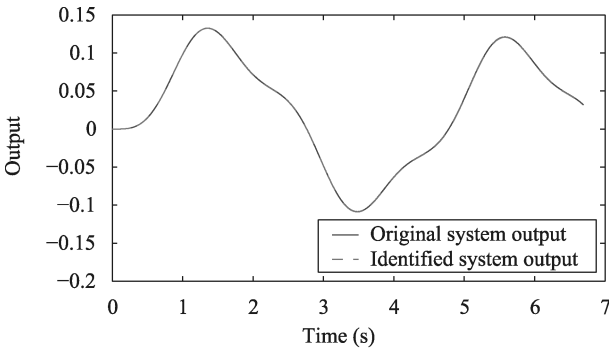

Fig. 4. Comparisons of the outputs between the ideal system and the identified one in Example 1.

With $N = 6693$ samples of the input/output data of the structure (22), parameter estimation is performed using the approaches proposed in Section III. Besides the order learning law is assumed as: $\alpha_{k+1} \triangleq \alpha_k + 0.01e_k(T)$. Fig. 5 presents the results of modeling of the actual and modeled outputs of the KiBaM system (22). The estimation results are:

$$f(u,\theta_n) = 1.496u^2$$
$$G(s) = \frac{1.9965s^{0.4988} + 1.2598}{2.9216s^{0.9976} + 0.9464s^{0.4988} + 1}.$$

Example 3: A real-world application of this paper is illustrated in this example. Given the structure of fractional order KiBaM model in Fig. 6, where R_s represents the nonlinear term, and a linear circuit is cascaded after it. Based on the standard of dynamic stress test (DST), the I/V data is applied to estimate those parameters, where the order of Warburg impedance $\beta = 0.5$ is assumed due to the property of Lithium-ion battery ($\beta = 0.25$ for fuel cells), then it, follows that

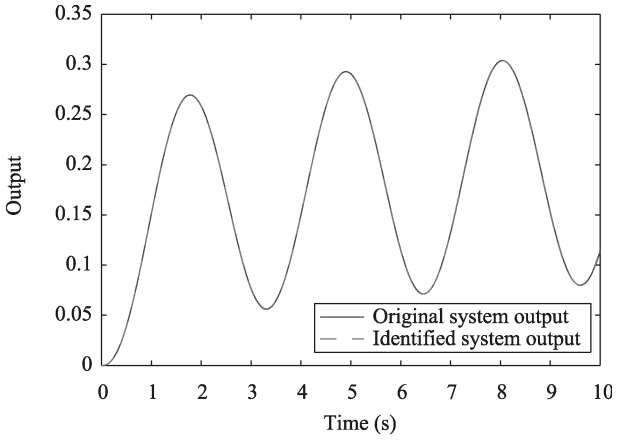

Fig. 5. Comparisons of the outputs between the ideal system and the identified one in Example 2.

$R_s = 19.1$ MΩ, $R_p = 15.6$ MΩ, $Y_0 = 1.24$ mho, $Y_1 = 370$ mho, α=0.665. Thus the electrochemical impedance spectroscopy is

$$G(s) = R_s + \frac{R_p Y_0 s^{\alpha} + R_p Y_1 s^{\beta} + 1}{R_p Y_0 Y_1 s^{\alpha+\beta} + Y_1 s^{\beta}}$$

where the nonlinear term Rs is varying according to the error e. The measured fractional order and fractional order KiBaM outputs are compared in Fig. 7, where $R_s \in [34, 37.7]$. It can be seen that the nonlinear term plays crucial role in the above real-time traffic test.

Remark 3: It should be noted that, given an accurate α, many methods can derive accurate models for sure, such as the prony technique. This α is a key parameter in FO KiBaM model which reveals a number of physical, chemical and distributed characteristics as introduced in Section II-C.

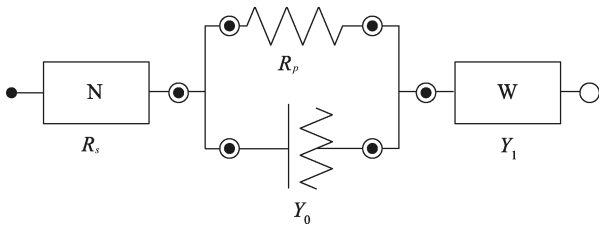

Fig. 6. Structural information of a fractional order KiBaM model.

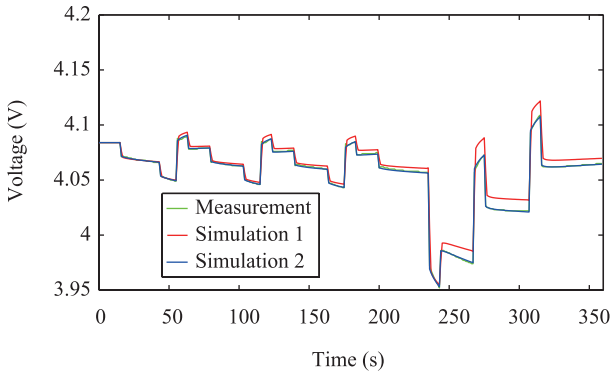

Fig. 7. Comparisons of the outputs between the fractional order model (FOM) and the discussed fractional order KiBaM model in Example 3.

Besides, the computations of other parameters are accordingly related to the fractional order derivatives of certain variables. Thus how to find α accurately, and how to compute fractional order derivatives accurately become primary tasks. In Example 1, the interval partition method failed to work because a too different α was obtained. But, the proposed method in Section III-C is adaptive to those linear parameters, and can converge to the real α in terms of other parameters in less than 20 iterations, where the errors are set to be in range $0.01-0.05$. The similar convergence accuracy and speed can be observed in various other simulations that we have done previously which are excluded in this paper such as more complicated nonlinear terms [25], [47]−[53].

Lastly, in the above examples, the small gain learning law guarantees the convergence, but sacrifices the convergence speed. But fortunately, this learning gain can be tuned from a small enough one to a large and optimized one so that the convergence speed is improved accordingly, where the whole tuning process is unidirectional. By doing so, the accuracy and convergence speed can be improved simultaneously.

V. Conclusions and Future Works

This paper deals with the parameter and order identifications of commensurate fractional order KiBaM systems in continuous-time domain. The least square method is applied to the linear and nonlinear parameters identification. A P-type order learning law associated with the terminal value of system error is applied to identify the system order accurately. The performance of the proposed algorithms has been testified by illustrative examples. Based on results presented in this paper, it is anticipated that the proposed identification algorithms will lead to more precise construction of fractional order quasi-linear systems. The other part of this work is the identification of variable order system which is another representation of nonlinearity.

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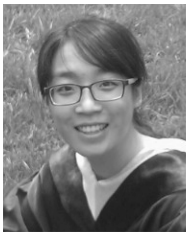

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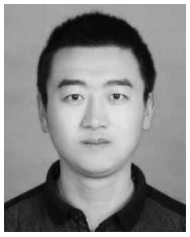

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