Artificial Bee Colony Algorithm-based Parameter Estimation of Fractional-order Chaotic System with Time Delay

Wenjuan Gu, Yongguang Yu, and Wei Hu

***Abstract*—It is an important issue to estimate parameters of fractional-order chaotic systems in nonlinear science, which has received increasing interest in recent years. In this paper, time delay and fractional order as well as system's parameters are concerned by treating the time delay and fractional order as additional parameters. The parameter estimation is converted into a multi-dimensional optimization problem. A new scheme based on artificial bee colony (ABC) algorithm is proposed to solve the optimization problem. Numerical experiments are performed on two typical time-delay fractional-order chaotic systems to verify the effectiveness of the proposed method.**

***Index Terms*—Artificial bee colony (ABC) algorithm, fractional-order chaotic system, parameters estimation, time delay.**

I. INTRODUCTION

FRACTIONAL calculus is a branch of mathematics which deals with differentiation and integration of arbitrary orders and is as old as calculus [1]. Although the classical calculus has been playing a dominant role in explaining and modeling dynamical processes observed in real world, the fractional calculus has gradually attracted the attention of scientists during the last decades because of its capability in describing important phenomena of non-local dynamics and memory effects. It has been introduced into various engineering and science domains, such as image processing [2], robotics [3], diffusion [4], mechanics [5], and others [1].

Time delay is commonly encountered in real systems, such as chemistry, climatology, biology, economy and crypto systems [6]. Time-delay differential equation is a differential equation in which the derivative of the function at any time depends on the solution at previous time. Introduction of time delay in the model can enrich its dynamics and provide a precise description of real life phenomenon [7]. Particularly, since Mackey and Glass [8] firstly detected chaos in time-delay systems, control and synchronization of time-delay chaotic systems have obtained increasing attention [9], [10], which can produce more complex and adequate dynamic behavior than those free of time delays.

Recently, chaotic behavior has been found in time-delay fractional-order systems, such as fractional-order financial system [11], fractional-order Chen system [12], fractional-order Liu system [13] and so on. Many control methods are valid for the fractional-order chaotic systems with known parameters and time-delays [14]−[16]. However, in some applications such as secure communications and chaos synchronization, the chaotic system is partially known. It means that the form of differential equation is known, but some or all of the time delays, fractional orders and system's parameters are unknown. Therefore, estimating the unknown parameters of time-delay fractional-order chaotic system is of vital significance in controlling and utilizing chaos.

Up to now, for the parameter estimation of chaotic systems, considerable methods have been put forward, such as the least-squares method [17], the symbolic time series analysis-based method [18], the adaptive control method [19]. Besides, by transforming the parameter estimation in dynamical systems as a multi-dimensional optimization problem, many evolutionary algorithms have been proposed to deal with the problem, such as differential evolution (DE) [20], particle swarm optimization (PSO) [21], cuckoo search (CS) [22], biogeography-based optimization (BBO) [23]. However, most of the works mentioned so far are involved mainly with integer-order chaotic systems or fractional-order chaotic systems without time delays. That is, very few have addressed the estimation problem on fractional-order chaotic systems with time delay.

Artificial bee colony (ABC) algorithm is a relatively new optimization technique which is developed by Karaboga in 2005 based on simulating the foraging behavior of honey-bee swarm. It has been shown to be competitive to other population-based algorithms for global numerical optimization problem with the advantage of employing fewer control parameters [24]−[26]. For example, apart from the maximum iteration number and population size, a standard GA has three more control parameters (crossover rate, mutation rate, generation gap) [27], a standard DE has at least two control parameters (crossover rate, scaling factor) [28] and a basic PSO has three control parameters (cognitive and social factors, inertia weight) [29]. Besides, limit values for the velocities $v_{\max}$ have a significant effect on the performance of PSO. The ABC algorithm has only one control parameter (limit)

Manuscript received September 6, 2015; accepted January 13, 2016. This work was supported by National Natural Science Foundation of China (11371049). Recommended by Associate Editor Dingyü Xue.

Citation: W. J. Gu, Y. G. Yu, and W. Hu, "Artificial bee colony algorithm-based parameter estimation of fractional-order chaotic system with time delay," *IEEE/CAA Journal of Automatica Sinica*, vol. 4, no. 1, pp. 107−113, Jan. 2017.

W. J. Gu is with the School of Economics and Management, Beijing Jiaotong University, Beijing 100044, China (e-mail: guwj0423@163.com).

Y. G. Yu and W. Hu are with the School of Science, Beijing Jiaotong University, Beijing 100044, China (e-mail: ygyu@bjtu.edu.cn; 13121527@bjtu.edu.cn).

Digital Object Identifier 10.1109/JAS.2017.7510340

apart from colony size and maximum cycle number. Although it uses less control parameters, the performance of ABC algorithm is better than or similar to that of these algorithms and it can be efficiently used for solving multimodal and multidimensional optimization problems.

Based on the above discussion, in this paper, a scheme based on artificial bee colony algorithm is firstly proposed to estimate the parameters of unknown time-delay fractional-order chaotic system. Numerical simulations are performed to estimate two well-known fractional-order chaotic systems with time delay. The simulation results demonstrate the good performance of the ABC algorithm, and thus the ABC algorithm proves to be a promising candidate for parameter estimation of time-delay fractional-order chaotic systems.

The rest of the paper is organized as follows. In Section II, the Caputo fractional-order derivative is introduced. In Section III, the problem of parameter estimation for time-delay fractional-order chaotic system is formulated from the view of optimization. In Section IV, a parameter estimation scheme is proposed after briefly introducing the ABC algorithm. Numerical simulations and conclusions are given in Sections V and VI.

II. Caputo Fractional-order Derivative

In general, three best-known definitions of fractional-order derivatives are widely used: Grunwald-Letnikov, Riemann-Liouville and Caputo definitions [1]. In particular, the main advantage of Caputo fractional-order derivative is that it owns same initial conditions with integer-order derivatives, which have well-understood features of physical situations and more applicable to real world problems. Thus, the Caputo fractional-order derivative is employed in this paper.

Definition 1 (Caputo fractional-order derivative): The Caputo fractional-order derivative of order $\alpha > 0$ for a function $f(t) \in C^{n+1}([t_0, +\infty), R)$ is defined as

$$ {}_{t_0}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \tag{1}$$

where $\Gamma(\cdot)$ denotes the gamma function and n is a positive integer such that $n-1 < \alpha \leq n$.

Property 1: When C is any constant, ${}_{t_0}D_t^{\alpha} C = 0$ holds.

Property 2: For constants μ and ν, the linearity of Caputo fractional-order derivative is described by

$$ {}_{t_0}D_t^{\alpha}(\mu f(t) + \nu g(t)) = \mu \ {}_{t_0}D_t^{\alpha} f(t) + \nu \ {}_{t_0}D_t^{\alpha} g(t). $$

III. Problem Formulation

The problem formulation of parameter estimation for time-delay fractional-order chaotic systems is presented in this section.

Let us consider the following time-delay fractional-order chaotic system described by delay differential equation

$$ {}_0D_t^{\alpha} Y(t) = f(Y(t), Y(t-\tau), Y_0, \theta) \tag{2}$$

where $Y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \in \mathbb{R}^n$ denotes the state vector of system (2), $Y_0 = Y(0)$ denotes the initial value for $t \leq \tau$, $\theta = (\theta_1, \theta_2, \ldots, \theta_m)^T$ denotes the set of system's parameters, $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ $(0 < \alpha_i < 1$, $i = 1, 2, \ldots, n)$ is the fractional derivative orders. $f(Y(t), Y(t-\tau), Y_0, \theta) = (f_1(Y(t), Y(t-\tau), Y_0, \theta)$, $f_2(Y(t), Y(t-\tau), Y_0, \theta)$, $\ldots$, $f_n(Y(t), Y(t-\tau), Y_0, \theta))^T$. In this paper, the delay time τ and fractional order α are treated as additional parameters to be estimated.

Suppose the structure of system (2) is known, then the corresponding estimated system can be written as

$$ {}_0D_t^{\tilde{\alpha}} \tilde{Y}(t) = f(\tilde{Y}(t), \tilde{Y}(t-\tilde{\tau}), Y_0, \tilde{\theta}) \tag{3}$$

where $\tilde{Y}(t) = (\tilde{y}_1(t), \tilde{y}_2(t), \ldots, \tilde{y}_n(t))^T \in \mathbb{R}^n$ is the state vector of the estimated system (3), $\tilde{\theta} = (\tilde{\theta}_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_m)^T$ is a set of estimated systematic parameters, $\tilde{\alpha} = (\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n)^T$ is the estimated fractional orders, and $\tilde{\tau}$ is the estimated time delay. Besides, systems (2) and (3) have the same initial conditions Y_0.

Based on the measurable state vector $Y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \in \mathbb{R}^n$, we define the following objective function or fitness function

$$ \begin{aligned} J(\tilde{\alpha}, \tilde{\theta}, \tilde{\tau}) &= \arg \min_{(\tilde{\alpha}, \tilde{\theta}, \tilde{\tau}) \in \Omega} F \\ &= \arg \min_{(\tilde{\alpha}, \tilde{\theta}, \tilde{\tau}) \in \Omega} \sum_{k=1}^{N} \|Y_k - \tilde{Y}_k\|_2 \end{aligned} \tag{4}$$

where $k = 1, 2, \ldots, N$ is the sampling time point and N denotes the length of data used for parameter estimation. Y_k and $\tilde{Y}_k$ respectively denote the state vector of the original system (2) and the estimated system (3) at time kh. h is the step size employed in the predictor-corrector approach for the numerical solutions of time-delay fractional-order differential equations [7]. $\|\cdot\|$ is Euclid norm. Ω is the searching area suited for parameters $\tilde{\alpha}$, $\tilde{\theta}$ and $\tilde{\tau}$.

Obviously, the parameter estimation for system (2) is multi-dimensional continuous optimization problem, where the decision vectors are $\tilde{\alpha}$, $\tilde{\theta}$ and $\tilde{\tau}$. The optimal solution can be achieved by searching suitable $\tilde{\alpha}$, $\tilde{\theta}$ and $\tilde{\tau}$ in the searching space Ω such that the objective function (4) is minimized. In this paper, a novel scheme based on artificial bee colony algorithm is proposed to solve this problem.

The time-delay fractional-order chaotic systems are not easy to estimate because of the unstable dynamics of the chaotic system and the complexity of the fractional-order nonlinear systems. Besides, due to multiple variables in the problem and multiple local search optima in the objective functions, it is easily trapped into local optimal solution and the computation amount is great. So it is not easy to search the global optimal solution effectively and accurately using the traditional general methods. Therefore, we aim to solve this problem by the effective artificial bee colony algorithm in this paper. The general principle of parameter estimation by ABC algorithm is shown in Fig. 1.

IV. A Novel Parameter Estimation Scheme

A. An Overview of the Original Artificial Bee Colony Algorithm

In the natural bee swarm, there are three kinds of honey bees to search foods generally, which include the employed bees, the onlookers and the scouts (both onlookers and the scouts are also called unemployed bees). The employed bees search the food around the food source in their memory. At the same time, they pass their food information to the onlookers. The onlookers tend to select good food sources from those found by the employed bees, then further search the foods around the selected food source. The scouts are transformed from a few employed bees, which abandon their food sources and search new ones. In short, the food search of bees is collectively performed by the employed bees, the onlookers and the scouts.

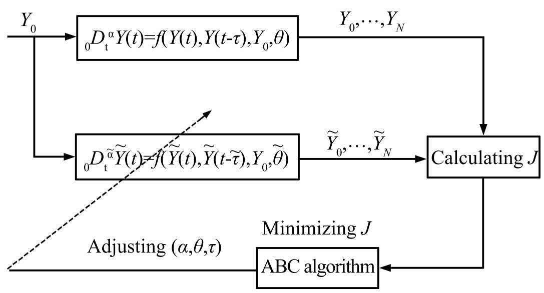

Fig. 1. The general principle of parameter estimation by ABC algorithm.

By simulating the foraging behaviors of honey bee swarm, Karaboga proposed a competitive optimization technique called artificial bee colony (ABC) algorithm [24]−[26]. In the original ABC algorithm, each cycle of the search consists of three steps: moving the employed and onlooker bees onto the food sources and calculating their nectar amounts; and determining the scout bees and directing them onto possible food sources. A food source position represents a possible solution to the problem to be optimized. The amount of nectar of a food source corresponds to the quality of the solution represented by that food source. Onlookers are placed on the food sources by using a probability based selection process. As the nectar amount of a food source increases, the probability value with which the food source is preferred by onlookers increases, too. Every bee colony has scouts that are the colony's explorers. The explorers do not have any guidance while looking for food. They are primarily concerned with finding any kind of food source. As a result of such behavior, the scouts are characterized by low search costs and a low average in food source quality. Occasionally, the scouts can accidentally discover rich, entirely unknown food sources. In the case of artificial bees, the artificial scouts could have the fast discovery of the group of feasible solutions as a task. In this work, one of the employed bees is selected and classified as the scout bee. The selection is controlled by a control parameter called limit. If a solution representing a food source is not improved by a predetermined number of trials, then the food source is abandoned by its employed bee and the employed bee is converted to a scout. The number of trials for releasing a food source is equal to the value of limit which is an important control parameter of ABC. In a robust search process, exploration and exploitation process must be carried out together. In the ABC algorithm, while onlookers and employed bees carry out the exploitation process in the search space, the scouts control the exploration process. Besides, the number of employed bees is equal to the number of onlooker bees which is also equal to the number of food sources. The detailed searching process is described as following:

At the first step, the ABC algorithm produces a randomly distributed initial population with the following equation:

$$x_{i,j} = x_{\min,j} + \mathrm{rand}(0,1)(x_{\max,j} - x_{\min,j}) \tag{5}$$

where $i = 1,2,\ldots,SN$, $j = 1,2,\ldots,D$. SN is the size of the solutions (food sources), D is the dimension of the optimization parameters. $x_{\min,j}$ and $x_{\max,j}$ are the lower and upper bounds for the dimension j, respectively.

After initialization, the population of the food sources (solutions) is subjected to repeated cycles. An employed bee makes a modification on the position in her memory depending on the local information as

$$v_{i,j} = x_{i,j} + \phi_{i,j}(x_{i,j} - x_{k,j}) \tag{6}$$

where $k = 1,2,\ldots,SN$ and $j = 1,2,\ldots,D$. k and j are randomly generated, and k must be different from i. $\phi_{i,j}$ is a random number in $[-1,1]$. Then, the employed bee tests the nectar amount of the new source. If the nectar amount of the new one is higher than that of the previous one in her memory, the bee memorizes the new position and forgets the old one. Otherwise, she keeps the position of the previous one in her memory.

Then, an onlooker bee evaluates the nectar information taken from all employed bees and chooses a food source with a probability related to its nectar amount and calculated as

$$p_i = \frac{fit_i}{\sum\limits_{j=1}^{SN} fit_j} \tag{7}$$

where fit_i denotes the fitness value of solution X_i. As in the case of the employed bee, she produces a modification on the position in her memory and checks the nectar amount of the candidate source. Besides, the fitness value fit_i is defined as follows:

$$fit_i = \begin{cases} \dfrac{1}{1+f(X_i)}, & \text{if } f(X_i) \geq 0 \\ 1 + |f(X_i)|, & \text{if } f(X_i) < 0 \end{cases} \tag{8}$$

where $f(X_i)$ represents the objective function value of the decision vector X_i.

In ABC, if a position cannot be improved further through a predetermined number of cycles (called limit), then that food source is assumed to be abandoned. The corresponding employed bee becomes a scout bee and a new food source is generated with (5).

Some more details can be found from [24]−[26] and the main steps of the original ABC algorithm are described in Algorithm 1 (see the top of next page).

B. A Novel Parameter Estimation Scheme

As far as we are concerned, little research has been done for the parameter estimation of time-delay fractional-order chaotic systems. Thus, in this paper, the parameter estimation

Algorithm 1 The main procedure of the original artificial bee colony algorithm

Step 0: Predefine some parameters: SN (population size number), D (searching dimension), $LOWER$ (lower bound), $UPPER$ (upper bound), $limit$ (control parameter), MCN (maximum cycle number)
Step 1: The population initialization phase:
  Step 1.1: Randomly generate $0.5 \times SN$ points in the search space to form an initial population via (5).
  Step 1.2: Evaluate the objective function values of population.
  Step 1.3: cycle=1.
Step 2: The employed bees phase:
  For $i = 1$ **to** $0.5 \times SN$ **do**
    Step 2.1:
      Step 2.1.1: Generate a candidate solution V_i by **(6)**.
      Step 2.1.2: Evaluate $f(V_i)$.
    Step 2.2: If $f(V_i) < f(X_i)$, set $X_i = V_i$, otherwise, set $trial_i = trial_i + 1$.
  End For
Step 3: Calculating the probability values p_i by (7), set $t = 0$, $i = 1$.
Step 4: The onlooker bees phase:
  While $t \leq 0.5 \times$ SN, **do**
    Step 4.1:
      If $\text{rand}(0, 1) < p_i$
        Step 4.1.1: Generate a candidate solution V_i by **(6)**.
        Step 4.1.2: Evaluate $f(V_i)$.
        Step 4.1.3: If $f(V_i) < f(X_i)$, set $X_i = V_i$, otherwise, set $trial_i = trial_i + 1$.
        Step 4.1.4: Set $t = t + 1$.
      End If
    Step 4.2: Set $i = i + 1$, if $i = 0.5 \times SN$, set $i = 1$.
  End While
Step 5: The scout bees phase:
  If $\max(trial_i) > limit$, replace X_i with a new candidate solution generated via (5).
Step 6: Set $cycle = cycle + 1$, and **if** $cycle > MCN$, then stop and output the best solution achieved so far, otherwise, go to Step 2.

for time-delay fractional-order chaotic systems is studied. It is converted into a nonlinear optimization problem via a functional extreme model in Section III. In Section IV-A, the artificial bee colony algorithm is described in details. In this subsection, a method based on ABC algorithm is firstly proposed and applied to estimate the unknown parameters of the time-delay fractional-order chaotic systems. The procedure of the new method for parameter estimation of time-delay fractional-order chaotic systems is outlined in Algorithm 2 (see the top of next page).

V. Simulations

To test the effectiveness of ABC algorithm, two typical time-delay fractional-order chaotic systems are selected to show the performance. The simulations were done using MATLAB 7.1 on Intel (R) Core (TM) i5-3470 CPU, 3.2 GHz with 4 GB RAM. The predictor-corrector approach for the numerical solutions of time-delay fractional-order differential equations is used, which can be found in [7]. It is obvious that if the population and the maximum cycle number are larger, the corresponding probability of finding the global optimum is larger as well. However, a larger population and maximum cycle number need a larger number of function evaluations. In the following simulations, for the ABC algorithm, the population size (SN) and maximum cycle number (MCN) are set as: $SN = 100$, $MCN = 300$. Besides, the control parameter limit is chosen as 15. The ABC algorithm is run for 15 independent times for each example, and all runs are terminated after the predefined maximum number of iterations is reached.

Example 1: Fractional-order financial system with time-delay [11] is described as:

$$\begin{cases} {}_0D_t^{\alpha_1}x(t) = z(t) + (y(t-\tau) - a)x(t) \\ {}_0D_t^{\alpha_2}y(t) = 1 - by(t) - x^2(t-\tau) \\ {}_0D_t^{\alpha_3}z(t) = -x(t-\tau) - cz(t) \end{cases} \tag{9}$$

when $(\alpha_1, \alpha_2, \alpha_3) = (0.76, 1, 1)$, $(a, b, c) = (3, 0.1, 1)$, $\tau = 0.08$ and initial point is $(0.1, 4, 0.5)$, system (9) is chaotic. In order to demonstrate the performance of ABC algorithm clearly, the true values of fractional order α_1, system's parameter c and time delay τ are assumed as unknown parameters which need to be estimated. The searching spaces of the unknown parameters are set as $(\alpha_1, c, \tau) \in [0.4, 1.4] \times [0.5, 1.5] \times [0.01, 0.1]$. The No. of samples is set as $N = 250$ and the step size $h = 0.01$.

The corresponding objective function can be written as

$$F(\tilde{\alpha}_1, \tilde{c}, \tilde{\tau}) = \sum_{k=1}^{N} \|Y_k - \tilde{Y}_k\|_2 \tag{10}$$

therefore, the parameter estimation of system (9) is converted into a nonlinear function optimization problem as (10). In particular, the smaller F is, the better combination of parameters (α_1, c, τ) is. The distribution of the objective function value for the time-delay fractional-order financial system (9) is shown in Fig. 2. As viewed in different colors in Fig. 2, it can be found that the objective function values are smaller in the neighborhood of the point $(\alpha_1, c, \tau) = (0.76, 1, 0.08)$ than those in other places.

To show the performance of ABC algorithm, the statistical results in terms of the best, the mean, and the worst estimated parameters over 15 independent runs are listed in Table I, it can be easily seen that the estimated value obtained via the ABC algorithm is close to the true parameter value, implying that it can estimate the unknown parameters of the time-delay fractional-order chaotic system accurately. The evolutionary

Algorithm 2 A novel parameter estimation method based on ABC algorithm

Step 1: The initialization phase:
  Step 1.1: Initialize the parameters for ABC algorithm and time-delay fractional-order chaotic system (2).
  Step 1.2: Generate the initial population in the feasible domain Ω referred to in Section III.
Step 2: The optimization phase:
  Repeat
    Optimize the function (4) by ABC algorithm (Algorithm 1).
  Until Termination condition is satisfied.

curves of the parameters and fitness values estimated by ABC algorithm are shown in Figs. 3 and 4 in a single run, which can also illustrate the effectiveness of the proposed method.

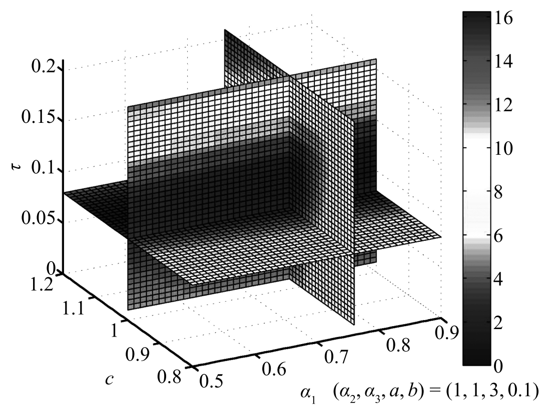

Fig. 2. The distribution of the objective function values for system (9).

TABLE I
SIMULATION RESULTS FOR SYSTEM (9) OVER 15 INDEPENDENT RUNS

	Best	Mean	Worst
α_1	0.7599999995	0.7600001772	0.7600028911
$\frac{\lvert\alpha_1 - 0.76\rvert}{0.76}$	5.06E−10	1.77E−07	2.89E−06
c	0.9999999791	0.9999994842	0.9999723488
$\frac{\lvert c-1\rvert}{1}$	2.09E−08	5.16E−07	2.77E−05
τ	0.0794338076	0.0796012039	0.0844809056
$\frac{\lvert\tau - 0.08\rvert}{0.08}$	5.66E−04	3.99E−04	4.48E−03
F	1.57E−07	2.39E−05	1.41E−04

Example 2: Fractional-order Chen system with time-delay [12] is described as:

$$\begin{cases} {}_0D_t^{\alpha_1}x(t) = a(y(t) - x(t-\tau)) \\ {}_0D_t^{\alpha_2}y(t) = (c-a)x(t-\tau) - x(t)z(t) + cy(t) \\ {}_0D_t^{\alpha_3}z(t) = x(t)y(t) - bz(t-\tau) \end{cases} \tag{11}$$

when $\alpha_1 = \alpha_2 = \alpha_3 = 0.94 = \alpha$, $(a, b, c) = (35, 3, 27)$, $\tau = 0.009$ and initial point is $(0.2, 0, 0.5)$, system (11) is chaotic. In this example, the fractional order α, system parameter b and time delay τ are treated as unknown parameters to be estimated. The searching spaces of the unknown parameters are set as $(\alpha, b, \tau) \in [0.4, 1.4] \times [2.5, 3.5] \times [0.001, 0.015]$.

The No. of samples is set as $N = 250$ and the step size $h = 0.001$. Similarly, the corresponding objective function can be written as

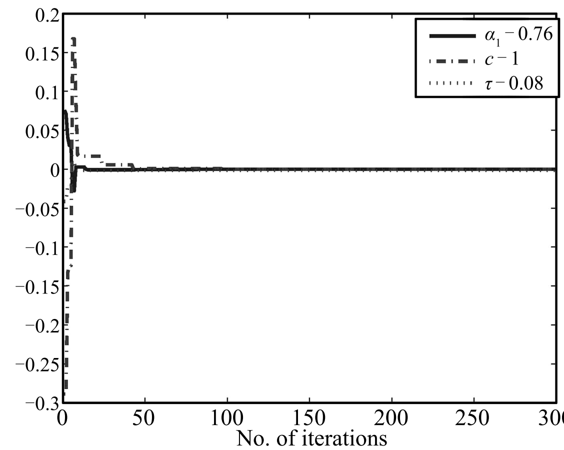

Fig. 3. Evolutionary curve in terms of estimated error values with the ABC algorithms on system (9) in a single run.

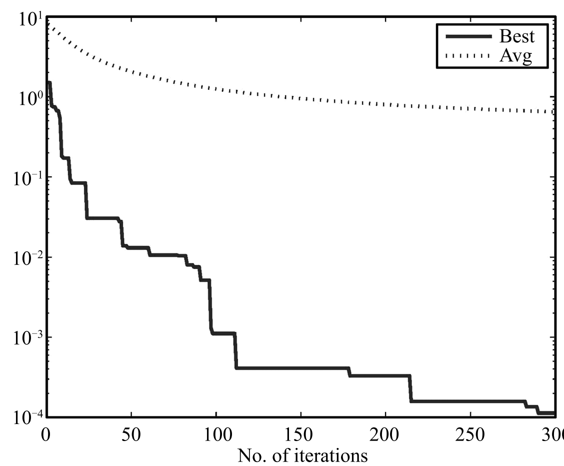

Fig. 4. Evolutionary curve in terms of fitness values with the ABC algorithms on system (9) in a single run.

$$F(\tilde{\alpha}, \tilde{b}, \tilde{\tau}) = \sum_{k=1}^{N} \|Y_k - \tilde{Y}_k\|_2 \tag{12}$$

therefore, the parameter estimation of system (11) is converted into a nonlinear function optimization problem as (12). Fig. 5 shows the distribution of the objective function value for the time-delay fractional-order Chen system (11).

The statistical results of the best, the mean and the worst estimated parameters with their corresponding relative error values over 15 independent runs are displayed in Table II. From Table II, it can be seen that the ABC algorithm can efficiently estimate the parameters of system (11). Figs. 6 and 7 depict the convergence profile of the evolutionary processes of the estimated parameters and the fitness values. From the figures, it can be seen that ABC algorithm can converge to the optimal solution rapidly.

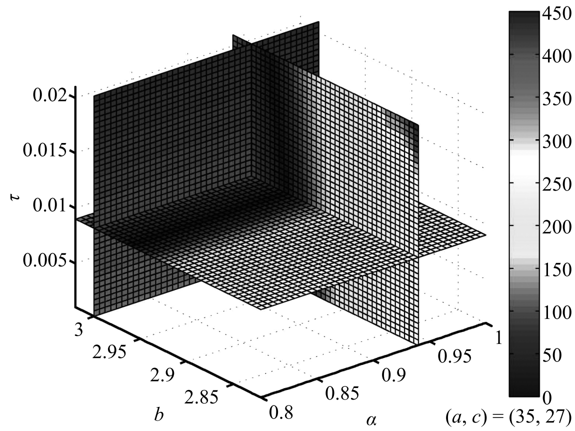

Fig. 5. The distribution of the objective function values for system (11).

TABLE II
SIMULATION RESULTS FOR SYSTEM (11) OVER 15 INDEPENDENT RUNS

	Best	Mean	Worst
α	0.9400000256	0.9400899098	0.9415600675
$\frac{\vert\alpha-0.94\vert}{0.94}$	2.56E−08	8.99E−05	1.56E−03
b	2.9999503453	2.9925101632	3.2099938577
$\frac{\vert b-3\vert}{3}$	4.97E−05	7.49E−03	2.10E−01
τ	0.0089214355	0.0089894034	0.0099489787
$\frac{\vert\tau-0.009\vert}{0.009}$	7.86E−05	1.06E−05	9.49E−04
F	1.48E−04	1.21E−01	1.11E+00

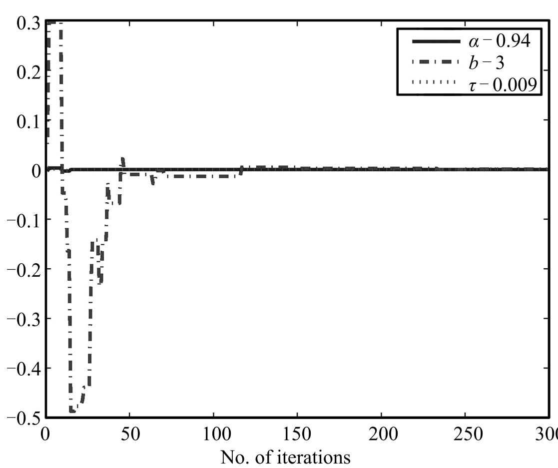

Fig. 6. Evolutionary curve in terms of estimated error values with the ABC algorithms on system (11) in a single run.

VI. CONCLUSIONS

In this paper, the parameter estimation of time-delay fractional-order chaotic systems is concerned by converting it into an optimization problem. A method based on artificial bee colony algorithm is proposed to deal with the optimization problem via functional extreme model. In simulations, the proposed method is applied to identify two typical time-delay fractional-order chaotic systems. And the simulation results show that the fractional order, the time delay and the system's parameter of chaotic system can be successfully estimated with the proposed scheme.

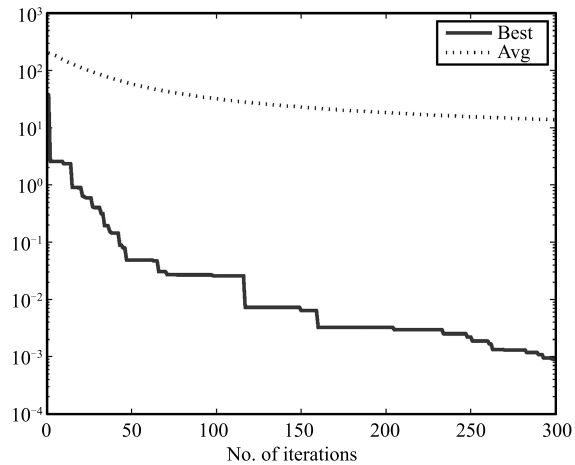

Fig. 7. Evolutionary curve in terms of fitness values with the ABC algorithms on system (11) in a single run.

The aim of this paper is to design a scheme based on ABC algorithm to estimate the unknown fractional orders, system's parameters and time delays. The proposed method can avoid the design of parameter update law in synchronization analysis of the time-delay fractional-order chaotic systems with unknown parameters. Though it is not good enough, we hope this method will contribute to the application of chaos control and synchronization for the time-delay fractional-order chaotic systems.

REFERENCES

[1] I. Podlubny, *Fractional Differential Equations*. San Diego: Academic Press, 1998.

[2] B. Mathieu, P. Melchior, A. Oustaloup, and C. Ceyral, "Fractional differentiation for edge detection," *Signal Proc.*, vol. 83, no. 11, pp. 2421−2432, Nov. 2003.

[3] M. F. Silva, J. A. T. Machado, and A. M. Lopes, "Fractional order control of a hexapod robot," *Nonlinear Dyn.*, vol. 38, no. 1−4, pp. 417−433, Dec. 2004.

[4] W. Chen, L. J. Ye, and H. G. Sun, "Fractional diffusion equations by the Kansa method," *Comput. Math. Appl.*, vol. 59, no. 5, pp. 1614−1620, Mar. 2010.

[5] M. A. E. Herzallah and D. Baleanu, "Fractional-order Euler-Lagrange equations and formulation of Hamiltonian equations," *Nonlinear Dyn.*, vol. 58, no. 1−2, pp. 385−391, Oct. 2009.

[6] M. Lakshmanan and D. V. Senthilkumar, *Dynamics of Nonlinear Time-Delay Systems*. Berlin Heidelberg: Springer, 2011.

[7] S. Bhalekar and V. Daftardar-Gejji, "A predictor-corrector scheme for solving nonlinear delay differential equations of fractional order," *J. Fract. Calculus Appl.*, vol. 1, no. 5, pp. 1−9, Jan. 2011.

[8] M. C. Mackey and L. Glass, "Oscillation and chaos in physiological control systems," *Science*, vol. 197, no. 4300, pp. 287−289, Jul. 1977.

[9] E. M. Shahverdiev and K. A. Shore, "Synchronization in multiple time delay chaotic laser diodes subject to incoherent optical feedbacks and incoherent optical injection," *Nonlinear Anal. Real World Appl.*, vol. 12, no. 6, pp. 3114−3124, Dec. 2011.

[10] H. Wang, X. Wang, X. J. Zhu, and X. H. Wang, "Linear feedback controller design method for time-delay chaotic systems," *Nonlinear Dyn.*, vol. 70, no. 1, pp. 355−362, Oct. 2012.

[11] Z. Wang, X. Huang, and G. D. Shi, "Analysis of nonlinear dynamics and chaos in a fractional order financial system with time delay," *Comput. Math. Appl.*, vol. 62, no. 3, pp. 1531−1539, Aug. 2011.

[12] V. Daftardar-Gejji, S. Bhalekar, and P. Gade, "Dynamics of fractional-ordered Chen system with delay," *Pramana*, vol. 79, no. 1, pp. 61−69, Jul. 2012.

[13] S. Bhalekar and V. Daftardar-Gejji, "Fractional ordered Liu system with time-delay," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 15, no. 8, pp. 2178−2191, Aug. 2010.

[14] Z. M. Odibat, N. Corson, M. A. Aziz-Alaoui, and C. Bertelle, "Synchronization of chaotic fractional-order systems via linear control," *Int. J. Bifurc. Chaos*, vol. 20, no. 1, pp. 81−97, Jan. 2010.

[15] X. Y. Wang, X. P. Zhang, and C. Ma, "Modified projective synchronization of fractional-order chaotic systems via active sliding mode control," *Nonlinear Dyn.*, vol. 69, no. 1−2, pp. 511−517, Jul. 2012.

[16] S. Bhalekar and V. Daftardar-Gejji, "Synchronization of different fractional order chaotic systems using active control," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 15, no. 11, pp. 3536−3546, Nov. 2010.

[17] R. Konnur, "Synchronization-based approach for estimating all model parameters of chaotic systems," *Phys. Rev. E*, vol. 67, no. 2, pp. 027204, Feb. 2003.

[18] N. Q. Li, W. Pan, L. S. Yan, B. Luo, M. F. Xu, N. Jiang, and Y. L. Tang, "On joint identification of the feedback parameters for hyperchaotic systems: an optimization-based approach," *Chaos Solit. Fract.*, vol. 44, no. 4−5, pp. 198−207, May 2011.

[19] M. F. Hu, Z. Y. Xu, R. Zhang, and A. H. Hu, "Parameters identification and adaptive full state hybrid projective synchronization of chaotic (hyper-chaotic) systems," *Phys. Lett. A*, vol. 361, no. 3, pp. 231−237, Jan. 2007.

[20] Y. G. Tang, X. Y. Zhang, C. C. Hua, L. X. Li, and Y. X. Yang, "Parameter identification of commensurate fractional-order chaotic system via differential evolution," *Phys. Lett. A*, vol. 376, no. 4, pp. 457−464, Jan. 2012.

[21] Y. Huang, F. Guo, Y. L. Li, and Y. F. Liu, "Parameter estimation of fractional-order chaotic systems by using quantum parallel particle swarm optimization algorithm," *PLoS One*, vol. 10, no. 1, pp. e0114910, Jan. 2015.

[22] Z. Sheng, J. Wang, S. D. Zhou, and B. H. Zhou "Parameter estimation for chaotic systems using a hybrid adaptive cuckoo search with simulated annealing algorithm," *Chaos* , vol. 24, no. 1, pp. 013133, Mar. 2014.

[23] J. Lin, "Parameter estimation for time-delay chaotic systems by hybrid biogeography-based optimization," *Nonlinear Dyn.*, vol. 77, no. 3, pp. 983−992, Aug. 2014.

[24] D. Karaboga, "An idea based on honey bee swarm for numerical optimization," Tech. Rep. TR06, Erciyes University, Engineering Faculty, Computer Engineering Department, Jan. 2005.

[25] D. Karaboga and B. Akay, "A comparative study of artificial bee colony algorithm," *Appl. Math. Comput.*, vol. 214, no. 1, pp. 108−132, Aug. 2009.

[26] D. Karaboga and B. Basturk, "On the performance of artificial bee colony (ABC) algorithm," *Appl. Soft Comput.*, vol. 8, no. 1, pp. 687−697, Jan. 2008.

[27] K. S. Tang, K. F. Man, S. Kwong, and Q. He, "Genetic algorithms and their applications," *IEEE Signal Proc. Mag.*, vol. 13, no. 6, pp. 22−37, Nov. 1996.

[28] K. Price, R. M. Storn, and J. A. Lampinen, *Differential Evolution: A Practical Approach to Global Optimization*. Berlin Heidelberg: Springer, 2005.

[29] J. Kennedy, "Particle swarm optimization," in *Encyclopedia of Machine Learning*, C. Sammut and G. I. Webb, Eds. US: Springer, 2010, pp. 760−766.

Wenjuan Gu received her B.S. degree at the School of Mathematics and Statistics, Ludong University, China in 2013. She is currently working toward the MEcons degree at the School of Economics and Management, Beijing Jiaotong University, China. Her research interests include intelligent optimization algorithms and its applications, logistic optimization, parameter estimation, data mining and analysis, applied statistics.

Yongguang Yu received his M.S. degree in the Department of Mathematical Science, Inner Mongolia University, China in 2001, and his Ph.D. degree at the Institute of Applied Mathematics, Academy of Mathematics and System Sciences, Chinese Academy of Sciences, China in 2004. From 2007 to 2009, he was a research fellow at City University of Hong Kong, China. Since 2010, he has been a professor with the Department of Mathematics, School of Science, Beijing Jiaotong University, China. His research interests include chaotic dynamics, chaos control and synchronization, complex networks, nonlinear control, and multi-agent systems. Corresponding author of this paper.

Wei Hu received his B.S. degree at the School of Mathematics and Statistics, Ludong University, China in 2013. He is currently working toward the M.S. degree in the Department of Mathematics, Beijing Jiaotong University, China. His research interests include parameter estimation of chaotic systems, chaos control and synchronization, and coordination control of multi-agent systems.