Set-point Filter Design for a Two-degree-of-freedom Fractional Control System

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***Abstract*—This paper focuses on a new approach to design (possibly fractional) set-point filters for fractional control systems. After designing a smooth and monotonic desired output signal, the necessary command signal is obtained via fractional input-output inversion. Then, a set-point filter is determined based on the synthesized command signal. The filter is computed by minimizing the 2-norm of the difference between the command signal and the filter step response. The proposed methodology allows the designer to synthesize both integer and fractional set-point filters. The pros and cons of both solutions are discussed in details. This approach is suitable for the design of two degree-of-freedom controllers capable to make the set-point tracking performance almost independent from the feedback part of the controller. Simulation results show the effectiveness of the proposed methodology.**

***Index Terms*—Fractional control systems, two-degree-of-freedom control, set-point following, system inversion.**

I. INTRODUCTION

FRACTIONAL systems have been proven to be effective in the design of control systems because of their capability to model complex phenomena and to achieve more challenging control specifications[1−12].

Actually, one of the main issues in a control system is often to achieve a satisfactory performance in the load disturbance rejection and in the set-point following tasks at the same time. An effective solution to this problem is the use of a two degree-of-freedom control system[13], where a suitable set-point filter should be designed in order to recover the set-point following performance independently from the employed feedback controller. Indeed, this approach has been proven to be effective also in the fractional framework. For example, in [14] the use of a set-point weight for fractional-order proportional-integral-derivative controllers is discussed. The use of a Davidson-Cole filter has then been proposed in [15]. In any case, it has to be stressed that such a kind of filter cannot decrease the rise time of the step response but it can just effectively reduce the overshoot[16].

By following another approach, the set-point following performance can be improved by using a suitably designed feedforward control law. In particular, the command signal to be applied to the closed-loop system is determined by exploiting the input-output inversion concept[17−19], that is, is computed in such a way it causes a desired smooth monotonic process variable transition, which is selected as a transition polynomial[20]. In this context, constraints on the control and process variables can be explicitly considered. This technique has been extended successfully also to fractional control systems[16] but it has the drawback that the use of a complex feedforward command signal might lead to implementation problems, especially considering the memory allocation issue.

Thus, in order to simplify significantly the implementation of this strategy by using a standard two-degree-of-freedom control scheme, in this paper, which is an extended version of [21], a methodology to design a set-point filter based on the inversion technique is proposed.

Indeed, the set-point filter is determined as the system that minimizes the 2-norm of the difference between its step response and the synthesized command signal. For this purpose, the differintegrals of both the transition polynomial and the command signal are determined. Then, two techniques to determine either a fractional-order or an integer-order filter are proposed. The advantages of both techniques will be discussed in detail: the integer filter is easier to implement on a commercial off-the-shelf control system, but may become unstable for a small transition time and cannot cope with uncompensated long fractional tails. On the contrary, the fractional filter (which is more complex to implement) is stable for every desired output transition time and works properly independently from the feedback controller tuning.

In this way, the achieved performance is close to the one that would have been obtained by using the synthesized command signal, without the memory allocation problems that would arise from the use of a complex feedforward signal. Moreover, the performance is still independent from the chosen controller and, finally, the filter can be fed with a simple step signal, that is, the overall control system can be implemented in any control setup.

The effectiveness of the proposed methodologies is proven through a series of illustrative examples.

Summarizing, the contribution of the paper is in the design of a (possibly) fractional set-point filter that can be employed in a standard two-degree-of-freedom control scheme and allows the achievement of high performance in terms of low settling time and low overshoot at the same time. This is different from the standard design of set-point filtering that uses a low-pass filtering approach that allows the reduction of the overshoot at the expense of the rise time.

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The paper is organized as follows. In Section II the problem is formalized and, in Section III, the design technique of the command signal is reviewed. The fractional differintegral of both transition polynomial and command signal is obtained in Section IV, while the filter design methodologies are presented in Section V and their use is discussed in Section VI. Illustrative examples are given in Section VII and conclusions are drawn in Section VIII.

Notation. $C^{(i)}$ denotes the space of the scalar real functions which are continuous till the ith time derivative. D^i denotes the ith derivative operator. Finally $[x]$ with $x \in \mathbf{R}$ is the biggest integer lower than x (note that, when $x \in \mathbf{R}\backslash\mathbf{N}$, this is the well-known integer part of x).

II. Problem Formulation

Consider the two degree-of-freedom control system shown in Fig. 1 where the process is a linear time-invariant commensurate strictly proper fractional system, L is the delay term and $\bar{G}(s)$ is minimum-phase.

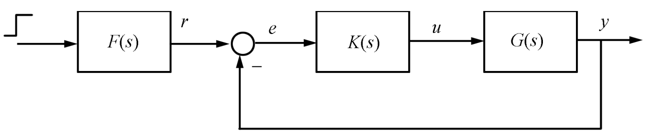

Fig. 1. The two degree-of-freedom unity-feedback control scheme.

$$G(s) = \bar{G}(s)\mathrm{e}^{-Ls} \tag{1}$$

The closed-loop systems transfer function is

$$T(s) = \frac{K(s)G(s)}{1+K(s)G(s)} \tag{2}$$

and it is assumed to be strictly proper.

It is also assumed that the controller has been designed in order to make the considered feedback loop internally stable.

The goal here is to design a filter $F(s)$ such that process output behaves well. Namely, to obtain, independently from the chosen controller $K(s)$, an output transition as close as possible to a desired output function which exhibits a smooth and monotonic transition from an initial steady-state value to a new one in a finite time interval τ, given a set of bounds on the control and process variables and their derivatives.

In order to do that a suitable command signal $r(t)$ is first synthesized, according to the technique proposed in [16], to obtain a perfect tracking of the desired output function.

Then, a linear (possibly fractional) filter $F(s)$ whose step response is the closest in terms of 2-norm to the determined command signal $r(t)$ is found.

It is worth stressing that in this way, once a suitable filter has been designed and implemented, the control system can be fed directly with a simple step signal instead of a complex command signal $r(t)$, that would require a significant precomputation and memory storage. Moreover, this allows the user to design the feedback controller $K(s)$ independently from the set-point following performance, hence, for example, by better addressing the performance/robustness trade-off (such as focusing the feedback controller design on robustness and/or disturbance rejection).

III. Command Signal Synthesis

For the reader's convenience, the technique proposed in [16] to design $r(t)$ is briefly revisited here. The command signal design problem can be formalized as follows:

Problem 1. Starting from null initial conditions and given a new steady-state output value y_e, design a "sufficiently smooth" τ-parametrized desired output $\bar{y}(\cdot;\tau)$ such that $\bar{y}(0;\tau) = 0$ and $\bar{y}(t;\tau) = 1\ \forall t \geq \tau$, and $\bar{y}(\cdot;\tau) \in C^{(k)}$ for some $k \in \mathbf{N}$. Then, find $r(\cdot;\tau)$ such that, for the τ-parametrized couple $(r(\cdot;\tau), \bar{y}(\cdot;\tau))$, it holds that

$$\mathcal{L}[\bar{y}(t-L;\tau)] = T(s)\mathcal{L}[r(t;\tau))]. \tag{3}$$

Moreover, determine the minimum time τ^* such that $u(t;\tau^*)$ and the first $l \in \mathbf{N}_0$ ($v \in \mathbf{N}$, respectively) derivatives of $u(t;\tau^*)$ ($\bar{y}(t;\tau^*)$), are bounded:

$$\begin{array}{l}|D^i u(t;\tau^*)| < u_M^i,\ \forall t > 0,\ i = 0,1,\ldots,l;\\ |D^i \bar{y}(t;\tau^*)| < y_M^i,\ \forall t > 0,\ i = 1,2,\ldots,v.\end{array} \tag{4}$$

Note that the requirements of null initial conditions and unitary transition are without loss of generality in view of the system linearity.

The simple and computationally efficient τ-parametrized transition polynomial proposed in [20] is chosen as desired output function. It has the nice property of being monotonic, which implies that neither overshoots nor undershoots occur. In the interval $[0,\tau]$ the desired output function is therefore selected as a polynomial

$$\bar{y}(t) := c_0 + c_1 t + \cdots + c_{2n+1}t^{2n+1}, \tag{5}$$

where the coefficients c_i $(i = 0,1,\ldots,2n+1)$ are obtained by solving the following system:

$$\left\{\begin{array}{l}\bar{y}(0) = 0, D\bar{y}(0) = 0, \ldots, D^n\bar{y}(0) = 0;\\ \bar{y}(\tau) = 1, D\bar{y}(\tau) = 0, \ldots, D^n\bar{y}(\tau) = 0.\end{array}\right. \tag{6}$$

Eventually, the solution of the previous systems leads to the desired output function

$$\bar{y}(t;\tau) := \left\{\begin{array}{lll}0, & \text{if} & t<0;\\ \frac{(2n+1)!}{n!\tau^{2n+1}}\sum\limits_{r=0}^{n}\frac{(-1)^{n-r}\tau^r t^{2n-r+1}}{r!(n-r)!(2n-r+1)}, & \text{if} & 0\le t\le\tau;\\ 1, & \text{if} & t>\tau.\end{array}\right. \tag{7}$$

Note that, by construction, $\bar{y}(t;\tau)$ allows an arbitrarily smooth transition between 0 and 1; indeed, it is possible to show that $\bar{y}(t;\tau) \in C^{(n)}$[20].

Consider a commensurate minimum-phase fractional system $H(s)$ of commensurate order $\nu \in \mathbf{R}$. By polynomial division the inverse of its transfer function can be always represented as

$$H^{-1}(s) = \gamma_{q-m}s^{\rho} + \gamma_{q-m-1}s^{\rho-\nu} + \cdots + \gamma_1 s^{\nu} + \gamma_0 + H_0(s), \tag{8}$$

where $q\nu$ and $m\nu$, with $q,m \in \mathbf{N}$, are, respectively, the numerator and the denominator orders, $\rho \in \mathbf{R}$ is the relative order and $H_0(s)$ is the zero dynamics of $H(s)$.

By polynomial division it can be shown that $H_0(s)$ is always stable and strictly proper and that it can be represented as

$$H_0(s) = \sum_{i=1}^{m} \frac{g_i}{(s^\nu - \lambda_i)^{k_i+1}}. \tag{9}$$

As a consequence, in the time domain, its impulse response $\eta_0(t)$ can be described as a linear combination of Mittag-Leffler functions[16, 22], that is:

$$\eta_0(t) = \sum_{i=1}^{m} \frac{g_i}{k_i!} \varepsilon_{k_i}(t, \lambda_i; \nu, \nu), \tag{10}$$

where

$$\varepsilon_k(t, \lambda; \alpha, \beta) := t^{k\alpha+\beta-1} \frac{\mathrm{d}^k}{\mathrm{d}(\lambda t^\alpha)^k} E_{\alpha,\beta}(\lambda t^\alpha), \tag{11}$$

with

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)} \ \alpha > 0, \beta > 0, \tag{12}$$

The following lemma solves the problem of computing the input signal such that a perfect tracking of the desired output is obtained for the system $H(s)$.

Proposition 1[23]**.** Consider $\bar{y}(t;\tau)$ defined in (7). If $n \geq [\rho] + 1$ then

$$\begin{aligned} u(t;\tau) \ &= \ \gamma_{q-m} D^\rho \bar{y}(t;\tau) + \gamma_{q-m-1} D^{\rho-\nu} \bar{y}(t;\tau) + \cdots \\ &+ \ \gamma_1 D^\nu \bar{y}(t;\tau) + \gamma_0 \bar{y}(t;\tau) + \textstyle\int_0^t \eta_0(t-\xi)\bar{y}(\xi;\tau)\mathrm{d}\xi. \end{aligned} \tag{13}$$

Eventually, for Problem 1, the command signal can be computed by applying Proposition 1 to the delay-free part of the open-loop transfer function, i.e., by defining $H(s) = K(s)\bar{G}(s)$, yielding the signal $r_{ol}(t;\tau)$. Then, a correction term $r_c(t;\tau) = \bar{y}(t - L;\tau)$ must be considered, so that the command signal is

$$r(t;\tau) = r_{ol}(t;\tau) + r_c(t;\tau). \tag{14}$$

Finally, it can be proven that the existence of a suitable command signal is guaranteed under the following condition:

$$n \geq [\rho_{K\bar{G}}] + 1, \tag{15}$$

where $\rho_{k\bar{G}}$ is the relative order of the open-loop transfer function.

$$\begin{cases} n \geq \max\{v; [\rho_{\bar{G}}] + 1 + l\}, \\ \tau \geq \max\{\tau_i^*; \tau_o^*\}, \end{cases} \tag{16}$$

where τ_o^* is the minimum transition time satisfying the output constraints, whereas τ_i^* is minimum transition time such that the input constraints are satisfied for each $\tau \geq \tau_i^*$.

IV. Command Signal Differintegrals

In this section, the differintegral of both the transition polynomial and the command signal are analytically obtained. Indeed, they are necessary to achieve the final result of designing an inversion-based set-point filter.

A. Transition Polynomial Fractional Differintegral

Considering that

$$x^n = (x - \tau + \tau)^n = \sum_{j=0}^{n} \binom{n}{j} (x-\tau)^{n-j}\tau^j \tag{17}$$

the transition polynomial can be represented as

$$\bar{y}(t;\tau) = \begin{cases} 0, & \text{if } t < 0, \\ \dfrac{(2n+1)!}{n!\tau^{2n+1}} \displaystyle\sum_{r=0}^{n} \frac{(-1)^{n-r}\tau^r t^{2n-r+1}}{r!(n-r)!(2n-r+1)}, & \text{if } 0 \leq t \leq \tau, \\ \dfrac{(2n+1)!}{n!\tau^{2n+1}} \displaystyle\sum_{r=0}^{n} \frac{(-1)^{n-r}\tau^r}{r!(n-r)!(2n-r+1)} & \\ \quad \times [t^{2n-r+1} - \displaystyle\sum_{j=0}^{2n-r+1} \binom{2n-r+1}{j} & \\ \quad \times (t-\tau)^{2n-r+1-j}\tau^j] + 1(t-\tau), & \text{if } t > \tau, \end{cases} \tag{18}$$

where $1(\cdot)$ is the Heaviside function. The previous expression can be further simplified considering that the transition polynomial is $C^{(n)}$ by construction. Hence, the summation of all the terms that by differentiating till the order n the transition polynomial would lead to impulse-like behaviors at $t = \tau$, is null. Thus, the summation over j can be truncated at $n - r$.

Now consider the fractional differintegral of the transition polynomial. By virtue of the previous reasoning, considering that $D^\alpha x^n = \frac{n!}{\Gamma(n+1-\alpha)} x^{n-\alpha}$, $\alpha \in \mathbf{R}$ and expanding the binomial coefficients in (18), the differintegral of the transition polynomial is finally obtained for $-\infty < \alpha \leq n + 1$:

$$D^\alpha \bar{y}(t;\tau) = \begin{cases} 0, & \text{if } t < 0; \\ \frac{(2n+1)!}{n!\tau^{2n+1}} \displaystyle\sum_{r=0}^{n} \frac{(-1)^{n-r}\tau^r(2n-r+1)!}{r!(n-r)!(2n-r+1)\Gamma(2n-r+2-\alpha)} & \\ \quad \times t^{2n-r+1-\alpha}, & \text{if } 0 \leq t \leq \tau; \\ \frac{(2n+1)!}{n!\tau^{2n+1}} \displaystyle\sum_{r=0}^{n} \frac{(-1)^{n-r}\tau^r(2n-r+1)!}{r!(n-r)!(2n-r+1)} & \\ \quad \times \left(\frac{t^{2n-r+1-\alpha}}{\Gamma(2n-r+2-\alpha)} - \displaystyle\sum_{j=0}^{n-r} \frac{\tau^j t^{2n-r+1-j-\alpha}}{j!\Gamma(2n-r+2-j-\alpha)} \right), & \text{if } t > \tau. \end{cases} \tag{19}$$

It is worth stressing the previous equation can also be used for a direct computation of the transition polynomial by selecting $\alpha = 0$.

B. Command Signal Fractional Differintegral

In order to integrate and differentiate the command signal, the following signals must be differintegrated: 1) the transition polynomial (r_c in (14)), 2) the fractional derivatives of the transition polynomial appearing in (13) and 3) the convolution integral appearing in (13).

In order to solve the first point, (19) can be used directly.

However, (19) cannot be applied straightforwardly to the second point since, in general, fractional operators do not commutate[22]. In particular, when using Caputo fractional derivatives, $D^m D^\alpha y(\cdot) \neq D^{m+\alpha} y(\cdot), m \in \mathbf{N}, \alpha \in \mathbf{R}$, unless $D^i y(0) = 0$ $(i = 0, \ldots, m)$. Nevertheless, when differentiating the fractional derivatives of the transition polynomial, in order to guarantee the existence of all the derivatives till a given order m, a sufficient condition is

$$n \geq m + [\rho]. \tag{20}$$

Hence, (19) can be applied. On the contrary, when integrating the fractional derivatives of the transition polynomial, Riemann-Liouville and Grünwald-Letnikov fractional operators do not commutate, that is $D^{-m}D^{\alpha}y(\cdot) \neq D^{-m+\alpha}y(\cdot)$, $m \in \mathbf{N}$, $\alpha \in \mathbf{R}$ unless $D^i y(0) = 0$ $(i = 0, \ldots, [\alpha])$. This condition, considering the transition polynomial, would lead to $n \geq [\rho]$, and it is automatically satisfied by the condition of existence for the inverting signal $n \geq [\rho] + 1$. Evidently, all these conditions must be applied to the specific inverting signal, that is $\rho = \rho_{K\bar{G}}$.

Finally, consider the differintegration of the convolution integral appearing in (13). In this case the operators commutation is guaranteed, independently from the adopted definition because of the strict properness of zero order dynamics, provided (20) is satisfied.

In $[0, \tau]$, considering that the Laplace transform of the convolution integrals equals the product of the Laplace transforms and that $\mathcal{L}[t^{\alpha}] = \Gamma(\alpha+1)\frac{1}{s^{\alpha+1}}$, starting from (19) its differintegral can be derived as an explicit expression in terms of Mittag-Leffler functions by exploiting the following equality:

$$\mathcal{L}^{-1}\left[\frac{k! s^{\alpha-\beta}}{(s^{\alpha} \pm \lambda)^{k+1}}\right] = \varepsilon_k(t, \mp\lambda; \alpha, \beta). \tag{21}$$

For $t > \tau$ a similar result is achievable by considering that the transition polynomial (19) can be represented as the summation of a polynomial and a delayed one. Hence, the same reasoning previously applied can be used by considering that $\mathcal{L}[(t-\tau)^{\alpha}] = \Gamma(\alpha+1)\frac{1}{s^{\alpha+1}}\mathrm{e}^{-\tau s}$, that is, the integration of a polynomial function, possibly delayed, that can be solved again in terms of Mittag-Leffler functions, leading to

$$\begin{aligned}
&D^{\alpha}\int_0^t \eta_0(t-\xi)y(\xi;\tau)\mathrm{d}\xi \\
&= \sum_{i=1}^{m}\frac{g_i}{k_i!}\frac{(2n+1)!}{n!\tau^{2n+1}}\sum_{r=0}^{n}\frac{(-1)^{n-r}\tau^r}{r!(n-r)!(2n-r+1)}(2n-r+1)! \\
&\times\Bigg[\varepsilon_{k_i}(t,\lambda_i;\nu,2n-r+2+\nu-\alpha) \\
&-\begin{cases} 0, & \text{if } 0 \leq t \leq \tau \\ \sum\limits_{j=0}^{n-r}\frac{\tau^j}{j!} & \\ \times\varepsilon_{k_i}(t-\tau,\lambda_i;\nu,2n-r+2-j+\nu-\alpha), & \text{if } t > \tau \end{cases}\Bigg].
\end{aligned} \tag{22}$$

Again, it is worth mentioning that the previous equation can be used for a direct computation of the convolution integral appearing in (13) in terms of Mittag-Leffler functions by selecting $\alpha = 0$.

It is noteworthy that the computation of (13) by means of (22) only requires the computation of the Mittag-Leffler function, that is widely treated in the literature (see for example [22, 24]). Note that, in the fractional framework, this is a basic requirement since the Mittag-Leffler function plays for fractional systems the same role that the exponential function plays for integer systems.

V. Least-squares Filter Design

In this section, two methodologies will be proposed to obtain the set-point filter. The first one will lead to a fractional-order filter, while the second one to an integer-order one. Also, pros and cons of the two approaches will be discussed in details.

A. Transition Polynomial-based Filter

The first methodology proposed exploits the design of a transfer function whose step response is as close as possible (in terms of 2-norm) to the transition polynomial. In this case, the following transfer function structure is proposed:

$$\tilde{F}(s) = \frac{1}{\sum\limits_{i=1}^{o} a_i s^i + 1}. \tag{23}$$

First $o = n + 1$ is selected, so that the filter step response exhibits the same degree of regularity of the transition polynomial. Then, by sampling at each Δt the transition polynomial and its derivatives obtained via (19), the following matrices are created

$$A = \begin{bmatrix} D^o\bar{y}(0;\tau) & \cdots & D^1\bar{y}(0;\tau) \\ \vdots & \ddots & \vdots \\ D^o\bar{y}(t-\Delta t;\tau) & \cdots & D^1\bar{y}(t-\Delta t;\tau) \\ D^o\bar{y}(t;\tau) & \cdots & D^1\bar{y}(t;\tau) \\ D^o\bar{y}(t+\Delta t;\tau) & \cdots & D^1\bar{y}(t+\Delta t;\tau) \\ \vdots & \ddots & \vdots \\ D^o\bar{y}(3\tau;\tau) & \cdots & D^1\bar{y}(3\tau;\tau) \end{bmatrix}, \tag{24}$$

$$\mathbf{B} = \begin{bmatrix} 1(0) - \bar{y}(0;\tau) \\ \vdots \\ 1(t-\Delta t) - \bar{y}(t-\Delta t;\tau) \\ 1(t) - \bar{y}(t;\tau) \\ 1(t+\Delta t) - \bar{y}(t+\Delta t;\tau) \\ \vdots \\ 1(3\tau) - \bar{y}(3\tau;\tau) \end{bmatrix}. \tag{25}$$

Finally the coefficients vector $\Theta = [a_o \ \cdots \ a_1]^{\mathrm{T}}$ is obtained as $\Theta = A^{\mathrm{T}}(AA^{\mathrm{T}})^{-1}B$. Note that, the transfer function (23) designed in this way has, by construction, unitary dc-gain. Now, using (23) and the process dynamics, the set-point filter can be designed as

$$F(s) = \tilde{F}(s)(\mathrm{e}^{-Ls} + (K(s)\bar{G}(s))^{-1}). \tag{26}$$

It is worth noting that, in this case, the obtained filter is fractional. Hence, it may be difficult to implement with standard industrial control hardware. In order to overcome this problem, in the next subsection a second methodology to design the set point filter is proposed.

B. Command Signal Filter

The second methodology is based on direct design of a filter whose step response is the closest, in terms of 2-norm, to the command signal.

Actually, a double approach to solve this problem is proposed. The first approach consists in identifying a suitable filter using directly the command signal (14). The second approach is based on the separate identification of a transfer function for the transition polynomial and a transfer function for the inverting part of the command signal $r_{ol}(t,\tau)$. The response of the first transfer function can therefore be arbitrarily delayed and, by selecting the system delay, a signal close to $r_c(t;\tau)$ is obtained.

When the first approach is used, the proposed filter structure is

$$F(s)=\frac{\sum_{j=1}^{o-p} b_j s^j+1}{\sum_{i=1}^{o} a_i s^i+\mu}, \tag{27}$$

where μ is the closed-loop dc-gain and

$$p=n-[\rho_{K\bar{G}}]. \tag{28}$$

Note that the relative order of the filter is chosen in such a way that forces the filter step response to have the greater degree of regularity equal to or smaller than the one of the command signal. Indeed, considering the possibly fractional nature of the considered control systems, the degree of regularity of the command signal is $n-\rho_{K\bar{G}}$. Also, note that the chosen value of p guarantees the accomplishment of condition (20), hence the existence of the derivative of the command signal independently from the adopted definition of fractional operator.

In this case, $o\in\mathbf{R}$ is a design parameter, to be chosen large enough to give to the filter a sufficient number of degrees of freedom. In this case, the identification would require $o-p$ differentiations of the step signal. In order to overcome this problem an integral approach is adopted integrating $o-p$ times both the step signal and the command signal.

Then, by sampling at each Δt the command signal and its integrals obtained via (13), (19) and (22) the following matrices are created

$$A=\begin{bmatrix} D^p r(0;\tau) & \cdots & D^{-o+p+1}r(0;\tau) & -1(0) & \cdots & -\frac{1}{(o-p+1)!}0^{(o-p+1)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ D^p r(t-\Delta t;\tau) & \cdots & D^{-o+p+1}r(t-\Delta t;\tau) & -1(t-\Delta t) & \cdots & -\frac{1}{(o-p+1)!}(t-\Delta t)^{(o-p+1)} \\ D^p r(t;\tau) & \cdots & D^{-o+p-1}r(t;\tau) & -1(t) & \cdots & -\frac{1}{(o-p+1)!}t^{(o-p+1)} \\ D^p r(t+\Delta t;\tau) & \cdots & D^{-o+p+1}r(t+\Delta t;\tau) & -1(t+\Delta t) & \cdots & -\frac{1}{(o-p+1)!}(t+\Delta t)^{(o-p+1)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ D^p r(3\tau;\tau) & \cdots & D^{-o+p+1}r(3\tau;\tau) & -1(\psi\tau) & \cdots & -\frac{1}{(o-p+1)!}(\psi\tau)^{(o-p+1)} \end{bmatrix}, \tag{29}$$

$$B=\begin{bmatrix} \frac{1}{(o-p)!}0^{(o-p)}-\mu D^{-o+p}r(0;\tau) \\ \vdots \\ \frac{1}{(o-p)!}(t-\Delta t)^{(o-p)}-\mu D^{-o+p}r(t-\Delta t;\tau) \\ \frac{1}{(o-p)!}t^{(o-p)}-\mu D^{-o+p}r(t;\tau) \\ \frac{1}{(o-p)!}(t+\Delta t)^{(o-p)}-\mu D^{-o+p}r(t+\Delta t;\tau) \\ \vdots \\ \frac{1}{(o-p)!}(\tau)^{(o-p)}-\mu D^{-o+p}r(\psi\tau;\tau) \end{bmatrix}, \tag{30}$$

where $\psi\in\mathbf{R}$ is a design parameter that must be big enough to capture a sufficient part of the command signal transient (made of action and postaction, see [16] for details) in order to obtain a satisfactory filter. Finally the coefficients vector $\boldsymbol{\Theta}=[a_o\ \cdots\ a_1\ b_{o-p}\ \cdots b_1]^{\mathrm{T}}$ is obtained as $\Theta=A^{\mathrm{T}}(AA^{\mathrm{T}})^{-1}B$.

The second approach uses the same filter structure (27) of the first one, but in order to identify the filter parameters it uses $r_{ol}(t;\tau)$ instead of the whole command signal (14) to build the matrices (29) and (30). In order to do that, the following procedure should be used:

1) If the open-loop transfer function $K(s)G(s)$ has a finite dc-gain μ_{ol}, then substitute μ with μ_{ol} both in (27) and (30). Then, use them to compute a filter $\bar{F}(s)$ having the same structure of (27) following the standard procedure;

2) If the open-loop transfer function has an integral behavior of order $\lambda\in\mathbf{R}$, then eliminate from (29) the last $[\lambda]-1$ columns, eliminate from (30) the integrals of the Heaviside function, set $\mu=1$ in (30) and use the following filter structure:

$$\bar{F}(s)=\frac{\sum_{j=[\lambda]}^{o-p} b_j s^j}{\sum_{i=1}^{o} a_i s^i+1}, \tag{31}$$

where $\Theta=[a_o\ \cdots\ a_1\ b_{o-p}\ \cdots b_{[\lambda]}]^{\mathrm{T}}$.

Then, the technique proposed in the Subsection V-A is employed to design a transfer function $\tilde{F}(s)$ whose step response is close to the transition polynomial $\bar{y}(\cdot)$. Finally, the filter is obtained as

$$F(s)=\bar{F}(s)+\tilde{F}(s)\mathrm{e}^{-Ls}. \tag{32}$$

VI. Discussion

Two methodologies have been proposed in Section V. The first one generates fractional set-point filters, while the second one can be used to obtain different integer-order set-point filters.

Clearly, when able to guarantee the same set-point tracking performance, an integer order filter is preferable for its ease of implementation.

Nevertheless, the second methodology is not always usable. In particular it may present two different problems:

1) When the required transition time τ is too small it may lead to unstable filters. In this case the first methodology i.e., the fractional filter, offers a great advantage. Indeed, the first technique gives the same results independently from the chosen transition time. Actually, when varying the transition time, the transition polynomial is just scaled along the time axis (i.e., it is selfsimilar). So, once a stable filter for a given

τ has been identified, it is possible to obtain many others just scaling its coefficients in such a way that the filter Bode plot is rigidly shifted along the $j\omega$ axes without changing its shape;

2) When the control loop presents uncompensated fractional dynamics (i.e., it is not properly tuned) the integer filter may lead to undershoot or overshoot long time after the application of the step signal. This depends on the fact that the transient response generated by the filter has already expired while the loop dynamics exhibits a slow non exponential decay typical of uncompensated fractional dynamics[25]. It is well known that it is not possible to match this kind of fractional power-law decay using integer systems, hence, in this case, the use of a fractional filter is mandatory in order to obtain a satisfactory result.

Summarizing, the first methodology always guarantees the same level of performance and can be successfully employed on a broader class of control systems, but its use should be carefully evaluated for the intrinsic complexity of implementation that fractional order systems have. Indeed, when usable, the second approach is preferable since, independently from the adopted approach, it leads to integer-order filters, which are easy to implement on off-the-shelf control setups. This issue will be further illustrated in the following section.

VII. Simulation Examples

In this section the proposed techniques will be tested via simulation examples in order to highlight the benefits and the problem that may arise from the use of these set-point filters.

For the purpose of simulation, the fractional-order dynamics has been approximated in the frequency domain by using the well-known Oustaloup approximation[26]. In order to obtain a precise approximation of the real fractional system, a high number of poles and zeros has been used, namely, 20 cells in a frequency band $[0.0001\ \ 10000]$.

A. Example 1

As a first illustrative example consider an unstable fractional system with the following transfer function[16]:

$$G(s)=\frac{3s^{0.5}+1}{s^{1.5}-1}\mathrm{e}^{-0.1s}, \tag{33}$$

whose commensurate order is, evidently, 0.5. A very simple stabilizing controller can be used, indeed a satisfactory set-point tracking performance can be obtained independently from the chosen feedback controller by using a suitable set-point filter. A proportional controller $K(s)=2$ is used here.

The control requirement is to obtain a smooth transition of the output from 0 to 1 constraining both the amplitude and the slew rate of control and process variables (note that these are common requirements in practical applications).

Accordingly, considering that the relative order of the system $\bar{G}(s)$ is $\rho=1$, $n=3$ is chosen, that is sufficient (not necessary) to satisfy conditions (15) and (16), and the transition polynomial $\bar{y}(t;\tau)$ is computed via (7):

$$\bar{y}(t;\tau)=-\frac{20}{\tau^7}t^7+\frac{70}{\tau^6}t^6-\frac{84}{\tau^5}t^5+\frac{35}{\tau^4}t^4. \tag{34}$$

Then, the technique proposed in Section III is applied. The zero dynamics of $K(s)\bar{G}(s)$ is obtained as

$$H_0(s)=\frac{-0.5185}{3s^{0.5}+1} \tag{35}$$

and its time domain version as

$$\eta_0(t)=\frac{-0.5185}{3}\varepsilon_{k_i}\left(t,\frac{1}{3};0.5,0.5\right). \tag{36}$$

Subsequently, the inversion-based part $r_{ol}(t;\tau)$ of the command signal $r(t;\tau)$ can be computed via (13), (19) and (22):

$$\begin{aligned} r_{ol}(t;\tau)\ &= 0.1667D^1y(t;\tau)-0.0556D^{0.5}\bar{y}(t;\tau)+0.0185\\ &+\int_0^t\eta_0(t-\xi)y(\xi;\tau)\mathrm{d}\xi. \end{aligned} \tag{37}$$

Now consider the following set of constraints:

$$\begin{aligned} &u_M^0\leq 1.5,\ \ u_M^1\leq 5,\\ &y_M^1\leq 5. \end{aligned} \tag{38}$$

The minimum transition time can be found by using, for instance, a simple bisection algorithm. It turns out that the most tightening constraint is the one imposed on the derivative of the control variable and the minimum transition time is $\tau^*=\tau_i^*=0.72$.

Once the command signal has been computed, the set-point filter is designed. First, the technique proposed in Subsection V-A is used to identify the parameters of (23) leading to

$$\tilde{F}(s)= \\ 1/(0.0002416s^4+0.004027s^3+0.05374s^2+0.3418s+1). \tag{39}$$

The step response of $\tilde{F}(s)$ is represented in Fig. 2 where it immediately shows the effectiveness of its design. Then, the fractional set-point filter $F(s)$ is obtained via (26) as

$$\begin{aligned} F(s)=&\,(s^{1.5}+6s\mathrm{e}^{-0.1s}+2\mathrm{e}^{-0.1s}-1)/(0.001450s^{4.5}\\ &+0.0004839s^4+0.02416s^{3.5}+0.008054s^3+0.3224s^{2.5}\\ &+0.1075s^2+2.0509s^{1.5}+0.6836s+6s^{0.5}+2). \end{aligned} \tag{40}$$

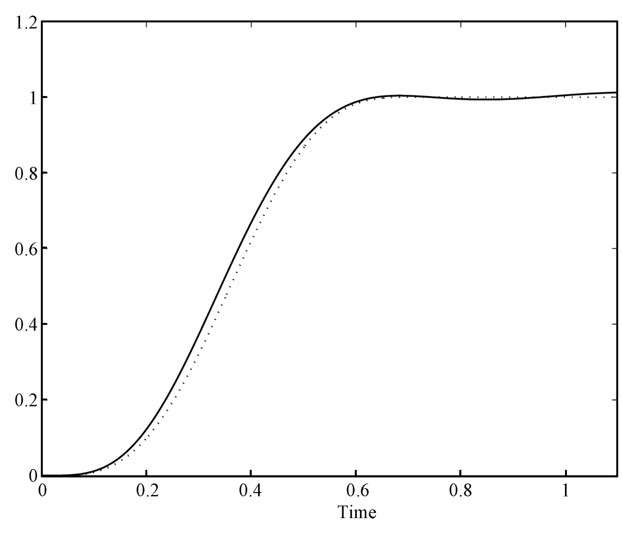

Fig. 2. Transition polynomial (dotted line) and $\tilde{F}(s)$ step response obtained by using the technique of Subsection V-A (solid line) for the set of constraints (38). – Example 1.

Now, also the first methodology proposed in Subsection V-B is implemented selecting $m = 3$, leading to the following integer-order filter

$$F(s) = (0.272s^2 + 15.28s + 1)/(0.001041s^5 + 0.03607s^4 + 0.6633s^3 + 5.473s^2 + 17.91s + 2). \quad (41)$$

Fig. 3 shows the responses obtained with the command signal, the proposed (fractional- and integer-order) set-point filters and, for the sake of comparison, the step command signal (scaled by the closed-loop dc-gain). Indeed, the response using the proposed fractional-order filter is close to the optimal one obtained with the inversion-based command signal and the constraints are almost satisfied. On the contrary, the step response does not respect the constraints and the system is very sluggish. Moreover, because of the long memory of the fractional dynamics, the 2 % settling time has the unacceptable value of 3800. Fig. 3 also reveals that the response of the integer-order filter is not capable to capture the long tail that the fractional dynamics exhibits, causing an unacceptable undershoot. Indeed, because of the very simple controller, the control loop exhibits a sluggish behavior with an uncompensated slow fractional dynamics and a settling time approximately close to the one obtained without the filter.

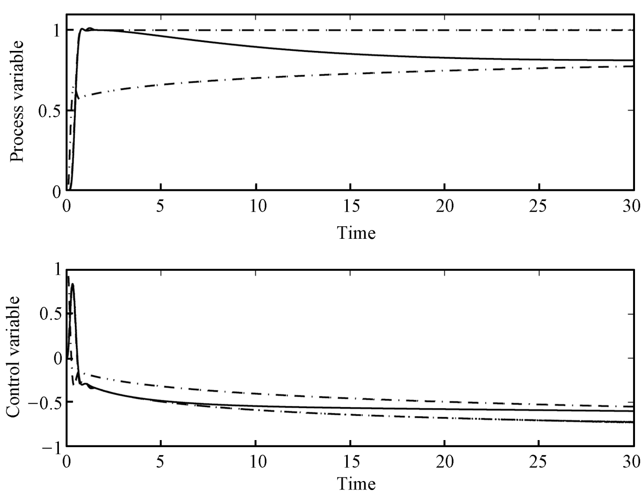

Fig. 3. Process variable (top) and control variable (bottom) obtained by using the command signal (dotted line), the filter designed with the technique of Subsection V-A (dashed line) and Subsection V-B (solid line) and a step command signal (dash-dot line) for the set of constraints (38). – Example 1.

In this context, the fractional-order filter is the only one that is capable to completely compensate this phenomena guaranteeing very good performance despite the simple (detuned) controller. This behavior is even clearer by analyzing the filter responses compared to the ideal command signal, as shown in Fig. 4. By observing Fig. 5, it turns out that the integer-order filter cannot match the whole fractional power-law tail, but it is only capable to match the required command signal only in the first part of the transient response. As a consequence, also the performance of the control system is satisfactory only in the first part of the transient response, as Fig. 6 shows. This depends on the incapability of integer-order systems to match power law decays[25]. It is worth stressing that this is a structural problem that cannot be solved by increasing m in (29) and (30). Moreover, an excessive growth of m would cause a loss of information in the first part of the transient with a consequent decay of the filter performance also in describing that part, which is usually the most exciting for the system dynamics.

Finally, a second simulation has been performed, this time

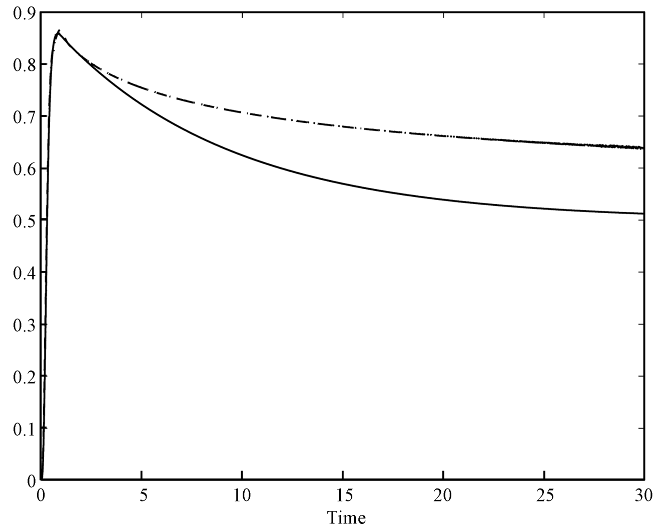

Fig. 4. Command signal (dotted line) and filter step response obtained by using the technique of Subsection V-A (dashed line) and Subsection V-B (solid line) for the set of constraints (38). – Example 1.

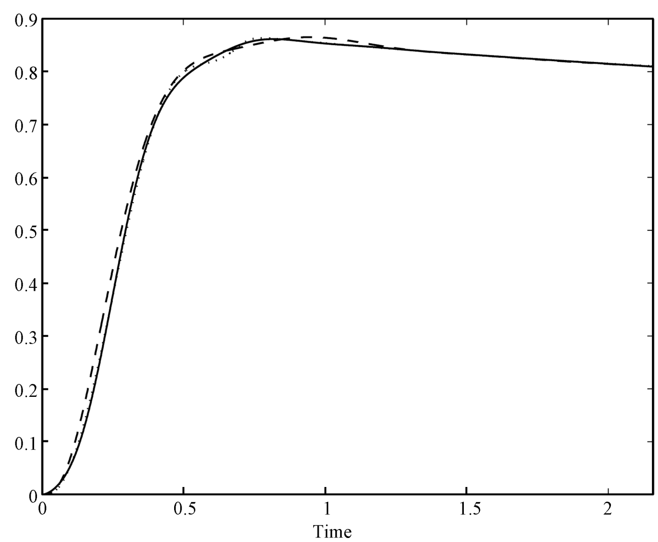

Fig. 5. Zoom of the first part of the command signal (dotted line) and filter step response obtained by using the technique of Subsection V-A (dashed line) and Subsection V-B (solid line) for the set of constraints (38). – Example 1.

neglecting the constraints and reducing the transition time to $\tau = 0.3$. Using this transition time the second methodology cannot be applied since it leads to an unstable filter. On the contrary, the first technique gives the same results independently from the chosen transition time. Indeed, when varying the transition time, the transition polynomial is just scaled along the time axis (i.e., it is selfsimilar). Thus, the Bode plot of the transfer function (23) identified again is identical to the previous one, but just rigidly shifted along the ω axis, as show in Fig. 7. The transfer function is

$$\tilde{F}(s) = 1/(7.28 \cdot 10^{-6}s^4 + 0.0002913s^3 + 0.009328s^2 \\ +0.1424s + 1) \tag{42}$$

and the associated fractional set-point filter is

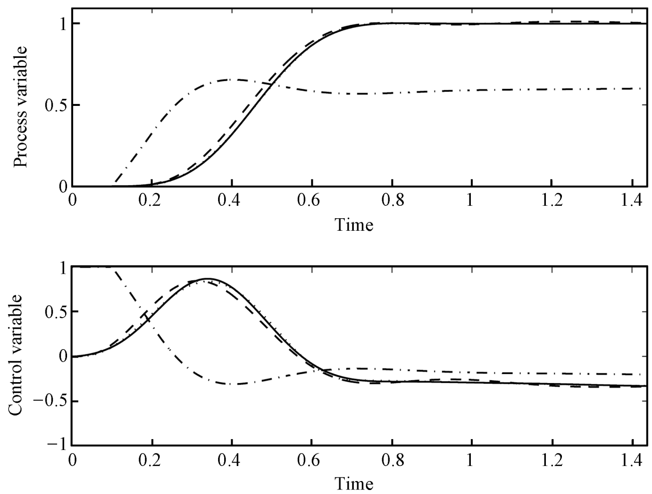

Fig. 6. Zoom of the first part of the process variable (top) and control variable (bottom) obtained by using the command signal (dotted line), the filter designed with the technique of Subsection V-A (dashed line) and Subsection V-B (solid line) and a step command signal (dash-dot line) for the set of constraints (38). – Example 1.

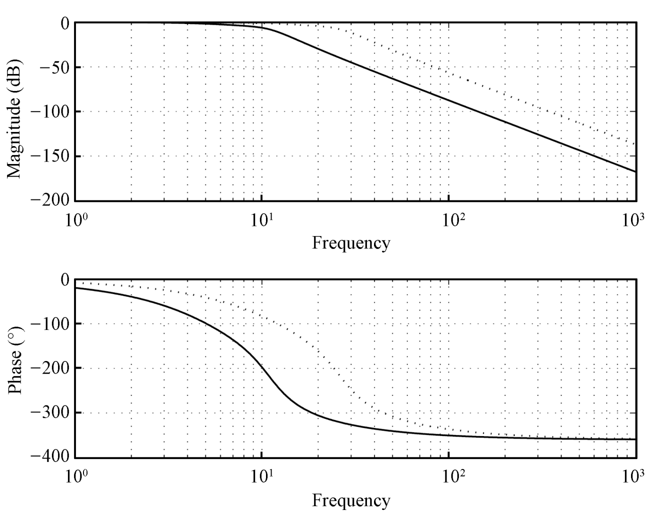

Fig. 7. Bode diagram of $\tilde{F}(s)$ for $\tau = 0.72$ (dotted line) and $\tau = 0.3$ (solid line). – Example 1.

$$F(s) = (s^{1.5} + 6s\mathrm{e}^{-0.1s} + 2\mathrm{e}^{-0.1s} - 1)/(0.00004368s^{4.5} \\ +0.00001456s^4 + 0.001749s^{3.5} + 0.0005826s^3 \\ +0.05597s^{2.5} + 0.01865s^2 + 0.8545s^{1.5} + 0.2848s \\ +6s^{0.5} + 2). \tag{43}$$

Again, the filter is computed via (26), and its step response is quite close to the ideal command signal, as shown in Fig. 8. Finally, Fig. 9 shows that, despite the strong transition time reduction, the process response remains smooth and almost monotonic, close again to the one obtained with the ideal command signal.

B. Example 2

As a second example consider a unity feedback control system where the process and the controller are the ones proposed in [27] and already used as a benchmark in [16]. The controlled process has the transfer function

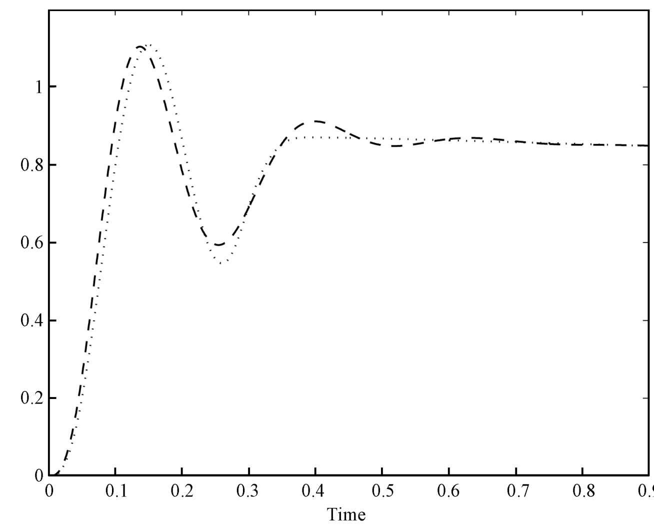

Fig. 8. Command signal (dotted line) and filter step response obtained by using the technique of Subsection V-A (dashed line) for the unconstrained solution. – Example 1.

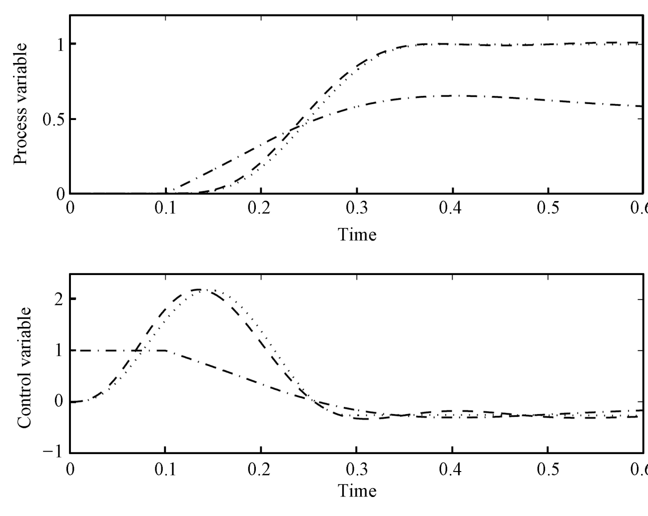

Fig. 9. Process variable (top) and control variable (bottom) obtained by using the command signal (dotted line), the filter designed with the technique of Subsection V-A (dashed line) and a step command signal (dash-dot line) for the unconstrained solution. – Example 1.

$$G(s) = \frac{0.25}{s(s+1)} \tag{44}$$

and the proposed controller is a fractional-order PID tuned in order to achieve the isodamping property:

$$K(s) = 3.8159 + \frac{2.1199}{s^{0.6264}} + 2.2195s^{0.809}. \tag{45}$$

Using the same reasoning proposed in [16], the actual controller is approximated with the following commensurate one

$$\tilde{K}(s) = 3.8159 + \frac{2.1199}{s^{0.6}} + 2.2195s^{0.8}, \tag{46}$$

leading to a control system (only used for design purposes) with commensurate order $\nu = 0.2$. A constraint on the maximum control variable has been considered:

$$u_M^0 \leq 10. \tag{47}$$

Note that, in the case of a servomotor, this is a common choice that means avoiding to saturate the current loop. In order to select the transition polynomial the relative order of the approximate closed-loop transfer function $\rho_{\tilde{T}} = 1.2$ and the relative order of the system $\rho_{\tilde{G}} = 2$ have been considered. Applying (15) and (16), the necessary and sufficient condition $n = 2$ is obtained. This choice also satisfies (16) and leads to the following transition polynomial:

$$\bar{y}(t;\tau) = \frac{6}{\tau^5}t^5 - \frac{15}{\tau^4}t^4 + \frac{10}{\tau^3}t^3. \tag{48}$$

Applying the command signal design technique (details are not given for the sake of brevity, they can be found in [16]) it turns out that a transition time $\tau = 1.8$ is sufficient to guarantee the constraint satisfaction.

Finally the filter design methodologies proposed in Section V have been employed, again selecting $m = 3$. It turns out that:

$$\tilde{F}(s) = 1/(0.068s^3 + 0.27s^2 + 0.8819s + 1) \tag{49}$$

that leads to the fractional filter

$$\begin{aligned} F(s) = {} & (s^{2.6} + s^{1.6} + 0.5549s^{1.4} + 0.9540s^{0.6} + 0.5300)/ \\ & (0.0377s^{4.4} + 0.0649s^{3.6} + 0.1498s^{3.4} + 0.0360s^3 \\ & +0.2576s^{2.6} + 0.4893s^{2.4} + 0.1431s^2 + 0.8413s^{1.6} \\ & +0.5549s^{1.4} + 0.4674s + 0.9540s^{0.6} + 0.5300), \end{aligned} \tag{50}$$

while the resulting integer-order filter transfer function (obtained by using the first approach of Section V-B) is

$$\begin{aligned} F(s) = {} & (0.01071s^4 - 0.01051s^3 + 1.133s^2 + 1.222s + 1)/ \\ & (0.003553s^5 + 0.0363s^4 + 0.2631s^3 + 0.8111s^2 \\ & +1.819s + 1). \end{aligned} \tag{51}$$

Again, both filters have been tested, as well as a step command signal and the ideal one. It is worth stressing that the tests have been done using the actual controller and not the approximated one.

Fig. 10 shows that both step responses of the filters are quite close to the ideal command signal. Indeed, in this case, as the fractional slow decay has been well compensated, the integer-order filter step response remains close to ideal command signal also after a long time.

Finally, in Fig. 11 the simulation results are shown. It appears evidently that both methodologies are able to provide responses close to the one obtained using the ideal command signal, notably improving the performance despite the already well-tuned controller.

Among the benefits that both the proposed methodologies provide, a smaller rise and settling times have to be mentioned, as well as a continuous control signal. In particular, this allows the avoidance of a very high peak (or saturation) of the control variable due to the so called "derivative kick" phenomenon[13].

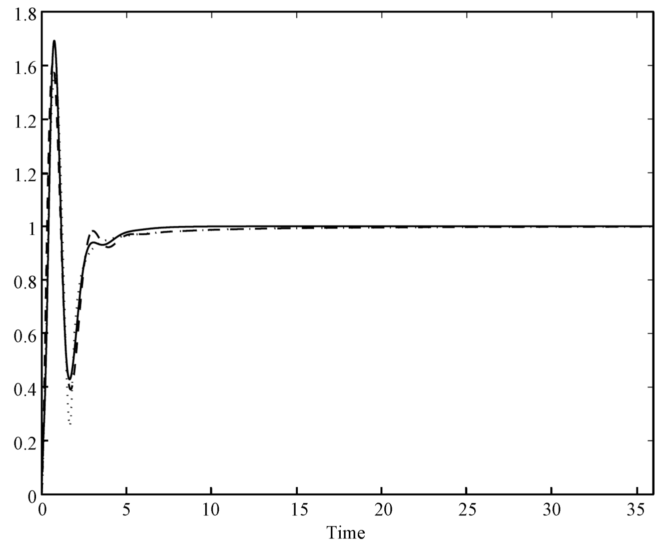

Fig. 10. Command signal (dotted line) and filter step response obtained by using the technique of Subsection V-A (dashed line) and Subsection V-B (solid line) for the set of constraints (47). – Example 2.

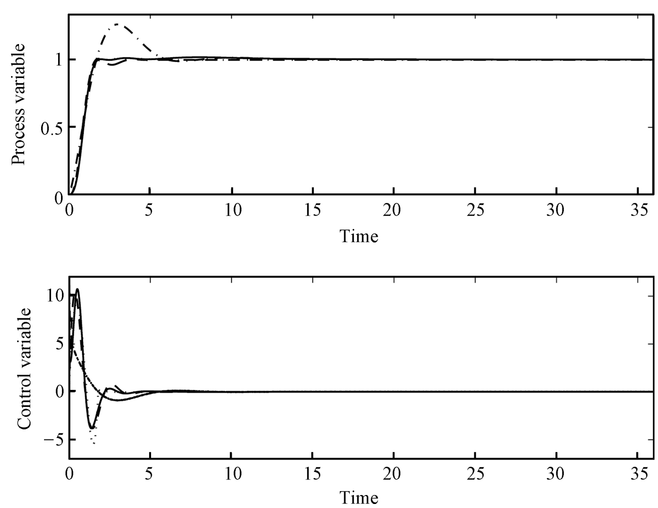

Fig. 11. Process variable (top) and control variable (bottom) obtained by using the command signal (dotted line), the filter designed with the technique of Subsection V-A (dashed line) and Subsection V-B (solid line) and a step command signal (dash-dot line) for the set of constraints (47). – Example 2.

C. Example 3

As a third example consider the following delay-dominant fractional plant

$$G(s) = \frac{1}{s^{1.8}+1}\mathrm{e}^{-3s} \tag{52}$$

and the proportional-integral (PI) controller

$$K(s) = 0.12\left(1+\frac{1}{0.65s}\right), \tag{53}$$

tuned in order to achieve a phase margin of approximately $60°$. The system, because of the fractional order of 1.8, exhibits an oscillatory behavior. It is well known that a PI controller is not sufficient to achieve a high performance when dealing with underdamped systems, but in many cases the use of such a controller is in force (in particular in the industry). Bearing in mind this idea, it is shown here how to significantly improve the set-point following performance by using a two-degree-of-freedom controller with suitable set-point filters. Also, note that an integrator is absolutely necessary in the controller in order to reject possible disturbances, since the proportional gain must be very small in order to avoid oscillations because of the strong delay of the plant.

Considering that no constraints are imposed, the condition $n = 2$ is sufficient to guarantee the existence of a command signal. Hence, the transition polynomial (48) is obtained. Then, by applying the command signal design procedure, the following results are obtained

$$\begin{aligned} H_0(s) \; &= \tfrac{2.1281}{s^{0.2}+1.0900} + \tfrac{3.3834+1.7277\mathrm{i}}{s^{0.2}+0.3368+1.0366\mathrm{i}} \\ &+ \tfrac{3.3834-1.7277\mathrm{i}}{s^{0.2}+0.3368+1.0366\mathrm{i}} + \tfrac{5.4145+1.0678\mathrm{i}}{s^{0.2}-0.8818+0.6407\mathrm{i}} \\ &+ \tfrac{5.4145-1.0678\mathrm{i}}{s^{0.2}-0.8818-0.6407\mathrm{i}}, \end{aligned} \tag{54}$$

$$\begin{aligned} r_{ol}(t;\tau) \; &= 8.3333D^{1.8}\bar{y}(t;\tau) - 12.8205D^{0.8}\bar{y}(t;\tau) \\ &+ 8.3333\bar{y}(t;\tau) + \textstyle\int_0^t \eta_0(t-\xi)\bar{y}(\xi;\tau)\mathrm{d}\xi, \end{aligned} \tag{55}$$

where the impulse response of the zero-order dynamics is not reported for the sake of readability, but can be easily obtained following the procedure proposed in [16].

After selecting the very small transition time $\tau = 1$ (note that it is considerably smaller than the time delay), the technique of Subsection V-A and the second one of Subsection V-B have been applied. It results

$$\tilde{F}(s) = 1/(0.01155s^3 + 0.08333s^2 + 0.4886s + 1). \tag{56}$$

Then, the associated fractional-order filter is determined as

$$\begin{aligned} &F(s) = (s^{2.8} + s + 1.2s\mathrm{e}^{-3s} + 0.1846\mathrm{e}^{-3s})/(0.001386s^4 \\ &+0.01213s^3 + 0.07402s^2 + 0.2102s + 0.1846), \end{aligned} \tag{57}$$

while, using the integer-order approach we obtain:

$$\begin{aligned} &\bar{F}(s) = (-0.05786s^4 + 4.801s^3 + 1.347s^2 + 5.127s)/ \\ &(0.0003913s^5 + 0.0132s^4 + 0.09208s^3 + 0.5454s^2 \\ &+1.117s + 1). \end{aligned} \tag{58}$$

and the associated integer-order filter is:

$$\begin{aligned} &F(s) = (-0.0006684s^7 + 0.05064s^6 + 0.3873s^5 + 2.459 \\ &+s^4 + 0.01155s^3\mathrm{e}^{-3s} + 5.886s^3 + 0.08333s^2\mathrm{e}^{-3s} \\ &+3.852s^2 + 0.4886s\mathrm{e}^{-3s} + 5.127s + \mathrm{e}^{-3s})/ \\ &(4.52\times10^{-6}s^8 + 0.000185s^7 + 0.002354s^6 + 0.02081s^5 \\ &+0.1165s^4 + 0.463s^3 + 1.175s^2 + 1.606s + 1). \end{aligned} \tag{59}$$

It is worth stressing that here the parameter $m = 5$ has been used, because of the large process delay and the small transition time. Indeed, the previous examples choice $m = 3$ would not be able to capture the first part of the postaction.

Since here a slow decay tail does not appear, both the techniques work properly. In particular, Fig. 12 shows that both the filters are capable to satisfactorily match the command signal.

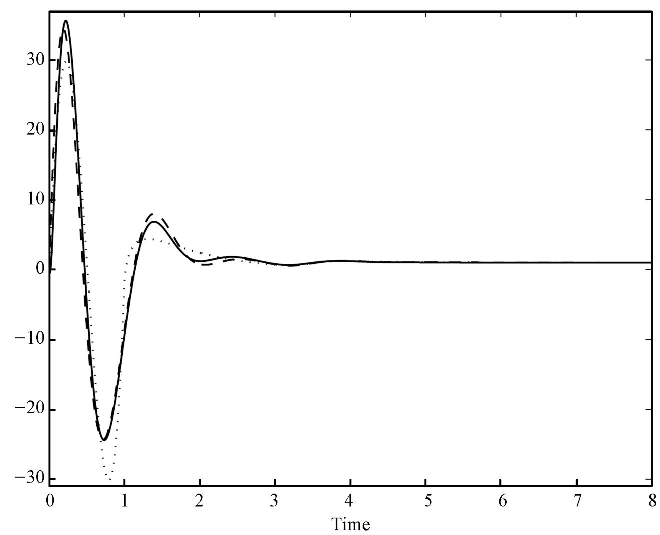

Fig. 12. Command signal (dotted line) and filter step responses obtained by using the technique of Subsection V-A (dashed line) and Subsection V-B (solid line) for the unconstrained problem. – Example 3.

Finally, Fig. 13 shows that both the integer filter and the fractional one are capable to strongly decrease the rise and the settling time contemporarily, guaranteeing a clear improvement of the set-point tracking performance despite the significant delay.

VIII. CONCLUSIONS

In this paper, a novel technique to design a set-point filter for a unity-feedback fractional control loop has been proposed.

It is based on a two-step procedure. First, an ideal command signal is synthesized in such away that a smooth and monotonic process output would have been obtained. Then, a linear filter is designed so that its step response is as close as possible, in terms of 2-norm, to the ideal command signal.

Two approaches are proposed, the first one based on a fractional-order filter and the second one on an integer-order one. Summarizing, the use of an integer-order filter should be limited to those cases where the feedback loop is tuned in such a way that no long fractional tails appear (note that this does not prevent the control system to exhibit a fractional dynamics as Examples 2 and 3 show) and the transition time is big enough to guarantee a stable filter. On the contrary, the fractional filter is always usable and guarantees a satisfactory performance, at the price of an increased implementation complexity.

The proposed technique is suitable for the design of two degree-of-freedom control structures and allows the user to design the feedback controller almost independently from the set-point tracking performance, that, on the contrary, mostly depends on the set-point filter.

Simulation results have demonstrated the effectiveness of the proposed methodology.

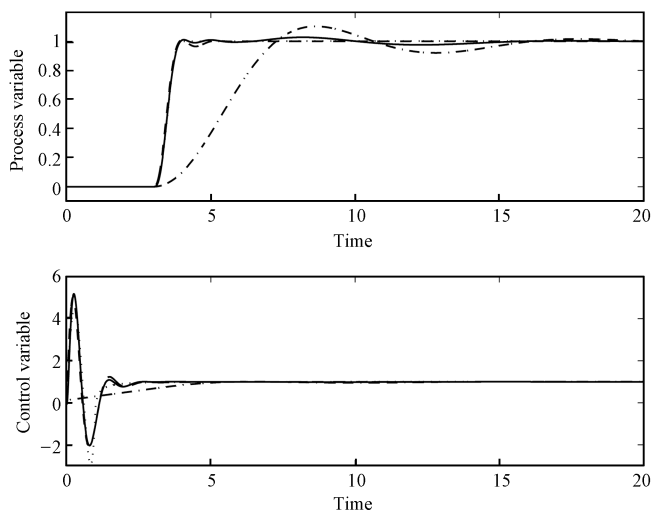

Fig. 13. Process variable (top) and control variable (bottom) obtained by using the command signal (dotted line), the filter designed with the technique of Subsection V-A (dashed line) and Subsection V-B (solid line) and a step command signal (dash-dot line) for the unconstrained problem. – Example 3.

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